Prof. Makdisi
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**Math 202 — Spring 2002**
**Differential Equations, sections 11–15**
**Final Exam, June 18, 2002 — Duration: 2 hours**

**NAME:** **AUB ID#:**

**Section:** 11: Th2, N323 12: F2, N412 13: Th12, B105 14: Th1, B105 15: F2, B104

**GRADES:**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | TOTAL/100 |
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**INSTRUCTIONS:**

1. **Write your NAME and AUB ID number above, and circle your SECTION.**
2. **Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit.**
3. **You may use the back of each page for scratchwork OR for solutions.**
4. **There are three extra blank sheets at the end, for extra scratchwork or solutions.**
5. **Do as much of the exam as you can, and budget your time carefully. THERE ARE 10 PROBLEMS ON THIS EXAM. Each problem is worth 10 points.**
6. **No calculators, books, or notes allowed. Turn off any cell phones or beepers.**

**GOOD LUCK!**

1. Use the Laplace transform to solve the integrodifferential equation

$$y' + 2y + 10\int_{\tau=0}^{t} y(\tau)\,d\tau = 2, \qquad y(0) = 1.$$

2. Use Laplace transforms to find the solution of the initial-value problem

$$y'' - 4y' + 3y = 4f(t), \quad \text{where } f(t) = \begin{cases} 0, & \text{for } t < 1 \\ e^{t-1}, & \text{for } t \geq 1. \end{cases} \quad \text{and } y(0) = 2, \quad y'(0) = 6.$$

3. Find the general (implicit) solution of the differential equation

$$y' - \frac{y}{x} = \frac{-5}{2}x^2y^3 \ln x.$$

4. Find the general solution of the differential equation $x^2y'' - xy' + y = \dfrac{x}{\ln x}$.

5. Find the general solution of $y''' + 3y'' + 9y' - 13y = 8\cos x + 6e^x$.

6. Find the (implicit) solution of the initial-value problem

$$(1 + 3x\sin y)\,dx - x^2\cos y\,dy = 0, \quad y(1) = \frac{\pi}{2}.$$

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7. a) Using a series centered at $x = 0$, find the general solution of the equation:

$$(1 + x^2)y'' + 2xy' - 2y = 0.$$

b) Bonus (2 points): Express your answer in terms of familiar functions.

8. **Note:** The different parts are NOT related.

a) Given that $\mathcal{L}\{J_0(t)\} = \dfrac{1}{\sqrt{s^2+1}}$, find the function $f(t)$ which is given by

$$f(t) = \int_{\tau=0}^{t} J_0(\tau)J_0(t-\tau)\,d\tau.$$

b) Given that $\Gamma(1/2) = \sqrt{\pi}$, what is the value of $\Gamma(7/2)$?

c) Write down the differential equation whose general solution is $y = AJ_{1/3}(2x) + BJ_{-1/3}(2x)$, with $A, B$ arbitrary constants. (If you don't already know the answer, try making the substitution $t = 2x$.)

d) Identify the singular points of the equation below, and specify for each singular point whether it is regular or irregular. **DO NOT SOLVE.**

$$x^2(x-5)^2y'' + 4xy' + (x^2 - 25)y = 0.$$

9. Write the following system in matrix form, and find its general solution:

$$\begin{cases} dx/dt &= 3x - 4y + 2z \\ dy/dt &= -y \\ dz/dt &= -2x + 2y - 2z \end{cases}$$

10. Find the solution of the following initial-value system of differential equations:

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} -1 & -4 \\ 2 & 3 \end{pmatrix}\vec{\mathbf{X}}, \qquad \vec{\mathbf{X}}\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$