Determine all the currents and voltages in the circuit using superposition and mark them on the circuit diagram.



Solution:





 Find the Thevenin equivalent of the circuit shown in figure 10.

a)
$$V_{th} = 10V$$
 and $R_{th} = 1K$
b) $V_{th} = 0V$ and $R_{th} = 0.1K$
c) $V_{th} = 10V$ and $R_{th} = 2K$

d)
$$V_{th} = 1V$$
 and $R_{th} = 1K$

e) None of the above



 In the circuit of Figure 1, the Thevenin resistance as seen from terminals ab is;



- 100/9Ω
- c. 50/90
- d. 10Ω
- e. None of the above



In the circuit of Figure 8, the Thevenin equivalent resistance, across terminals a-b, is:







a)5.357A b)11.25A c)7.5A d)22.5A e)NOA

Problem 1 (10 pts)

Consider the circuit shown below.



1. We, first, set the current sources 5 mA and 10 mA to zero. Determine the equivalent resistance seen by the 20 mA current source. (5 pts)

R₁ = (4 + 1)||4 = 20/9 kΩ; R₂ = 20/9 + 2 = 38/9 kΩ, R_{eq} = 2||(38/9) = 19/14= 1.36 kΩ

2. Write the node voltage equations by inspections (do not solve) (5 pts)

1.75V ₁	-	0.25V ₂	_	V_3	=	20
-0.25V ₁	+	V ₂	_	0.25 <i>V</i> ₃	=	5
- <i>V</i> 1	_	0.25V ₂	+	1.25 <i>V</i> ₃	=	5

Problem 2 (10 pts)

Consider the circuit shown below



1. We, first, set the 4V source and the 10V source to zero. Determine the equivalent resistor seen by the 8V voltage source. (5 pts)

 R_{eq} = 5 + (2||4||4||1) = 5 + 0.5 = 5.5 Ω

2. Write the mesh-current equations. (do not solve). (5 pts)

7 <i>I</i> 1	-	21 ₂	-	0/ ₃	-	0 <i>I</i> 4	=	8
-2/ ₁	+	6 <i>1</i> ₂	_	4 / 3	-	0 <i>1</i> 4	=	4
0/ ₁	_	4 <i>I</i> ₂	+	5 <i>1</i> ₃	-	<i>I</i> ₄	=	-10
0/ ₁	_	0 <i>1</i> 2	_	<i>I</i> ₃	+	5 <i>1</i> 4	=	-4



21. Determine the resistance between terminals a b in figure 20.



 Determine the equivalent resistance between terminals a and b, given that all resistances are 1 Ω.

A. 5 Ω
B. 4.5 Ω
C. 4 Ω
D. 3 Ω

E. None of the above

Solution: The resistances not connected directly to

terminals a and b form a balanced bridge. Hence the resistance across the bridge does not carry any current and can be replaced by an open circuit or a short circuit. If replaced by an open circuit, $R_{eq} = 1 + 2||2 + 1 = 3 \Omega$.



Find the power consumed by the 50 20ohms ohm resistor in the circuit shown below 10v 20ohms + A) P= 0.246 W B) P= 0.692 W 50ohms V_0 C) P= 2.358 W 20ohms 5v D) P= 5.100 W E) None of the — 20ohms above



Given the circuit above, determine the current I_x if voltage-controlled current source has B=0.2:

- \rightarrow A) 0
 - B) 0.285
 - C) 1.285
 - D) 0.5
 - E) None of the above



7

What is I₀?

A) 1 A → B) -1 A C) 2 A D) -2 A E) None of the above



The following values are given: I_A= 1 mA, I_B= 2 mA, I_C = 4 mA, R₁ = R₂ = R₃ = 1 kΩ, What is the value of V_{bc} ?

→ A) 4 V B) -2 V C) 1 V D) 5 V E) None of the above



Find I_x.

A) 4.2 A B) 3.5 A → C) 2.25 A D) 4.75 A E) None of the above

Problem 10



Find the Thevenin equivalent across terminals a and b.

A) $R_{TH} = 40\Omega$, $V_{TH} = 6.67 V$ B) $R_{TH} = 40\Omega$, $V_{TH} = -6.67 V$ \rightarrow C) $R_{TH} = 80\Omega$, $V_{TH} = 12 V$ D) $R_{TH} = 80\Omega$, $V_{TH} = -12 V$ E) None of the above



Find the current, I_0 , flowing through the dependant source.



Problem 12



Find the voltage, V, across the 2A source.

A) 400 V → B) 140 V C) 6 V D) -300 V E) None of the above

P3.1.25 Determine V_o in Fig. P3.1.25 using nodevoltage analysis.







 $V_d - V_c = 2V_x = 2V_b - 2V_a$, or $2V_a - V_c + V_d = 40$. Solving, $V_a = 40$ V, $V_c = 40$ V, $V_d = V_0 = 0$.

P3.1.26 Determine V_0 in Fig. P3.1.25 using mesh-current analysis.

Solution For mesh 2: $-2I_1 + 6I_2 - 2I_4 =$ $-2V_x$; substituting $I_1 = 10$ and $V_x = -2I_2$: $I_2 - I_4 = 10$. For mesh 3: $-4I_1 + 8I_3 - 4I_4 =$ $2V_x$; $I_2 + 2I_3 - I_4 = 10$. For mesh 4: $-2I_2 - 4I_3 + 6I_4 = -20$, or $-I_2 2I_3 + 3I_4 = -10$. Solving, $I_2 =$ 10 A, $I_3 = 0$, and $I_4 = 0$, which gives $V_0 = 0$.



P3.2.12 Determine V_0 in Fig. P3.1.19 using superposition and calculate the power dissipated in the 5 Ω resistor.



Figure P3.1.19

Solution: With the 2 A source acting alone, the circuit becomes as shown. The source current flows through the 5 Ω resistor, so that $V_{01} = 10$ V. Similarly, when the 4 A source is applied alone, $V_{02} = 20$ V. From superposition, $V_0 = V_{01} + V_{02} = 30$ V. The dependent source does not contribute to V_0 .

Power dissipated in the 5 Ω resistor is

$$\frac{(30)^2}{5} = 180 \text{ W}.$$



Determine Io P3.2.13 *I*₀ in Fig. P3.1.21 using 5 S superposition 4 S 4 S and calculate \ge 2 S 2 S < 4 V 2 V the power + dissipated in 3/₀ the 5 S resistor.



Solution: With the 2 V source acting alone, and the 4 V source replaced by a short circuit, the circuit becomes as shown. The source voltage is applied across the 5 S resistor, so that $I_{O1} = 10$ A. Similarly,



when the 4 V source is applied alone, $I_{O2} = 20$ A. From superposition, $I_O = I_{O1} + I_{O2} = 30$ A. The dependent source does not contribute to I_O . Power dissipated in the 5 S resistor is

$$\frac{(30)^2}{5} = 180$$
 W.

P4.1.8 Derive TEC between terminals ab in Fig. P4.1.8.

Solution: Let us first remove the 20 Ω resistor and reapply it later. On open circuit, each 1 A source produces a 10 V drop across the resistor in parallel with it. Hence $V_{oc1} = 20$ V. On short circuit, $I_{sc} = 1$ A, so that $R_{Th1} = 20 \Omega$. When the 20 Ω resistor is added at terminals ab, $V_{Th} = 20 \times (20/40) = 10$ V and $R_{Th} = (20||20) = 10 \Omega$



Figure P4.1.8

P4.1.9 Derive TEC between terminals ab in Fig. P4.1.9.

Solution: On open circuit, KVL around the upper mesh gives $20I_x = 20$, or $I_x = 1$ A. It follows that $V_{Th} = V_{oc} = 20 + 20I_x = 40$ V.

On short circuit, the current in the 10 Ω resistor is 2 A. KVL around the upper mesh gives: $20 = 5(I_x - I_{sc}) + 5(3I_x - I_{sc})$, or $2I_x - I_{sc} = 2$; from KCL at node c: $3I_x - I_{sc} + 2 = I_x$, or $2I_x - I_{sc} = -2$. This means that I_x and I_{sc} are indeterminate. This suggests that $R_{Th} = 0$, which would make I_{sc} indeterminate. To verify this, we apply a test



source of 1 A, with the voltage source set to zero. Then, $4I_x + 2 = 0$, so that $I_x = -0.5$ A and $V_T = 2(-0.5) + 1 = 0$. Hence $R_{Th} = 0$.



P4.1.10 Derive NEC between the short-circuited terminals ab in Fig. P2.1.4.

Solution: If a test voltage is applied, KCL at node a gives $I_{\phi} = I_T + 3$. From KCL at node c, $I_T + 3 + 10 = 5$, or $I_T = -8$ A, irrespective of V_T . It follows that $I_N = 8$ A. Since I_T is independent of V_T , it means that there is no resistance in parallel with the current source I_N .

С



P4.1.11 Determine V_0 in Fig. P3.1.7 using TEC.





Solution: Open-circuit voltage: When only the

10 V source is applied, $V_{Th1} = \frac{10}{30} \times 10 = \frac{10}{3}$ V. When only the 20 V source is applied $V_{Th2} = 20$ V. Hence, $V_{Th} = 70/3$ V. With both sources set to zero, $R_{Th} = 10||20 = 20/3 \Omega$. It follows that V_0

$$= \frac{40}{40+20/3} \times \frac{70}{3} = 20 \,\mathrm{V}.$$



P4.1.12 Determine *I*₀ in Fig. P3.1.9 using NEC.





Solution: With the 10 A source acting

alone, $I_{N1} = \frac{10}{30} \times 10 = \frac{10}{3}$ A. With the 20 A source acting alone, $I_{N2} = 20$ A. Hence, $I_N = 70/3$ A. The conductance between terminals bc is $\frac{20 \times 10}{20 + 10} = \frac{20}{3}$ S. It follows from NEC that $I_0 = \frac{40}{40 + 20/3} \times \frac{70}{3} = 20$ A.



P4.1.15 Determine V₀ in Fig. P3.1.15 using TEC.





Solution: On open circuit, $I_x = 0$, and the dependent source becomes an open circuit. It follows that $V_{Th} = 10$ V. On short circuit, the circuit becomes as shown, where $I_x = I_{SC}$ and the dependent source becomes $5I_{SC}$. It follows from

KCL that: $I_{SC} = 5I_{SC} + \frac{10}{4}$, which gives	
$I_{SC} = -\frac{5}{8}$ A, and $R_{SC} = -16 \Omega$. Hence	

$$V_0 = \frac{4}{4 - 16} \times 10 = -\frac{10}{3}$$
 V.



P4.1.16 Determine I_0 in Fig. P3.1.17 using NEC.





Solution: On open circuit, 10 A flows through the 4 S resistor, so that $V_{oc} = 5V_{oc}$ $+\frac{10}{4}$, which gives $V_{oc} = -\frac{5}{8}$ A. On short circuit, $V_x = 0$ and the dependent source is zero, so that $I_N = 10$ A. This makes $G_N = -16$ S. It follows that $I_0 = \frac{4}{4-16} \times 10 = -\frac{10}{3}$ A.

P4.1.17 Determine V_0 in Fig. P3.1.19 using NEC.

Solution: If the 5 Ω resistor is replaced by an open circuit, the circuit is invalid, as two unequal current sources will be connected in series through the 2 Ω resistors, and $V_0 \rightarrow \infty$. If a test source is applied in place of the 5 Ω resistor and the current sources replaced by open circuits, the resistance seen by the source is infinite. If the 5 Ω resistor is





replaced by a short circuit, $I_{SC} = 6$ A. It follows that the circuit does not possess a TEC between the specified terminals, only an NEC consisting of an ideal current source of 6 A. This gives V_o = 30 V. **P4.1.18** Determine I_0 in Fig. P3.1.21 using TEC.



circuits, the resistance seen by the source is zero. If the 5 S resistor is replaced by an open circuit, $V_{Th} = 6$ V. It follows that the circuit does not possess an NEC between the specified terminals, only a TEC consisting of an ideal voltage source of 6 V. This gives $I_0 = 30$ A.

P4.1.19 Determine I_0 in Fig. P3.1.23 using NEC.





Solution: With the 4 S resistor replaced by a short circuit, I_0 can be obtained from mesh-current analysis. The mesh current equations are the same as those for P3.1.24 but with a coefficient of 0.5 for I_4 in the equation for mesh 4. The equations are:

 $l_1 - 0.5l_2 = 9$; $-0.5l_1 + 0.75l_2 + 0.5l_4 = -1$; and $2l_1 - l_2 + l_4 = -4$. Solving, $l_4 = l_0 = -22$ A.

If a test source is substituted for the 4 S resistor, with the independent source set to zero, it is seen from KCL at the middle node that $I_T = 0$, which means that the source resistance is infinite. The circuit does not possess a TEC between the specified terminals, only an NEC.











Ą

 $V_x +$ b Solution: If a current source is applied at node d, а + with the independent sources set to zero, it is seen 2Ω that $V_{ac} = V_x$, so that $V_{bd} = 0$ and $V_0 = 0$. In other V_{x} 2Ω 2Ω 2*V*, words the source sees a short circuit and $R_{src} = 0$. If the resistor between node d and the reference node С d + is replaced by an open circuit, the node-voltage 4Ω V_{0} equation at node a is: $V_a - 0.5V_c = 20$, and the node voltage equation at node c is: $-0.5 V_a + 0.75 V_c = -I_y$, where $I_{y} =$ b а $0.5(V_c + 2V_x - 20) =$ 2Ω $0.5V_c + V_a - 30$. Substituting for 2Ω 2Ω $2V_x$ I_{v} : 0.25 V_{a} + V_{c} = 30. Solving, gives $V_a = V_c = 40$ V. Hence, V_x 20 V 10 A d + = 20 V and V_d = 0. In other 4Ω V_{O} words, TEC and NEC are just short circuits,

- 1. Determine V_0 assuming I_{SRC} = 0.25 A.
 - A. 4 V
 - B. 1 V
 - C. 5 V
 - D. 2 V
 - E. 3 V

Solution: The two 5 Ω resistances can be combined in parallel to give a 2.5 Ω resistance, and the two 10 Ω resistances can be combined in parallel to give a 5 Ω resistance carrying a current of 2*I*_x, as shown. It follows that *I*_{SRC} – 2*I*_x =

0.5
$$I_x$$
, or $I_x = \frac{I_{SRC}}{2.5}$ and $V_0 = 10I_x$, so that
 $V_0 = 4I_{SRC}$.



- 2. Determine I_{SRC} assuming V_{SRC} = 2 V and all resistances are 2 Ω .
 - A. 1.5 A
 - B. 3 A
 - C. 2.5 A
 - D. 2A
 - E. 1A

Solution: From symmetry the two currents I_x are equal and sum to zero. Hence, $I_x = 0$ and the two resistors can be removed. The equivalent resistance seen by the source is $(2 + 2)||(2 + 2) = 2 \Omega$. It follows that $I_{SRC} = V_{SRC}/2$.





- 3. Determine V_O assuming $I_{SRC} = 1$ A.
 - A. 7.5 V
 - B. 12.5 V
 - C. 5 V
 - D. 15 V
 - E. 10 V



Vo

/\/\\ 10 Ω

 $> 5 \Omega$

+

5Ω

Solution: The two current sources are equivalent to a current source I_{SRC} connected as shown, since KCL is the same at the two nodes. The resistance seen by the source is $10||(5 + 5) = 5 \Omega$. Hence, $V_0 = 5I_{SRC}$.

4. Determine Thevenin's resistance looking into terminals ab, assuming $\alpha = 10$.

- A. 50Ω
- B. 25Ω
- C. 100 Ω
- D. 200 Ω
- Ε. 20 Ω

Solution: When a test source V_T is applied at terminals ab, with the independent voltage source set to zero, it follows from the circuit that:

$$I_x = -\frac{2V_0}{2} = -V_0 = -V_T \text{ mA. } I_T = -\alpha I_x =$$

$$\alpha V_T$$
 mA. Hence, $\frac{V_T}{I_T} = \frac{1}{\alpha} k\Omega \equiv \frac{1000}{\alpha} \Omega.$





- 5. Determine V_2 so that $V_x = 0$, assuming $V_1 = 4 \text{ V}$.
 - A. 8 V
 - B. 6 V
 - C. 6.5 V
 - D. 7.5 V
 - E. 7 V

Solution: The 6 Ω and 3 Ω resistors do not carry any current. They can removed from the circuit, with nodes a and b being at the same voltage. V_1 can be transformed to a current source $V_1/4$ A in parallel with a 4 Ω resistor. The total current is $(0.25V_1 + 2)$ A in parallel with 2 Ω . V_2 is the voltage of node a, which gives: $V_2 = 2(0.25V_1 + 2) = (0.5V_1 + 4)$ V.



6. Derive the mesh current equations in terms of I_1 , I_2 , and I_3 . DO NOT SOLVE THE EQUATIONS

Solution: Considering the voltage drop V_{ab} as a unit, the equation for mesh 1 is: $(10 + 5)I_1 - 5I_3 = 12 - V_{ab}$ The mesh-current equation for mesh 2 is: $(20 + 5)I_2 - 5I_3 = V_{ab}$ Adding these two equations: $15I_1 + 25I_2 - 10I_3 = 12$ The remaining equations are: $I_3 = 6$, and $I_2 - I_1 = 2I_x = 2(I_2 - I_3)$, or

 $I_1 + I_2 - 2I_3 = 0$



Note that if the 15 Ω resistor is denoted by *R* and the conventional mesh-current procedure is applied, the term in *R* cancels out. Thus, for mesh 1:

 $(10 + 5 + R)I_1 - RI_2 - 5I_3 = 12 - V_x$, where V_x is the voltage drop across dependent current source in the direction of I_1 . For mesh 2, $-RI_1 + (20 + 5 + R)I_2 - 5I_3 = V_x$. Adding these two equations gives the same equation as before.

If these equations are solved, $I_1 = 22.8 \text{ A}$, $I_2 = -10.8 \text{ A}$, $I_x = -16.8 \text{ A}$, $V_x = -804 \text{ V}$.

7. Determine Thevenin's 21_x equivalent circuit seen 6Ω 2Ω 4Ω between terminals a and b ⊖ a I_x 6 A 3Ω 10 V $\geq 4 \Omega$ 2Ω 1V Ob Solution: Method 1: Leave the circuit as it is. Considering the mesh on the RHS, $1 = 3I_x +$ $2I_x$ $4I_x + V_{cb}$, where $V_{cb} = 10 + 10$ 6Ω 2Ω 4Ω оa $2I_{x}$. Substituting for V_{cb} gives I_{x} 6 A $I_x = -1$ A, so that $V_{Th} = 4$ V. 3Ω Applying a test $\leq 4 \Omega$ $2 \Omega \ge V_{cb}$ 10 V source with the independent 1V sources set to zero, the branch containing the 6 A Оb source is open circuited. The 6 Ω and 4 Ω resistors are in parallel I_T 4Ω а with one terminal at node b and the other I_x terminal connected to an open circuit. They do not carry any current and can be 1 $2I_x$ 2Ω **>**3Ω V_{T} removed. The circuit reduces to that shown. $V_T = 4I_x + 2I_x = 6I_x$, and $I_T = I_x +$ $V_T/3$. Substituting for I_x gives $V_T/I_T = R_{Th} =$ b

2Ω.

If terminals ab are short circuited, KVL around the outermost loop gives: $10 + 2I_x + 4I_x = 0$, so that I_x = -5/3 A; $I_{sc} = -I_x + 1/3 = 2$ A. It follows that $R_{Th} = 4/2$ $= 2 \Omega$.



<u>Method 2</u>: If the branch consisting of the 1 V source in series with 3 Ω is removed, $I_x = 0$, the dependent source becomes a short circuit, and the open-circuit voltage between terminals a and b is the same as that of the 10 V source. Hence $V_{Th1} = 10$ V.

If a test current source I_T is applied between terminals a and b, with the independent sources set to zero, as before, and the 2 Ω resistor removed because it is in parallel with the $2I_x$ ideal voltage source and is redundant as far as V_{ab} is concerned, the circuit reduces to that shown. The $2I_T$ CCVS is equivalent to a 2 Ω resistor, which in series with the 4 Ω resistor gives $R_{Th1} = 6 \Omega$.

When the branch between terminals a and b is reintroduced, the circuit becomes as shown. With terminals a and b open circuited, the current in the circuit is 1. A in the direction of the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the circuit is 1. A in the direction of the current in the

and b open circuited, the current in the circuit is 1 A in the direction shown and $V_{ab} = 4$ V. If the voltage sources are set to zero, the resistance seen between terminals ab is (6||3) = 2 Ω . Hence, $V_{Th} = 4$ V and $R_{Th} = 2 \Omega$.





Find I_x .



Problem 4

Find V_x .



- A) 130.9V
- B) -43.64V
- →C) 43.64V
 - D) -130.9V
 - E) None of the above

Find the power associated with the current source.



- A) 256W
- B) -200W
- C) 200W
- D) -256W
- \rightarrow E) None of the above **176.9 W absorbed**

Problem 6

In the circuit below R_1 is chosen such that $I_3 = 1A$. Find R_1 .



- A) 12.5Ω
- B) 16Ω
- C) 25Ω
- \rightarrow D) 6.25 Ω
 - E) None of the above

Find I_x .



- → A) -6A
 - B) 6A
 - C) 16A
 - D) -16A
 - E) None of the above

Problem 8

Find I_0 .



- A) 1.15A
- →B) -0.65A
 - C) -1.15A
 - D) 0.65A
 - E) None of the above

Find the Thevenin equivalent resistance between a and b.



- →A) 333.33Ω
 - B) 250Ω
 - C) 83.33Ω
 - D) 740.46Ω
 - E) None of the above

Problem 10

Find the Thevenin equivalent voltage between a and b (V_{ab}) .



Find the Thevenin equivalent resistance between a and b of the previous figure.

- A) $6K\Omega$
- B) $8K\Omega$
- C) $4.5 \mathrm{K}\Omega$
- →D) 1.5KΩ
 - E) None of the above

Problem 12

Find the Thevenin equivalent resistance between a and b.



- A) 6Ω
- B) 8.52Ω
- C) 14.28 Ω
- \rightarrow D) 10.52 Ω
 - E) None of the above

Find V_1 .



- →A) -1.22V
 - B) 1.22V
 - C) -1.57V
 - D) 1.57V
 - E) None of the above

Find the Thevenin Equivalent Voltage between A and B (V_{AB}) .



Problem 6

Find the Norton equivalent resistance between A and B.



- A) 3R
- B) 3R/2
- \rightarrow C) R/3
 - D) R
 - E) None of the above

Find the Norton equivalent current source between A and B.



- A) -2.28I(A)
- **→**B) -1.24I(A)
 - C) -3.21I(A)
 - D) -6.42I(A)
 - E) None of the above