MATHEMATICS 202

(1st Semester, 1994 - 95)

Time : 60 mins. **QUIZ I** (closed books)

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1. Solve the following DE's

   a) $(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0$

   b) $y''' + 2y'' + 5y' - 26y = 0$

   c) $x(y')^3 - y(y')^2 + 1 = 0$

2. Solve the IVP

   $(y^2\sin x)dx + (1/x - y/x)\,dy = 0\ ;\ \ y(\pi) = 1$

3. Given one solution $y_1(x) = x$ for

   $y'' - xy' + y = 0$

   What is the other solution ?

. Over what interval we are assured that there is a unique solution to the IVP:

   $y''' + x^2y'' + (\ln x)y' - (\cos x)y = (2-x)^{-1}$;

   $y(1) = 3, \quad y'(1) = -2, \quad y''(1) = -6$

   Support your answer with an explanation.

Solution
Quiz1 1994-95

1)

a) $(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0$

we can conclude that $N = x^2y$ is an integrating factor .

$(5x^4y^2 + 6x^5y^3 + 4x^3y^3)dx + (2x^5y + 3x^6y^2 + 3x^4y^2)dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 10x^4 + 18x^5y^2 + 12x^3y^2$$

=> exact equation

$$f(x,y) = \int M dx = x^5y^2 + x^6y^3 + x^4y^3 + g(y)$$

$$\frac{\partial f(x,y)}{\partial y} = N$$

=> $g'(y) = 0$ => $g(y) = c$

$$f(x,y) = x^5y^2 + x^6y^3 + x^4y^3 + c = c_1$$

b) $y''' + 2y'' + 5y' - 26y = 0$

$m^3 + 2m^2 + 5m - 26 = 0$

=> $m = 2$ is a solution

$(m - 2)(m2 + 4m + 13)$

$\Delta' = 4 - 13 = -9$

$m = -2 + 3i$ & $m = -2 - 3i$

$$y = c_1e^{2x} + c_2e^{-2x}\sin 3x + c_3e^{-2x}\cos 3x$$

c) $x(y')^3 - y(y')^2 + 1 = 0$

$$xy' - y = \frac{-1}{y'^2}$$

=> $y = c_1x + (1/c^2)$ $x = -2/t^3$

$$y = \frac{+1}{t^2} - \frac{2}{t^2} = \frac{-1}{t^2}$$

2) $(y^2\sin x)dx + (1/x - y/x)\,dy = 0$; $y(\pi) = 1$ (separable)

$$x\sin x\,dx = \frac{-(1-y)}{y^2}dy$$

$$-x\cos x + \cos x + c = y^{-1} + \ln|y|$$

3) $y'' - xy' + y = 0$ $\qquad\qquad y_1(x) = x$

$$y_2 = y_1 \int \frac{e^{-\int -x dx}}{(y_1)^2} dx = x \int \frac{e^{x^2/2}}{x^2} dx$$

Note : This integration is to be solved with the help of series expansion from Math 201 .

4) The interval is :

$x > 0 \,;\, x \neq 2$ $\qquad\qquad I =]0, 2[$

The interval ~~and~~ should contain 1

the coefficient should be determined .