M202 - Differential Equations — Sample Quizz 1

Reminder: 1st Quizz on 22nd March, 12:05–12:55 in Nicely 500

1. Which of the following sets of functions on $\mathbb{R}$ are linearly independent:
   (a) $\{1\ ,\quad x\ ,\quad \cos(3x)\}$
   (b) $\{\cos^2(x)\ ,\quad \sin^2(x)\ ,\quad x-1\ ,\quad x^2-1\ ,\quad x^2+x\}$
   (c) $\{\sqrt{|x|}+2\ ,\quad x\ ,\quad 1\}$
2. How many linearly independent solutions has the differential equation $y''' + \cos(x^3)y' = 0$?
3. Let y be a function on $\mathbb{R}$ satisfying the differential equation $e^y(x)y'(x) = 1 + e^{2y(x)}$ for all $x \in \mathbb{R}$ and such that $y(63) = 1$. Compute $y(64)$.
4. Find a fundamental system for the differential equation $y'' + y' - y = 0$.
5. Let y be a solution of the differential equation $xy' + y = \frac{9}{1+x}$ for $x > 0$ with $y(1) = 3$. Compute $y(27)$.
6. Find an implicit solution of the equation $2xydx + (2x^2 + \sin(y))dy = 0$.
7. Find the general solution of the equation $y'''(x) + 8y''(x) + 20y'(x) + 16y(x) = x$
   Hint: $(-2)^3 + 8 * (-2)^2 + 20 * (-2) + 16 = 0$
8. Find the general solution of the equation $y''(x) - 6y'(x) + 10y(x) = e^x + 5x$.
9. Find the general solution of the equation $xy''(x) + (2x-1)y'(x) - 2y(x) = 0$ for $x > 0$.
   Hint: $y_1(x) = e^{-2x}$ *is a solution. You do not need to prove this*
10. Let f be the function on $\mathbb{R}$ such that

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \text{ or } 2 \le x \le 3 \\ 0 & \text{else} \end{cases}$$

    Let y be a function on $\mathbb{R}$ with $y(-3) = 0$ and such that $y'(x) + 3y(x) = f(x)$. Compute $y(27)$.