Math 202–Final Exam (Spring 05)

B. Shayya

- Please write your **section number** on your booklet.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on pages **1**, **2**, and **3** of the booklet.)
(15 pts each) Solve the following differential equations.

$$\text{(a)}\qquad \frac{dy}{dx} - \frac{4}{x}y = x^5 e^x$$

$$\text{(b)}\qquad (y^2 + yx)\,dx - x^2\,dy = 0$$

$$\text{(c)}\qquad \left(\frac{dy}{dx}\right)^3 = -\frac{x^3 y}{(1+x^2)^5}$$

Problem 2 (answer on page **4** of the booklet.)
(10 pts) Find the general solution of the Bessel equation

$$x^2y'' + xy' + \left(9x^2 - \frac{1}{4}\right)y = 0$$

on the interval $(0, \infty)$.

Problem 3 (answer on pages **5**, **6**, and **7** of the booklet.)
(38 pts) Find two linearly independent series solutions of the equation

$$xy'' + 3(1-x)y' + 3y = 0$$

about the regular singular point $x = 0$. (It is enough to find the first 4 terms of each series. **GRR**, **RRI**, and **RRII** must be clearly stated.)

Problem 4 (answer on pages **8** and **9** of the booklet.)
(25 pts) Use the Laplace transform to solve the IVP

$$\begin{cases} y'' + 4y' + 5y = \delta(t - 2\pi) \\ y(0) = 0, \quad y'(0) = 0. \end{cases}$$

Problem 5 (answer on pages **10** and **11** of the booklet.)
(8 pts each) Let a be a constant. Find the following transforms.

(i) $L\left\{\int_0^t \tau e^{a\tau}\cos(t-\tau)\,d\tau\right\}$ (ii) $L\{(at-3)u(t-1)\}$ (iii) $L^{-1}\left\{\frac{e^{-5s}}{(s-a)^3}\right\}$

Problem 6 (answer on pages **12**, **13**, and **14** of the booklet.)
(28 pts) Find the general solution of the system

$$X' = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} X$$

on the interval $(-\infty, \infty)$. (Don't forget to write your answer in both **complex** and **real** forms.)

Problem 7 (answer on pages **15** and **16** of the booklet.)
(10 pts) Prove the following theorem in the case $n = 2$:

Theorem. Let $y_1, y_2, \ldots, y_n$ be solutions of the nth-order linear homogeneous ODE

$$a_n(x)\frac{d^n y}{dx^n} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

on an interval I where $a_0(x), a_1(x), \ldots, a_n(x)$ are continuous and where $a_n(x)$ is never zero. Then

$$y_1, y_2, \ldots, y_n \text{ linearly independent on } I \quad \Rightarrow \quad W(y_1, y_2, \ldots, y_n) \neq 0 \text{ everywhere in } I.$$