Fall 2012-2013	Final Exam	Date:
K.Yaghi	Math 201- Sections 5-8	Duration: 2 hours

Problem 1 *(answer on pages* 1 & 2 *of the booklet)*

Which of the following series converge, and which diverge? (5 *pts each*)

a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\,(\ln n)^{100}}$ b) $\sum_{n=2}^{\infty} \frac{\sqrt{2+\frac{1}{n^2}}-\sqrt{2}}{\sqrt{2+\frac{1}{n^2}}}$ c) $\sum_{n=1}^{\infty} (\cos[n\, ln\,(1+\frac{\pi}{2n})])^n$

Problem 2 (*answer on pages* 3 & 4 *of the booklet*)

Consider the function $f(x,y) = x^2 + 2y^2 - \frac{y^3}{3}$

a) Find all local maxima, local minima and saddle point for $f(x,y)$.(13 *pts*)
b) In this part, we constrain (x,y) to lie in the disk $x^2 + y^2 = 1$. At what points of the disk does f attain its absolute maximum and minimum values? (12 *pts*)
c) Find the equation of the tangent plane and normal line to the surface $z = f(x,y)$ at the point $P(1,1,32/3)$. (12 *pts*)

Problem 3 (*answer on pages* 5 & 6 *of the booklet*)

Show that the differential form in the following integral is exact, the evaluate the integral. (24 *pts*)

$$\int_{(0,1,1)}^{(0,3,2)} (e^x yz)\, dx + (e^x z + 2yz)dy + (e^x y + y^2 + 1)dz$$

Problem 4 (*answer on pages* 7 & 8 *of the booklet*)

Use the transformation

$$u = x^2 + y^2 \quad and \quad v = x^2 + y^2 - 2y$$

to rewrite

$$\int_0^{1/2} \int_{\sqrt{2y-y^2}}^{\sqrt{1-y^2}} x\, e^y dx dy$$

as an integral over an appropriate region G in the uv – plane. Then evaluate the uv integral.(25 *pts*)

Problem 5 *(answer on pages* 9 & 10 *of the booklet)*

Let D be the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{3x^2 + 3y^2}$. Set up, but <u>do not evaluate</u>, the iterated triple integral that gives the volume of D in the order: (8 *pts* each)

(i) $dzdrd\theta$ (ii) $drdzd\theta$ (iii) $d\rho d\emptyset d\theta$ (iv) $d\emptyset d\rho d\theta$

Problem 6 *(answer on page* 11 *of the booklet)*

Find the volume of the region in the first octant bounded by the coordinate planes, the plane $2x + 3z = 12$ and the surface $y = \frac{1}{2}z^2$. (10 *pts*)

Problem 7 *(answer on pages 12 & 13 of the booklet)*

Given a vector field $F = x^2\vec{j}$. Let C_1 be the region of the circle $x^2 + y^2 = 4$ in the first quadrant and let D be the straight line of equation $y + x = 2$. Finally let R be the region enclosed by the C_1 and D in the first quadrant and oriented counterclockwise.

a) Calculate the work integral $\oint_{C1} \vec{F}.\vec{T}ds$, by finding a suitable parametrization for C_1. (8 *pts*)

b) Calculate the flux integral $\oint_{R} \vec{F}.\vec{n}ds$, by finding a suitable parametrization for R. (8 *pts*)

c) Use Green's theorem to recalculate the integrals in parts (a) and (b). (8 *pts*)

Problem 8 *(answer on page 14 of the booklet)*

Integrate $f(x, y) = \frac{x}{\sqrt{16x^6+1}}$ over the curve $C : y = x^4 + 1$ from (0,1) to (1,2). (10 *pts*)

Problem 9 *(answer on pages 15 & 16 of the booklet)*

Consider the function $f(x) = e^{x^2}$

a) Write a power series expansion for $f(x)$ about the point x = 0. Then find the taylor polynomials p1(x) and p2(x) generated by $f(x)$ about x = 0. (5 *pts*)

b) Use taylor's theorem to estimate $\int_0^1 e^{x^2}\, dx$ with an error of magnitude no more than 10^{-6}. (8 *pts*)

c) In this part we consider the function $g(x) = 2xe^{x^2}$.

(i) Use part (a) to find a power series expansion of g(x) about x = 0. (3 *pts*)

(ii) Use power series expansion of g(x) about the point x = 0 to prove that $\int g(x)\, dx = f(x)$ (7 pts)

Good Luck !

K. Yaghi