MATHEMATICS 202

(1st Semester, 1994 - 95)

Time: 60 mins. **QUIZ I** Novembe 24,1994

1. Solve the following:

(a) $(1 + 2e^x)dx - (1 + c^x)(2y + \cos y)dy = 0; \; y(0) = 0.$

(b) $(y - \sqrt{x^2 - y^2})\,dx - xdy = 0$

(c) $\frac{dy}{dx} + \frac{2y}{x} = xy^2$

(d) $\frac{dy}{dx} = x + y$ using the substitution $y = ve$

2. Determine A so that

$$(3x^2y^2 + Axy)dx + (2x^3y + 2x^2)dy = 0 \qquad \text{is exact.}$$

If y is a solution of the resulting equation satisfying :

$y(1) = 2$ and $y(2) > 0$, find $y(2)$.

3. If $y = u + x$ satisfies the differential equation

$$\frac{dy}{dx} + \frac{1}{x} = x,$$

find u.

4. If $y = x$ and $y = \sin x$ are solutions of

$$y'' + ay' + by = 0,$$

determine the functions a and b.

Solution
Quiz1 1994-95

1)

a) $(1 + 2e^x)dx - (1+e^x)(2y + \cos y)dy = 0$

$$\Rightarrow \frac{1 + 2e^x}{1 + e^x} dx = (2y + \cos y)\, dy$$

$$\frac{1 + e^x + e^x}{1 + e^x} dx = (2y + \cos y)\, dy$$

$$(1 + \frac{e^x}{1 + e^x})\, dx = (2y + \cos y)\, dy$$

$x + Ln(1 + e^x) + C = y^2 + \sin y$

$y(0) = 0$

$\Rightarrow 0 + Ln\, 2 + C = 0 + 0$

$\Rightarrow C = - Ln\, 2$

$y2 + \sin y = x + Ln(1 + e^x) - Ln2$

b) $(y - \sqrt{x^2 - y^2})\, dx - xdy = 0$

$y = ux \Rightarrow dy = udx + xdu$

$(ux - \sqrt{x^2 - u^2 x^2})\, dx - x(udx + xdu) = 0$

$x (u - \sqrt{1 - u^2})\, dx - x (udx + xdu) = 0$

$\Rightarrow - \sqrt{1 - u^2}\, dx - xdu = 0$

$$\frac{du}{\sqrt{1 - u^2}} = - \frac{dx}{x} \Rightarrow \text{ARC} \sin u = - \text{Ln}|x| + C$$

$$\Rightarrow u = - \sin(\text{Ln}|x| + C)$$

c)

$$\frac{dy}{dx} + \frac{2y}{x} = xy^2$$

$$\Rightarrow y^{-2} \frac{dy}{dx} + \frac{2}{x} y^{-1} = x$$

Let $w = y^{-1} \Rightarrow \frac{dw}{dx} = -y^{-2} \frac{dy}{dx}$

$$\Rightarrow -\frac{dw}{dx} + \frac{2}{x} w = x$$

$\frac{dw}{dx} - \frac{2}{x} w = -x$ Linear equation.

$$m = e^{-\int \frac{2}{x} dx} = x^{-2}$$

$$\Rightarrow x^{-2} \frac{dw}{dx} - \frac{2}{x^3} w = - x^{-1}$$

$$\frac{d}{dx}(x^{-2} w) = - \frac{1}{x}$$

$$x^{-2} w = - \text{Ln} x + C$$

$$\Rightarrow w = - \frac{\text{Ln} x}{x^{-2}} + C x^2 = y^{-1}$$

d)

$$\frac{dy}{dx} = x + y$$

$$y = ve^x$$

$$\Rightarrow \frac{dy}{dx} = v\,e^x + e^x\frac{dv}{dx}$$

$$ve^x + e^x\,\frac{dv}{dx} = x + ve^x$$

$$\Rightarrow e^x\,\frac{dv}{dx} = x$$

$$\Rightarrow dv = e^{-x}\,x\,dx$$

$$v = \int(e^{-x}\,x\,dx) \quad = -xe^{-x} + \int e^{-x}dx\;C$$

$u = x \qquad dv = e^{-x}$

$du = dx \qquad v = -\,e^{-x}$

$$v = -xe^{-x} - e^{-x} + C = {}^{y}\!/_{e^x}$$

$$\Rightarrow y = -x - 1 + C = -x + C$$

2)

$$(3x^2y^2 + Axy)\,dx + (2x^3y + 2x^2)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 6x^2 + Ax$$

$$\frac{\partial N}{\partial x} = 6x^2y + 4x$$

$$\Rightarrow A = 4$$

$$\frac{\partial f(x,y)}{\partial x} = 3x^2y^2 + 4xy$$

$$f(x,y) = x^3y^2 + 2x^2y + g(y)$$

$$\frac{\partial f}{\partial y} = 2x^3y + 2x^2 + g'(y) = N$$

$$\Rightarrow g'(y) = 0 \quad \therefore g(y) = C$$

$$\Rightarrow f(x,y) = x^3 y^2 + 2x^2y + C = C_1$$

$y(1) = 2$

$$\Rightarrow C_2 = -(4+4) = -8$$

$$x^3y^2 + 2x^2y - 8 = 0$$

$$y(2) = 8y^2 + 8y - 8$$

$$\Delta = 64 + 4(64) \quad \Rightarrow \sqrt{\Delta} = 8\sqrt{5}$$

$$y(2) = \frac{-8 + 8\sqrt{5}}{16} = \frac{-1 + \sqrt{5}}{2} \qquad (y(2) > 0)$$

3.

$$\frac{dy}{dx} = \frac{du}{dx} + 1$$

$$\frac{du}{dx} + 1 + \frac{1}{x} = x$$

$$\Rightarrow du = (x - \frac{1}{x} - 1)\,dx$$

$$u = \frac{x^2}{2} - Ln|x| - x + C$$

4.

$y = x \qquad\qquad y = \sin x$

$y'' + ay' + by = 0$ .... (0)

$y = x \Rightarrow y' = 1$ & $y'' = 0$

$0 + a + bx = 0$ .... (1)

$\Rightarrow a = -bx$

$y = \sin x \Rightarrow y' = \cos x \;\&\; y'' = -\sin x$

$-\sin x + a\cos x + b\sin x = 0$

$-\sin x - bx\cos x + b\sin x = 0$

$\Rightarrow -\sin x + b(\sin x - x\cos x) = 0$

$$\Rightarrow b = \frac{\sin x}{\sin x - x\cos x}$$

$$a = -x\frac{\sin x}{\sin x - x\cos x}$$