Math 202 — Spring 2011
Differential Equations
Final exam, June 8 — Duration: 2 hours 15 minutes

GRADES (each problem is worth 10 points):

1	2	3	4	5	6	7	8	9	10	TOTAL/100

YOUR NAME: ____________________________

YOUR AUB ID NUMBER: __________________

YOUR SECTION NUMBER (SEE BELOW FOR A LIST): ______________

Section 1 (**Brock**)
Lecture MWF 11:00
Recitation F 1:00
Ms. Jaber

Section 2 (**Brock**)
Lecture MWF 11:00
Recitation F 12:00
Ms. Jaber

Section 4 (**Brock**)
Lecture MWF 11:00
Recitation F 9:00
Ms. Jaber

Section 8 (**Raji**)
Lecture MWF 2:00
Recitation Th 3:30
Ms. Nassif

Section 9 (**Raji**)
Lecture MWF 2:00
Recitation Th 2:00
Ms. Mroue

Section 10 (**Raji**)
Lecture MWF 2:00
Recitation Th 9:30
Ms. Mroue

Section 11 (**Raji**)
Lecture MWF 2:00
Recitation Th 12:30
Ms. Mroue

Section 12 (**Kobeissi**)
Lecture MWF 12:00
Recitation F 2:00
Dr. Kobeissi

Section 13 (**Kobeissi**)
Lecture MWF 12:00
Recitation F 3:00
Ms. Mroue

Section 14 (**Kobeissi**)
Lecture MWF 12:00
Recitation F 4:00
Ms. Mroue

Section 15 (**Kobeissi**)
Lecture MWF 12:00
Recitation F 5:00
Ms. Mroue

Section 16 (**Makdisi**)
Lecture MWF 8:00
Recitation Tu 12:30
Prof. Makdisi

Section 17 (**Makdisi**)
Lecture MWF 8:00
Recitation Tu 3:30
Ms. Jaber

Section 18 (**Makdisi**)
Lecture MWF 8:00
Recitation Tu 5:00
Ms. Jaber

Section 19 (**Makdisi**)
Lecture MWF 8:00
Recitation Tu 2:00
Ms. Jaber

INSTRUCTIONS:

1. **Write your NAME, your AUB ID number, your SECTION above.**
2. **Solve the problems inside the booklet. Explain your reasoning precisely and clearly to ensure full credit. Partial solutions will receive partial credit.**
3. **You may use the back of each page for scratchwork OR for solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.**
4. **Closed book and notes. Calculators are not allowed. Turn off and put away any cell phones.**

GOOD LUCK!

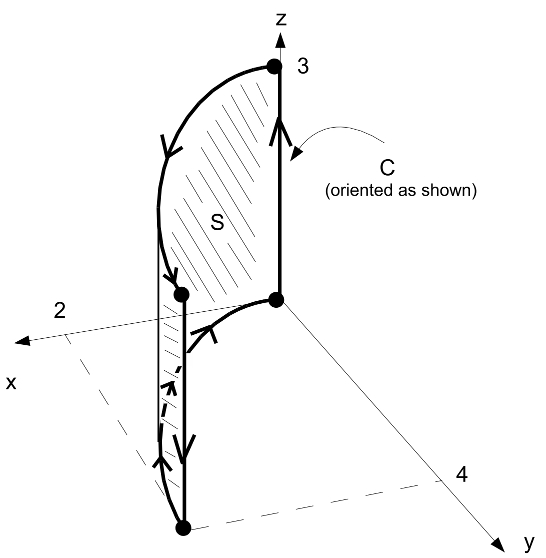
An overview of the exam problems. Each problem is worth 10 points. The problems are arranged according to the chapters covered in the course (i.e., chapters 16, 2, 4, 6, 7, 8), NOT according to difficulty. Take a few minutes to look at all the questions, BEFORE you start solving.

PLEASE ANSWER THE PROBLEMS INSIDE THE BOOKLET. EACH PROBLEM HAS ITS OWN PAGE.

1. Use Stokes' theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is the vector field $\mathbf{F} = y\mathbf{i} + z\mathbf{j} - 2x\mathbf{k}$, and the curve C is the boundary of the surface S given by

$$S : y = x^2, \quad 0 \le x \le 2, \quad 0 \le z \le 3, \qquad\qquad C = \text{boundary of } S$$

with the orientation shown in the picture (counterclockwise if viewed from the positive y-axis).



2. Find the general solution of the differential equation $\quad y' = \dfrac{y}{x} + \dfrac{e^{-2(\frac{y}{x})}}{(\frac{y}{x})}.$

3. Use an integrating factor μ to find the general solution of the differential equation

$$(ye^y \cos(xy) + e^x)\, dx + (xe^y \cos(xy) - e^x)\, dy = 0.$$

4. Find the general solution of the equation

$$y''' + 6y'' + 10y' = 30x^2 + 16x + 14.$$

(Note: find the exact constants in y_P, not just the form of the particular solution.)

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PLEASE ANSWER THE PROBLEMS INSIDE THE BOOKLET.
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5. Find the general solution of the Cauchy-Euler equation $x^2y'' - 3xy' + 4y = \dfrac{x^2}{1+(\ln x)^2}$.
(Note: put the DE into standard form before finding y_P.)

6. We wish to find a series solution centered at $x = 0$ for the differential equation

$$(x^2+2)y'' + 6xy' + 6y = 0. \qquad (*)$$

a) (1 point) BEFORE solving $(*)$, what radius of convergence do you expect the series solution to have? Why?
b) (9 points) Find the general solution of $(*)$ in the form $y = c_0y_1 + c_1y_2$. Note: it is enough to find just the first four nonzero terms in the series for y_1 and y_2.

7. a) (9 points) Using a Frobenius series centered at $x = 0$, find ONE solution y_1 (corresponding to the LARGER indicial root r_1) for the following differential equation.

$$3xy'' + y' - y = 0.$$

Note: it is enough to find just the first four nonzero terms in the series.
b) (1 point) WITHOUT SOLVING, does the second solution y_2 contain $\ln x$? Why or why not?

8. Use Laplace transforms to solve the Volterra integral equation

$$f(t) - \int_{\tau=0}^{t} (t-\tau)e^{-2(t-\tau)}f(\tau)\,d\tau = 2e^t.$$

(Note: there is a table of Laplace transforms at the end of this booklet.)

9. Find the solution of $y'' + 2y' + 10y = \delta(t-2) + 13e^t$, $\quad y(0) = 0, \quad y'(0) = 13.$
(Note: there is a table of Laplace transforms at the end of this booklet.)

10. Find the general solution of the following linear system:

$$\mathbf{X}' = \begin{pmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{pmatrix}\mathbf{X}.$$

1. Use Stokes' theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is the vector field $\mathbf{F} = y\mathbf{i} + z\mathbf{j} - 2x\mathbf{k}$, and the curve C is the boundary of the surface S given by

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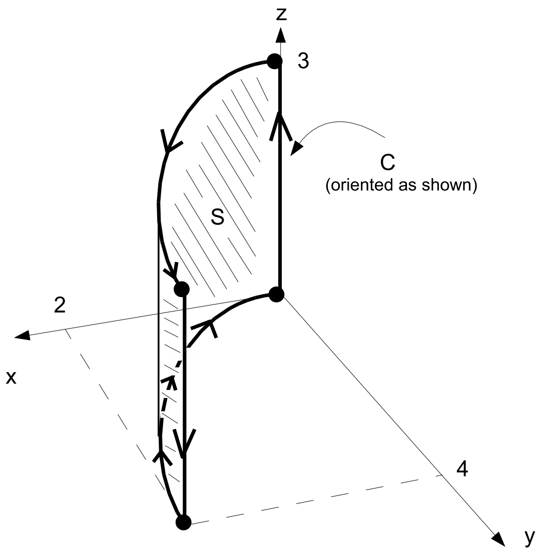

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(Answer on this page and the next.)

Your solution for question 3, continued.

4. Find the general solution of the equation

$$y''' + 6y'' + 10y' = 30x^2 + 16x + 14.$$

(Note: find the exact constants in y_P, not just the form of the particular solution.)

5. Find the general solution of the Cauchy-Euler equation $x^2y'' - 3xy' + 4y = \dfrac{x^2}{1 + (\ln x)^2}$.

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(Answer on this page and the next.)

Your solution for question 5, continued.

6. We wish to find a series solution centered at $x = 0$ for the differential equation

$$(x^2 + 2)y'' + 6xy' + 6y = 0. \tag{$*$}$$

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Your solution for question 6, continued.

7. a) (9 points) Using a Frobenius series centered at $x = 0$, find ONE solution y_1 (corresponding to the LARGER indicial root r_1) for the following differential equation.

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(Answer on this page and the next.)

Your solution for question 8, continued.

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Your solution for question 9, continued.

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(Answer on this page and the next.)

Your solution for question 10, continued.