MATHEMATICS 202
SPRING SEMESTER 2005-06
QUIZ I

Time: 70 MINUTES.
Date: March 11, 2006.
Name:
ID Number:
Section Number:
Course Instructors: Prof. Abdallah Lyzzaik and Dr. Hassan Yamani

| Question | Grade |
| :---: | :---: |
| 1 | $/ 14$ |
| 2 | $/ 14$ |
| 3 | $/ 14$ |
| 4 | $/ 14$ |
| 5 | $/ 14$ |
| 6 | $/ 12$ |
| 7 | $/ 18$ |
| TOTAL | $/ 100$ |

Answer The Following Seven Questions On The Page Allocated For Each Question (You May Use The Back Of The Pages If Needed).

1. Solve the initial-value problem
(14 points)

$$
\frac{d y}{d x}=\frac{y^{2}-x^{2}}{y x}, \quad y(1)=-\sqrt{2}
$$

2. Solve the differential equation

$$
\left(y^{3}+x+1\right) d x+3 y^{2} d y=0
$$

by finding an appropriate integrating factor to make it exact.
3. Solve the initial-value problem

$$
\left(3 x-y^{2}\right) d x-4 x y d y=0, \quad y(1)=2 .
$$

4. Solve the differential equation
(14 points)

$$
(x+y)^{2} \frac{d y}{d x}=1
$$

5. Find the general solution of the differential equation (14 points)

$$
x^{2} y^{\prime \prime}-\left(x^{2}+2 x\right) y^{\prime}+(x+2) y=0
$$

provided that $y=x$ is a known solution.
6. Sketch the regions in the $x y$-plane for which the initial-value problem

$$
\frac{d y}{d x}=\sqrt{\frac{x^{2}-1}{1-y^{2}}}, \quad y\left(x_{0}\right)=y_{0}
$$

possesses real and unique solutions; justify your answer.
(12 points)
7. The differential equation

$$
x^{2} y^{\prime \prime}-4 x y^{\prime}+6 y=0
$$

is known to have solutions $y_{1}=x^{2}$ and $y_{2}=x^{3}$.
(a) Show that the Wronskian $W\left(y_{1}, y_{2}\right)=0$ for every real $x$.
(b) Show that the set $\left\{y_{1}, y_{2}\right\}$ is linearly independent on $(-\infty, \infty)$. (6 points)
(c) Do the results in (a) and (b) contradict? Justify your answer.
(6 points)

