MATHEMATICS 202
(2nd Semester , 1992-93)
Quiz I

Time : 60 mins. March 27 , 1993

1) Find the general solution of :

$$(2x^3 - y + x^2 \sin y)dx + (x^3 \cos y + x)dy = 0$$ (10 pts.)

2) Consider the differential equation :

$$2ww' + x = \frac{e^{-3/4(2w^2+x^2)}}{x^2+1} \quad \text{.... (1)}$$

a) Use the substitution $y = e^{w^2}$ to transform equation (1) to Bernoulli type equation (2) .

b) Solve (2) and deduce the solution of (1) .

c) Find the part solution that satisfies w(0) = 0 (10 pts.)

3) Find the general solution of the differential equation :

$$y'' - 3y' + 2y = \cos(e^{-x})$$ (10 pts.)

4) Given the initial value problem :

$$y' + 2y^2 - 3y + 1 = 0 \ ; \qquad y(0) = 3/2 \quad \text{...} \quad (1)$$

Provide the solution of (1) , that must not contain any arbitrary constant .

Hint : Find the constant k, so that the substitution y = kz'/z will produce a second order differential equation any Math 202 student may handle . (10 pts.)

50

5) Find the general solution of the following integro-differential equation :

$$y'+4y+3\int_0^x y(t)dt = \sin^2 x + e^{-x}$$

Hint : Take the derivative of both sides .

Solution
Quiz1 1992-93

1)

$$(2x^3 - y + x^2 \sin y)dx + (x^3 \cos y + x)dy = 0$$

$$\frac{\partial M}{\partial y} = -1 + x^2 \cos y$$

$$\frac{\partial N}{\partial x} = 3x^2 \cos y + 1$$

Then $\mu = 1/x^2$ is an integrating factor .

$$\frac{\partial \mu N}{\partial y} = \frac{\partial \mu M}{\partial x} = \cos y - \frac{1}{x^2}$$

Exact equation :

$$f(x,y) = \int (2x - \frac{1}{x^2}y + \sin y)dx = x^2 + \frac{1}{x}y + x\sin y + g(y)$$

$$\frac{\partial f(x,Y)}{\partial y} = \mu N$$

$(1/x) + x\cos y + g'(y) = x\cos y + 1/x$

=> $g'(y) = 0$ => $g(y) = c$

=> $(1/x) + x\cos y = c_2$

2) $$2ww' + x = \frac{e^{-3/4(2w^2+x^2)}}{x^2+1} \quad .... \ (1)$$

$y = e^{w^2}$ => $y' = 2ww'e^{w^2}$

$$\frac{y'}{y} + x = \frac{y^{-3/2}e^{(-3/4)x^2}}{x^2+1}$$ which is Bernoulli

$$\frac{y'}{y^{-1/2}} + y^{3/2}x = \frac{e^{-(3/4)x^2}}{x^2+1}$$

$z = y^{3/2} \Rightarrow z' = \frac{3}{2}y^{-1/2}y' \Rightarrow z' + \frac{3}{2}zx = \frac{e^{-\frac{3}{4}x^2}}{x^2+1}$ linear.

substituting and solving we get :

$A = e^{\int \frac{3}{2}xdx}$ integrating factor

$$\Rightarrow ze^{\frac{3}{4}x^2} = \int \frac{3}{2}(\frac{1}{x^2+1})dx$$

$$ze^{(3/4)x^2} = \frac{3}{2} Arctgx + c$$

$$z = \frac{3 Arctagx}{2e^{(3/4)x^2}} + \frac{c}{e^{(3/4)x^2}}$$

$$e^{(3/2)w^2} = \frac{3 Arctagx}{2e^{(3/4)x^2}} + \frac{c}{e^{(3/4)x^2}} \qquad w(0) = 0$$

$$e^{(3/2)w^2} = \frac{3 Arctagx}{2e^{(3/4)x^2}}$$

3) $y'' - 3y' + 2y = \cos(e^{-x}) = f(x)$

$y'' - 3y' + 2y = 0$ is an auxilliary equation :

$m2 - 3m + 2 = 0$

$m = 1$ & $m = 2$

$y = c_1 e^x + c_2 e^{2x}$ $w = e^{3x}$ Wromst

$$u'_1 = \frac{-e^{2x} f(x)}{w}, \quad u'_2 = -\frac{e^{-x} f(x)}{w}$$

particular solution :

$u'_1 = e^{-x} \cos(e^{-x})$

$u'_2 = e^{-2x} \cos(e^{-x})$

$\Rightarrow$ $u_1 = \sin(e^{-x})$ & $u_2 = \int e^{-x}(e^{-x}\cos(e^{-x}))dx$

solving for u_2 by integration by parts we get :

$v = e^{-x}$ $du = e^{-x} \cos e^{-x} dx$

$u_2 = e^{-x} \sin e^{-x} - \cos e^{-x}$

$y_p = u_1 y_1 + u_2 y_2$

$= e^{-x} \sin e^{-x} + e^{-2x}(e^{-x} \sin e^{-x} - \cos e^{-x})$

4) $y' + 2y2 - 3y + 1 = 0$; $y(0) = 3/2$

$$y = \frac{kz'}{z}$$

$$y' = \frac{kz''}{z} - \frac{kz'^2}{z^2}$$

$$\frac{kz''}{z} - \frac{kz'^2}{z^2} + 2(\frac{kz'}{z})^2 - 3\frac{kz'}{z} + 1 = 0$$

$2k^2 - k = 0 \Rightarrow k = 0$ uneccaptable, $k = 1/2$.

Let $k = 1/2$

$$\frac{1}{2}z'' - \frac{3}{2}z' + z = 0$$

$$m^2 - 3m + 2 = 0$$

$m = 1 \qquad \& \qquad m = 2$

$$z = c_1e^x + c_2e^{2x} \Rightarrow y = k\frac{z'}{z} = \frac{1}{2}\frac{(C_1e^x + 2C_2e^{2x})}{C_1e^x + c_2e^{2x}}$$

$$y = \frac{\frac{C_1e^x}{2} + C_2e^{2x}}{C_1e^x + C_2e^{2x}} \qquad y(0) = \frac{3}{2}$$

$$\frac{3}{2} = \frac{\frac{C_1}{2} + C_2}{C_1 + C_2} = C_1 + 2C_2 = 3C_1 + 3C_2 \Rightarrow C_2 = -2C_1$$

y'=-2y2 +3y -1 y'(0) = -2(9/4) + (3×3)/2 -1 = -1

By substitution we get another equation.

5)

$$y' + 4y + 3\int_0^x y(t)dt = \sin^2 x + e^{-x}$$

$y'' + 4y' + 3y = 2 \sin x \cos x - e^{-x}$

$y'' + 4y' + 3y = \sin 2x - e^{-x}$

This is a homogeneous equation

$y = c_1e^{-x} + c_2e^{-3x}$

$y_p = A\sin 2x + B\cos 2x + Ce^{-x}$

A,B,&C can be found by the method of undetermined coefficients