

MATHEMATICS 202

(2nd Semester, 1993-94)

March 26, 1994

QUIZ I

1. (a) Use an appropriate substitution and solve:

$$(x+2y-1) dx + 3(x+2y) dy = 0$$

Subject to: $y(1) = -1$

- (b) Solve the initial value problem:

$$y(x^2+y^2)dx + x(3x^2-5y^2)dy = 0; \quad y(2) = 1$$

(8 + 10 = 18 pts.)

2. (a) Find r so that x^r is an integrating factor of the equation:

$$(3xy^2+4y)dx + (4x^2y + 8x)dy = 0$$

then find the general solution.

- (b) Solve: $y' = y^2 - y - 2; \quad y(0) = 1$

(9 + 9 = 18 pts.)

3. Given the differential equation:

$$y'' + Ay' + By = 0 \dots \quad (1)$$

- (a) Find constants A and B so that $y_1 = ex\sin x$ is a solution of (1)
- (b) Find a second linearly independent solution y_2 of (1)
- (c) Using the result of parts (a) and (b) above, determine a particular solution of :

$$y'' + Ay' + By = 1 \dots \quad (2)$$

by inspection. Then determine the general solution of (2).

(5 + 6 + 3 = 14 pts.)

Solution
Quiz 1 1993-94

1)

a) $(x+2y-1)dx + 3(x+2y)dy = 0 \quad y(1) = -1$

$$\text{Let } u = x + 2y \Rightarrow \frac{du}{dx} = 1 + 2\frac{dy}{dx} \Rightarrow u' = 1 + 2y'$$

$$u - 1 + 3/2(u)(u' - 1) = 0$$

$$3/2(uu') - 1/2(u) - 1 = 0$$

$$3uu' - u - 2 = 0$$

$$3udu - (u+2)dx = 0$$

$$\frac{3u}{u+2} du = dx \Rightarrow$$

$$\int \frac{3u}{u+2} du = x + c$$

integrating we get :

$$3u - 6\ln|u+2| = x + c$$

$$3(x+2y) - 6\ln|x+2y+2| = x + c$$

$$y(1) = -1 \Rightarrow c = -4 \Rightarrow$$

$$3(x+2y) - 6\ln|x+2y+2| = x - 4$$

b) $y(x^2+y^2)dx + x(3x^2-5y^2)dy = 0; \quad y(2) = 1$

This is a homogeneous equation of order 3 .

$$\text{Let } y = ux \Rightarrow dy = xdu + udx$$

$$ux(x^2 + u^2x^2)dx + x(3x^2 - 5u^2x^2)(xdu + udx) = 0$$

$$x^3u(u+1)dx + x^4(3-5u^2)du + x^3(3-5u^2)udx = 0$$

$$x^3(-5u^3 + u^2 + 4u)dx + x^4(3-5u^2)du$$

$$\int \frac{5u^2 - 3}{-5u^3 + u^2 + 4u} du = \ln|x| + c$$

by partial differentiation we get :

$$\int \frac{5u^2 - 3}{u(-5u^2 + u + 4)} du = \int \left(\frac{-4}{3u} + \frac{(5/4)u + 4/3}{-5u^2 + u + 4} \right) du$$

integrating we get :

$$\ln|x| + c = \frac{-4}{3} \ln\left|\frac{y}{x}\right| - \frac{1.5}{12} \ln\left|\frac{-5y^2}{x^2} + \frac{y}{x} + 4\right| - \frac{7}{3} \ln\left|\frac{y}{x} - 1\right|^{1/5} - \frac{7}{3} \ln\left|\frac{y}{x} - \frac{4}{5}\right|^{-1/5}$$

it is easy to find c by replacing x & y by (2 & 1).

2)

a) $u = x^r$

$$(3xy^2 + 4y)dx + (4x^2y + 8x)dy = 0$$

$$\mu \times M = 3x^{r+1}y^2 + 4x^ry$$

$$\mu \times N = 4x^{2+r}y + 8x^{r+1}$$

$$\frac{\partial NM}{\partial y} = 6x^{r+1}y + 4x^r$$

$$\frac{\partial \mu N}{\partial x} = 4(2+r)x^{r+1}y + 8(r+1)x$$

$$6x^{r+1}y + 4x^r = 4(2+r)x^{r+1}y + 8(r+1)x$$

$$6 = 8 + 4r \Rightarrow r = \frac{-1}{2}$$

verification :

$$4 = 8(-1/2 + 1)$$

$$(3x^{1/2}y^2 + 4x^{-1/2}y)dx + (4x^{3/2}y + 8x^{1/2})dy = 0$$

Exact - Equation

$$\int M dx = 2y^2x^{3/2} + 8yx^{1/2} + gy$$

$$\frac{\partial (Mdx)}{\partial y} = N$$

$$\Rightarrow 4yx^{3/2} + 8x^{1/2} + g(y) = 4x^{3/2}y + 8x^{1/2}$$

$$g(y) = 0 \Rightarrow g(y) = c_1$$

$$2y^2x^{3/2} + 8yx^{1/2} + c_1 = c_2$$

b)

$$y' = y^2 - y - 2$$

$$y(0) = 1$$

$$y' + y - y^2 + 2 = 0$$

Ricard, Van basken, Gwillik

$$\text{Let } y = y_1 + u$$

$$y_1 = 2$$

$$y = 2 + u$$

$$y' = u'$$

$$\Rightarrow u' - u_1 + 3u = 0$$

Bernoulli

$$\frac{u'}{u^2} + \frac{3}{u} = 1$$

$$\text{Let } z = 1/u$$

$$\Rightarrow z' = -u'/u^2$$

linear equation

$$N = e^{-3x}$$

$$z = \frac{1}{3} + \frac{c}{e^{-3x}} = \frac{1}{u} = \frac{1}{y-2}$$

substituting values of x & y we get : $c = -4/3$

3)

$$y'' + Ay' + By = 0$$

a) $y_1 = e^x \sin x \Rightarrow y' = e^x \sin x + \cos x (e^x)$
 $y'' = 2e^x \cos x$
 $2e^x \cos x + A(e^x \sin x + \cos x (e^x)) + Be^x \sin x = 0$
 $A = -2 \quad A = -B \quad \Rightarrow \quad B = 2$

b) $y'' - 2y' + 2y = 0$
 $y_2 = y_1 \int \frac{e^{-\int -2dx}}{(y_1)^2} dx = -e^x \sin x \cot gx$

c) $y'' - 2y' + 2y = 1$
 $y = 1/2$ is a particular solution

$$c_1 e^x \sin x + c_2 e^x \sin x \cot gx + \frac{1}{2} = 0 \quad (\text{General solution})$$