Fall 2013-2014 **Final Exam** **Date:**

K.Yaghi **Math 201- Sections 5-8** **Duration: 2 hours**

Problem 1 *(answer on pages* 1 & 2 *of the booklet)*

Which of the following series converge, and which diverge? (5 *pts each*)

a) $\sum_{n=2}^{\infty}\frac{\sin(n)}{n^n}$ b) $\sum_{n=2}^{\infty}\arctan(\frac{1}{1+n+n^2})$ c) $\sum_{n=1}^{\infty}\ln n\ (e^{\frac{1}{n}}-1)^{13}$

Problem 2 (*answer on pages* 3 & 4 *of the booklet*)

Consider the function $f(x,y)=12x^2+12y^2+(x+y)^3$

a) Find all local maxima, local minima and saddle point for $f(x,y)$.(13 *pts*)
b) In this part, we constrain (x,y) to lie in the disk $x^2+y^2=4$. At what points of the disk does f attain its absolute maximum and minimum values? (12 *pts*)
c) In what direction does f increases most rapidly at the point $P(1,1)$? Also find the directions of zero change at $P(1,1)$. (12 *pts*)

Problem 3 (*answer on pages* 5 & 6 *of the booklet*)

Show that the vector field

$$\vec{F}(x,y,z)=\left(\frac{y}{1+x^2y^2}\right)\vec{i}+\left(\frac{x}{1+x^2y^2}+e^z\cos y\right)\vec{j}+(e^z\sin y)\vec{k}$$

is conservative, then evaluate the work done by $\vec{F}$ along a curve joining $(\frac{1}{\pi},\pi,1)$ and $(\frac{2}{\pi},\frac{\pi}{2},\ln 3)$. (24 *pts*)

Problem 4 (*answer on pages* 7 & 8 *of the booklet*)

Let R be the region in the first quadrant bounded by the curves $y=x^2$, $y=\frac{x^2}{5}$, $xy=2$, and $xy=4$.

Use the transformation

$$u=\frac{x^2}{y}\quad and\quad v=xy$$

to rewrite $\iint_R dA(x,y)$ as an integral over an appropriate region G in the uv – plane. Then evaluate the uv integral.(25 *pts*)

Problem 5 *(answer on pages* 9 & 10 *of the booklet)*

Let D be the region bounded above by the sphere $x^2+y^2+z^2=4$ and below by plane $z=1$ and on the sides by the cone $z=\sqrt{x^2+y^2}$. Set up, but <u>do not evaluate</u>, the iterated triple integral that gives the volume of D in the order: (8 *pts* each)

(i) $dzdrd\theta$ (ii) $drdzd\theta$ (iii) $d\rho d\emptyset d\theta$ (iv) $d\emptyset d\rho d\theta$

Problem 6 *(answer on page* 11 *of the booklet)*

Find the volume of the region in the space bounded below by the xy plane , on the sides by the cylinder $y=x^2$ and above by the plane $y+z=1$. Use the order $dydxdz$ (10 *pts*)

Problem 7 *(answer on pages* 12 & 13 *of the booklet)*

Given a vector field $F = (1 + y^2)\vec{\imath}$. Consider the region R enclosed by $x - axis$, $x = 1$ and $y = x^3$.Travelling in a counterclockwise direction along the boundary R, call C_1 the portion of R that goes from (0,0) to (1,0), C_2 the portion of R that goes from (1,0) to (1,1), and C_3 the portion of R that goes from (1,1) to (0,0).

a) Use Green's theorem to calculate the work integral $\oint_R \vec{F}.\vec{T}ds$ and the flux integral $\oint_R \vec{F}.\vec{n}ds$. (10 *pts*)

b) Calculate the work of F along C_1 and C_2 directly.(8 *pts*)

c) Use parts (a) and (b) to calculate the work of F along C_3. (5 *pts*)

Problem 8 *(answer on page* 14 *of the booklet)*

Integrate $f(x,y) = \frac{x}{\sqrt{16x^6+1}}$ over the curve $C : y = x^4 + 1$ from (0,1) to (1,2). (10 *pts*)

Problem 9 *(answer on pages* 15 & 16 *of the booklet)*

a) Use the proof of the "integral test" to show that $\ln(n!) \geq n \ln n - n + 1 \quad for \;\; n > 1$. (8 *pts*)

b) Use part (a) to show that $ln(n!) \geq n \ln n$ for $n \geq 10$. (7 pts)

c) Decide if $\sum_{n=2}^{\infty} \frac{\ln(n!)}{n^2 \ln n}$ converge or diverge? (5 *pts*)

Good Luck !

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