

MATHEMATICS 202  
( 1st Semester, 1994-95 )  
Quiz II

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Time : 60 mins.

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(closed book)

I.     Solve the following DE's.

(a)     $x^2 y'' - 4xy' + 6y = \ln x^2$

(b)     $xy'' + (1 - x)y' = 0$

II.    Use the substitution  $y = \sqrt{x} u(x)$  to solve

$$x^2 y'' + (x^2 + 1/4)y = 0$$

III.   Represent  $J_{-\frac{5}{2}(x)}$  in terms of elementary functions .

IV.   Find the Laplace transform of  $f(t)$  and of  $f(t) U(t-2)$  when

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t^3, & t \geq 1 \end{cases}$$

*Solution*  
*Quiz II 1994-95*

1)

a)  $x^2y'' - 4xy' + 6y = \ln x^2$

$$x^2y'' - 4xy' + 6y = 0$$

0 singular point

$$r(r-1) - 4r + 6 = 0$$

$$r^2 - 5r + 6 = 0 \Rightarrow r = 1 \quad \& \quad r = 6$$

but we can easily see that this is a cauchy-euler equation =>

$$y = c_1x + c_2x^6$$

$$y_p = ??$$

$$w = 5x^6 \quad w_1 = -2x^6 \ln x \quad w_2 = 2x \ln x \quad u'_1 = \frac{w_1}{w}$$

$$u'_1 = \frac{-2}{5} \ln x \Rightarrow u_1 = 2x \ln x - 2x \quad u'_2 = \frac{w}{w_2} = \frac{2}{5} x^{-5} \ln 2$$

$$y_{1p} = \frac{2}{5}(x \ln x - x)x$$

$$y_{2p} = \frac{2}{5} \int (x^{-5} \ln x) dx$$

by parts we get :

$$y_{2p} = 2\left(\frac{x^{-4}}{-4} \ln x - \frac{x^{-4}}{-4}\right)x^6$$

b)  $xy'' + (1-x)y' = 0$

Let  $y' = z \Rightarrow y'' = z'$

$$xz' + (1-x)z = 0$$

$$P_0 = \frac{(1-x)}{x} x = 1-x \Rightarrow 0 \text{ is a singular point}$$

$$r(r-1) + r = 0 \Rightarrow r = 0$$

$$z = \sum C_n x^{n+r}, \quad z' = \sum (n+r)C_n x^{n+r-1}$$

ex :

$$x^r \left[ \sum_1 (n+r)C_n x^n - \sum C_n x^n + \sum C_n x^{n+1} \right]$$

$$= \sum_{k=0} [(k+r)C_k x^k - C_k] x^k + \sum_{k=1} C_{k-1} x^k$$

$$rC_0 = 0 \quad C_0 = C_0$$

$$C_k = \frac{C_{k-1}}{k-1}, \quad 0C_1 = 0 = C_0$$

$$z = 0 \Rightarrow y_1 = C_0$$

$$y_2 = C_0 \int \frac{e^{-\int \frac{(1-x)}{x} dx}}{C_0^2} dx = \frac{1}{C_0} \int \frac{e^x}{2} dx$$

2)  $y = \sqrt{x} u(x)$

$$x^2 y'' + (x^2 + 1/4)y = 0 \quad \dots (1)$$

$$y' = \frac{1}{2\sqrt{x}} u(x) + u'(x)\sqrt{x}$$

$$y'' = \frac{u'(x)}{2\sqrt{x}} + \frac{1}{4} u(x)(x^{-3/2}) + u''(x)\sqrt{x} + \frac{1}{2\sqrt{x}} u'(x)$$

substituting in (1) & multiplying by  $x^{-1/2}$  we get :

$$x^2 u''(x) + xu'(x) + x^2 u(x) = 0$$

which is a Bessel equation where  $v = 0$ ,  $y = C_1 J_0 x + C_2 J_1 x$

3)  $J_{-\frac{5}{2}}(x)$

$$2J_v = xJ_{v+1} + xJ_{v-1}$$

$$J_{-3/2} = \frac{2J_{-1/2} - xJ_{1/2}}{x} \quad J_{-\frac{1}{2}} = \sqrt{\frac{2}{\pi x}} \sin x$$

$$J_{-5/2} = \frac{2J_{-3/2} - xJ_{-1/2}}{x} \quad J_{-\frac{3}{2}} = \sqrt{\frac{2}{\pi x}} \cos x$$

4)  $f(t) = \begin{cases} 0, & 0 < t < 1 \\ t^3, & t \geq 1 \end{cases} = U(t-1)t^3.$

$$\zeta - U(t-1)t^3 = \frac{e^{-s} \times 3!}{s^4} - \frac{e^{-s}}{s} - \left( \frac{3e^{-s}}{s^2} \right)$$