

Problem 1 (answer on page 1 of the booklet)

Find the domain and the range of the function $f(x, y, z) = \sqrt[2]{-2[(x-1)^2 + y^2 + (z-3)^2 - 4]}$ determine if the domain of f is an open region, a closed region or neither? Also, determine if the domain is bounded or unbounded. Also, describe the level curves of f . (10 pts)

Problem 2 (answer on page 2 of the booklet)

Consider the function $f(x, y, z) = x^3z + x^2y^2 + \sin(yz)$

- (i) Find the tangent plane and normal line to the surface $f(x, y, z) = -3$ at the point $(-1, 0, 3)$. (10 pts)
(ii) Suppose that the equation $f(x, y, z) - xy = \ln(z)$ defines z implicitly as a function of (x, y) .

Find $\frac{\partial z}{\partial x}$ at the point $(2, \pi, \pi)$ (4 pts)

Problem 3 (answer on page 3 of the booklet)

Find all local maxima, local minima and saddle points for $f(x, y) = 2x^3 - 3y^3 + 6xy^2 - 150x$. (10 pts)

Problem 4 (answer on page 4 of the booklet)

For each of the following limits, say if it exists or no, justifying your answer. (5 pts each)

a) $\lim_{(x,y) \rightarrow (0,0)} \cos\left(\frac{x^6y^5}{x^{10}+y^2}\right)$ b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^6y^2}{x^{10}+y^5}$ c) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy(e^x-1)}{y-x}$ d) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^2}{x^4+y^2}$

Problem 5 (answer on page 5 of the booklet)

Is $f(x, y) = \begin{cases} x \sin \frac{1}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ continuous at $(0, 0)$? (5 pts)

Problem 6 (answer on page 6 and the last page of the booklet)

Suppose that the derivative of a function $f(x, y, z)$ at the point $(2, -3, 1)$ decreases most rapidly in the direction of $A = 3i - 2j + k$, and that in this direction the value of the derivative is $-2\sqrt{14}$. Also suppose that $f(3, 1, 0) = 7$, $f(5, -2, 4) = 20$, $\nabla f(3, 1, 0) = 3i - j$ and $\nabla f(5, 2, -4) = 4i - 3j + k$.

$f(2, -3, 1) = 4$.

Let $x = 3r - s$, $y = r - 4s$, $z = r^2s$ and $w = f(x, y, z)$

- (i) Find the derivative of f at the point $(3, 1, 0)$ in the direction of $i + j + \sqrt{2}k$. (5 pts)
(ii) Is there a unit vector u such that $D_u f(3, 1, 0) = \sqrt{10}$? (3 pts)
(iii) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $(r, s) = (1, 0)$. (7 pts)
(iv) Find the directions of zero change in w at the point $(r, s) = (1, 0)$ (4 pts)
(v) Find a line normal to the surface $w(r, s) = 4$ in the rs - plane. (8 pts)
(vi) Find a plane tangent to the surface $w(r, s) = 7(r - s)$ in the rs - plane. (7 pts)
(vii) Find the normal line to the surface $w(r, s) = 7 - \frac{1}{t}$ in the rst - space. (7 pts)