MATH 202: FINAL EXAM, JUNE 2ND 2004

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Problem #1. Find the general solution of the given system

$$X' = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} X$$

Problem #2. Use the Laplace transform to solve the given integrodifferential equation.

$$\begin{cases} 9y'(t) + \int_0^t y(u)du = \delta(t-2) \\ y(0) = 0. \end{cases}$$

Problem #3. Use the Laplace transform to solve the given IVP.

$$y'' + 4y = f(t), \quad y(0) = y'(0) = 1.$$

Here the function f is given by

$$f(t) = \begin{cases} e^{4t} & \text{if } t < 1, \\ 0 & \text{if } 1 \leq t \leq 2, \\ 2e^{4t} & \text{if } t > 2. \end{cases}$$

Problem #4. Solve the BVP:

$$\begin{cases} y'' + y = \dfrac{1}{\cos^3 x} \\ y(0) = \dfrac{1}{6} \\ y(\dfrac{\pi}{6}) = \dfrac{1}{2\sqrt{3}}. \end{cases}$$

Problem #5. Use a power series centered at $x = 0$ to solve the following differential equation. You should find all the coefficients in the general solution until the term involving x^8 (inclusive). You do not have to find the general formula for the coefficients.

$$y'' - xy = 2.$$

Problem #6. Use the substitution $y = x^{-1}z(x)$ to transform the differential equation

$$4x^2y'' + 12xy' + (4x^2 + 3)y = 0$$

into a Bessel's differential equation. Then express the general solution of the above equation in terms of Bessel functions.

Problem #7. It is known that there exists n such that $\mu(x, y) = (x + y)^n$ is the integrating factor for

$$(4x + y + 3)dx + (3x + 3)dy = 0.$$

Find n and use it to solve the given equation.

Problem #8.

(1) Find at least two solutions of the IVP:

$$\begin{cases} y' = 3y^{\frac{2}{3}} \\ y(0) = 0. \end{cases}$$

(2) State the existence and uniqueness theorem related to the IVP given in this problem.

(3) Explain why the fact that our IVP has more than one solution does not contradict the theorem.