Quiz 3 – December 18, 2010

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1. A current $i = I_m\cos(\omega t + 30°)$ flows through an impedance $(5 – j5)$ Ω. Determine the rms phasor voltage across the impedance if I_m = 2.5 A.

Solution: The phasor current is $I_m\angle 30°$ A. The impedance is $5\sqrt{2}\angle -45°$ Ω. The phasor voltage is $5I_m\sqrt{2}\angle -15°$ V peak value or $5I_m\angle -15°$ V rms.

Version 1: I_m = 2.5 A, **V** = $12.5\angle -15°$ V rms

Version 2: I_m = 5 A, **V** = $25\angle -15°$ V rms

Version 3: I_m = 7.5 A, **V** = $37.5\angle -15°$ V rms

Version 4: I_m = 10 A, **V** = $50\angle -15°$ V rms

Version 4: I_m = 12.5 A, **V** = $62.5\angle -15°$ V rms.

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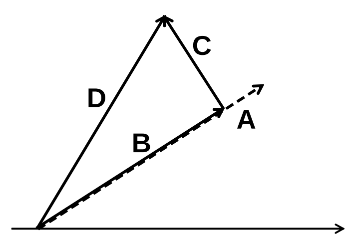
2. In the phasor diagram shown, phasor **A** is a current, the other phasors **B**, **C**, and **D** are voltages. To which of the following combinations of circuit elements does this phasor diagram apply?



   A. *R* in series with *L*
   B. *R* in series with *C*
   C. *R* in parallel with *C*
   D. *L* in series with *C*
   E. *L* in parallel with *C.*

Solution: The phasor diagram represents a series *RL* circuit, Phasor **B** is the voltage across *R*. Phasor **C** is the voltage across *L*, leading the current by 90°, and phasor **D** is the voltage across the series combination.

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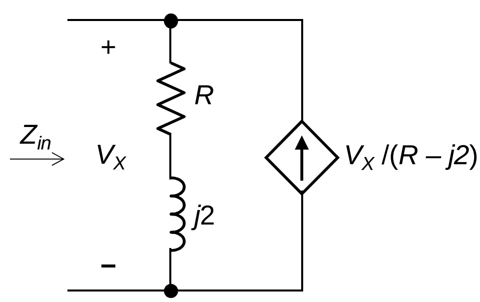
3. Determine the input impedance Z_{in} assuming R = 2 Ω.



Solution: If a test voltage source $\mathbf{V_T}$ is applied, $\mathbf{I_T}$ = $\left(\frac{1}{R + j2} - \frac{1}{R - j2}\right)\mathbf{V_T}$, or Z_{in} =

$$\frac{1}{\left(\frac{1}{R+j2}-\frac{1}{R-j2}\right)}=\frac{(R+j2)(R-j2)}{R-j2-R-j2}=\frac{R^2+4}{-j4}=j\left(1+\frac{R^2}{4}\right)$$ Ω. Alternatively, the dependent source is equivalent to an impedance -(R – j2) and Z_{in} = $(R+j2)\,||\,[-(R-j2)]$ Ω.

Version 1: R = 2 Ω; Z_{in} = j2 Ω

Version 2: R = 3 Ω; Z_{in} = j3.25 Ω

Version 3: R = 4 Ω; Z_{in} = j5 Ω

Version 4: R = 5 Ω; Z_{in} = j7.25 Ω

Version 5: R = 6 Ω; Z_{in} = j10 Ω.

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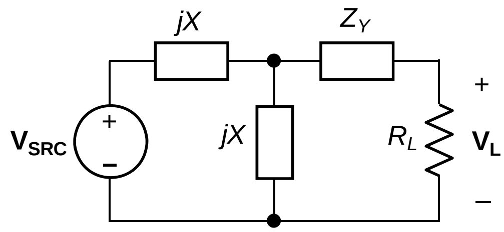
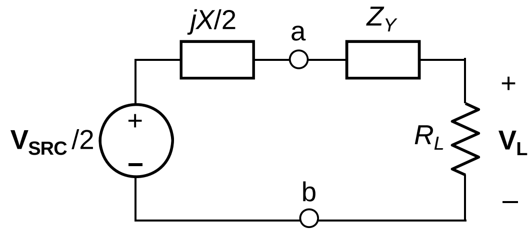
4. Determine Z_Y so that $\mathbf{V_L}$ is in phase with $\mathbf{V_{SRC}}$, assuming X = -5 Ω with R_L and $\mathbf{V_{SRC}}$ unknown.





Solution: TEC as seen from terminals a and b will have Z_{Th} = jX/2. For $\mathbf{V_L}$ to be in phase with $\mathbf{V_{SRC}}$, Z_Y = -jX/2.

Version 1: X = -5 Ω; Z_Y = j2.5 Ω

Version 2: X = 5 Ω; Z_Y = -j2.5 Ω

Version 3: X = -10 Ω; Z_Y = j5 Ω

Version 4: X = 10 Ω; Z_Y = -j5 Ω

Version 5: X = -15 Ω; Z_Y = j7.5 Ω.

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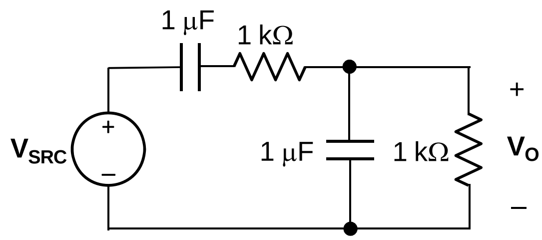
5. Determine $\mathbf{V_O}$ if ω = 1 krad/s and $\mathbf{V_{SRC}}$ = 3 V.



Solution: ωCR = 1;

$$\frac{\mathbf{V_O}}{\mathbf{V_{SRC}}}=\frac{R/(1+j\omega CR)}{R/(1+j\omega CR)+R-j/\omega C}=$$

$$\frac{R/(1+j)}{R/(1+j)+R-j/\omega C}=\frac{1/(1+j)}{1/(1+j)+1-j}=\frac{1}{1+(1+j)(1-j)}=\frac{1}{1+1-j^2}=\frac{1}{3}$$

Version 1: $\mathbf{V_{SRC}}$ = 3 V; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}$/3 = 1 V

Version 2: $\mathbf{V_{SRC}}$ = 4.5 V; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}$/3 = 1.5 V

Version 3: $\mathbf{V_{SRC}}$ = 6 V; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}$/3 = 2 V

Version 4: $\mathbf{V_{SRC}}$ = 7.5 V; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}$/3 = 2.5 V

Version 5: $\mathbf{V_{SRC}}$ = 9 V; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}$/3 = 3 V.

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6. Two coils are wound on a core of high permeability. Coil 1 has 100 turns and coil 2 has 400 turns. A current of 1 A in coil 1, with coil 2 open circuited, produces a core flux of 0.5 mWb. Determine the magnitude of the core flux produced by a current of 0.8 A in coil 2, with coil 1 open circuited.

Solution: From the definition of mutual inductance, $\frac{N_2\phi_{21}}{i_1} = \frac{N_1\phi_{12}}{i_2}$, so that $\phi_{12} = \frac{i_2}{i_1}\frac{N_2}{N_1}\phi_{21} =$

$\frac{0.8}{1}\frac{400}{100}\phi_{21} = 3.2\phi_{21}$.

Version 1: ϕ_{21} = 0.5 mWb; ϕ_{12} = 3.2×0.5 = 1.6 mWb

Version 2: ϕ_{21} = 0.6 mWb; ϕ_{12} = 3.2×0.6 = 1.92 mWb

Version 3: ϕ_{21} = 0.7 mWb; ϕ_{12} = 3.2×0.7 = 2.24 mWb

Version 4: ϕ_{21} = 0.8 mWb; ϕ_{12} = 3.2×0.8 = 2.56 mWb

Version 5: ϕ_{21} = 0.9 mWb; ϕ_{12} = 3.2×0.9 = 2.88 mWb.

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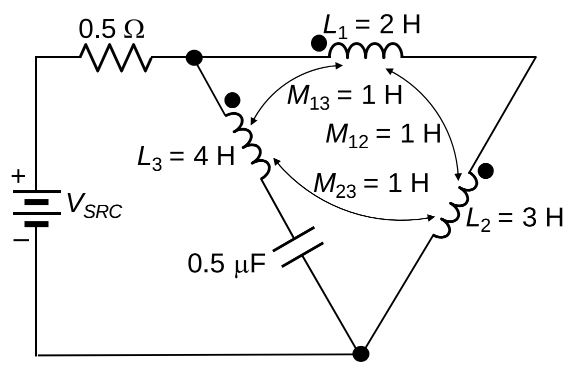
7. Determine the stored **magnetic energy** under dc conditions, assuming V_{SRC} = 1 V.



Solution: The branch containing the capacitor carries no current under dc conditions. The current in the other two branches is V_{SRC} /0.5 A and the stored magnetic energy is W =

$\frac{1}{2}\left(L_1 I^2 + L_1 I^2 + 2MI^2\right) =$

$\frac{1}{2}\left(\frac{V_{SRC}}{0.5}\right)^2\left(2 + 3 + 2\times 1\right) = 14V_{SRC}^2$.

Version 1: V_{SRC} = 1 V; W = 6×1 = 14 J

Version 2: V_{SRC} = 1.5 V; W = 6×2.25 = 31.5 J

Version 3: V_{SRC} = 2 V; W = 6×4 = 56 J

Version 4: V_{SRC} = 2.5 V; W = 6×6.25 = 87.5 J

Version 5: V_{SRC} = 3 V; W = 6×9 = 126 J.

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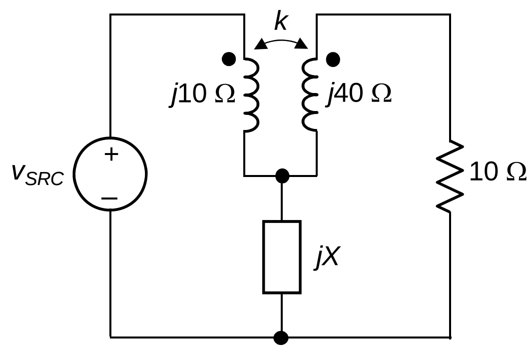
8. Given $v_{SRC} = 6\cos\omega t$, where ω is unknown and $k = 0.97$. Determine X so that no power is dissipated in the circuit.

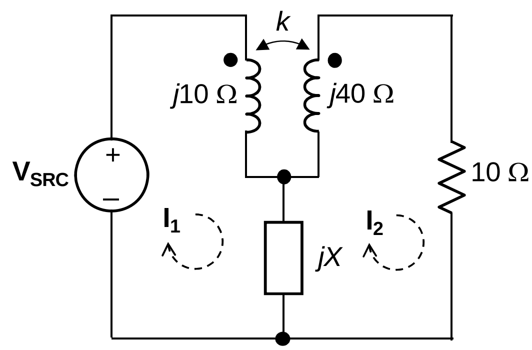


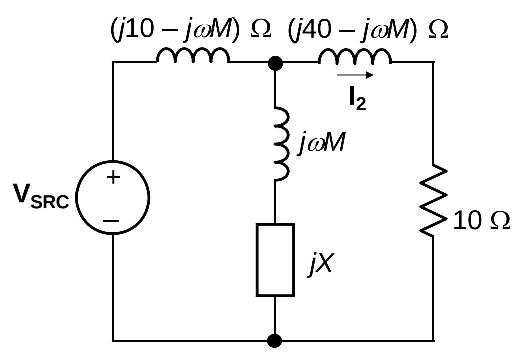
Solution: Considering mesh currents $\mathbf{I_1}$ and $\mathbf{I_2}$, then for no power dissipation, $\mathbf{I_2} = 0$. The mesh-current equation for mesh 2 is:

$$-(j\omega M + jX)\mathbf{I_1} + (j40 + jX + 10)\mathbf{I_2} = 0$$

For $\mathbf{I_2}$ to be zero, $\omega M + X = 0$, or $X = -\omega M =$
$-\omega k\sqrt{L_1L_2} = -k\sqrt{(\omega L_1)(\omega L_2)} = -k\sqrt{400} = -20k$.

Alternatively, it follows from the T-equivalent circuit that if $X = -\omega M$ the shunt branch will have zero impedance so $\mathbf{I_2} = 0$.





Version 1: $k = 0.97$; $X = -0.97\times20 = -19.4$ Ω

Version 2: $k = 0.96$; $X = -0.96\times20 = -19.2$ Ω

Version 3: $k = 0.95$; $X = -0.95\times20 = -19$ Ω

Version 4: $k = 0.94$; $X = -0.94\times20 = -18.8$ Ω

Version 5: $k = 0.93$; $X = -0.93\times20 = -18.6$ Ω.

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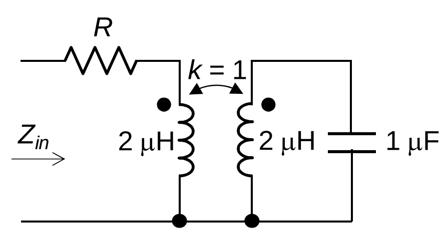
9. Determine $\boldsymbol{Z_{in}}$, assuming $R = 10$ Ω, and $\omega =$ 1 Mrad/s.



Solution: $M = k\sqrt{L\times L} = 2$ μH. It follows that the series branches of the T-equivalent circuit are zero and the shunt branch is $j\omega M = j2$ Ω; $-\dfrac{j}{\omega C} = -j$. The parallel impedance of $j2$ and $-j$ Ω is $\dfrac{2}{j2 - j} = -j2$ Ω. $Z_{in} = R - j2$ Ω.

Version 1: $R = 10$ Ω; $Z_{in} = 10 - j2$ Ω

Version 2: $R = 12$ Ω; $Z_{in} = 12 - j2$ Ω

Version 3: $R = 15$ Ω; $Z_{in} = 15 - j2$ Ω

Version 4: $R = 20$ Ω; $Z_{in} = 20 - j2$ Ω

Version 5: $R = 22$ Ω; $Z_{in} = 22 - j2$ Ω.

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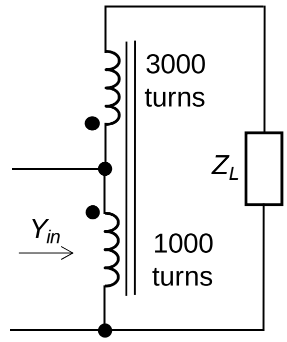
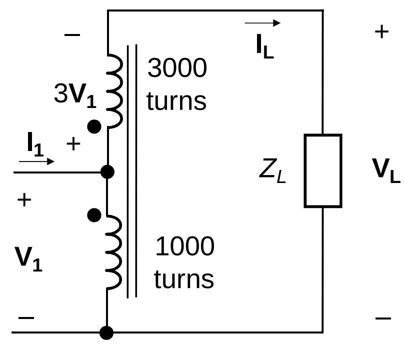
10. Determine the input admittance Y_{in} assuming $Z_L = 10\angle 45^\circ\ \Omega$.



Solution: It follows from the circuit shown that $\mathbf{V_L} = -2\mathbf{V_1}$, so that $\frac{\mathbf{V_L}}{\mathbf{V_1}} = -2$. Since the ideal autotransformer does not dissipate power, and $\mathbf{V_L} = -2\mathbf{V_1}$, $\mathbf{V_L} \times \mathbf{I_L} = \mathbf{V_1} \times \mathbf{I_1}$, $\frac{\mathbf{I_L}}{\mathbf{I_1}} = -\frac{1}{2}$. It follows that

$$Y_{in} = \frac{\mathbf{I_1}}{\mathbf{V_1}} = \frac{4}{Z_L}\text{ S.}$$



Version 1: $Z_L = 10\angle 45^\circ\ \Omega$; $Y_{in} \equiv \frac{4}{10\angle 45^\circ} = 0.4\angle -45^\circ$ S

Version 2: $Z_L = 10\angle -45^\circ\ \Omega$; $Y_{in} \equiv \frac{4}{10\angle -45^\circ} = 0.4\angle 45^\circ$ S

Version 3: $Z_L = 20\angle 45^\circ\ \Omega$; $Y_{in} \equiv \frac{4}{20\angle 45^\circ} = 0.2\angle -45^\circ$ S

Version 4: $Z_L = 20\angle -45^\circ\ \Omega$; $Y_{in} \equiv \frac{4}{20\angle -45^\circ} = 0.2\angle 45^\circ$ S

Version 5: $Z_L = 40\angle 45^\circ\ \Omega$; $Y_{in} \equiv \frac{4}{40\angle 45^\circ} = 0.1\angle -45^\circ$ S.

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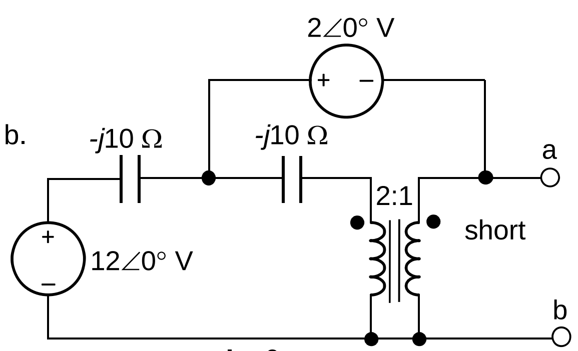
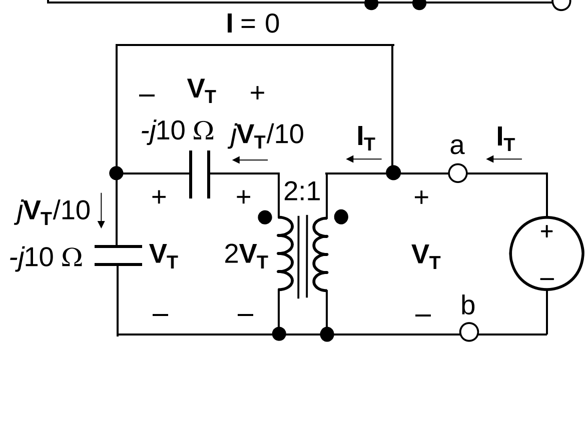
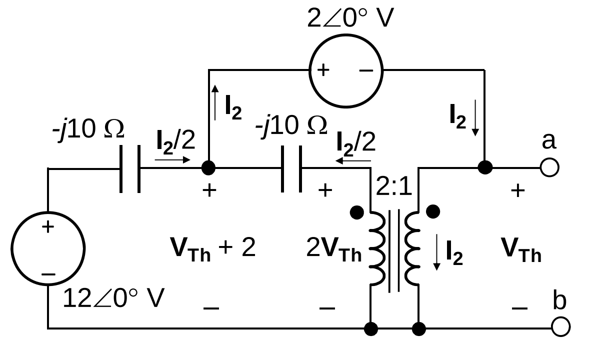
11. Determine Z_{Th} looking into terminals a and b.



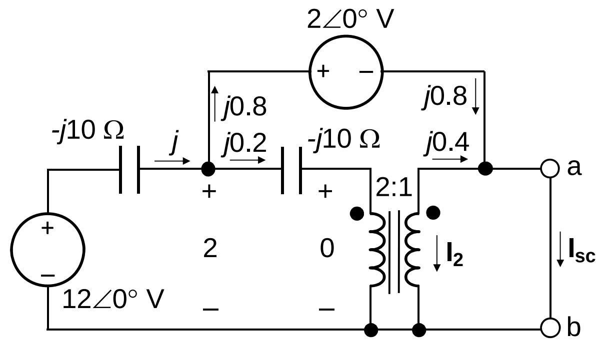
Solution-*Method 1*: Apply a test voltage $\mathbf{V_T}$, with the independent sources replaced by circuits. The secondary voltage is $\mathbf{V_T}$ and the primary voltage is $2\mathbf{V_T}$. The current through each capacitor is $j\mathbf{V_T}/10$ as shown, so that the current in the short-circuit replacing the 2 V source is zero. For the ideal transformer, $\mathbf{I_T} = j2\mathbf{V_T}/10$. It follows that $\frac{\mathbf{V_T}}{\mathbf{I_T}} = \frac{5}{j} = -j5\ \Omega$.



Method 2: Under open circuit conditions, the currents and voltages will be as shown. The voltages across the two capacitors are of equal magnitude but opposite polarity. It follows that $2\mathbf{V_{Th}} = 12$ V, and $\mathbf{V_{Th}} = 6$ V.



Under open circuit conditions, the currents and voltages will be as shown. It follows that $I_{SC} = j1.2$ A. Hence, $Z_{Th} = 6/j1.2 = -j5\ \Omega$.



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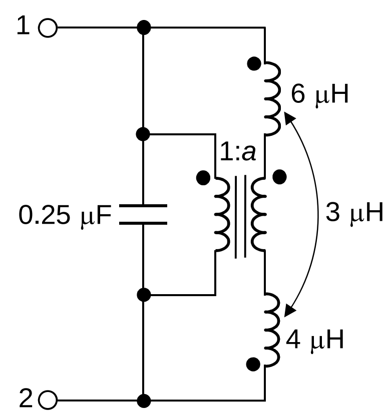
12. Determine the turns ratio *a* so that Norton's admittance looking into terminals 1 and 2 is zero, assuming $\omega = 1$ Mrad/s.



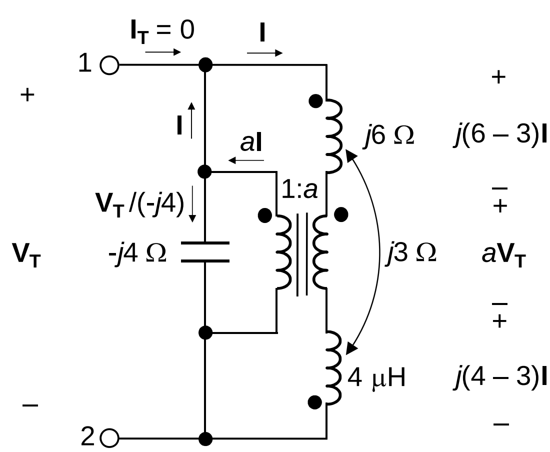
Solution-*Method 1:* The impedances are: $j\omega 6 = j6\ \Omega$, $j\omega 4 = j4\ \Omega$, $j\omega 3 = j3\ \Omega$, and $1/j\omega C = 1/(j10^6 \times 0.25 \times 10^{-6}) = -j4\ \Omega$.

When a test source $\mathbf{V_T}$ is applied, the test current $\mathbf{I_T}$ should be zero. The voltages and currents in the circuit will be as shown.

From KVL in the branch containing the coupled coils,

$\mathbf{V_T} = j(6 - 3)\mathbf{I} + a\mathbf{V_T} + j(4 - 3)\mathbf{I} = a\mathbf{V_T} + j4\mathbf{I}$ V,

or $\quad (1 - a)\mathbf{V_T} = j4\mathbf{I} \qquad (1)$



From KCL at node 1, $a\mathbf{I} = \mathbf{I} + \dfrac{\mathbf{V_T}}{-j4}$, or

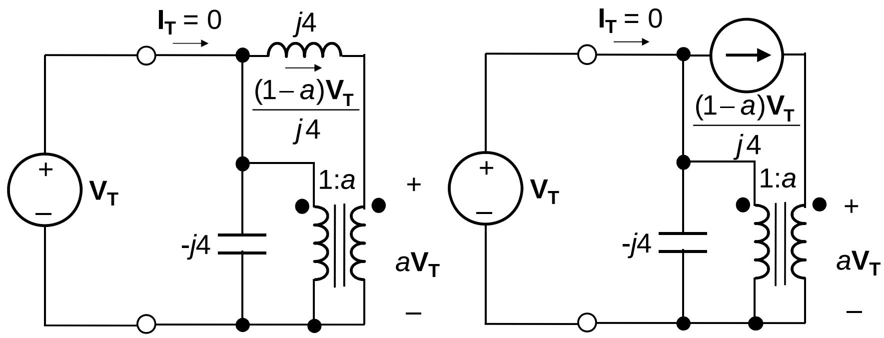
$$\frac{\mathbf{V_T}}{j4} = (1 - a)\mathbf{I} \qquad (2)$$

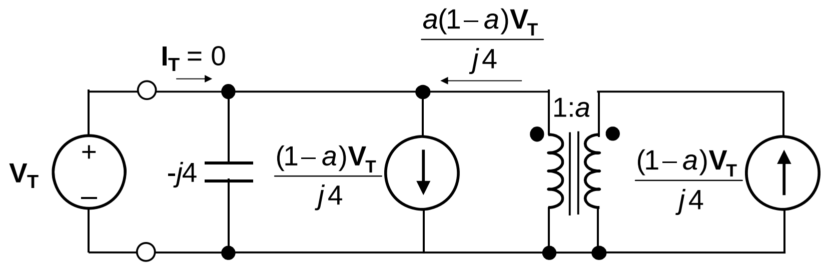
Dividing Equation 1 by Equation 2:

$j4(1-a) = \dfrac{j4}{(1-a)}$, $(a - 1)^2 = 1$; $a = 1 \pm 1$,

which gives $a = 2$.

Method 2: The circuit can be redrawn as shown, where the two coupled coils in series have been replaced by the equivalent inductance ($j6 + j4 - j6$) = $j4$ Ω. This inductance carries a current $\frac{(1-a)}{j4}\mathbf{V_T}$ and can be replaced by a current source of this value in accordance with the substitution theorem. This source can then be rearranged as two sources, as shown. From KCL,





$$\frac{a(1-a)}{j4}\mathbf{V_T} = \frac{(1-a)}{j4}\mathbf{V_T} + \frac{\mathbf{V_T}}{-j4}$$, $a - a^2 = 1 - a - 1$, which gives $a = 0$ or 2, so a must equal 2.

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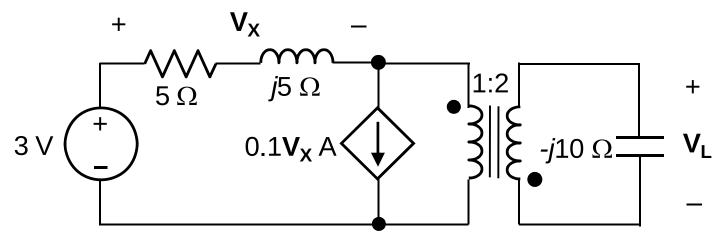
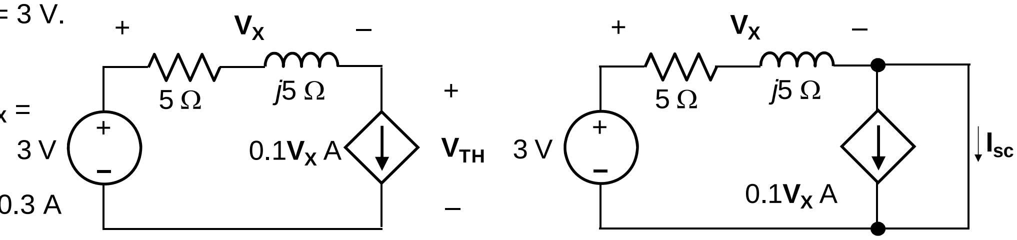
13. Determine $\mathbf{V_L}$.

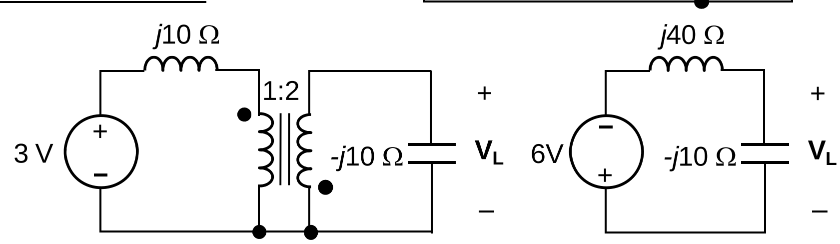


Solution-*Method 1*: Derive TEC at the primary terminals. $\mathbf{V_x} = 0.1\mathbf{V_X}(5 + j5)$, which gives $\mathbf{V_x} = 0$ and $\mathbf{V_{TH}} = 3$ V. On short circuit, $\mathbf{V_X} = 3$ V and $0.1\,\mathbf{V_X} = 0.3$ A From KCL.



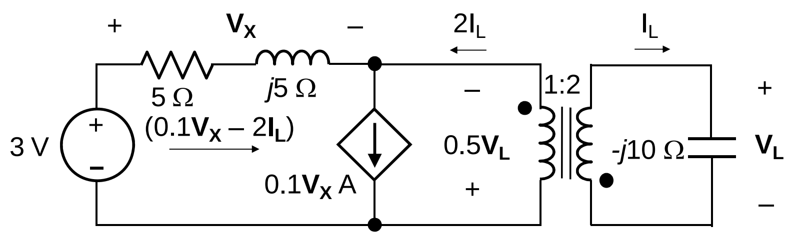
$$\frac{3}{5 + j5} = 0.3 + \mathbf{I_{SRC}}$$

which gives, $\mathbf{I_{SRC}} = -j0.3$ A and $Z_{Th} = \frac{3}{-j0.3} = j10\,\Omega$. TEC becomes as shown. Reflecting this to the secondary, it follows from voltage division that



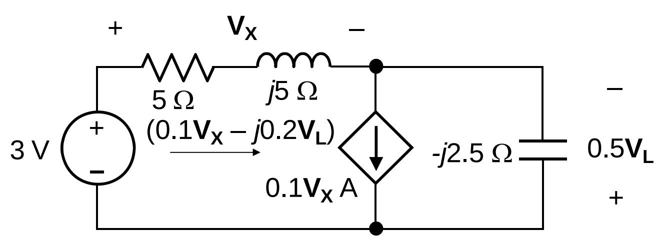
$$\mathbf{V_L} = -\frac{-j10}{j30} \times 6 = 2 \text{ V}.$$

Method 2: No reflections. The primary voltage and current will be as shown in the figure. From KCL, the current in the (5 +*j*5) Ω impedance is $(0.1\mathbf{V_X} - 2\mathbf{I_L})$,



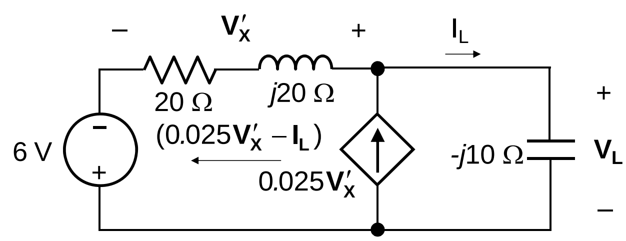
so that $\mathbf{V_X} = (0.1\mathbf{V_X} - 2\mathbf{I_L})(5 + j5)$ and $\mathbf{I_L} = \mathbf{V_L}/(-j10) = j0.1\mathbf{V_L}$. Substituting for $\mathbf{I_L}$, $\mathbf{V_X} = (0.1\mathbf{V_X} - j0.2\mathbf{V_L})(5 + j5) = 0.5\mathbf{V_X} + j0.5\mathbf{V_X} - j(1 + j)\mathbf{V_L}$, or, $0.5(1 - j)\mathbf{V_X} = (1 - j)\mathbf{V_L}$, which gives, $\mathbf{V_X} = 2\mathbf{V_L}$. From KVL on the primary side, $0.5\mathbf{V_L} + 3 - \mathbf{V_X} = 0$; substituting for $\mathbf{V_X}$, $1.5\mathbf{V_L} = 3$ and $\mathbf{V_L} = 2$ V.

Method 3: The capacitive branch is reflected to the primary side together with $\mathbf{V_L}$. From KCL, the current through the (5 +*j*5) Ω impedance is $0.1\mathbf{V_X} - 0.5\mathbf{V_L}/\text{-}j2.5 = 0.1\mathbf{V_X} - j0.2\mathbf{V_L}$,



so that $\mathbf{V_X} = (0.1\mathbf{V_X} - j0.2\mathbf{V_L})(5 + j5) = (0.5\mathbf{V_X} - j\mathbf{V_L})(1 + j) = 0.5\mathbf{V_X} + j0.5\mathbf{V_X} - j\mathbf{V_L}(1 + j)$, or $0.5(1 - j)\mathbf{V_X} = (1 - j)\mathbf{V_L}$, which gives $\mathbf{V_X} = 2\mathbf{V_L}$. From KVL on the primary side, $0.5\mathbf{V_L} + 3 - \mathbf{V_X} = 0$; substituting for $\mathbf{V_X}$, $1.5\mathbf{V_L} = 3$ and $\mathbf{V_L} = 2$ V.

Method 4: The primary circuit is reflected to the secondary side. The (5 + *j*5) Ω is multiplied by 4. The source voltage and $\mathbf{V'_X}$ are multiplied by 2 and reversed in



sign. To maintain the same dependency relation, with the dependent source still on the primary side, *k* is divided by 2 and reversed in sign. When reflected to the secondary side, this current source must be divided by 2. The overall effect is to divide *k* by 4 and reverse the sign of the current source as shown. It follows that $\mathbf{V'_X} = (0.025\mathbf{V'_X} - \mathbf{I_L})(20 + j20) = (0.5\mathbf{V'_X} - j2\mathbf{V_L})(1 + j) = 0.5\mathbf{V'_X} + j0.5\mathbf{V'_X} + 2\mathbf{V_L}(1 - j)$, or, $0.5\mathbf{V'_X}(1 - j) = 2\mathbf{V_L}(1 - j)$, which gives $\mathbf{V'_X} = 4\mathbf{V_L}$. From KVL, $\mathbf{V_L} - \mathbf{V'_X} + 6 = 0$. Substituting for $\mathbf{V'_X}$, $\mathbf{V_L} = 2$ V.