MATHEMATICS 202
SPRING SEMESTER 2005-06
QUIZ II

Time: 70 MINUTES.
Date: April 8, 2006.
Name:-
ID Number:
Section Number:
Course Instructors: Prof. Abdallah Lyzzaik and Dr. Hassan Yamani

| Question | Grade |
| :---: | :---: |
| 1 | $/ 16$ |
| 2 | $/ 16$ |
| 3 | $/ 16$ |
| 4 | $/ 16$ |
| 5 | $/ 100$ |
| 6 |  |
| TOTAL |  |

Answer The Following Six Questions On The Page Allocated For Each Question (You May Use The Back Of The Pages If Needed).

## 2

1. Find the general solution of the differential equation (16 points)

$$
x(x+1) y^{\prime \prime}+\left(2-x^{2}\right) y^{\prime}-(x+2) y=(x+1)^{2}
$$

whose complementary solution is $y=c_{1} e^{x}+c_{2} x^{-1}$ for all real $x \neq 0$.
2. Find the general solution of the differential equation (16 points)

$$
y^{\prime \prime}-2 y^{\prime}+y=x e^{x} .
$$

3. Suppose $m_{1}=1$ and $m_{2}=1+\sqrt{3} i$ are the characteristic roots each of multiplicity 2 of the characteristic equation of a homogeneous linear differential equation. Write down the general solution of the differential equation if it is
(a) an equation of constant coefficients.
(8 points)
(b) a Cauchy-Euler differential equation.
4. Classify each singular point of the differential equation

$$
x^{4}\left(x^{2}-36\right)(x-3)^{2} y^{\prime \prime}+x(1-\cos x) y^{\prime}+2 x^{2} y=0
$$

as regular or irregular, and find the least possible radius of convergence for the series solution of the differential equation at each regular point. Justify your answers.
(16 points)
5. Use the substitution $x=e^{t}$ to transform the Cauchy-Euler differential equation

$$
2 x^{2} y^{\prime \prime}+5 x y^{\prime}-2 y=2 \ln x, \quad(x>0),
$$

to a differential equation with constant coefficients, then solve the original equation.
6. Show that the indicial roots of the differential equation

$$
2 x^{2} y^{\prime \prime}+\left(x^{2}-x\right) y^{\prime}+y=0
$$

at the regular singular point $x=0$ are $1 / 2$ and 1 , and find the solution of the differential equation associated with the root $1 / 2$. ( 20 points)

