Math 202 - Final (Summer 11)

T. Tlas

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- Please answer questions 4, 5 and 6 on the same sheet of paper on which they are written. Questions 1, 2 and 3 each have an extra sheet for you to write your answer on them. Any part of your answer written on the wrong page will not be graded.
- There are 6 problems in total. Some questions have several parts. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$Q5$	
$Q6$	
$TOTAL$	

Problem 1

(4 points each) Solve the following IVPs:

i-

$$y' = yx - x + y - 1 \qquad ; \qquad y(0) = 2$$

ii-

$$y'' - 3y' + 2y = 6e^{-x} \qquad ; \qquad y(0) = 0 \qquad ; \qquad y'(0) = -1$$

iii-

$$y'' + y = \delta(t-1) \qquad ; \qquad y(0) = 0 \qquad ; \qquad y'(0) = 0$$

iv-

$$y' + 6y + 9\int_0^t y(\tau)d\tau = \delta(t-1) \qquad ; \qquad y(0) = 0$$

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2

(10 points) Solve the following BVP:

$$4xy'' + 2y' + y = 0 \qquad ; \qquad y(0) = 0 \qquad , \qquad y\left(\frac{\pi^2}{4}\right) = 1$$

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER

Problem 3

(10 points) Solve the following IVP:

$$\dot{\mathbf{X}} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 0 \\ t \end{pmatrix} \qquad ; \qquad \mathbf{X}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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ADDITIONAL SHEET FOR PROBLEM 3 ANSWER

Problem 4

(6 points) Prove the following identity involving the Bessel function:

$$2\lambda J_\lambda(x) = xJ_{\lambda+1}(x) + xJ_{\lambda-1}(x)$$

where the series expression for the Bessel function is:

$$J_\lambda(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(1+\lambda+n)} \left(\frac{x}{2}\right)^{2n+\lambda}$$

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Problem 5

(10 points) Let $\vec{F}$ be the vector field given by

$$\overrightarrow{F(x,y)} = -\frac{y}{x^2+y^2}\,\vec{\mathbf{i}} + \frac{x}{x^2+y^2}\,\vec{\mathbf{j}}$$

Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve made of the straight line segment going from $(1,0)$ to $(1,1)$ followed by the straight line segment going from $(1,1)$ to $(-1,1)$, followed by the straight line segment going from $(-1,1)$ to $(-1,0)$.

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Problem 6

(8 points) Let V be the following vector field:

$$\overrightarrow{V(x,y,z)} = \vec{\mathbf{k}}$$

Let S be the upper unit hemisphere centered at the origin. Find the flux $\int_S \vec{V} \cdot d\vec{A}$, where the normal vector is 'outwards' (away from the origin).