Prof Makdisi
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Math 202 — Spring 2004
Differential Equations, sections 1–4
Final exam, June 2 — Duration: 2 hours

JUST A MINUTE — PLEASE READ THE INSTRUCTIONS BELOW FIRST

1. Write your name, AUB ID number, and section number ON THE FRONT COVER OF THE AUB EXAMINATION BOOKLET.
To remind you, the sections are as follows:

Section 1	Section 2	Section 3	Section 4
Recitation F 1	Recitation F 2	Recitation F 12	Recitation F 3
Professor Makdisi	Professor Makdisi	Dr. Yamani	Ms. Jaafar

2. PLEASE DO YOUR WORK FOR THE PROBLEMS IN ORDER IN YOUR EXAM BOOKLET. If you wish to temporarily skip a problem and come back to it later, you may want to LEAVE ONE OR TWO BLANK PAGES before continuing to the next problem.

3. Please write your solutions on the answer part of your booklet. Use the back of each page for scratch work only. If you need an extra booklet be sure to WRITE YOUR NAME, AUB ID, and SECTION NUMBER on the second booklet AND indicate on BOTH booklets that you used two booklets total.

4. THE OTHER INSTRUCTIONS (f is NEW, review c and e):

- a. **Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit. Each problem is worth 12 points.**
- b. **Do as much of the exam as you can, and budget your time carefully.**
- c. **If you cannot do a certain integral, just leave it as an integral in your solution for partial credit on the rest of the problem.**
- d. **No calculators, books, or notes allowed. Turn off and put away any cell phones or beepers.**
- e. **Problem 8 asks for series solutions. For full credit on this problem, you need to give a formula for the coefficients. However, just like in Quiz 2, you can get almost full credit if you give the first FOUR nonzero terms for any solution.**
- f. **REGARDING COMPLEX SOLUTIONS: For full credit, the solution to a differential equation should involve real numbers only. However, if you are short on time, leave the solution in complex form — you'll still get most of the credit on the problem.**

GOOD LUCK!

1. a) Find the general solution of the equation

$$\left(3x^2e^y + 2xy + 1\right)dx + \left(x^3e^y + x^2 + y^2\right)dy = 0.$$

b) (UNRELATED) Solve the IVP

$$y\sqrt{1-x^2}\,\frac{dy}{dx} - e^y = 0, \qquad y(0) = 0.$$

2. Use Laplace transforms to solve the IVP:

$$y'' - 4y' + 3y = \delta(t-7), \qquad y(0) = 1, \quad y'(0) = 0.$$

3. Use the substitution $x = \sqrt{t}$ (so $t = x^2$) to solve the following equation in terms of Bessel functions:

$$\frac{d^2y}{dx^2} + x^{-1}\frac{dy}{dx} + 4x^2y = 0.$$

Suggestion: write $\frac{d}{dx}$ entirely in terms of t. Please do NOT use any power series in the solution.

4. a) Solve the following IVP:

$$x^2y'' - 5xy' + 8y = 0, \qquad y(1) = 3, \quad y'(1) = 7.$$

b) Find numbers x_0, a, b such that the IVP has NO solution when we change the initial conditions to:

$$x^2y'' - 5xy' + 8y = 0, \qquad y(x_0) = a, \quad y'(x_0) = b.$$

5. Find the general solution of the system:

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \vec{\mathbf{X}}.$$

6. Find the general solution of $y' + y = \dfrac{y^{-3}}{e^x + 1}$.

7. Find the general solution of

$$9y'' - 6y' + y = x^2 + \frac{e^{x/3}}{x^2+1}.$$

8. Find the general solution of the following equation using a series centered at $x = 0$:

$$x^2y'' - 2x^3y' - 2y = 0.$$

Suggestion: it is possible to solve this using only the smaller indicial root, BUT be careful when calculating c_3.

9. a) Write the following function $f(t)$ in terms of the unit step function $\mathcal{U}$, and find its Laplace transform $F(s)$:

$$f(t) = \begin{cases} t, & t < 3 \\ 0, & 3 \le t < 4 \\ e^{-t}, & t \ge 4 \end{cases}$$

b) (UNRELATED) Solve the following integral equation:

$$y(t) = e^{3t}\sin t + \int_{\tau=0}^{t} e^{\tau} y(t-\tau)\, d\tau.$$

10. Write the following system in terms of matrices and solve it:

$$\begin{cases} dx/dt = x - y, \\ dy/dt = x + 3y, \end{cases} \qquad \begin{array}{l} x(\ln 2) = 1 \\ y(\ln 2) = 0 \end{array}$$