AMERICAN UNIVERSITY OF BEIRUT — FACULTY OF ARTS AND SCIENCES
Math 202, Differential Equations — Spring semester 1999/2000
FINAL EXAM — June 7, 2000 — Duration: 2 hours

Lecturer for sections 1–6: Professor K. Khuri-Makdisi
Lecturer for sections 7–12: Professor N. Haidar

Sec. 1: F 10	Sec. 2: W 11	Sec. 3: W 12	Sec. 4: W 2	Sec. 5: M 2	Sec. 6: M 3
Sec. 7: F 11	Sec. 8: M 11	Sec. 9: M 12	Sec. 10: M 2	Sec. 11: F 12	Sec. 12: F 1

Instructions: *Write your name, AUB ID number,* **and section number** *on the front cover of your exam booklet.* **Please work on the problems IN ORDER in your exam booklet.** *If you wish to temporarily skip a problem and come back to it later, you may want to leave one or two blank pages before continuing to the next problem.* **Please write your solutions on the answer part of your booklet.** *Use the back of each page for scratch work only. The total number of points on the exam is 90.* *GOOD LUCK!*

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1. [6 pts] Solve the IVP
$$y' + 2xy = x; \qquad y(0) = 3.$$

2. [10 pts] By using a suitable integrating factor, find the general solution of:
$$(3xy^4 + 2y)\,dx + (2x^2y^3 - x)\,dy = 0.$$

3. [8 pts] Determine the general solution of the equation
$$y' = \frac{x}{y}e^{-2\left(\frac{y}{x}\right)} + \frac{y}{x}.$$

4. [10 pts] Find the general solution of
$$y''' - y' = \sin x + xe^x.$$

5. [7 pts] Find the general solution of
$$(x-2)^2y'' + 7(x-2)y' + 13y = 0.$$

6. [7 pts] Find the general solution of
$$y'' - 2y' + y = \frac{e^x}{x^2+1}.$$

7. [10 pts] Use a power series centered at $x = 0$ to solve the following DE. You should find all the coefficients in the general solution until the term involving x^7 (inclusive). You do not have to find the general formula for the coefficients.
$$y'' + xy = 0.$$

8. [14 pts] Find the regular Frobenius series solution of the following differential equation (i.e., the solution that does **NOT** involve $\ln x$):
$$x^2y'' + (5x - x^2)y' + (3 + x)y = 0.$$

9. [11 pts] Solve the following integrodifferential equation:
$$y' + 3y + 2\int_{\tau=0}^{t} y(\tau)\,d\tau = f(t); \qquad y(0) = 1,$$
where $f(t)$ is the function defined by
$$f(t) = \begin{cases} 0, & \text{if } t < 4 \\ t, & \text{if } t \geq 4. \end{cases}$$

10. [7 pts] Solve the following IVP system **ONLY** for $y(t)$.
$$\begin{cases} y' = 2y - 2z + \delta(t-1); & y(0) = 0 \\ z' = 4y - 2z; & z(0) = 1 \end{cases}$$

Math 202
Final Examination A, February 12, 2002
F. Abi-Khuzam

1. (20 points)
   (a) (10 points) Find the inverse Laplace transform of $\frac{s-1}{(s-2)(s^2-9)}$.
   (b) (10 points) Solve the integral equation: $y(t) = e^{-t} - 3t^2 + \int_0^t y(\tau)e^{t-\tau}d\tau$.
2. (30 points) Use Laplace transforms to solve the problem:
$$y'' + 6y' + 18y = \mathcal{U}(t-2) + \delta(t-4),\ y(0) = 0, y'(0) = 1.$$
Here δ is the Dirac delta-function, and $\mathcal{U}$ is the unit step function.
3. (30 points)
   a. (15 points) Find all eigenvalues and corresponding eigenvectors for the matrix
$$A = \begin{pmatrix} 1 & 4 & 3 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$
   Verify your answers by showing they satisfy the definitions of eigenvalue and eigenvector.
   b. (15 points) Find the solution of the problem $\begin{aligned} x' &= x + 4y + 3z \\ y' &= z \\ z' &= 2y + z \end{aligned}$,where $x(0) = 1, y(0) = 0, z(0) = 1$.
4. (15 points) Solve the equation: $\frac{dy}{dx} = y(xy^3 - 1)$.
5. (25 points) Find the general solution of the differential equation:
$$y^{(5)} + y^{(4)} - y^{(3)} + 7y'' - 20y' + 12y = 16 + 12x.$$
6. (30 points)
   a. (20 points) Let $y = \sum_{n=0}^{\infty} c_n x^n$ be a solution of the differential equation $y' - 5x^4 y = 3x^9$, satisfying $y(0) = 1$. Find c_n as a function of n, then find y and express it, as far as possible, in closed form.
   b. (10 points) Solve the differential equation in part (a) directly, without the use of power series.
7. (10 points) If J_1 and J_0 are Bessel functions of the first kind of orders 1 and 0 respectively, prove that $(xJ_1(x))' = xJ_0(x)$.