FINAL EXAMINATION, MATH 202

JUNE 8, 2005; 3:00-5:00 P.M.

Name: Signature:

Number: Section: 13, 14, 15, 16

1. Instructions:

- No calculators are allowed.
- There are two types of questions: **PART I** consists of seven subjective questions, and **PART II** consists of five multiple-choice questions of which each has exactly one correct answer.
- Give detailed solutions for the problems of **PART I** in the provided space and circle the appropriate answers for the problems of **PART II**.

2. Grading policy:

- 10 points for each of the problems of **PART I**.
- 5 points for each correct answer of **PART II**.
- 0 point for no answer or more than one answer of **PART II**.

GRADE OF PART I:

GRADE OF PART II:

TOTAL GRADE:

Part I(1). Find the general solution of the system $X' = AX$ where

$$A = \begin{pmatrix} 7 & -2 & 4 \\ -4 & 0 & -2 \\ 0 & 0 & -1 \end{pmatrix}.$$

(2). Find the general solution of the differential equation

$$y'' + y = 2\sin x + 5e^{2x}.$$

Part I(3). Use Laplace transform to solve the initial value problem

$$t\,y'' - y' = t^2, \quad y(0) = 0.$$

Part I(4). Find the general solution of the differential equation

$$\left[\frac{x}{\sqrt{x^2+y^2}} - \frac{y}{x^2}\right] dx + \left[\frac{y}{\sqrt{x^2+y^2}} + \frac{1}{x}\right] dy = 0.$$

Part I(5). Show that $y = \sqrt{x}\, J_\nu(x)$ is a solution for the differential equation

$$x^2\, y'' + \left(x^2 - \nu^2 + \frac{1}{4}\right) y = 0.$$

Part I(6). Solve the differential equation

$$x^2\, y' + 2x\, y = y^3.$$

Part I(7). Find the general solution of the differential equation

$$x^2 y'' - xy' + y = 4x \ln x, \quad x > 0.$$

Part II: Multiple Choice Problems.

Part II(1). The solution of the initial-value problem

$$y' + 1 = e^{-(x+y)} \sin x, \quad y(0) = 0,$$

passes through the point

(a) $(x = \pi/2, y = \pi/2 - \ln 2)$.

(b) $(x = \pi/2, y = -\pi/2 - \ln 2)$.

(c) $(x = \pi/2, y = \pi/2 + \ln 2)$.

(d) $(x = \pi/2, y = -\pi/2 + \ln 2)$.

(e) None of the above.

Part II(2). Knowing that the differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

has a regular singular point at $x = 0$ with an indicial root r, then an application of Frobenius method to find the associated series solution $\sum_{n=0}^{\infty} c_n x^{n+r}$ yields for the coefficients c_n, $n \geq 1$ the recurrence relation

(a) $c_n = c_{n-1}/[(n + r + 1)(2n + 2r - 3)]$.

(b) $c_n = -c_{n-1}/[(n + r - 1)(2n + 2r - 3)]$.

(c) $c_n = -c_{n-1}/[(n + r - 1)(2n + 2r + 3)]$.

(d) $c_n = c_{n-1}/[(n + r - 1)(2n + 2r - 3)]$.

(e) None of the above.

. **Part II**(3). Answer TRUE (T) or (FALSE) only:

(a) —- If m is a positive integer and J_m is Bessel's function of the first kind of order m, then $J_{-m} = (-1)^m J_m$.

(b) —- Bessel's function of the first kind of order ν is defined by

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(\nu+n)} \left(\frac{x}{2}\right)^{2n+\nu}.$$

(c) —- The Gamma function is defined by

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt, \quad x > 0.$$

(d) —- Picard's theorem guarantees a unique solution for the initial value problem

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

if f and f_x are continuous functions in some open rectangle containing (x_0, y_0).

(e) —- The convolution of functions f and g on $[0,\infty)$ is defined by

$$(f * g)(t) = \int_0^t f(t) g(\tau + t)\, d\tau.$$

Part II(4).

$$\mathcal{L}^{-1}\left\{e^{-2s} + \ln\left(\frac{s-1}{s+1}\right)\right\}.$$

(a) $\delta(t+2) + (e^t - e^{-t})/t$.

(b) $\delta(t+2) - (e^t - e^{-t})/t$.

(c) $\delta(t-2) + (e^t - e^{-t})/t$.

(d) $\delta(t-2) - (e^t - e^{-t})/t$.

(e) None of the above.

Part II(5). The solution of the initial-value problem

$$y'(t) = \cos t + \int_0^t y(\tau)\cos(t-\tau)d\tau, \quad y(0) = 1,$$

satisfies

(a) $y(2) = 5$.

(b) $y(2) = 2$.

(c) $y(2) = 3$.

(d) $y(2) = 4$.

(e) None of the above.

Part II(6). The indicial roots of the differential equation

$$2xy'' - (1 + 2x^2)y' - xy = 0$$

are

(a) $1, 3/2$.

(b) $0, 3/2$.

(c) $-1, 2/3$.

(d) $0, -3/2$.

(e) None of the above.