M202 - Differential Equations — Sample Quizz 3

Reminder: Final Exam for Sections 5-8 at 8:00 AM on 7th June 2006 in WHCR

A series solution or an explicit solution is accepted for full credit if and only if this is indicated in the problem.

You may use any method you like to find a solution.

For full credit you need to state your result clearly!

1. Solve the differential equation $(y')^3 = -x^3 y/(1+x^2)^5)$.

2. Find a solution $y = y(x)$ of $y' = 1 + \sqrt{x+y}$. It suffices to find an **implicit solution**.

3. Find the general solution $y = y(x)$ of
$$y''' + y' = \cos(x) \ .$$

4. Find the general solution of
$$y'' - 4y' + 3y = \frac{1}{1+e^{-2x}} \ .$$

5. Find the solution $y = y(x)$ of
$$y'' + (2 + x + x^2)y' + y = 0 \quad \text{with} \quad y(0) = 3 \quad \text{and} \quad y'(0) = 5 \ .$$
It suffices to specify a power series solution here. For full credit, give a complete recursive definition of the coefficients of your series. Compute the first 3 coefficients explicitely.

6. Let $(y_1, y_2) = (y_1(t), y_2(t))$ be the solution of
$$\begin{aligned} y_1' &= y_1 + y_2 + t \\ y_2' &= -y_1 + 2y_2 \end{aligned}$$
with $y_1(0) = y_2(0) = 1$. Compute $y_2(t)$ for $t \geq 0$.

7. Find the inverse Laplace transforms of
$$\frac{s+5}{s(s^2+s+21)} \quad \text{and} \quad \arctan\left(\frac{2}{s-7}\right) \ .$$

8. Find the solution $Y = Y(t) = (y_1(t), y_2(t))$ of the differential equation
$$Y' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} Y \quad \text{with} \quad Y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
For full credit you must find an explicit formula for the coefficients $y_1(t), y_2(t), y_3(t)$.

9. Find the solution $Y = Y(t) = (y_1(t), y_2(t), y_3(t))$ of the differential equation
$$Y' = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 7 \end{pmatrix} Y \quad \text{with} \quad Y(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$
For full credit you must find an explicit formula for the coefficients $y_1(t), y_2(t), y_3(t)$.

10. Find the eigenvalues and eigenvectors of
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$
What is the general solution of $Y' = AY$, $Y = Y(t) = (y_1(t), y_2(t))$.