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Math 202 — Spring 2005
Differential Equations, sections 1–4
Final exam, June 8 — Duration: 2 1/2 hours

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JUST A MINUTE — PLEASE READ THE INSTRUCTIONS BELOW FIRST

1. Write your name, AUB ID number, and section number ON THE FRONT COVER OF THE AUB EXAMINATION BOOKLET.
To remind you, the sections are as follows:

Section 1	Section 2	Section 3	Section 4
Recitation F 1	Recitation F 2	Recitation F 12	Recitation F 3
Ms. Zantout	Ms. Zantout	Ms. Zantout	Dr. Yamani

2. PLEASE DO YOUR WORK FOR THE PROBLEMS ON THE CORRECT PAGES IN YOUR EXAM BOOKLET, as indicated for each problem. If you need more space, ask us for a new booklet (see point 3 below). Please INDICATE where you have continued your solution to a problem.

3. Please write your solutions on the answer part of your booklet. Use the back of each page for scratch work only. If you need an extra booklet be sure to WRITE YOUR NAME, AUB ID, and SECTION NUMBER on the second booklet AND indicate on BOTH booklets that you used two booklets total.

4. THE OTHER INSTRUCTIONS (f is NEW, review c and e):

a. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit. Each problem is worth 12 points, and there are 11 problems for a TOTAL 132 points.
b. Do as much of the exam as you can, and budget your time carefully. THE PROBLEMS ARE ARRANGED ACCORDING TO THE CHAPTERS COVERED IN THE BOOK (i.e., chapters 2, 4, 6, 7, 8); just because a problem comes later does NOT mean that it is harder than the earlier problems.
c. If you cannot do a certain integral, just leave it as an integral in your solution for partial credit on the rest of the problem.
d. No calculators, books, or notes allowed. Turn off and put away any cell phones or beepers.
e. Problems 5 and 6 ask for series solutions. For full credit on these problems, you need to give a formula for the coefficients. However, just like in Quiz 2, you can get almost full credit if you give the first FOUR nonzero terms for any solution.
f. REGARDING COMPLEX SOLUTIONS: For full credit, the solution to a differential equation should involve real numbers only. However, if you are short on time, leave the solution in complex form — you'll still get most of the credit on the problem.

GOOD LUCK!

(Remember, each problem is worth 12 points for a total of 132)

1. **(Please answer on page 1 of the booklet):**
   a) Find the general solution of $\dfrac{dy}{dx} = \dfrac{3x^2+1}{y^2 \ln y \sqrt{x^3+x+1}}$.
   b) (UNRELATED) Find the general solution of $y' + \dfrac{8}{x}y = \dfrac{4\tan x}{x^2} y^{3/4}$.

2. **(Please answer on page 2 of the booklet):**
   Use an integrating factor μ to find the general solution of $(4y^3+3x^2y^2)dx + (3xy^2+x^3y)dy = 0$.

3. **(Please answer on page 3 of the booklet):**
   Find the general solution of $y'' - 2y' + y = x^2 + \dfrac{e^x}{x^2+1}$.

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4. (Please answer on page 4 of the booklet):
Solve the following IVP. (Note the THIRD derivative y''' appears in the DE.)

$$x^3y''' - 8xy' + 8y = 0, \qquad y(1) = 1, \quad y'(1) = 7, \quad y''(1) = 0.$$

5. (Please answer on pages 5 and 6 of the booklet):

Use a power series of the form $y = \sum_{n=0}^{\infty} c_n x^n$ to find the general solution of $y'' + x^2y = 0$.

6. (Please answer on pages 7 and 8 of the booklet):

Use a generalized (i.e., Frobenius) power series of the form $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ to find ONE solution of $3x^2y'' + 2xy' - xy = 0$, corresponding to the LARGER indicial root.

7. (Please answer on page 9 of the booklet):
Use Laplace transforms to solve $y'' - 4y = e^t + \delta(t-3)$, $\quad y(0) = 0, \quad y'(0) = 2$.

8. (Please answer on page 10 of the booklet):
Three UNRELATED questions on the Laplace transform:

a) Find the inverse transform $\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+4s+13}\right\}$.

b) Write the following function $f(t)$ in terms of the unit step function $\mathcal{U}$, and find its Laplace transform $F(s)$:

$$f(t) = \begin{cases} 1, & \text{if } t < 2, \\ t, & \text{if } t \geq 2. \end{cases}$$

c) Use Laplace transforms and convolutions to evaluate $h(t)$, which is given by the integral below (do not try to simplify the factorials in your answer):

$$h(t) = \int_{\tau=0}^{t} \tau^{202}(t-\tau)^{2005}\, d\tau.$$

9. (Please answer on pages 11 and 12 of the booklet):
Find the general solution of the following system. (Hint: there are multiple eigenvalues, but in the easy case where you can find enough eigenvectors. You do NOT need any solutions involving $te^{\lambda t}$.)

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} 0 & 0 & 2 \\ 2 & -1 & 4 \\ 1 & 0 & 1 \end{pmatrix} \vec{\mathbf{X}}.$$

10. (Please answer on pages 13 and 14 of the booklet):
Find the general solution of the following system:

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} -2 & 6 \\ -3 & 4 \end{pmatrix} \vec{\mathbf{X}}.$$

11. (Please answer on pages 15 and 16 of the booklet):

In this problem, we will study the partial fraction decomposition $\frac{b_1x^2 + b_2x + b_3}{(x^2+2x+5)(x+1)} = \frac{a_1x + a_2}{x^2+2x+5} + \frac{a_3}{x+1}$. Here b_1, b_2, b_3 are KNOWN constants, and a_1, a_2, a_3 are UNKNOWN constants.

a) Put both sides over a common denominator to find the equations satisfied by a_1, a_2, a_3, BUT DO NOT SOLVE THESE EQUATIONS YET. Show that your equations can be written as

$$A\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \qquad \text{where} \qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 5 \end{pmatrix}.$$

b) Find the determinant $\det A$ and use this to show that A is invertible.

c) Calculate the inverse matrix A^{-1}.

d) Use your answer from part (c) to write a_1, a_2, a_3 in terms of b_1, b_2, b_3.