FINAL EXAMINATION, MATH 202

August 25, 1997; 11:00 A.M.-1:00 P.M.

Name: Signature:

Student number:

Instructor: Abdallah Lyzzaik

1. Instructions:

- No calculators are allowed.

- There are two types of questions: **PART I** consisting of seven subjective questions, and **PART II** consisting of seven multiple-choice questions of which each has exactly one correct answer.

- GIVE DETAILED SOLUTIONS FOR THE PROBLEMS OF **PART I** IN THE PROVIDED SPACE AND CIRCLE THE APPROPIATE ANSWERS FOR THE PROBLEMS OF **PART II**.

2. Grading policy:

- 11 points for each problem of **PART I**.

- For **PART II**, 7 points for problem 6 and 6 points for other problems.

- -1 point (penalty) for each wrong answer of **PART II**.

- 0 point for no answer or more than one answer of **PART II**.

GRADE OF PART I:

GRADE OF PART II:

TOTAL GRADE/120:

Part I(1). Find the general solution of the differential equation

$$(5x+1)^2 y'' + 5(5x+1)y' + 25y = 0,$$

by using the change of variable $u = 5x + 1$.

Part I(2). Use Laplace transform to find the set of all solutions $y(t)$ of the initial value problem

$$ty'' + 2ty' + 2y = 0,\ y(0) = 0.$$

Part I(3). One solution of the differential equation

$$xy'' - 2(x+1)y' + (x+2)y = 0$$

is $y_1(x) = e^x, x \in (0, \infty)$. Find a second solution that is linearly independent with $y_1$in the interval $(0, \infty)$.

Part I(4). Find

$$\mathcal{L}^{-1}\left\{\frac{2}{s} - 3\frac{e^{-s}}{s^2} + \ln\frac{s-2}{s+2}\right\}$$

Part I(5). Find the general solution of the differential equation

$$\frac{d}{dx}[xy'] + (x - \frac{\pi}{x})y = 0. \ (x > 0)$$

Part I(6). Find the general solution of the differential equation

$$y'' - 2y' + 2y = e^x \cot x.$$

Part I(7). Solve by the method of undetermined coefficients the differential equation

$$y'' + 4y = \sin^2 x.$$

Part II: Multiple Choice Problems.

Part II(1). If

$$t - 2f(t) = \int_0^t f(t-\tau)(e^{\tau} - e^{-\tau})d\tau$$

then the value of $f(1)$ is

(a) 0.

(b) 1/4.

(c) 1/3.

(d) 5/12.

(e) None of the above.

Part II(2). The solution of the initial value problem

$$\frac{dy}{dx} = (x+y+1)^2, \ y(0) = 0$$

passes through the point

(a) $x = \pi/4$, $y = \pi/4 + 1$.

(b) $x = \pi/4$, $y = \pi/4 - 1$.

(c) $x = -\pi/4$, $y = \pi/4 + 1$.

(d) $x = -\pi/4$, $y = \pi/4 - 1$.

(e) None of the above.

Part II(3). The solution of the initial value problem

$$-xy' + y = (y' + 1)^2,\ y(0) = 0$$

satisfies

(a) $x = 0,\ y = 1.$

(b) $x = 0,\ y = -1.$

(c) $x = 1,\ y = 0.$

(d) $x = 1,\ y = -1.$

(e) None of the above.

Part II(4). The indicial roots of the differential equation

$$3x^2y'' + xy' - (1 + x)y = 0$$

are

(a) -1, -1/3.

(b) 1, 1/3.

(c) 1, -1/3.

(d) -1, 1/3.

(e) None of the above.

Part II(5). The solution of the initial value problem

$$x^3y''' + xy' - y = 0,\ y(1) = y'(1) = 0, y''(1) = 2$$

passes through the point

(a) $x = y = e^2$.

(b) $x = e,\ y = 1/e$.

(c) $x = 1/e,\ y = e$.

(d) $x = y = 1/e$.

(e) None of the above.

Part II(6). The differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

has indicial roots $r_1 = 3/2$ and $r_2 = 1$. If $y = \sum_{n=0}^{\infty} c_n x^{n+3/2}$, with c_0 arbitrary, is a solution of the differential equation, then the recurrence relation of the coefficients c_n, $n \geq 1$, is given by

(a) $c_n = \frac{1}{n(2n-1)} c_{n-1}$.

(b) $c_n = \frac{1}{n(2n+1)} c_{n-1}$.

(c) $c_n = -\frac{1}{n(2n-1)} c_{n-1}$.

(d) $c_n = -\frac{1}{n(2n+1)} c_{n-1}$.

(e) None of the above.

Part II(7). Let y_1 and y_2 be solutions over an interval I for the differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0,$$

where a_0, a_1, and a_2 are continuous on I. Which of the following statements is **TRUE**?

(a) The Wronskian $W(y_1, y_2)$ is zero for some values of x and nonzero for other values, of the interval I.

(b) If $a_2(x)$ is never zero on I, $y_1(x_0) = y_2(x_0)$ and $y_1'(x_0) = y_2'(x_0)$ then y_1 and y_2 are identical on I.

(c) If $a_2(x_0) = 0$ for some $x_0 \in I$, then there always exist infinitely many solutions $y = y(x)$ with prescribed values for $y(x_0)$ and $y'(x_0)$.

(d) y_1 and y_2 can always be written as power series about any point x_0 interior to I.