Math 202 - Final (Fall 12)

T. Tlas

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- Write the answer to question 5 on its sheet (note that a sheet of paper has two sides, you can write on both of them). The first four questions have extra sheets for you to write your answers on them. Any part of your answers written on the wrong sheet will not be graded.
- There are 5 problems in total. Most of the questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$Q5$	
$TOTAL$	

Problem 1

(8 points each) Solve the following IVPs:

i-

$$y' = ye^x \qquad ; \qquad y(0) = e$$

ii-

$$y' = \frac{1}{x + y + e^x e^y} - 1 \qquad ; \qquad y(1) = -1$$

iii-

$$y' = \frac{-y}{x} + x \qquad ; \qquad y(1) = 1$$

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2

(8 points each) Solve the following IVPs:

i-

$$y'' - 3y' + 2y = -e^x \qquad ; \qquad y(0) = 0 \quad , \quad y'(0) = 2$$

ii-

$$y'' + 4y = \delta(t-3) \qquad ; \qquad y(0) = 1 \quad , \quad y'(0) = 0$$

iii-

$$\dot{\mathbf{X}} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{X} \qquad ; \qquad \mathbf{X}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER

Problem 3

(18 points) Solve the following IVP:

$$y'' - 2xy' + 8y = 0 \qquad y(0) = 1 \quad , \quad y'(0) = 0$$

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ADDITIONAL SHEET FOR PROBLEM 3 ANSWER

Problem 4

Let

$$\overrightarrow{F(x,y,z)} = \Big(-\frac{y}{x^2+y^2} + x \ , \ \frac{x}{x^2+y^2} + y \ , \ z \Big)$$

Find

a- $\int_C \overrightarrow{F} \cdot \overrightarrow{dr}$ if C is:

i- (6 points) The straight line segment from $(0,1,1)$ to $(0,3,1)$.

ii- (6 points) The circle of center $(0,0,1)$, radius 10, in the plane $z = 1$, traversed counterclockwise if looking from above.

b- $\int_S \overrightarrow{F} \cdot \overrightarrow{dA}$ if S is:

i- (6 points) The sphere of center $(10,10,10)$ whose radius is 4.

ii- (6 points) The cube whose corners are
$(10,10,10), (11,10,10), (10,11,10), (10,10,11), (11,11,10), (11,10,11), (10,11,11), (11,11,11)$

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ADDITIONAL SHEET FOR PROBLEM 4 ANSWER

Problem 5

(10 points) Suppose f is a function defined in space (i.e. it is a function of three variables). Let $\overrightarrow{F} = \overrightarrow{\nabla f}$, where $\overrightarrow{\nabla f} = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$. Let $A = (0, 10, 13)$ and $B = (134, 23, 1)$ be two points in space and suppose that $f(A) = 1$ and $f(B) = 5$. Let C be a curve which goes from A to B. Find, with justification, $\int_C \overrightarrow{F} \cdot \overrightarrow{dr}$.

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