Math 202

Final Examination A, February 12. 2002

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1. (20 points)

   (a) (10 points) Find the inverse Laplace transform of $\frac{s+1}{(s+2)(s^2+9)}$.

   (b) (10 points) Solve the integral equation: $y(t) = e^{-t} - 3t^2 + \int_0^t y(\tau)e^{t-\tau}\,d\tau$.

2. (30 points) Use Laplace transforms to solve the problem:

$$y'' + 6y' + 18y = \mathcal{U}(t-2) + \delta(t-4), \;\; y(0) = 0, y'(0) = 1.$$

   Here δ is the Dirac delta-function, and $\mathcal{U}$ is the unit step function.

3. (30 points)

   a. (15 points) Find all eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

   Verify your answers by showing they satisfy the definitions of eigenvalue and eigenvector.

   b. (15 points) Find the solution of the problem
$$\begin{aligned} x' &= x + 4y + 3z \\ y' &= z \\ z' &= 2y + z \end{aligned}$$,where

   $x(0) = 1, y(0) = 0, z(0) = 1$.

4. (15 points) Solve the equation: $\frac{dy}{dx} = y(xy^3 - 1)$.

5. (25 points) Find the general solution of the differential equation:

$$y^{(5)} + y^{(4)} - y^{(3)} + 7y'' - 20y' + 12y = 16 + 12x.$$

6. (30 points)

   a. (20 points) Let $y = \sum_{n=0}^{\infty} c_n x^n$ be a solution of the differential equation $y' - 5x^4 y = 3x^9$, satisfying $y(0) = 1$. Find c_n as a function of n, then find y and express it, as far as possible, in closed form.

   b. (10 points) Solve the differential equation in part (a) directly, without the use of power series.

7. (10 points) If J_1 and J_0 are Bessel functions of the first kind of orders 1 and 0 respectively, prove that $(xJ_1(x))' = xJ_0(x)$.