EECE 210 – Final Exam

January 20, 2012

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1. Determine *Vy*, assuming *VSRC* = 1 V.
2. 2 V
3. 4 V
4. 5 V
5. 1 V
6. 3 V

**Solution:** *Vx* = -2*Vx*×1, which makes *Vx* = 0. It follows that *Vy* = *VSRC*.

**Version 1:** *VSRC =* 1 V, *Vy* = *VSRC =* 1 V

**Version 2:** *VSRC =* 2 V, *Vy* = *VSRC =* 2 V

**Version 3:** *VSRC =* 3 V, *Vy* = *VSRC =* 3 V

**Version 4:** *VSRC =* 4 V, *Vy* = *VSRC =* 4 V

**Version 5:** *VSRC =* 5 V, *Vy* = *VSRC =* 5 V.

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1. Determine *Vac,* assuming all resistances are 10 Ω and *VSRC* = 2 V.
2. 3 V
3. 1 V
4. 5 V
5. 4 V
6. 2 V

**Solution**: From symmetry, nodes aa′ are at the same voltage, as are nodes bb′ and cc′. These nodes can therefore be joined together, which connects four resistors in parallel in the two sets of resistors in the middle, and results in the circuit shown. Adding the two 2.5 Ω resistors and splitting the resulting 5 Ω resistor into two 10 Ω resistors in parallel gives *Vac* = *VSRC*/2.

**Version 1:** *VSRC =* 2 V, *Vac* = *VSRC*/2 *=* 1 V

**Version 2:** *VSRC =* 4 V, *Vac* = *VSRC*/2 *=* 2 V

**Version 3:** *VSRC =* 6 V, *Vac* = *VSRC*/2 *=* 3 V

**Version 4:** *VSRC =* 8 V, *Vac* = *VSRC*/2 *=* 4 V

**Version 5:** *VSRC =* 10 V, *Vac* = *VSRC*/2 *=* 5 V.

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1. Given a load impedance 0.1(4 + *j*3) Ω. Determine the susceptance of the load.
2. 0.6 S
3. -0.6 S
4. -1.2 S
5. -0.3 S
6. 1.2 S

**Solution:** The load admittance is . Hence,  S.

**Version 1:** *k* = 0.1, S

**Version 2:** *k* = 0.2, S

**Version 3:** *k* = 0.3, S

**Version 4:** *k* = 0.4, S

**Version 5:** *k* = 0.5, S.

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**4.** If *vSRC* = 3cos*ωt* V, determine *ω* at which the voltage across the 10 Ω resistor is a maximum and specify this voltage.

1. 2 V, 2 krad/s
2. 4 V, 2 krad/s
3. 8 + *j*6 V, 1 krad/s
4. 8 – *j*6 V, 2 krad/s
5. 4 V, 4 krad/s

**Solution:** *vSRC* = *A*cos*ωt* V. The voltage across the 10 Ω resistor is maximum when the parallel impedance of *L* and *C* is infinite, that is, when *XL* = -*XC*, which makes ** → ∞. This occurs when **, or ** rad/s. The voltage is *VR* = 2*A*/3 V.

**Version 1:** *A* = 3, *VR* = 2*A*/3 = 2 V

**Version 2:** *A* = 6, *VR* = 2*A*/3 = 4 V

**Version 3:** *A* = 9, *VR* = 2*A*/3 = 6 V

**Version 4:** *A* = 12, *VR* = 2*A*/3 = 8 V

**Version 5:** *A* = 15, *VR* = 2*A*/3 = 10 V.

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**5.** Determine the total reactive power absorbed by *L* and *C* in Problem 4 at *ω* = 4 krad/s and with the 5 Ω resistor replaced by a short circuit.

1. -13 VAR
2. +6.75 VAR
3. -27 VAR
4. 13 VAR
5. -6.75 VAR

**Solution:** The total reactive power absorbed is ** ** VAR.

**Version 1:** *A* = 3, **-6.75 VAR

**Version 2:** *A* = 6, **-27 VAR

**Version 3:** *A* = 9, **-60.75 VAR

**Version 4:** *A* = 12, **-108 VAR

**Version 5:** *A* = 15, **-168.75 VAR.



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**6.** Determine *X* so that the current in the 5 Ω resistor is zero, given that *a* = 2, and for the linear transformer, *ωL*1 = *ωM* = 10 Ω and *ωL*2 = 20 Ω.

1. 2.22 Ω
2. -2.22 Ω
3. 2.5 Ω
4. -2.5 Ω
5. 1.88 Ω

**Solution:** To have zero current in the resistor, the open-circuited output of the linear transformer should be **V**. The input current is **V/***jωM*, and the input voltage is **. The primary voltage of the ideal transformer is **V/***a*, and the primary current is *a***V/***jωM*. It follows that **.

**Version 1:** *a* = 2, ** Ω

**Version 2:** *a* = 3, ** Ω

**Version 3:** *a* = 4, ** Ω

**Version 4:** *a* = 5, ** Ω

**Version 5:** *a* = 6, ** Ω.

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1. A load absorbs a real power of 12 kW and a capacitive, reactive power of 5 kVAR. Determine the power factor of the load.
2. 0.8
3. 0.92
4. 0.86
5. 0.83
6. 0.89

**Solution:** The complex power is **. The p.f. is ** leading.

**Version 1:** *Q* = 5 kVAR, p.f. = ** = 0.92 leading

**Version 2:** *Q* = 6 kVAR, p.f. = ** = 0.89 leading

**Version 3:** *Q* = 7 kVAR, p.f. = ** = 0.86 leading

**Version 4:** *Q* = 8 kVAR, p.f. = ** = 0.83 leading

**Version 5:** *Q* = 9 kVAR, p.f. = ** = 0.8 leading.

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1. The load of Problem 7 is supplied at 250 V rms, 50 Hz. Determine the inductance that must be placed in series with the load to make the total power factor of the load and the inductor equal to unity.
2. 7.22 mH
3. 7.65 mH
4. 6.63 mH
5. 7.96 mH
6. 5.89 mH

**Solution:** The complex power in the preceding problem is *SL* = 12,000 – *j*1,000*Q* VA. Considering the load voltage to be *VL*∠0° V rms, the load current is *IL* = ** A. ** A2. Hence, **, or *L* = **.

**Version 1:** *Q* = 5 kVAR, *L* = ** = 5.89 mH

**Version 2:** *Q* = 6 kVAR, *L* = ** = 6.63 mH

**Version 3:** *Q* = 7 kVAR, *L* = ** = 7.22 mH

**Version 4:** *Q* = 8 kVAR, *L* = ** = 7.65 mH

**Version 5:** *Q* = 9 kVAR, *L* = ** = 7.96 mH.

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**9.** If *Rsrc* = 2 Ω and *RL* and *XL* are variable over an arbitrary range, determine *RL* and *XL* that will maximize power transfer to *RL*.

1. 8 Ω, -*j*10 Ω
2. 10 Ω, *j*10 Ω
3. 12 Ω, -*j*10 Ω
4. 8 Ω, *j*10 Ω
5. 10 Ω, -*j*10 Ω

**Solution:** *XLm* = -*j*10 Ω so as to cancel the +*j*10 Ω, and *RLm* = 4*Rsrc* so as to match the source and load resistances.

**Version 1:** *Rsrc =* 2 Ω, *RLm* = 4*Rsrc* = 8 Ω

**Version 2:** *Rsrc =* 2.5 Ω, *RLm* = 4*Rsrc* = 10 Ω

**Version 3:** *Rsrc =* 3 Ω, *RLm* = 4*Rsrc* = 12 Ω

**Version 4:** *Rsrc =* 3.5 Ω, *RLm* = 4*Rsrc* = 14 Ω

**Version 5:** *Rsrc =* 4 Ω, *RLm* = 4*Rsrc* = 16 Ω.

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**10.** In the circuit of Problem 9, *Rsrc* = 20 Ω, *RL* = 3 Ω, and *XL* = -*j*6 Ω. Determine the ratio of primary to secondary turns that will maximize power transfer to *RL*.

1. 4
2. 5
3. 3
4. 2
5. 6

**Solution:** Let the primary to secondary turns ratio be *a*. The secondary load impedance is *ZL* = 3 + *j*10 – *j*6 = 3 + *j*4 Ω. When reflected to the primary side, this becomes (3 + *j*4)*a*2, of magnitude 5*a*2. For maximum power transfer, 5*a*2 = *Rsrc*, or .

**Version 1:** *Rsrc =* 20 Ω, 

**Version 2:** *Rsrc =* 45 Ω, 

**Version 3:** *Rsrc =* 80 Ω, 

**Version 4:** *Rsrc =* 125 Ω, 

**Version 5:** *Rsrc =* 180 Ω, .

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**11.** If the primary to secondary turns ratio in problem 10 is 4 and *vSRC* = 16sin(100*t*) V, determine the power dissipated in the 3 Ω resistor.

1. 0.70 W
2. 0.18 W
3. 0.11 W
4. 0.48 W
5. 0.30 W

**Solution:** The magnitude of the source voltage referred to the secondary is 16/4 = 4 V, and the source resistance is *Rsrc* /16 Ω. The magnitude of the load current is  A rms, and the power dissipated is  ** W.

**Version 1:** *Rsrc =* 20 Ω, **= 0.70 W

**Version 2:** *Rsrc =* 45 Ω, **= 0.48 W

**Version 3:** *Rsrc =* 80 Ω, **= 0.3 W

**Version 4:** *Rsrc =* 125 Ω, **= 0.18 W

**Version 5:** *Rsrc =* 180 Ω, **= 0.11 W.

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1. The exponential form of the Fourier series expansion of a periodic function is . Determine the second harmonic in trigonometric form.
2. 0.5cos(*nω0t* + 60°)
3. 2cos(*nω0t* + 60°)
4. cos(*nω0t* + 60°)
5. 2.5cos(*nω0t* + 60°)
6. 1.5cos(*nω0t* + 60°)

**Solution:** The ac component is . , hence  and . The trigonometric form of the *n*th harmonic is  = = . The second harmonic is .

**Version 1:** *A* = 1, 

**Version 2:** *A* = 2, 

**Version 3:** *A* = 3, 

**Version 4:** *A* = 4, 

**Version 5:** *A* = 5, .

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1. The function shown has *a*0 = 0 and for *n* = 1, 2, 3, etc.:
2. *an* = 0, *bn* = 0 for even *n* only and can be determined by integrating the function from

*t* = 0 to *t* = *T*/2

1. *an* = 0, *bn* = 0 for even *n* only and can be determined by integrating the function from

*t* = 0 to *t* = *T*/4

1. *an* = 0, *bn* = 0 for odd *n* only and can be determined by integrating the function from

*t* = 0 to *t* = *T*/2

1. *an* = 0, *bn* = 0 for odd *n* only and can be determined by integrating the function from

*t* = 0 to *t* = *T*/4

1. *an* ≠ 0, *bn* ≠ 0, for all *n*, and both can be determined by integrating the function from *t* = 0 to *t* = *T/2*

**Solution:** The waveform is half-wave symmetric, is not quarter-wave symmetric, and is neither odd nor even. Hence, *an* = 0, *bn* = 0 for even *n*, and can be determined by integrating the function from *t* = 0 to *t* = *T*/2.

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**14.** If *Frms* is the rms value of a half-wave symmetric, periodic waveform, determine the rms value of the same function but with the negative half-waves missing.

1. **
2. **
3. **
4. **
5. **

**Solution:** ** + ** = **. Hence, **

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**15.** Specify how the magnitude of the *n*th harmonic varies with *n* for the periodic function shown.

1. *n*
2. *n*2
3. 1/*n*
4. 1/*n*2
5. 1/*n*3

**Solution:** Since the function is discontinuous, the magnitude of the *n*th harmonic varies as 1/*n*.

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**16.** Determine Thevenin’s equivalent circuit between terminals ab, assuming **VSRC** = 5∠0° V. Express **VTh** in polar coordinates and *ZTh* in rectangular coordinates.

**Solution:** The current in the *LC* branch is **VX**/*j*5 = -*j*0.2**Vx**, and the current in the middle branch is 0.2(1 – *j*)**Vx**. From KVL, **VSRC** – 5×0.2(1 – *j*)**Vx** = **VX** + (-*j*2.5)(-*j*0.2)**Vx** = 0.5**Vx**, or **Vx** = **VSRC**/(1.5 – *j*), **VTh** = 0.5**VX** =** ** V. When terminals ab are short circuited, the current source is zero, and **ISC** =** A. It follows that *ZTh* = **(1.15 + *j*0.77) Ω.

**Version 1:** **VSRC** = 5∠0° V, ** V

**Version 2:** **VSRC** = 10∠0° V, ** V

**Version 3:** **VSRC** = 15∠0° V, ** V

**Version 4:** **VSRC** = 20∠0° V, ** V

**Version 5:** **VSRC** = 25∠0° V, ** V.

**11%**

**17.** Determine *vO*(*t*), assuming *vSRC* = 2cos1000t V.

**Solution:** The impedances of the reactive elements are: mH → *j*8 Ω, 1 mH → *j* Ω, 2 mH → *j*2 Ω, and 0.5 mF → -*j*2 Ω. The circuit in the frequency domain becomes as shown, with the linear transformer replaced by its T-equivalent circuit. The parallel impedance to the right of terminals ab is ** ** Ω, and **Vab** = **= **= -*j*0.6**VSRC**. It follows that **= -0.3(1 + *j*)**VSRC** =** V, and *vO*(*t*) = ** V.

**Version 1:** *Vm* = 2, *vO*(*t*) = ** = -0.85cos(1000*t* + 45°) V

**Version 2:** *Vm* = 3, *vO*(*t*) = ** = -1.27cos(1000*t* + 45°) V

**Version 3:** *Vm* = 4, *vO*(*t*) = ** = -1.70cos(1000*t* + 45°) V

**Version 4:** *Vm* = 5, *vO*(*t*) = ** = -2.12cos(1000*t* + 45°) V

**Version 5:** *Vm* = 6, *vO*(*t*) = ** = -2.55cos(1000*t* + 45°) V.

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**18.** *RL* is variable in the range (1, 8 Ω). Determine *RL* for maximum power transfer and the maximum power transferred to it, assuming *B* = 4.

**Solution:** Thevenin’s voltage for the *B*sin2*ω*0*t* source, as seen by *RL*, is *B*sin2*ω*0*t* V. From superposition, Thevenin’s voltage for the cos*ω*0*t* sources is (2 + 10×0.2)cos*ω*0*t* = 4cos*ω*0*t* V. *RTh* = 10 Ω. Hence, for maximum power transfer *RL* = 8 Ω. The power dissipated in *RL* is *PL* = ** W.

**Version 1:** *B =* 4, *PL* = (*B*2 + 16)/81 = 32/81 = 0.396 W

**Version 2:** *B =* 6, *PL* = (*B*2 + 16)/81 = 52/81 = 0.642 W

**Version 3:** *B =* 8 *PL* = (*B*2 + 16)/81 = 80/81 = 0.988 W

**Version 4:** *B =* 10, *PL* = (*B*2 + 16)/81 = 116/81 = 1.432 W

**Version 5:** *B =* 12, *PL* = (*B*2 + 16)/81 = 160/81 = 1.975 W.

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**19.** Derive the first four terms of the trigonometric Fourier series expansion of the periodic function *f*(*t*) = *A*sin*t*, -*π*/2 ≤ *t* ≤ *π*/2, with *A* = 1. Note that:









**Solution:** The function is odd and of zero average. Hence *a*0 = 0, *an* =0 for all integer *n*, and **, where *T* = *π* and *ω*0 = 2*π*/*T* = 2. Using the trigonometric relation for the product of two sine functions, **=**=**. Hence,

**

**Version 1:** *A =* 1, **

**Version 2:** *A =* 2, **

**Version 3:** *A =* 3, **

**Version 4:** *A =* 4, **

**Version 5:** *A =* 5, **.

**11%**

**20.** Determine the rms value of the periodic function shown, assuming *A* = 1.

**Solution:** The function has an average value of 0.25 A, because the areas under the positive and negative triangles cancel out, and the areas under the rectangles are 0.5 *A* – 0.25*A*. Alternatively, it can be argued that if 0.25A is subtracted from *f*(*t*), the function becomes symmetrical with respect to the horizontal axis The ac component of *f*(*t*) = *A*(2*t* + 0.75), 0 ≤ *t* ≤ 0.25 s. The area under the square of the function over this time interval is = ==  The mean square is obtained by dividing by 0.25, which gives . The square of the rms value of *f*(*t*) is = and the rms value is .

**Version 1:** *A* = 1, rms value is = 1.041

**Version 2:** *A* = 2, rms value is = 2.082

**Version 3:** *A* = 3, rms value is = 3.122

**Version 4:** *A* = 4, rms value is = 4.163

**Version 5:** *A* = 5, rms value is = 5.204.