FINAL EXAMINATION, MATH 202

June 7, 2006; 8:00-10:00 P.M.

VERSION I

Name: Signature:

Number: Section: 13, 14, 15, 16

1. Instructions:

- No calculators are allowed.
- There are two types of questions: **PART I** consists of **SIX** subjective questions, and **PART II** consists of **EIGHT** multiple-choice questions of which each has exactly one correct answer.
- Give detailed solutions for the problems of **PART I** in the provided space.
- 10 points for each question of **PART I**.
- Circle one answer for each question of **PART II**.
- 5 points for each correct answer of **PART II**.
- 0 point for no answer or more than one answer of **PART II**.

GRADE OF PART I	**/60**
GRADE OF PART II	**/40**
TOTAL GRADE	**/100**

Part I(1). solve, without using Laplace transforms, the initial-value problem

$$\begin{aligned} \frac{dx}{dt} &= 2x + 4y \\ \frac{dy}{dt} &= -x + 6y \end{aligned}$$

with initial conditions $x(0) = -1, y(0) = 6$.

Extra Page

Part I(2). Use Laplace transforms to find the solution of the initial-value problem

$$y'' + y = f(t), \quad f(t) = \begin{cases} 1, & 0 \le t < \pi/2 \\ 0, & \pi/2 \le t \le \infty \end{cases}, \quad y(0) = 1, y'(0) = 0.$$

Part I(3). Use the substitution $u = \sqrt{x}\, y$ to find the general solution of the differential equation

$$\frac{d^2u}{dx^2} + \left(1 - \frac{\nu^2 - 1/4}{x^2}\right) u = 0, \quad x > 0.$$

Part I(4). Show that the differential equation

$$4x^2\,y'' - 2x(x-2)\,y' - (3x+1)\,y = 0,$$

has a regular point and indicial roots $r_1/2$ and $r_2 = -1/2$ at $x = 0$, and use Frobenius method to find a series solution about $x = 0$ associated with $r_1 = 1/2$.

Extra Page

Part I(5). Find the general solution of the differential equation

$$x\,\frac{dy}{dx} = (1+x)\,y + x\,y^2.$$

by an appropriate substitution.

Part I(6). Find the general solution of the differential equation

$$x^2\, y'' - x\, y' + y = x^3, \quad x > 0.$$

Extra Page

Part II: Multiple Choice Problems.

Part II(1). The solution of the initial-value problem

$$y'(t) = 1 - \sin t - \int_0^t y(\tau)d\tau, \quad y(0) = 0,$$

satisfies

(a) $y(t) = \cos t + \sin t * \cos t$.

(b) $y(t) = \cos t - \sin t * \cos t$.

(c) $y(t) = \sin t + \sin t * \cos t$.

(d) $y(t) = \sin t - \sin t * \cos t$.

(e) None of the above.

Part II(2).

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2} + \tan s\right\} =$$

(a) $(t+1)\mathcal{U}(t+1) + (\sin t)/t$.

(b) $(t-1)\mathcal{U}(t-1) - (\sin t)/t$.

(c) $(t-1)\mathcal{U}(t-1) - (\cos t)/t$.

(d) $(t+1)\mathcal{U}(t+1) - (\sin t)/t$.

(e) None of the above.

Part II(3). The solution of the initial-value problem

$$\frac{dy}{dx} = -\frac{1}{2}(x+y-1)^3 - 1, \quad y(1) = 1,$$

satisfies

(a) $y(4) = -5/2$.

(b) $y(4) = 5/2$.

(c) $y(3) = 5/2$.

(d) $y(3) = -5/2$.

(e) None of the above.

Part II(4). The solution of the initial-value problem

$$\left[\frac{\sin 2x}{y} + x\right] dx + \left[y - \frac{\sin^2 x}{y^2}\right] dy, \quad y(0) = 1,$$

is given implicitly by the equation $x^2 y + 2\sin^2 x = h(y)$, where

(a) $h(1) = 0$.

(b) $h(1) = 2$.

(c) $h(1) = 4$.

(d) $h(1) = 6$.

(e) None of the above.

Part II(5). The differential equation

$$(x^2 + 1)y'' - 4xy' + 6y = 0$$

has an ordinary point at $x = 0$ and two power series solutions $\sum_{n=0}^{\infty} c_n x^n$,

whose coefficients c_n, $n = 0, 1, 2, \cdots$, satisfy the recurrence relation

(a) $c_{n+2} = \dfrac{(n-2)(n-3)}{(n+1)(n+2)} c_n, \quad n = 0, 1, 2, \cdots$.

(b) $c_{n+2} = -\dfrac{(n-2)(n-3)}{(n+1)(n+2)} c_n, \quad n = 0, 1, 2, \cdots$.

(c) $c_{n+2} = -\dfrac{(n-3)(n-4)}{(n+1)(n+2)} c_n, \quad n = 0, 1, 2, \cdots$.

(d) $c_{n+2} = \dfrac{(n-1)(n-2)}{(n+1)(n+2)} c_n, \quad n = 0, 1, 2, \cdots$.

(e) None of the above.

Part II(6). The solution of the initial-value problem

$$\left[x \cos^2\left(\frac{y}{x}\right) - y\right] dx + x\, dy = 0, \quad y(1) = \frac{\pi}{4}$$

satisfies

(a) $y(e^{-2}) = 0$.

(b) $y(e^{-1}) = 0$.

(c) $y(e) = 0$.

(d) $y(e^2) = 0$.

(e) None of the above.

Part II(7). Answer TRUE (T) or (FALSE) only:

(a) —— The Laplace transform of a function f is

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt.$$

(b) —- The convolution of functions f and g on $[0, \infty)$ is defined by

$$(f * g)(t) = \int_0^t f(\tau) g(t - \tau)\, d\tau.$$

(c) —- The general solution of the parametric Bessel equation

$$x^2\, y'' + x\, y' + (\lambda^2 x^2 - \nu^2) y = 0$$

is $y = c_1 J_\nu(\lambda x) + c_2 Y_\nu(\lambda x)$.

(d) —- If the set of two functions f_1 and f_2 is linearly independent on an interval I, then the Wronskian $W(f_1, f_2) \neq 0$ for all $x \in I$.

(e) —- The initial-value problem $dy/dx = \sqrt{y},\ y(x_0) = y_0$ has no solution for $y_0 < 0$.

Part II(8). If $y_1 = (\sin x)/x$ is one solution of the differential equation,

$$x\, y'' + 2\, y' + x\, y = 0,$$

then the solution of the associated boundary-value problem with boundary conditions $y(\pi/2) = 0$ and $y(\pi) = 1/2$ satisfies

(a) $y(\pi/4) = -\sqrt{2}$.

(b) $y(\pi/4) = \sqrt{2}$.

(c) $y(\pi/4) = -\sqrt{2}/2$.

(d) $y(\pi/4) = \sqrt{2}/2$.

(e) None of the above.