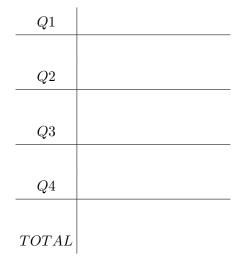
Math 202 - Exam 1 (Spring 09) T. Tlas

- Please answer questions 2,3 and 4 on the same sheet of paper on which they are written. Question 1 has an extra sheet for you to write your answer on it. Any part of your answer written on the wrong page will not be graded.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 4 problems in total. Some questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

Name :

ID # :

Section Number :



(10 points each) Solve the following IVPs:

i-

ii-

$$y' = x^2 + y^2 + 2xy$$
$$y' = y(y+1)x$$
$$y' = -\frac{y}{x} + e^x$$

iv-

iii-

$$y' = -\frac{2x}{1+x^2+y}$$

v-

$$y' = \frac{x^5 + x y^2}{y^2 + x^4}$$

The initial condition for each of the above equations is y(1) = 1.

ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

(4 points each) Calculate the circulation counterclockwise of the following vector field in the plane

$$\vec{V}(x,y) = \left(e^x \sin(e^x) + x^3 - 2 + y\right)\vec{i} + \left(2x + y\sin(y) + e^{y^2}\right)\vec{j}$$

around each of the following curves:

- i- The circle of radius $\sqrt{\frac{3}{\pi}}$ centered at (0 , 0).
- ii- The circle of radius $\sqrt{\frac{3}{\pi}}$ centred at (2, 10).
- iii- The square whose bottom left corner is located at (3.5 , 2) and whose side length is equal to $\sqrt{3}$.
- iv- The square whose area is equal to 3 and whose top right corner is located at (100,100).

v- The equilateral triangle whose vertices are (0,0), $(2\sqrt[4]{3}, 0)$, $(\sqrt[4]{3}, \sqrt[4]{27})$.

Hint: There is a shortcut.

(20 points) Consider the vector field

$$\vec{F}(x,y,z) = \left(\cos(y-1)\right)\vec{\mathbf{i}} + \left(3\,x^2\,y^3\right)\vec{\mathbf{j}} + \left(2\,x-1\right)\vec{\mathbf{k}}$$

as well as the cube whose eight vertices are located at the following points: (1, 1, -1), (-1, 1, -1), (1, -1, -1), (-1, -1, -1), (1, 1, 1), (-1, 1, 1), (1, -1, 1), (-1, -1, 1).

Calculate the flux of $\vec{F}(x, y, z)$ through the surface S which consists of the top face of the cube (the one satisfying z = 1) as well as the four side faces (i.e. the ones satisfying x=1, y=1, x=-1, y=-1). In other words calculate the flux of $\vec{F}(x, y, z)$ through the surface obtained from the cube by removing its bottom face. Choose the orientation where the unit normal vector points away from the origin.

(10 points) Consider the IVP

$$y' = y^2 - x$$
 , $y(0) = 0$

Show that if y(x) is a solution of the above IVP then

$$y(x) + \frac{x^2}{2} \ge 0$$

Hint: It can be shown that y(x) cannot be written in closed form in terms of the elementary functions, so don't waste your time trying to find a formula for it.