FINAL EXAMINATION, MATH 202

September 2, 1996; 11:00 A.M.-1:00 P.M.

Name: Signature:

Student number:

Section number (Encircle): 1 2 5

1. Instructions:

- No calculators are allowed.

- There are two types of questions: **PART I** consisting of seven subjective questions, and **PART II** consisting of seven multiple-choice questions of which each has exactly one correct answer.

- GIVE DETAILED SOLUTIONS FOR THE PROBLEMS OF **PART I** IN THE PROVIDED SPACE AND CIRCLE THE APPROPRIATE ANSWERS FOR THE PROBLEMS OF **PART II**.

2. Grading policy:

- 9 points for each of the problems 1, 3, 4, 5 and 7 **PART I**.

- 10 points for each of the problems 2 and 6 of **PART I**.

- 5 points for each correct answer of **PART II**.

- -1 point (penalty) for each wrong answer of **PART II**.

- 0 point for no answer or more than one answer of **PART II**.

GRADE OF PART I:

GRADE OF PART II:

TOTAL GRADE:

Part I(1). Find the general solution of the differential equation

$$(3x+2)^2 y'' + 3(3x+2)y' - 36y = 0,$$

by using a change of variable of the form $u = 3x + 2$.

Part I(2). Use Laplace transform to find the solution of the initial value problem

$$ty'' - y' = t^2,\ y(0) = 0.$$

Part I(3). One solution of the differential equations

$$(1-x^2)y'' - 2xy' + 2y = 0$$

is $y_1(x) = x, x \in (-1, 1)$. Find a second solution that is linearly independent with $y_1$in the interval $(-1, 1)$.

Part I(4). Find

$$\mathcal{L}^{-1}\left\{\frac{2}{s} - 3\frac{e^{-s}}{s^2} + \ln\frac{s-2}{s+2}\right\}$$

Part I(5). Find the general solution of the differential equation

$$x^2y'' + xy' + (16x^2 - 1/4)y = 0. \ (x > 0)$$

Part I(6). Find the general solution of the differential equation

$$y'' - 3y' + 2y = \cos e^{-x}.$$

Part I(7). Find the differential equation

$$\frac{dy}{dx} = \cos x - \frac{1}{2}\sin x \tan x + \frac{1}{2}(\sec x)y^2.$$

knowing that $y = \sin x$ is a solution.

Part II: Multiple Choice Problems.

Part II(1). If

$$f'(t) = \cos t + \int_0^t f(\tau)\cos(t-\tau)d\tau, \ f(0) = 1$$

then the value of $f(2)$ is

(a) 0.

(b) 2.

(c) 5.

(d) -1.

(e) None of the above.

Part II(2). The solution of the initial value problem

$$y\frac{dx}{dy} + 2x\ln x = xe^y; \ y(1) = 1$$

passes through the point

(a) $x = e^{e^2/4}$, $y = 2$.

(b) $x = e^{-e^2/4}$, $y = 2$.

(c) $x = e^{e^3/9}$, $y = 3$.

(d) $x = e^{-e^3/9}$, $y = 3$.

(e) None of the above.

Part II(3). If $y_1 = x^3$ is a solution of the differential equation

$$x^2y'' - 5xy' + 9y = 0,$$

and $y_2 = v(x)y_1(x)$, with $v(e) = 1$, is another solution that is linearly independent with y_1, then the value of $y_2(e^{-2})$ equals

(a) $e/3$.

(b) $-2e^{-6}$.

(c) $8e^3$.

(d) $2e^2/3$.

(e) None of the above.

Part II(4). If $x^m y^n$ is an integrating factor of the differential equation

$$x(x+y)dy - y^2dx = 0$$

then

(a) $m = 2$ and $n = 1$.

(b) $m = 2$ and $n = -1$.

(c) $m = -2$ and $n = 1$.

(d) $m = -2$ and $n = -1$.

(e) None of the above.

Part II(5). The solution of the initial value problem

$$x^3y''' + xy' - y = 0,\ y(1) = y'(1) = 0 and y''(1) = 2$$

passes through the point (a) $x = y = e$.

(b) $x = e,\ y = 1/e$.

(c) $x = 1/e,\ y = e$.

(d) $x = y = 1/e$.

(e) None of the above.

Part II(6). The differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

has indicial roots $r_1 = 3/2$ and $r_2 = 1$. If $y = \sum_{n=0}^{\infty} c_n x^{n+3/2}$, with c_0 arbitrary, is a solution of the differential equation, then the recurrence relation of the coefficients c_n, $n \geq 1$, is given by

(a) $c_n = \frac{1}{n(2n+1)} c_{n-1}$.

(b) $c_n = \frac{1}{n(2n-1)} c_{n-1}$.

(c) $c_n = \frac{-1}{n(2n+1)} c_{n-1}$.

(d) $c_n = \frac{-1}{n(2n-1)} c_{n-1}$.

(e) None of the above.

Part II(7). Let y_1 and y_2 be solutions over an interval I for the differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0,$$

where a_0, a_1, and a_2 are continuous on I. Which of the following statements is **FALSE**?

(a) The Wronskian $W(y_1, y_2)$ is either identical to zero or never zero there on I.

(b) If a_2 is never zero on I, $y_1(x_0) = y_2(x_0)$ and $y_1'(x_0) = y_2'(x_0)$ then y_1 and y_2 are identical on I.

(c) If $a_2(x_0) = 0$ for some $x_0 \in I$, then there may exist infinitely many solutions $y = y(x)$ with prescribed values for $y(x_0)$ and $y'(x_0)$.

(d) y_1 and y_2 can always be written as power series about any point x_0 interior to I.