Prof. Makdisi
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Not To Be Taken Out
Reserve Reading Room

Math 202 — Spring 2003
Differential Equations, sections 1–5
Final Exam, June 5, 2003 — Duration: 2 hours

NAME:

AUB ID#:

GRADES (each problem is worth 10 points):

1	2	3	4	5	6	7	8	9	10	TOTAL/100

INSTRUCTIONS:

1. Write your NAME and AUB ID number above.
2. Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit.
3. You may use the back of each page for scratchwork OR for solutions.
4. There are three extra blank sheets at the end, for extra scratchwork or solutions.
5. Do as much of the exam as you can, and budget your time carefully. THERE ARE 10 PROBLEMS ON THIS EXAM. Each problem is worth 10 points.
6. No calculators, books, or notes allowed. Turn off any cell phones or beepers.

GOOD LUCK!

1. Find the general solution of the equation $\dfrac{dy}{dx} = \dfrac{\sqrt{1+(2x+y)^3}}{(2x+y)^2} - 2$.

2. Find the general solution of $\left(\cos(x+y) + \dfrac{2\sin(x+y)}{x} + \dfrac{1}{x^3}\right) dx + \cos(x+y)\, dy = 0$.

3. a) Let $g(t)$ be defined by:

$$g(t) = \begin{cases} 3, & \text{if } t < 4 \\ 2t-5, & \text{if } t \geq 4. \end{cases}$$

Write g in terms of the unit step function $\mathcal{U}(t)$, and find the Laplace transform $G(s) = \mathcal{L}(g)$.

b) (unrelated) Solve the Volterra integral equation

$$y = t^2 + 2\int_{\tau=0}^{t} y(\tau)e^{-2(t-\tau)}\, d\tau.$$

4. Find the solution of $y'' + 2y' + 10y = \delta(t-2) + 13e^t$, $\quad y(0) = 0, \quad y'(0) = 13.$

5. Solve the following initial-value problem for a system of differential equations:

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \vec{\mathbf{X}}, \qquad \vec{\mathbf{X}}(0) = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}.$$

6. Find the general solution of the following system of differential equations:

$$\begin{cases} dx/dt &= x - 5y \\ dy/dt &= x - 3y \end{cases}$$

7. Given that $y_1 = x$ is a particular solution of the following equation, find the general solution:

$$y'' - \left(\frac{4+x}{x}\right) y' + \left(\frac{4+x}{x^2}\right) y = 0.$$

8. Find the general solution of $y'' + 9y = 7e^x + \cos 3x + \dfrac{3}{\cos 3x}$.

9. a) What second-order differential equation has as its general solution $y = AxJ_2(x) + BxY_2(x)$? (Here A and B are arbitrary constants.)

b) (unrelated) WITHOUT SOLVING the differential equation

$$(x-1)(x+3)y'' + \sqrt{x+2}\, y' + (\cos x)y = 0, \qquad y(0) = 5, \quad y'(0) = 7,$$

determine the range of x for which a unique solution is guaranteed to exist.

c) For the same equation as in part b), suppose we try to find a series solution of the form $y = \sum_{n=0}^{\infty} c_n x^n$. On what interval is this series guaranteed to converge?

10. a) Find ONE series solution (centered about $x_0 = 0$) of the equation

$$x^2 y'' - (3x + x^3)y' + 4y = 0.$$

b) Give the FORM ONLY of a second solution to the equation.

c) (Bonus, 2 points): Identify your solution to part a) in terms of familiar functions.