AUB - Math department
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MATH 202 **Hints for the final exam sample** June 5, 2007

Exercise 6. Use the following facts
1) $J_0' = -J_1$.
2) J_0 is a solution to $y'' + \frac{1}{x}y' + y = 0$.

Exercise 7. There are several ways to solve the problem

$$x' = -x + 2y \tag{1}$$
$$y' = -2x - y + e^{-t} \tag{2}$$
$$x(0) = y(0) = 0. \tag{3}$$

First method (substitution-elimination). Equation (1) implies that

$$y = \frac{1}{2}(x' + x) \quad \text{and so} \quad y' = \frac{1}{2}(x'' + x').$$

Replacing y and y' by their expressions in equation (2) and simplifying you should get

$$x'' + 2x + 5x = 2e^{-t}.$$

Furthermore, by equation (3), we have that $x'(0) = 0$. I am sure now that most of you can solve the initial value problem

$$x'' + 2x + 5x = 2e^{-t}$$
$$x(0) = x'(0) = 0$$

by many ways.

Second method (Laplace transforms). Applying the Laplace operator to equations (1) and (2) and using (3), you should get the algebraic system

$$(s+1)X - 2Y = 0$$
$$2X + (s+1)Y = \frac{1}{s+1},$$

where X and Y are the Laplace transforms of x and y respectively. There are several ways to solve this system (Cramer's, substitution, elimination).

Learn this trick which may save time and avoid errors

$$\frac{1}{(s+1)[(s+1)^2+4]} = \frac{1}{4}\frac{(s+1)^2+4-(s+1)^2}{(s+1)[(s+1)^2+4]} = \frac{1}{4}\left(\frac{1}{s+1} - \frac{s+1}{(s+1)^2+4}\right).$$

Third method (matrix method).

1) Find the general solution of the homogeneous system by finding the eigenvalues and the eigenvectors of the matrix $\begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}$.

2) Find a particular solution of the form $e^{-t}\begin{pmatrix} a \\ b \end{pmatrix}$.

Work out the details and compare your results.

Remark. In all the above methods you cannot escape from the polynomial $r^2 + 2r + 5 = (r + 1)^2 + 4$.

Exercise 8.

The integrals can be computed directly by parameterizing the boundary of the square (it is composed of four pieces). But if you recall Green's theorem you are led to compute the divergence and the curl of F.

Answer: zero.