Math 202 - Exam 1 (Spring 09)

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- Please answer questions 2,3 and 4 on the same sheet of paper on which they are written. Question 1 has an extra sheet for you to write your answer on it. Any part of your answer written on the wrong page will not be graded.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 4 problems in total. Some questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section Number :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$TOTAL$	

Problem 1

(10 points each) Solve the following IVPs:

i-

$$y' = x^2 + y^2 + 2\,x\,y$$

ii-

$$y' = y\,(y+1)\,x$$

iii-

$$y' = -\frac{y}{x} + e^x$$

iv-

$$y' = -\frac{2x}{1 + x^2 + y}$$

v-

$$y' = \frac{x^5 + x\,y^2}{y^2 + x^4}$$

The initial condition for each of the above equations is $y(1) = 1$.

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2

(4 points each) Calculate the circulation counterclockwise of the following vector field in the plane

$$\vec{V}(x,y) = \left(e^x \sin(e^x) + x^3 - 2 + y\right)\vec{\mathbf{i}} + \left(2\,x + y\sin(y) + e^{y^2}\right)\vec{\mathbf{j}}$$

around each of the following curves:

i- The circle of radius $\sqrt{\frac{3}{\pi}}$ centered at $(0\ ,\ 0)$.

ii- The circle of radius $\sqrt{\frac{3}{\pi}}$ centred at $(2\ ,\ 10)$.

iii- The square whose bottom left corner is located at $(3.5\ ,\ 2)$ and whose side length is equal to $\sqrt{3}$.

iv- The square whose area is equal to 3 and whose top right corner is located at (100,100).

v- The equilateral triangle whose vertices are $(0,0)$, $(2\sqrt[4]{3}\ ,\ 0)$, $(\sqrt[4]{3}\ ,\ \sqrt[4]{27})$.

Hint: There is a shortcut.

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Problem 3

(20 points) Consider the vector field

$$\vec{F}(x,y,z) = \Big(\cos(y-1)\Big)\vec{\mathbf{i}} + \Big(3\,x^2\,y^3\Big)\vec{\mathbf{j}} + \Big(2\,x-1\Big)\vec{\mathbf{k}}$$

as well as the cube whose eight vertices are located at the following points:
(1 , 1 , -1) , (-1 , 1 , -1) , (1 , -1 , -1) , (-1 , -1 , -1) , (1 , 1 , 1) , (-1 , 1 , 1) , (1 , -1 , 1) , (-1 , -1 , 1).

Calculate the flux of $\vec{F}(x,y,z)$ through the surface S which consists of the top face of the cube (the one satisfying $z=1$) as well as the four side faces (i.e. the ones satisfying x=1, y=1, x= -1, y=-1). In other words calculate the flux of $\vec{F}(x,y,z)$ through the surface obtained from the cube by removing its bottom face. Choose the orientation where the unit normal vector points away from the origin.

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Problem 4

(10 points) Consider the IVP

$$y' = y^2 - x \qquad , \qquad y(0) = 0$$

Show that if $y(x)$ is a solution of the above IVP then

$$y(x) + \frac{x^2}{2} \geq 0$$

Hint: It can be shown that y(x) cannot be written in closed form in terms of the elementary functions, so don't waste your time trying to find a formula for it.

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