M202 - Differential Equations   Sample Quizz 2

Reminder: 2nd Quizz on 3/5/2006, 12:00–13:00 in Nicely 500

1. Given the differential equation $x^2(1+x^2)(1-x)^3 y'' + y = 0$

   (a) which real numbers a are singular, regular-singular, regular points of the equation?

   (b) For $a = -1$ resp. $a = 1/2$ there is a fundamental system of power series solutions $y_i = \sum_{n=0}^{\infty} c_{i,n} x^n$, $i = 1, 2$. Give a lower bound for the radius of convergence of these series.

2. Find a solution $y = y(x)$ of $y'' + x^4 y = 0$ with $y(1) = 1$, $y'(1) = 0$.

3. Find the general solution of the equation $y'''(x) + 8y''(x) + 20y'(x) + 16y(x) = x$

   Hint: $(-2)^3 + 8*(-2)^2 + 20*(-2) + 16 = 0$

4. Find the general solution of $y'' - 2y' + y = -\ln(x) + \frac{1}{x}$.

5. Let $\sum_{n=0}^{\infty} c_n x^n$ be the Taylor series of the following functions f, around 0, i.e. $c_n = \frac{1}{n!} \frac{d}{dx} f(0)$. What is the radius of convergence of this series in the cases

$$ f(x) = \frac{1}{1+x+x^2} \quad \text{and} \quad f(x) = \cos\left(e^{\frac{1}{3+x^2}}\right) . $$