AUB - Math department
Prof. H. Gebran

MATH 202 **Sample of the final exam** June 5, 2007

Material: First lectures notes. Sections 4.1, 4.2, 4.3, 4.4, 4.6, 4.7, 6.1, 6.2, 6.3 (without Legendre's equations), 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 8.1, 8.2, 16.1, 16.2, 16.4.

Duration: 2 hours 30 minutes.

Remarks. You are allowed to bring with you six A4 papers on which you can write any useful formula or theorem. Calculators, computers, cellular phones are not allowed.

Exercise 1. (*10 points*)

Solve the initial value problem $y' - y = y^2, \;\; y(0) = 1.$

Exercise 2. (*15 points*)

Use the variation of parameters method to solve the equation

$$y'' + \frac{1}{x}y' - \frac{4}{x^2}y = x^2 \quad \text{for } x > 0.$$

Exercise 3. (*20 points*)

a) Find a function whose Laplace transform is $\frac{1}{(s^2+4)^2}$.

b) Find a function whose Laplace transform is $\frac{1}{(s^2+4)^3}$.

c) Solve the initial value problem

$$y'' + 4y = \sin(2t)$$
$$y(0) = 0, y'(0) = 1.$$

Exercise 4. (*20 points*)

a) Let $y_1 = \sum_{m=0}^{\infty} a_m x^{m-1}$ with $a_0 = 1$ be a solution to

$$x^2y'' + 3xy' + (1-x)y = 0, \; x > 0. \tag{E}$$

Show by induction that $a_m = \frac{1}{(m!)^2}$ for all nonnegative integers m.

b) Find another solution to (E) of the form $y_1 \ln(x) + \sum_{m=0}^{\infty} b_m x^{m-1}$ (write the first three terms of the series found).

Exercise 5. (*10 points*)

Express the general solution of $x^2y'' + 2xy' + 9x^2y = 0$ in terms of some Bessel functions.

Hint. Find an equation satisfied by $h(x) = x^{1/2}y(x)$.

Exercise 6. (*5 points*)

Let J_0 and J_1 be the Bessel functions of the first kind of order 0 and 1 respectively. Explain why J_0 and J_1 cannot vanish at the same point.

Exercise 7. (*13 points*)

Solve the system

$$\begin{aligned} x' &= -x + 2y \\ y' &= -2x - y + e^{-t} \\ x(0) &= y(0) = 0. \end{aligned}$$

Does x or y have any limit as $t \to \infty$?

Exercise 8. (*07 points*)

Find the outward flux and the circulation of the vector field $F(x, y) = (x + y, x - y)$ across the boundary of the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$.