American University of Beirut

Math 202-Differential Equations

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Final Exam- Time: 120 minutes

May 14, 2014

Your Name:..... and ID:.....

Grades:

1/8	2/8	3/21	4/8	5/8	6/8	7/24	8/15	Total/100

Notes:

- **Calculators are not allowed.**

PLEASE CIRCLE YOUR SECTION:

Section 1(N. Nahlus, TTh 1230, Recitation F 1),

Section 2(N. Nahlus, TTh 1230, Recitation F 12),

Section 3(N. Nahlus, TTh 1230, Recitation F 8),

Section 4(N. Nahlus, TTh 1230, Recitation F 9),

Section 5(N. Nahlus, TTh 2 Recitation W 8),

Section 6(N. Nahlus, TTh 2 Recitation W 12),

Section 7(N. Nahlus, Th 2 Recitation W 11),

Section 8(N. Nahlus, Th 2 Recitation W 2).

Section 9(H. Yamani, MWF 10, Recitation Th 3:30),

Section 10(H. Yamani, MWF 10, Recitation Th 11),

Section 11(H. Yamani, MWF 10, Recitation Th 9:30),

Section 12(H. Yamani, MWF 10, Recitation Th 8),

Section 13(M. Kobeissi, MWF 2, Recitation F 12),

Section 14(M. Kobeissi, MWF 2, Recitation F 3),

Section 15(M. Kobeissi, MWF 2, Recitation F 4),

Section 16(M. Kobeissi, MWF 2, Recitation F 5),

Section 17(K. Aziziheris, MWF 8, Recitation M 2),

Section 18(K. Aziziheris, MWF 8, Recitation F 2),

Section 19(K. Aziziheris, MWF 8, Recitation F 4),

Section 20(W. Raji, MWF 1, Recitation M 8),

Section 21(W. Raji, MWF 1, Recitation M 9),

Section 22(W. Raji, MWF 1, Recitation M 2),

Section 23(W. Raji, MWF 1, Recitation M 5).

Usual Laplace formulas and rules:

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$

- $\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t) dt$

- $\mathcal{L}\{f^{(n)}t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$

- $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

- $\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as} F(s)$

- $\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s}$

- $\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

- $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$

- $f * g = \int_0^t f(\tau) g(t-\tau) d\tau$

- $\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

- $\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$

Problem 1. (8 pts) Use Gauss divergence Theorem carefully to find the outward flux $\int\int_S F \cdot n d\sigma$ of the vector field

$$\overrightarrow{F} = \frac{1}{\sqrt{x^2+y^2+z^2}}(2x\,\overrightarrow{i} + y\,\overrightarrow{j} + 3z\,\overrightarrow{k}\,\sqrt{x^2+y^2+z^2})$$

across the surface $\rho = 3$.

Problem 2. (8 pts) Find the area of the surface cut form the plane $x+2y+2z=5$ by the cylinder whose walls are $x=y^2$ and $x=2-y^2$.

Problem 3. (21 pts, each part 7 pts) Solve the following DE's:

- $$x\frac{dy}{dx} + 2y = x^2y^2, y(1) = 1/5$$

- $$\frac{dy}{dx} = \frac{\sqrt{1+(2x+y)^3}}{(2x+y)^2} - 2$$

- $$x^2y'' - 2xy' + 2y = \frac{x^3}{\sqrt{1-x^2}}$$

Problem 4. (8 pts) Use Frobenius method to find ONE series solution (corresponding to the larger root) of the equation

$$x(x-1)y'' + 3y' - 2y = 0$$

about the regular singular point $x = 0$. (Hint: Only find the first three non-zero terms)

Problem 5. (8 pts) Use Laplace Transform and solve the following DE:

$$y'' + 2y' + 10y = \delta(t-2) + 13e^t, y(0) = 0, y'(0) = 13$$

Problem 6. (8 pts) Use Laplace Transform and solve the given equation:

$$\frac{-3}{8}(1+t-f(t)) = \int_0^t f(\tau)(t-\tau)^3 d\tau$$

Problem 7. (24 pts, each part 8 pts) Answer the following questions:

1. Find $f(t)$ given that

$$F(s) = \left(\frac{s}{s^2+25}\right)^{(4)},$$

where the exponent (4) is the fourth derivative.

2. Use the definition of Laplace Transform to find the value of

$$\int_0^\infty e^{-3t}\cos(4(t-5))\mathcal{U}(t-5)dt$$

3. Find the Laplace of

$$f(t) = \frac{1 - \cos t}{t}$$

Problem 8. (15 pts) Find the general solution of the given system:

$$X' = AX,$$

where

$$A = \begin{bmatrix} 7 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$