## EECE 210

## Quiz 2 - Nov 20, 2010

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1. The current components in a resistor $R$, as found from superposition, are -8 A and +4 A . Determine the power dissipated in the resistor if $R=2 \Omega$.

Solution: The magnitude of the current through $R$ is 4 A . The power dissipated in $R$ is $16 R$.
Version 1: $R=2 \Omega, P=16 R=32 \mathrm{~W}$
Version 2: $R=3 \Omega, P=16 R=48 \mathrm{~W}$
Version 3: $R=4 \Omega, P=16 R=64 \mathrm{~W}$
Version 4: $R=5 \Omega, P=16 R=80 \mathrm{~W}$
Version 5: $R=6 \Omega, P=16 R=96 \mathrm{~W}$.

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2. Determine $I_{O}$ if $V_{S}=1 \mathrm{~V}$. Solution: The two terminals of the $5 \Omega$ resistor are at the same voltage. Hence, this resistor carries no current and can be removed. The $1 \Omega$ in series with $V_{S}$ can be split into two parallel branches, each of $2 \Omega$ in series with $V_{s}$. The
 circuit can then be split into two halves as shown. It follows that $I_{O}=\frac{5-V_{S}}{4} \mathrm{~A}$.

Version 1: $V_{S}=1 \mathrm{~V}, I_{O}=\frac{5-1}{4}=1 \mathrm{~A}$
Version 2: $V_{S}=2 \mathrm{~V}, I_{O}=\frac{5-2}{4}=0.75 \mathrm{~A}$
Version 3: $V_{S}=3 \mathrm{~V}, I_{O}=\frac{5-3}{4}=0.5 \mathrm{~A}$
Version 4: $V_{S}=4 \mathrm{~V}, I_{O}=\frac{5-4}{4}=0.25 \mathrm{~A}$


Version 5: $V_{S}=5 \mathrm{~V}, I_{O}=\frac{5-5}{4}=0 \mathrm{~A}$.
3. Determine $R$ so Norton's current between terminals a and b is zero, given that $I_{S R C}=5 \mathrm{~A}$ and $\alpha=1 \mathrm{~V} / \mathrm{A}$.

Solution: If a and b are short-circuited and no current flows in the short circuit, $I_{\text {SRC }}$ flows through $R$. Applying $\mathrm{KVL}, R I_{S R C}=\alpha l_{\text {SRC }}$, which gives $R=\alpha \Omega$.

Version 1: $\alpha=1, R=1 \Omega$
Version 2: $\alpha=2, R=2 \Omega$
Version 3: $\alpha=3, R=3 \Omega$


Version 4: $\alpha=4, R=4 \Omega$
Version 5: $\alpha=5, R=5 \Omega$.

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4. Determine the power delivered by the source, assuming all resistances are $1 \Omega$ and $V_{S R C}=1 \mathrm{~V}$.
Solution: From symmetry, each top node along the same vertical line is at the same voltage as the corresponding bottom node. Joining these nodes together simplifies the circuit to that shown. The resistance on each side of the source is $0.5 \| 2 \Omega=$
 $0.4 \Omega$. Hence, $I_{S R C}=V_{S R C} / 0.8 \mathrm{~A}$ and the power delivered by the source is $\frac{V_{S R C}^{2}}{0.8} \mathrm{~W}$.

Version 1: $V_{S R C}=1 \mathrm{~V}, P=\frac{1}{0.8}=1.25 \mathrm{~W}$


Version 2: $V_{S R C}=1.2 \mathrm{~V}, P=\frac{(1.2)^{2}}{0.8}=1.8 \mathrm{~W}$
Version 3: $V_{S R C}=1.4 \mathrm{~V}, P=\frac{(1.4)^{2}}{0.8}=2.45 \mathrm{~W}$
Version 4: $V_{S R C}=1.6 \mathrm{~V}, P=\frac{(1.6)^{2}}{0.8}=3.2 \mathrm{~W}$
Version 5: $V_{S R C}=1.8 \mathrm{~V}, P=\frac{(1.8)^{2}}{0.8}=4.05 \mathrm{~W}$
5. Determine $V_{x}$ assuming $V_{S}=1 \mathrm{~V}$.

Solution: With the 4 V source applied alone, and $V_{s}$ set to zero, $V_{x 1}=\frac{2}{2+4 \| 4} \times 4=2 \mathrm{~V}$. With $V_{S}$ applied and the 4 V source set to zero,
$V_{x 2}=-\frac{2 \| 4}{4+2 \| 4} \times V_{s}=-\frac{4 / 3}{4+4 / 3} \times V_{s}=-\frac{V_{S}}{4} ; V_{x}$

$=2-\frac{V_{S}}{4} \mathrm{~V}$.
Version 1: $V_{S}=1 \mathrm{~V}, V_{x}=2-\frac{1}{4}=1.75 \mathrm{~V}$
Version 2: $V_{S}=2 \mathrm{~V}, V_{x}=2-\frac{2}{4}=1.5 \mathrm{~V}$
Version 3: $V_{s}=3 \mathrm{~V}, V_{x}=2-\frac{3}{4}=1.25 \mathrm{~V}$
Version 4: $V_{S}=4 \mathrm{~V}, V_{x}=2-\frac{4}{4}=1 \mathrm{~V}$
Version 5: $V_{S}=5 \mathrm{~V}, V_{x}=2-\frac{5}{4}=0.75 \mathrm{~V}$.

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6. Determine Norton's current at terminals a and b , assuming $I_{S R C}=0.5 \mathrm{~A}$.
Solution: When terminals ab are short circuited, $V_{X}=0$ and the dependent voltage
 source $0.5 V_{X}$ is zero, so that $I_{N}=I_{S R C}$.

Version 1: $I_{S R C}=0.5 \mathrm{~A}, I_{N}=0.5 \mathrm{~A}$
Version 2: $I_{S R C}=1 \mathrm{~A}, I_{N}=1 \mathrm{~A}$
Version 3: $I_{S R C}=1.5 \mathrm{~A}, I_{N}=1.5 \mathrm{~A}$
Version 4: $I_{S R C}=2 \mathrm{~A}, I_{N}=2 \mathrm{~A}$
Version 5: $I_{S R C}=2.5 \mathrm{~A}, I_{N}=2.5 \mathrm{~A}$.

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7. Determine Thevenin's voltage between terminals a and b , with $R_{L}$ removed, assuming $V_{S R C}=5 \mathrm{~V}$.


Solution: When terminals ab are open circuited, $I_{x}=0$. To make $I_{X}=0,2 V_{O}=V_{S R C}$, so that $V_{T h}=V_{S R C} / 2 \mathrm{~V}$.
Version 1: $V_{S R C}=5 \mathrm{~V}, V_{T h}=V_{S R C} / 2=2.5 \mathrm{~V}$
Version 2: $V_{S R C}=10 \mathrm{~V}, V_{T h}=V_{S R C} / 2=5 \mathrm{~V}$
Version 3: $V_{S R C}=15 \mathrm{~V}, V_{T h}=V_{S R C} / 2=7.5 \mathrm{~V}$
Version 4: $V_{S R C}=20 \mathrm{~V}, V_{T h}=V_{S R C} / 2=10 \mathrm{~V}$
Version 5: $V_{S R C}=25 \mathrm{~V}, V_{T h}=V_{S R C} / 2=12.5 \mathrm{~V}$.

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8. Determine the power delivered by $I_{S R C}$ assuming $I_{S R C}=0.5 \mathrm{~A}$.

Solution: To determine $V_{a b}$, the $5 \Omega$ and $1 \Omega$ resistances are redundant.


Transforming the voltage source to a current source, it follows that $V_{a b}=2\left(2+I_{S R C}\right)$ From the original circuit, the voltage rise across the current source is $V_{a b}+1 \times I_{S R C}=3 I_{S R C}+4$, and the power delivered is $P=I_{S R C}\left(3 I_{S R C}+4\right)$.
Version 1: $I_{S R C}=0.5 \mathrm{~A}, P=0.5(1.5+4)=2.75 \mathrm{~W}$


Version 2: $I_{S R C}=1 \mathrm{~A}, P=1(3+4)=7 \mathrm{~W}$
Version 3: $I_{S R C}=1.5 \mathrm{~A}, P=1.5(4.5+4)=12.75 \mathrm{~W}$
Version 4: $I_{S R C}=2 \mathrm{~A}, P=2(6+4)=20 \mathrm{~W}$
Version 5: $I_{S R C}=2.5 \mathrm{~A}, P=2.5(7.5+4)=28.75 \mathrm{~W}$.

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9. Determine $V_{T h}$ assuming $V_{S}=1 \mathrm{~V}$ and $R=2$ $\Omega$.

Solution: Let $V_{X}$ be the voltage drop $V_{d c}$.
Applying KCL to node d :
$\frac{V_{x}}{3 R}+\frac{V_{x}}{6 R}+\frac{V_{x}+8}{4}=2$; this gives $V_{x}=0$,
so that $V_{\text {ef }}=0$ and $V_{T h}=V_{S}$.
Alternatively, if the subcircuit between nodes c and d is removed, $V_{d c}$ in this circuit is zero,
 and $V_{d c}$ in the bridge is zero, which makes $V_{e f}=0$ and $V_{T h}=V_{S}$.


Version 1: $V_{S}=1 \mathrm{~V}, V_{T h}=1 \mathrm{~V}$
Version 2: $V_{S}=1.5 \mathrm{~V}, V_{T h}=1.5 \mathrm{~V}$
Version 3: $V_{S}=2 \mathrm{~V}, V_{T h}=2 \mathrm{~V}$
Version 4: $V_{S}=2.5 \mathrm{~V}, V_{T h}=2.5 \mathrm{~V}$
Version 5: $V_{S}=3 \mathrm{~V}, V_{T h}=3 \mathrm{~V}$.

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10. Determine $R_{T h}$ in the preceding problem. Solution: If a test source is applied with all independent sources set to zero, the circuit reduces to that shown, where the current in the $4 \Omega$ resistor is zero. It follows that $R_{T h}=$ $3 R \| 6 R=2 R$.

Version 1: $R=1 \Omega, R_{\text {Th }}=2 \Omega$
Version 2: $R=1.5 \Omega, R_{\text {Th }}=3 \Omega$
Version 3: $R=2 \Omega, R_{\text {Th }}=4 \Omega$
Version 4: $R=2.5 \Omega, R_{T h}=5 \Omega$
Version 5: $R=3 \Omega, R_{T h}=6 \Omega$.

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11. For the circuit shown, write the circuit equations in terms of the node voltages $V_{a}, V_{b}$, and $V_{c}$. You are not

## required to solve these equations.

Solution: The circuit is redrawn as shown with the redundant 2 S resistor removed and the three 3 S resistors combined into a 1 S resistor.

For node a: $V_{a}=0.5 I_{x}$, where, for node $b, I_{x}=5-$ $1 \times V_{b}$. Substituting and rearranging, $2 V_{a}+V_{b}=5$. For node $c$ : $-2 V_{a}+6 V_{c}=I_{x}=5-V_{b}$; or $-2 V_{a}+V_{b}+6 V_{c}=5$. The third equation is: $V_{b}-V_{c}=10$.

12. Use the substitution theorem and superposition to determine $V_{y}$, where $R$ is unknown.

Solution: In accordance with the substitution theorem, $R$ can be replaced by a 0.5 A current source. With this current source set to zero, $V_{y 1}=$ $-\frac{7.5}{17.5} \times 30=-\frac{225}{17.5} \mathrm{~V}$. With the 30 V source set to zero and the 0.5 A current
 source applied, $V_{y 2}=-\frac{10}{17.5} \times 0.5 \times 7.5=-\frac{37.5}{17.5} \mathrm{~V}$. It follows that $V_{y}=V_{y 1}+V_{y 2}=$ $-\frac{225+37.5}{17.5}=-\frac{262.5}{17.5}=-15 \mathrm{~V}$.

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13. Determine Thevenin's equivalent circuit between terminals ab, assuming all resistances are $5 \Omega$.
Solution: With the 5 A source applied
 alone, it follows from symmetry that $V_{T h 1}=0$. With the 12 V source applied alone, it follows from symmetry (as in Example 4.5.1a) that the middle nodes are all at the same voltage. The circuit reduces to that shown, where the effective resistance between nodes c and d is $10\left|\mid 20=20 / 3 \Omega\right.$, so that $V_{c d}=$ $\frac{20 / 3}{20 / 3+10} \times 12=4.8 \mathrm{~V}$, and $V_{T h}=\frac{10}{20} \times 4.8=2.4 \mathrm{~V}$.
 Alternatively, $V_{T h}$ can be found by scaling.

To determine $R_{T h}$, with the two sources set to zero, it follows again from Example 4.5.1a that the middle nodes are at the same voltage so that the horizontal resistor in the middle carries no current and can be removed. With the 12 V source replaced by a short circuit, the circuit reduces to that shown. It follows that the effective resistance between nodes c and d is 5 $\Omega$, and $R_{\text {Th }}=15 \| 10=6 \Omega$.


