Math 201 **Practice Problems** for Quiz I (from Previouses) N.Nahlus

1) Find the following limits. Justify your answer.

a) $\lim_{n\to\infty} \left(\frac{7n-1}{7n+5}\right)^n$ b) $\lim_{n\to\infty} \left(\frac{4n+1}{7n+5}\right)^n$ c) $\lim_{n\to\infty} (-1)^n \sin\left(\frac{1}{\sqrt{n}}\right) \cos(n!)$)

d) $\lim_{n\to\infty} 100^{3n} . \frac{5^{2n}}{n!}$ e) $\lim_{n\to\infty} \sqrt{9n+5}\,\sin\left(\frac{1}{\sqrt{n}}\right)$ f) $\lim_{n\to\infty} \frac{n^n}{e^{n\sqrt{n}}}$

g) $\lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^{n^2}}{e^n}$ (TRICKY!) h) $\lim_{n\to\infty} \frac{1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\ldots+\frac{1}{n}}{\ln n}$ (10.3 Integral Test Estimate**s**)

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2) Investigate convergence/divergence of the following series

Warning: The big hints/tricks below will not be given on the Quiz (but very easy to remember!)

a) $\sum_{n=1}^{\infty} \left(\frac{7n+2}{7n-1}\right)^n$ b) $\sum_{n=2}^{\infty} (\ln n)^{10} \sin\frac{1}{n^{0.7}}$ ($\sin x \approx x$ if $x \approx 0$)

c) $\sum_{n=2}^{\infty} \ln\left(1+\frac{5^n}{n!}.(8000)^n\right)$ ($\ln(1+x) \approx x$ if $x \approx 0$) (see p.602)

d) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n (n!)^2}{(2n!)}$ (Ratio Test) e) $\sum_{n=1}^{\infty} (-1)^n \frac{4^n (n!)^2}{(2n!)}$ ($p=1$!) (p. 565)

f) $\sum_{n=1}^{\infty} \left(e^{\frac{1}{\sqrt{n}}}-1\right)^p$ **(*$e^x \approx 1+x$ if $x \approx 0$*)** (***see p***. 602)

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g) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}-\sin\frac{1}{\sqrt{n}}\right)^{1/2}$ ($\sin x \approx x-\frac{x^3}{3!}$ if $x \approx 0$) (see p. 602)

h) $\sum_{n=1}^{\infty} \frac{5e^n+2n^3+2^n}{3e^n+n\ln n+n!}$ i) $\sum_{n=1}^{\infty} (\sqrt[n]{n}-1)$ j) TRICKY $\sum_{n=2}^{\infty} \frac{\ln(n!)}{n^{1.5}}$

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3) **Find the domain** of convergence of the following power series.

Also indicate absolutely or conditionally.

a) $\sum_{n=2}^{\infty} (-1)^n \frac{(x-2)^n}{3^n\, n\ln n}$

b) $\sum_{n=1}^{\infty} (-1)^n (x-5)^n \left(\frac{n!}{n^n}\right)^n$ c) $\sum_{n=1}^{\infty} (-1)^n (x-5)^n \left(\frac{n^n}{n!}\right)^n$

4, 5, 6,………..10 **(see next)**

4) Find the following Limits:

a) $\lim_{n\to\infty} \left(\frac{n+\ln 3}{n+\ln 5}\right)^{n+7}$

b) $\lim_{n\to\infty} n^{\frac{1}{\ln n}}$

c) $\lim_{n\to\infty} n(7^{\frac{1}{n}}-1)$

d) $\lim_{n\to\infty} (-1)^n \tan^{-1}\left(\frac{1}{\sqrt{n}}\right) \cos^3(n!)$

e) $\lim_{n\to\infty} \dfrac{1+\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+\dots\dots+\frac{1}{n\ln n}}{\ln\sqrt{\ln n}}$

f) $\lim_{n\to\infty} \dfrac{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots\dots+\frac{1}{\sqrt{n}}}{2\sqrt{n}+3\ln n}$

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5) Find the domain of convergence of the following power series.
Also indicate absolutely or conditionally.

a) $\sum_{n=2}^{\infty} (-1)^n \dfrac{(x-2)^n}{3^n\sqrt{n+1}}$

b) $\sum_{n=2}^{\infty} (-1)^n \dfrac{(x-2)^n}{6^n\, n\ln\sqrt{n}}$

c) $\sum_{n=2}^{\infty} (-1)^n \dfrac{(x-2)^n}{6^n\, n\sqrt{\ln n}}$

d) $\sum_{n=1}^{\infty} (-1)^n (x-5)^n \left(\dfrac{n^n}{e^{n\sqrt{n}}}\right)^n$

e) $\sum_{n=1}^{\infty} (-1)^n \dfrac{(n!)^2}{4^n\,(2n!)}(x-5)^{2n}$

(BIG Hint: Rescue at End points)

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6) Show that

a) $\dfrac{1}{(1-x)^2}=\sum_{n=1}^{\infty} n\, x^{n-1}$ for $|x|<1$

b) Find the exact value of $\sum_{n=1}^{\infty} \dfrac{n^2}{3^n}$

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7) Investigate convergence/divergence of the following series

a) $\sum_{n=1}^{\infty} \dfrac{1}{n^{0.1}(e^{2n}+n\ln n)}$

b) …………………c) ……………………..

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8) a) Find the Maclaurin series of $f(x)=\dfrac{1}{4+3x}$ by any method

b) Find the Taylor series of $f(x)=\dfrac{1}{4+3x}$ at $a=2$ by any method