1

Circuit Variables

Assessment Problems

AP 1.1 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

\$100 billion $= \$100 \times 10^9$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = \$3.17/\text{ms}$$

AP 1.2 First, we recognize that 1 ns = 10^{-9} s. The question then asks how far a signal will travel in 10^{-9} s if it is traveling at 80% of the speed of light. Remember that the speed of light $c = 3 \times 10^8$ m/s. Therefore, 80% of c is $(0.8)(3 \times 10^8) = 2.4 \times 10^8$ m/s. Now, we use a product of ratios to convert from meters/second to inches/nanosecond:

$$\frac{2.4 \times 10^8 \text{ m}}{1\text{s}} \cdot \frac{1 \text{ s}}{10^9 \text{ ns}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{(2.4 \times 10^8)(100)}{(10^9)(2.54)} = \frac{9.45 \text{ in}}{1 \text{ ns}}$$

Thus, a signal traveling at 80% of the speed of light will travel $9.45''$ in a nanosecond.

AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = -$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0 i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^{\infty} 20e^{-5000}\quad dx = \frac{20}{-5000}e^{-5000}\quad\Big|_0^{\infty} = \frac{20}{-5000}(e^{\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004\text{ C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = -$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-}\quad\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-}\quad\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-}\quad\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-}\quad - \alpha\frac{t}{\alpha}e^{-}\quad\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-}\quad\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-}$$

$$= te^{-}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-}\quad) = e^{-}\quad + t(-\alpha)e\quad = (1-\alpha t)e^{-}\quad = 0$$

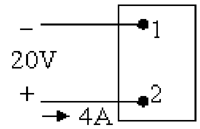
Since e^{-} never equals 0 for a finite value of t, the expression equals 0 only when $(1-\alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-}\qquad = \frac{1}{\alpha}e^{-1}$$

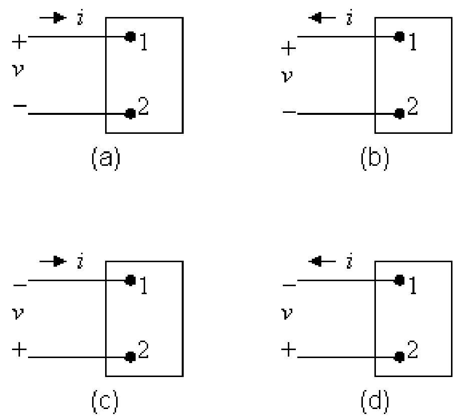
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10\text{ A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

(a) $v = -20\,\text{V}, \qquad i = -4\,\text{A};$ (b) $v = -20\,\text{V}, \qquad i = 4\,\text{A}$

(c) $v = 20\,\text{V}, \qquad i = -4\,\text{A};$ (d) $v = 20\,\text{V}, \qquad i = 4\,\text{A}$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. From Eq. (1.3), we know that power is the time rate of change of energy, or $p = —$. If we know the power, we can find the energy by integrating Eq. (1.3). To begin, find the expression for power:

$$p = vi = (10{,}000e^{-5000}\ )(20e^{-5000}\ ) = 200{,}000e^{-10\,000}\ = 2 \times 10^5 e^{-10\,000}\ \text{ W}$$

Now find the expression for energy by integrating Eq. (1.3):

$$w(t) = \int_0 p(x)\,dx$$

Substitute the expression for power, p, above. Note that to find the total energy, we let $t \to \infty$ in the integral. Thus we have

$$w = \int_0^\infty 2 \times 10^5 e^{-10\,000} \; dx = \frac{2 \times 10^5}{-10{,}000} e^{-10\,000} \Bigg|_0^\infty$$
$$= \frac{2 \times 10^5}{-10{,}000}(e^{-\infty} - e^0) = \frac{2 \times 10^5}{-10{,}000}(0 - 1) = \frac{2 \times 10^5}{10{,}000} = 20 \text{ J}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 To begin, we calculate the number of pixels that make up the display:

$$n_{\text{pixels}} = (1280)(1024) = 1{,}310{,}720 \text{ pixels}$$

Each pixel requires 24 bits of information. Since 8 bits comprise a byte, each pixel requires 3 bytes of information. We can calculate the number of bytes of information required for the display by multiplying the number of pixels in the display by 3 bytes per pixel:

$$n_{\text{bytes}} = \frac{1{,}310{,}720 \text{ pixels}}{1 \text{ display}} \cdot \frac{3 \text{ bytes}}{1 \text{ pixel}} = 3{,}932{,}160 \text{ bytes/display}$$

Finally, we use the fact that there are 10^6 bytes per MB:

$$\frac{3{,}932{,}160 \text{ bytes}}{1 \text{ display}} \cdot \frac{1 \text{ MB}}{10^6 \text{bytes}} = 3.93 \text{ MB/display}$$

P 1.2 $c = 3 \times 10^8$ m/s so $\frac{1}{2}c = 1.5 \times 10^8$ m/s

$$\frac{1.5 \times 10^8 \text{ m}}{1 \text{ s}} = \frac{5 \times 10^6 \text{ m}}{x \text{ s}} \qquad \text{so} \qquad x = \frac{5 \times 10^6}{1.5 \times 10^8} = 33.3 \text{ ms}$$

P 1.3 We can set up a ratio to determine how long it takes the bamboo to grow $10\ \mu$m First, recall that 1 mm $= 10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10\ \mu$m:

$$\frac{10/3456 \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \qquad \text{so} \qquad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s}$$

P 1.4 Volume $=$ area $\times$ thickness

$$10^6 = (10 \times 10^6)(\text{thickness})$$

$$\Rightarrow \text{ thickness } = \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm}$$

P 1.5
$$\frac{300 \times 10^9 \text{ dollars}}{1 \text{ year}} \cdot \frac{100 \text{ pennies}}{1 \text{ dollar}} \cdot \frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{1.5 \text{ mm}}{1 \text{ penny}} \cdot \frac{1 \text{ m}}{1000 \text{ mm}}$$
$$= 1426 \text{ m/s}$$

P 1.6 Our approach is as follows: To determine the area of a bit on a track, we need to know the height and width of the space needed to store the bit. The height of the space used to store the bit can be determined from the width of each track on the disk. The width of the space used to store the bit can be determined by calculating the number of bits per track, calculating the circumference of the inner track, and dividing the number of bits per track by the circumference of the track. The calculations are shown below.

$$\text{Width of track } = \frac{1 \text{ in}}{77 \text{ tracks}} \frac{25{,}400\mu\text{m}}{\text{in}} = 329.87\mu\text{m/track}$$

$$\text{Bits on a track } = \frac{1.4 \text{ MB}}{2 \text{ sides}} \frac{8 \text{ bits}}{\text{byte}} \frac{1 \text{ side}}{77 \text{ tracks}} = 72{,}727.273 \text{ bits/track}$$

$$\text{Circumference of inner track } = 2\pi(1/2'')(25{,}400\mu\text{m/in}) = 79{,}796.453\mu\text{m}$$

$$\text{Width of bit on inner track } = \frac{79{,}796.453\mu\text{m}}{72{,}727.273 \text{ bits}} = 1.0972\mu\text{m/bit}$$

$$\text{Area of bit on inner track } = (1.0972)(329.87) = 361.934\mu\text{m}^2$$

P 1.7 $\text{C/m}^3 = 1.6022 \times 10^{-19} \times 10^{29} = 1.6022 \times 10^{10}\,\text{C/m}^3$

$$\text{C/m} = (1.6022 \times 10^{10})(5.4 \times 10^{-4}) = 8.652 \times 10^{6}\,\text{C/m}$$

$$\text{Therefore, } (8.652 \times 10^{6})\frac{\text{C}}{\text{m}} \times \text{ave vel}\left(\frac{m}{s}\right) = i$$

$$\text{Thus, average velocity } = \frac{1400}{8.652} \times 10^{-6} = 161.81\,\mu\text{m/s}$$

P 1.8 **[a]** $n = \dfrac{20 \times 10^{-6}\ \text{C/s}}{1.6022 \times 10^{-19}\ \text{C/elec}} = 1.25 \times 10^{14}\ \text{elec/s}$

[b] $m = 3303\ \text{mi} \cdot \dfrac{5280\ \text{ft}}{1\ \text{mi}} \cdot \dfrac{12\ \text{in}}{1\ \text{ft}} \cdot \dfrac{2.54\ \text{cm}}{1\ \text{in}} \cdot \dfrac{10^4\ \mu\text{m}}{1\ \text{cm}} = 5.32 \times 10^{12}\ \mu\text{m}$

Therefore, $\dfrac{n}{m} = 23.5$

The number of electrons/second is approximately 23.5 times the number of micrometers between Sydney and San Francisco.

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{(0)}^{( )} dx = 20\int_0 \cos 5000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\Big|_0 = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4 \times 10^{-3}\sin 5000t\,\text{C} = 4\sin 5000t\,\text{mC}$

P 1.10 $w = qV = (1.6022 \times 10^{-19})(9) = 14.42 \times 10^{-19} = 1.442\ \text{aJ}$

P 1.11 $p = (6)(100 \times 10^{-3}) = 0.6$ W; $\quad 3\text{ hr} \cdot \dfrac{3600\text{ s}}{1\text{ hr}} = 10{,}800\text{ s}$

$$w(t) = \int_0 p\,dt \qquad w(10{,}800) = \int_0^{10\,800} 0.6\,dt = 0.6(10{,}800) = 6480\text{ J}$$

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the $+$ terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

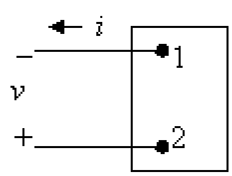
[a] $p = (20)(15) = 300$ W   300 W from A to B

[b] $p = (100)(-5) = -500$ W   500 W from B to A

[c] $p = (-50)(4) = -200$ W   200 W from B to A

[d] $p = (-25)(-16) = 400$ W   400 W from A to B

P 1.13 **[a]**



$p = vi = (-20)(5) = -100$ W
Power is being delivered by the box.

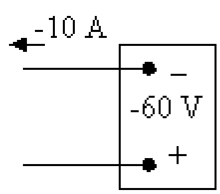
[b] Leaving

[c] Gaining

P 1.14 **[a]** $p = vi = (-20)(-5) = 100$ W, so power is being absorbed by the box.

[b] Entering

[c] Losing



P 1.15 **[a]** In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the $+$ terminal of the battery of Car A). Therefore using the passive sign convention, $p = vi = (-40)(12) = -480$ W.
Since the power is negative, the battery in Car A is generating power, so Car B must have the ”dead” battery.

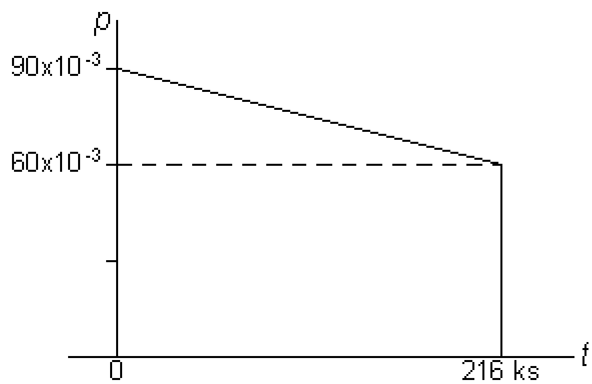
[b] $w(t) = \int_0 p\,dx;\qquad 1.5\text{ min} = 1.5\cdot\dfrac{60\text{ s}}{1\text{ min}} = 90\text{ s}$

$$w(90) = \int_0^{90} 480\,dx$$

$$w = 480(90-0) = 480(90) = 43{,}200\text{ J} = 43.2\text{ kJ}$$

P 1.16 $p = vi;\qquad w = \int_0 p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 60 hr = 216,000 s = 216 ks

$$p(0) = (6)(15\times 10^{-3}) = 90\times 10^{-3}\text{ W}$$

$$p(216\text{ ks}) = (4)(15\times 10^{-3}) = 60\times 10^{-3}\text{ W}$$

$$w = (60\times 10^{-3})(216\times 10^{3}) + \frac{1}{2}(90\times 10^{-3} - 60\times 10^{-3})(216\times 10^{3}) = 16.2\text{ kJ}$$

P 1.17 **[a]** To find the power at $625\,\mu$s, we substitute this value of time into both the equations for $v(t)$ and $i(t)$ and multiply the resulting numbers to get $p(625\,\mu\text{s})$:

$$v(625\,\mu\text{s}) = 50e^{-1600(625\times 10^{-6})} - 50e^{-400(625\times 10^{-6})} = 18.394 - 38.94 = -20.546\text{ V}$$

$$i(625\,\mu\text{s}) = 5\times 10^{-3}e^{-1600(625\times 10^{-6})} - 5\times 10^{-3}e^{-400(625\times 10^{-6})}$$

$$= 0.0018394 - 0.003894 = -0.0020546\text{ A}$$

$$p(625\,\mu\text{s}) = (-20.546)(-0.0020546) = 42.2\text{ mW}$$

[b] To find the energy at $625\,\mu$s, we need to integrate the equation for $p(t)$ from 0 to $625\,\mu$s. To start, we need an expression for $p(t)$:

$$p(t) = v(t)i(t) = (50)(5\times 10^{-3})(e^{-1600} - e^{-400})(e^{-1600} - e^{-400})$$

$$= \frac{1}{4}(e^{-3200} \;\; - 2e^{-2000} \;\; + e^{-800} \;\;)$$

Now we integrate this expression for $p(t)$ to get an expression for $w(t)$. Note we substitute x for t on the right side of the integral.

$$w(t) = \frac{1}{4}\int_0 (e^{-3200} \;\; - 2e^{-2000} \;\; + e^{-800} \;\;)dx$$

$$= \frac{1}{4}\left[\frac{e^{-3200}}{-3200} + \frac{e^{-2000}}{1000} - \frac{e^{-800}}{800}\right]\Bigg|_0$$

$$= \frac{1}{4}\left[\frac{e^{-3200}}{-3200} + \frac{e^{-2000}}{1000} - \frac{e^{-800}}{800} - \left(\frac{1}{-3200} + \frac{1}{1000} - \frac{1}{800}\right)\right]$$

$$= \frac{1}{4}\left[\frac{e^{-3200}}{-3200} + \frac{e^{-2000}}{1000} - \frac{e^{-800}}{800} + 5.625 \times 10^{-4}\right]$$

Finally, substitute $t = 625\,\mu$s into the equation for $w(t)$:

$$w(625\,\mu\text{s}) = \frac{1}{4}[-4.2292 \times 10^{-5} + 2.865 \times 10^{-4} - 7.5816 \times 10^{-4} + 5.625 \times 10^{-4}]$$

$$= 12.137\,\mu\text{J}$$

[c] To find the total energy, we let $t \to \infty$ in the above equation for $w(t)$. Note that this will cause all expressions of the form e^{-} to go to zero, leaving only the constant term 5.625×10^{-4}. Thus,

$$w_{\text{total}} = \frac{1}{4}[5.625 \times 10^{-4}] = 140.625\,\mu\text{J}$$

P 1.18 **[a]** $v(20\text{ ms}) = 100e^{-1}\sin 3 = 5.19$ V
$i(20\text{ ms}) = 20e^{-1}\sin 3 = 1.04$ A $\;p(20\text{ ms}) = vi = 5.39$ W

[b]

$$\begin{aligned}
p &= vi = 2000e^{-100} \;\; \sin^2 150t \\
&= 2000e^{-100} \;\; \left[\frac{1}{2} - \frac{1}{2}\cos 300t\right] \\
&= 1000e^{-100} \;\; - 1000e^{-100} \;\; \cos 300t \\
w &= \int_0^\infty 1000e^{-100} \;\; dt - \int_0^\infty 1000e^{-100} \;\; \cos 300t\,dt \\
&= 1000\,\frac{e^{-100}}{-100}\Bigg|_0^\infty \\
&\quad -1000\left\{\frac{e^{-100}}{(100)^2 + (300)^2}[-100\cos 300t + 300\sin 300t]\right\}\Bigg|_0^\infty \\
&= 10 - 1000\left[\frac{100}{1 \times 10^4 + 9 \times 10^4}\right] = 10 - 1 \\
w &= 9 \text{ J}
\end{aligned}$$

P 1.19 **[a]** $0\text{ s} \le t < 1\text{ s}$:

$v = 5$ V; $i = 20t$ A; $p = 100t$ W

$1\text{ s} < t \le 2\text{ s}$:

$v = 0$ V; $i = 20$ A; $p = 0$ W

$2\text{ s} \le t < 3\text{ s}$:

$v = 0$ V; $i = 20$ A; $p = 0$ W

$3\text{ s} < t \le 4\text{ s}$:

$v = -5$ V; $i = 80 - 20t$ A; $p = -400 + 100t$ W

$4\text{ s} \le t < 5\text{ s}$:

$v = -5$ V; $i = 80 - 20t$ A; $p = -400 + 100t$ W

$5\text{ s} < t \le 6\text{ s}$:

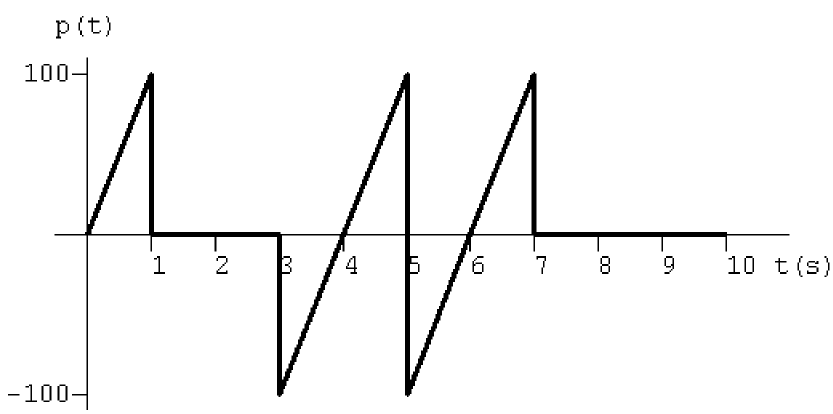
$v = 5$ V; $i = -120 + 20t$ A; $p = -600 + 100t$ W

$6\text{ s} \le t < 7\text{ s}$:

$v = 5$ V; $i = -120 + 20t$ A; $p = -600 + 100t$ W

$t > 7$ s:

$v = 0$ V; $i = 20$ A; $p = 0$ W



[b] Calculate the area under the curve from zero up to the desired time:

$$w(1) = \tfrac{1}{2}(1)(100) = 50\text{ J}$$

$$w(6) = \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) + \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) = 0\text{ J}$$

$$w(10) = \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) + \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) + \tfrac{1}{2}(1)(100) = 50\text{ J}$$

P 1.20 **[a]** $p = vi = (100e^{-500})(0.02 - 0.02e^{-500}) = (2e^{-500} - 2e^{-1000})$ W

$$\frac{dp}{dt} = -1000e^{-500} + 2000e^{-1000} = 0 \qquad \text{so} \qquad 2e^{-1000} = e^{-500}$$

$2 = e^{500}$ so $\ln 2 = 500t$ thus p is maximum at t = 1.4 ms

$p_{\max} = p(1.4 \text{ ms}) = 0.5$ W

[b] $w = \int_0^\infty [2e^{-500} - 2e^{-1000}]\,dt = \left[\frac{2}{-500}e^{-500} - \frac{2}{-1000}e^{-1000}\Big|_0^\infty\right]$

$= \frac{4}{1000} - \frac{2}{1000} = 2$ mJ

P 1.21 **[a]** $p = vi = 200\cos(500\pi t)4.5\sin(500\pi t) = 450\sin(1000\pi t)$ W

Therefore, $p_{\max} = 450$ W

[b] $p_{\max}$(extracting) $= 450$ W

[c]

$$p_{\text{avg}} = \frac{1}{4\times10^{-3}}\int_0^{4\times10^{-3}} 450\sin(1000\pi x)\,dx = \frac{450}{4\times10^{-3}}\left[\frac{-\cos 1000\pi t}{1000\pi}\right]_0^{4\times10^{-3}}$$

$$= \frac{-450}{4\pi}[\cos 4\pi - \cos 0] = \frac{-450}{4\pi}[1-1] = 0 \text{ W}$$

[d] $p_{\text{avg}} = \frac{-450}{4\pi}[\cos 15\pi - \cos 0] = \frac{-450}{4\pi}[-1-1] = \frac{900}{4\pi} = 71.62$ W

P 1.22 **[a]** q = area under i vs. t plot

$$= \frac{6(5000)}{2} + 6(5000) + \frac{6(10{,}000)}{2} + 8(15{,}000) + \frac{8(5000)}{2}$$

$$= 15{,}000 + 30{,}000 + 30{,}000 + 120{,}000 + 20{,}000 = 215{,}000 \text{ C}$$

[b] $w = \int p\,dt = \int vi\,dt$

$v = 0.2\times10^{-3}t + 8 \qquad 0 \le t \le 20$ ks

$0 \le t \le 5000s$

$$i = 20 - 1.2\times10^{-3}t$$

$$p = (8 + 0.2\times10^{-3}t)(20 - 1.2\times10^{-3}t)$$

$$= 160 - 5.6\times10^{-3}t - 2.4\times10^{-7}t^2$$

$$w_1 = \int_0^{5000}(160 - 5.6\times10^{-3}t - 2.4\times10^{-7}t^2)\,dt$$

$$= \left(160t - \frac{5.6\times10^{-3}}{2}t^2 - \frac{2.4\times10^{-7}}{3}t^3\right)\Bigg|_0^{5000} = 720 \text{ kJ}$$

$5000 \le t \le 15{,}000$s

$$i = 17 - 0.6\times10^{-3}t$$

$$p = (8 + 0.2\times10^{-3}t)(17 - 0.6\times10^{-3}t)$$

$$= 136 - 1.4\times10^{-3}t - 1.2\times10^{-7}t^2$$

$$w_2 = \int_{5000}^{15\,000}(136 - 1.4\times10^{-3}t - 1.2\times10^{-7}t^2)\,dt$$

$$= \left(136t - \frac{1.4\times10^{-3}}{2}t^2 - \frac{1.2\times10^{-7}}{3}t^3\right)\Bigg|_{5000}^{15\,000} = 1090 \text{ kJ}$$

$15{,}000 \le t \le 20{,}000\text{s}$

$$
\begin{aligned}
i &= 32 - 1.6 \times 10^{-3}t \\
p &= (8 + 0.2 \times 10^{-3}t)(32 - 1.6 \times 10^{-3}t) \\
&= 256 - 6.4 \times 10^{-3}t - 3.2 \times 10^{-7}t^2 \\
w_3 &= \int_{15\,000}^{20\,000} (256 - 6.4 \times 10^{-3}t - 3.2 \times 10^{-7}t^2)\, dt \\
&= \left(256t - \frac{6.4 \times 10^{-3}}{2}t^2 - \frac{3.2 \times 10^{-7}}{3}t^3\right)\Bigg|_{15\,000}^{20\,000} = 226{,}666.67 \text{ J} \\
w &= w_1 + w_2 + w_3 = 720{,}000 + 1{,}090{,}000 + 226{,}666.67 = 2036.67 \text{ kJ}
\end{aligned}
$$

P 1.23 **[a]**

$$
\begin{aligned}
p &= vi = [10^4t + 5)e^{-400}\ ][(40t + 0.05)e^{-400}\ ] \\
&= 400 \times 10^3t^2e^{-800}\ + 700te^{-800}\ + 0.25e^{-800} \\
&= e^{-800}\,[400{,}000t^2 + 700t + 0.25] \\
\frac{dp}{dt} &= \{e^{-800}\,[800 \times 10^3t + 700] - 800e^{-800}\,[400{,}000t^2 + 700t + 0.25]\} \\
&= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800}
\end{aligned}
$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$
so $p_{\max}$ occurs at $t = 1.68$ ms.

[b]

$$
\begin{aligned}
p_{\max} &= [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(\ 00168)} \\
&= 666.34 \text{ mW}
\end{aligned}
$$

[c]

$$
\begin{aligned}
w &= \int_0 p dx \\
w &= \int_0 400{,}000x^2e^{-800}\ \ dx + \int_0 700xe^{-800}\ \ dx + \int_0 0.25e^{-800}\ \ dx \\
&= \frac{400{,}000e^{-800}}{-512 \times 10^6}[64 \times 10^4x^2 + 1600x + 2]\Bigg|_0 + \\
&\quad \frac{700e^{-800}}{64 \times 10^4}(-800x - 1)\Bigg|_0 + 0.25\frac{e^{-800}}{-800}\Bigg|_0
\end{aligned}
$$

When $t \to \infty$ all the upper limits evaluate to zero, hence
$w = \frac{(400{,}000)(2)}{512 \times 10^6} + \frac{700}{64 \times 10^4} + \frac{0.25}{800} = 2.97$ mJ.

P 1.24 **[a]** We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$
\begin{aligned}
p &= 0 \quad t < 0, \qquad p = 0 \quad t > 40 \text{ s} \\
p &= vi = (t - 0.025t^2)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3 \text{ W} \qquad 0 < t < 40 \text{ s} \\
\frac{dp}{dt} &= 4 - 0.6t + 0.015t^2 = 0
\end{aligned}
$$

Use a calculator to find the two solutions to this quadratic equation:

$t_1 = 8.453$ s; $\qquad t_2 = 31.547$ s

Now we must find which of these two times gives the minimum power by substituting each of these values for t into the equation for $p(t)$:

$$p(t_1) = (8.453 - 0.025(8.453)^2)(4 - 0.2 \cdot 8.453) = 15.396 \text{ W}$$

$$p(t_2) = (31.547 - 0.025(31.547)^2)(4 - 0.2 \cdot 31.547) = -15.396 \text{ W}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\text{max}} = 15.396$ W (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{maxext}} = 15.396$ W (extracted)

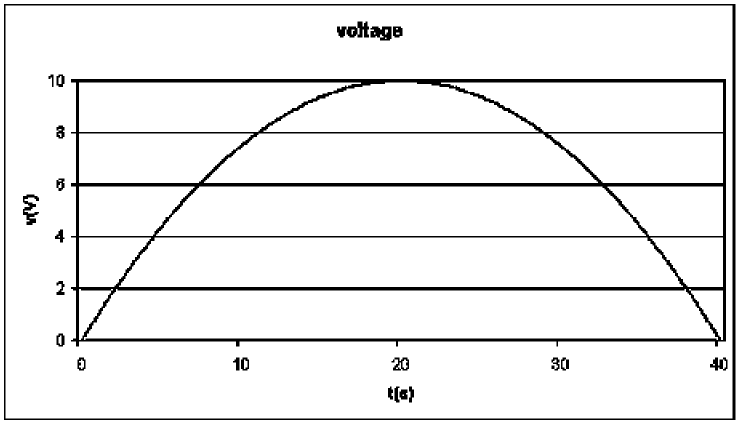
[e] $w = \int_0 p dx = \int_0 (4x - 0.3x^2 + 0.005x^3)dx = 2t^2 - 0.1t^3 + 0.00125t^4$

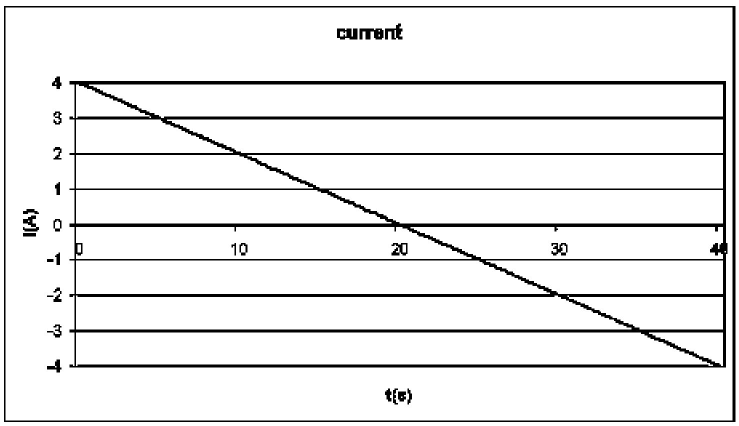
$w(0)$ = 0 J $\qquad w(30)$ = 112.50 J

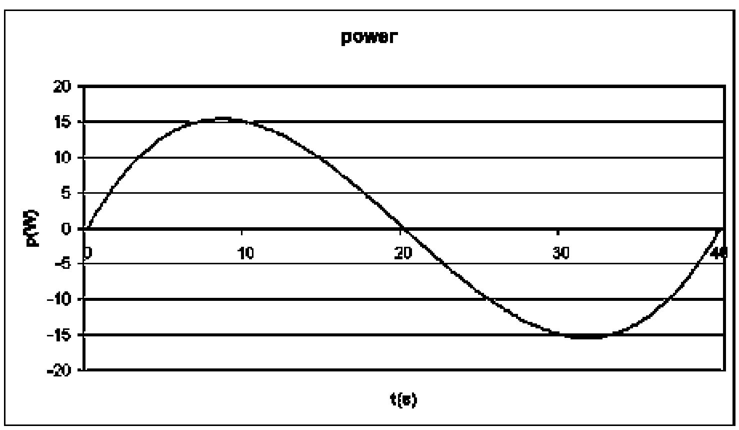
$w(10)$ = 112.50 J $\qquad w(40)$ = 0 J

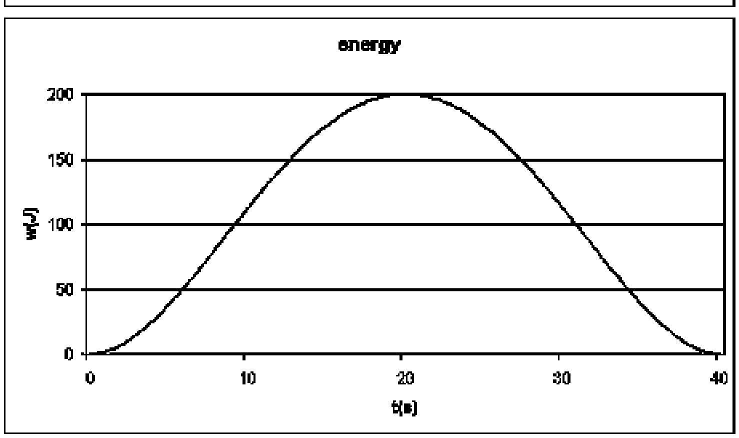
$w(20)$ = 200 J

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:


voltage
v(V)
10
8
6
4
2
0
0
10
20
30
40
t(s)


current
I(A)
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40
t(s)


power
p(W)
20
15
10
5
0
-5
-10
-15
-20
0
10
20
30
40
t(s)


energy
w(J)
200
150
100
50
0
0
10
20
30
40
t(s)

P 1.25 **[a]**

$$p = vi = (8 \times 10^4 te^{-500})(15te^{-500}) = 12 \times 10^5 t^2 e^{-1000} \text{ W}$$
$$\frac{dp}{dt} = 12 \times 10^5 [t^2(-1000)e^{-1000} + e^{-1000}(2t)]$$
$$= 12 \times 10^5 e^{-1000} [t(2 - 1000t)]$$
$$\frac{dp}{dt} = 0 \text{ at } t = 0, \qquad t = 2 \text{ ms}$$

We know p is a minimum at $t = 0$ since v and i are zero at $t = 0$.

[b] $p_{\max} = 12 \times 10^5 (2 \times 10^{-3})^2 e^{-2} = 649.61$ mW

[c]

$$w = 12 \times 10^5 \int_0^{\infty} t^2 e^{-1000} \, dt$$
$$= 12 \times 10^5 \left\{ \frac{e^{-1000}}{(-1000)^3} [10^6 t^2 + 2{,}000t + 2] \Big|_0^{\infty} \right\} = 2400 \, \mu\text{J}$$

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$p_a = -v_a i_a = -(-18)(-51) = -918 \text{ W}$$
$$p_b = v_b i_b = (-18)(45) = -810 \text{ W}$$
$$p_c = v_c i_c = (2)(-6) = -12 \text{ W}$$
$$p_d = -v_d i_d = -(20)(-20) = 400 \text{ W}$$
$$p_e = -v_e i_e = -(16)(-14) = 224 \text{ W}$$
$$p_f = v_f i_f = (36)(31) = 1116 \text{ W}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$\sum P_{\text{dev}} = 918 + 810 + 12 = 1740$ W;

$\sum P_{\text{abs}} = 400 + 224 + 1116 = 1740$ W

Thus, the power balances and the total power developed in the circuit is 1740 W.

P 1.27 **[a]** From the diagram and the table we have

$$\begin{aligned}
p_\text{a} &= -v_\text{a}i_\text{a} = -(900)(-22.5) = 20{,}250 \text{ W}\\
p_\text{b} &= -v_\text{b}i_\text{b} = -(105)(-52.5) = 5512.5 \text{ W}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(-600)(-30) = -18{,}000 \text{ W}\\
p_\text{d} &= v_\text{d}i_\text{d} = (585)(-52.5) = -30{,}712.5 \text{ W}\\
p_\text{e} &= -v_\text{e}i_\text{e} = -(-120)(30) = 3600 \text{ W}\\
p_\text{f} &= v_\text{f}i_\text{f} = (300)(60) = 18{,}000 \text{ W}\\
p_\text{g} &= -v_\text{g}i_\text{g} = -(585)(82.5) = -48{,}262.5 \text{ W}\\
p_\text{h} &= -v_\text{h}i_\text{h} = -(-165)(82.5) = 13{,}612.5 \text{ W}\\
\sum P_\text{del} &= 18{,}000 + 30{,}712.5 + 48{,}262.5 = 96{,}975 \text{ W}\\
\sum P_\text{abs} &= 20{,}250 + 5512.5 + 3600 + 18{,}000 + 13{,}612.5 = 60{,}975 \text{ W}
\end{aligned}$$

Therefore, $\sum P_\text{del} \neq \sum P_\text{abs}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$96{,}975 - 60{,}975 = 36{,}000$$

One-half of this difference is $18{,}000$W, so it is likely that p_c or p_f is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the top node, the current i_c should be 30 A, not -30 A!) If the sign of p_c is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\begin{aligned}
\sum P_\text{del} &= 30{,}712.5 + 48{,}262.5 = 78{,}975 \text{ W}\\
\sum P_\text{abs} &= 20{,}250 + 5512.5 + 18{,}000 + 3600 + 18{,}000 + 13{,}612.5 = 78{,}975 \text{ W}
\end{aligned}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.28

$$\begin{aligned}
p_\text{a} &= v_\text{a}i_\text{a} = (9)(1.8) = 16.2 \text{ W}\\
p_\text{b} &= -v_\text{b}i_\text{b} = -(-15)(1.5) = 22.5 \text{ W}\\
p_\text{c} &= -v_\text{c}i_\text{c} = -(45)(-0.3) = 13.5 \text{ W}\\
p_\text{d} &= -v_\text{d}i_\text{d} = -(54)(-2.7) = 145.8 \text{ W}\\
p_\text{e} &= v_\text{e}i_\text{e} = (-30)(-1) = 30 \text{ W}\\
p_\text{f} &= -v_\text{f}i_\text{f} = -(-240)(4) = 960 \text{ W}\\
p_\text{g} &= -v_\text{g}i_\text{g} = -(294)(4.5) = -1323 \text{ W}\\
p_\text{h} &= v_\text{h}i_\text{h} = (-270)(-0.5) = 135 \text{ W}
\end{aligned}$$

Therefore,

$$\sum P_{\text{abs}} = 16.2 + 22.5 + 13.5 + 145.8 + 3 - +960 + 135 = 1323 \text{ W}$$

$$\sum P_{\text{del}} = 1323 \text{ W}$$

$$\sum P_{\text{abs}} = \sum P_{\text{del}}$$

Thus, the interconnection satisfies the power check

P 1.29

$$p_{\text{a}} = v_{\text{a}}i_{\text{a}} = (-160)(-10) = 1600 \text{ W}$$
$$p_{\text{b}} = v_{\text{b}}i_{\text{b}} = (-100)(-20) = 2000 \text{ W}$$
$$p_{\text{c}} = -v_{\text{c}}i_{\text{c}} = -(-60)(6) = 360 \text{ W}$$
$$p_{\text{d}} = v_{\text{d}}i_{\text{d}} = (800)(-50) = -40{,}000 \text{ W}$$
$$p_{\text{e}} = -v_{\text{e}}i_{\text{e}} = -(800)(-20) = 16{,}000 \text{ W}$$
$$p_{\text{f}} = -v_{\text{f}}i_{\text{f}} = -(-700)(14) = 9800 \text{ W}$$
$$p_{\text{g}} = -v_{\text{g}}i_{\text{g}} = -(640)(-16) = 10{,}240 \text{ W}$$

$\sum P_{\text{del}} = 40{,}000$ W

$\sum P_{\text{abs}} = 1600 + 2000 + 360 + 16{,}000 + 9800 + 10{,}000 = 40{,}000$ W

Therefore, $\sum P_{\text{del}} = \sum P_{\text{abs}} = 40{,}000$ W

P 1.30 **[a]** From an examination of reference polarities, the following elements employ the passive convention: $a, c, e,$ and f.

[b]

$$p_{\text{a}} = -56 \text{ W}$$
$$p_{\text{b}} = -14 \text{ W}$$
$$p_{\text{c}} = 150 \text{ W}$$
$$p_{\text{d}} = -50 \text{ W}$$
$$p_{\text{e}} = -18 \text{ W}$$
$$p_{\text{f}} = -12 \text{ W}$$

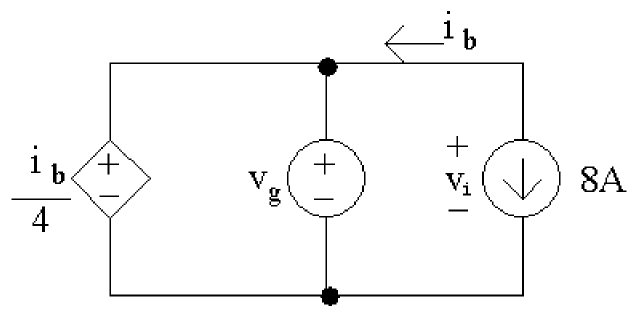
$\sum P_{\text{abs}} = 150$ W; $\sum P_{\text{del}} = 56 + 14 + 50 + 18 + 12 = 150$ W.

2

Circuit Elements

Assessment Problems

AP 2.1



[a] To find v write a KVL equation clockwise around the left loop, starting below the dependent source:

$$-\frac{i}{4} + v = 0 \qquad \text{so} \qquad v = \frac{i}{4}$$

To find i write a KCL equation at the upper right node. Sum the currents leaving the node:

$$i + 8\,\text{A} = 0 \qquad \text{so} \qquad i = -8\,\text{A}$$

Thus,

$$v = \frac{-8}{4} = -2\,\text{V}$$

[b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v. To do this, write a KVL equation clockwise around the left loop, starting below the voltage source:

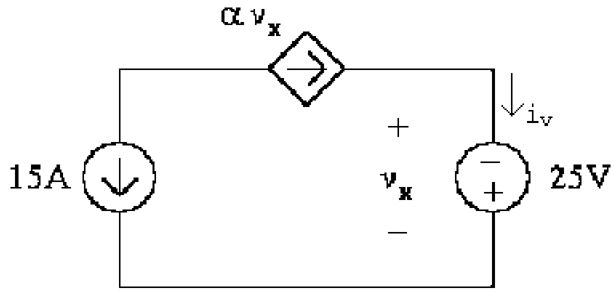
$$-v + v = 0 \qquad \text{so} \qquad v = v = -2\,\text{V}$$

Using the passive sign convention,

$$p = (8\,\text{A})(v) = (8\,\text{A})(-2\,\text{V}) = -16\,\text{W}$$

Thus the current source generated 16 W of power.

AP 2.2



[a] Note from the circuit that $v \ \ = -25$ V. To find α write a KCL equation at the top left node, summing the currents leaving:

$$15\,\text{A} + \alpha v \ \ = 0$$

Substituting for v ,

$$15\,\text{A} + \alpha(-25\,\text{V}) = 0 \qquad \text{so} \qquad \alpha(25\,\text{V}) = 15\,\text{A}$$

Thus $\qquad \alpha = \dfrac{15\,\text{A}}{25\,\text{V}} = 0.6$ A/V

[b] To find the power associated with the voltage source we need to know the current, i . To find this current, write a KCL equation at the top left node, summing the currents leaving the node:

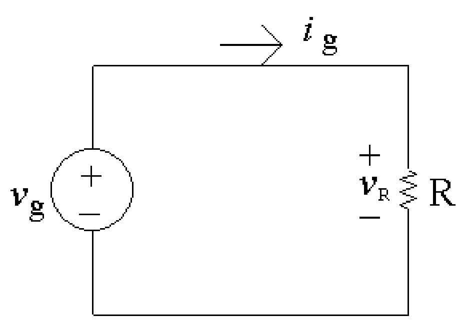
$$-\alpha v \ \ + i \ \ = 0 \qquad \text{so} \qquad i \ \ = \alpha v \ \ = (0.6)(-25) = -15\,\text{A}$$

Using the passive sign convention,

$$p \ \ = -(i \ )(25\,\text{V}) = -(-15\,\text{A})(25\,\text{V}) = 375\,\text{W}.$$

Thus the voltage source dissipates 375 W

AP 2.3



[a] A KVL equation gives

$$-v \ \ + v \ \ = 0 \qquad \text{so} \qquad v \ \ = v \ \ = 1\,\text{kV}$$

Note from the circuit that the current through the resistor is $i \ \ = 5$ mA. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v}{i} = \frac{1\,\text{kV}}{5\,\text{mA}} = 200\,\text{k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p \ \ = (v \ \ )(i \ ) = (1\,\text{kV})(5\,\text{mA}) = 5\,\text{W}$$

The resistor is dissipating 5 W of power.

[b] Note from part (a) the $v \quad = v$ and $i \quad = i$. The power delivered by the source is thus

$$p_{\text{source}} = -v\ i \qquad \text{so} \qquad v \quad = -\frac{p_{\text{source}}}{i} = -\frac{(-3\,\text{W})}{75\,\text{mA}} = 40\,\text{V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v}{i} = \frac{40\,\text{V}}{75\,\text{mA}} = 533.33\,\Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p \quad = -p_{\text{source}} = -(-3\,\text{W}) = 3\,\text{W}$$

[c] Again, note the $i \quad = i$. The power dissipated by the resistor can be determined from the resistor's current:

$$p \quad = R(i \quad)^2 = R(i \quad)^2$$

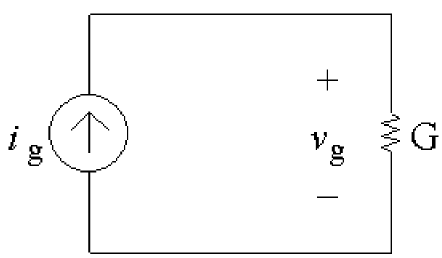
Solving for i ,

$$i^2 = \frac{p}{R} = \frac{480\,\text{mW}}{300\,\Omega} = 0.0016 \qquad \text{so} \qquad i \quad = \sqrt{0.0016} = 0.04\,\text{A} = 40\,\text{mA}$$

Then, since $v \quad = v$

$$v \quad = Ri \quad = Ri \quad = (300\,\Omega)(40\,\text{mA}) = 12\,\text{V} \qquad \text{so} \qquad v \quad = 12\,\text{V}$$

AP 2.4



[a] Note from the circuit that the current throught the conductance G is i , flowing from top to bottom (from KCL), and the voltage drop across the current source is v , positive at the top (from KVL). From a version of Ohm's law,

$$v \quad = \frac{i}{G} = \frac{0.5\,\text{A}}{50\,\text{mS}} = 10\,\text{V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v\ i \quad = -(10)(0.5) = -5\,\text{W}$$

Thus the current source delivers 5 W to the circuit.

[b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p = Gv^2 \qquad \text{so} \qquad G = \frac{p}{v^2} = \frac{9}{15^2} = 0.04\,\text{S} = 40\,\text{mS}$$

$$i = Gv = (40\,\text{mS})(15\,\text{V}) = 0.6\,\text{A}$$

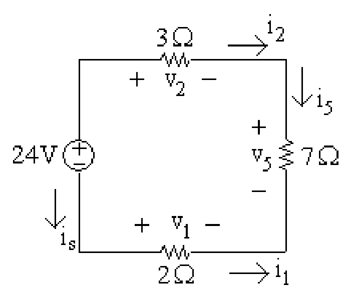
[c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p = Gv^2 \qquad \text{so} \qquad v^2 = \frac{p}{G} = \frac{8\,\text{W}}{200\,\mu\text{S}} = 40{,}000$$

$$\text{Thus} \qquad v = \sqrt{40{,}000} = 200\,\text{V}$$

$$i = Gv = (200\,\mu\text{S})(200\,\text{V}) = 0.04\,\text{A} = 40\,\text{mA}$$

AP 2.5 **[a]** Redraw the circuit with all of the voltages and currents labeled for every circuit element.



Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24\,\text{V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1$$

A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5:

$$v_2 = 3i_2 = 3i_5; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1 = -2i_5$$

Now substitute these expressions for the three voltages into the first equation:

$$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4\,\text{V}$

[c] $v_2 = 3i_5 = 3(2) = 6\,\text{V}$

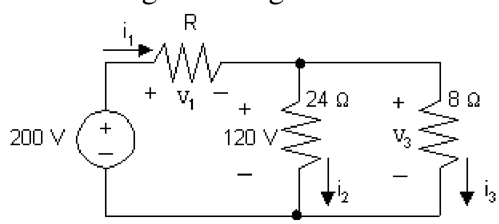
[d] $v_5 = 7i_5 = 7(2) = 14\,\text{V}$

[e] A KCL equation at the lower left node gives $i\ \ = i_1$. Since $i_1 = -i_5$, $i\ \ = -2\,\text{A}$. We can now compute the power associated with the voltage source:

$$p_{24} = (24)i\ \ = (24)(-2) = -48\,\text{W}$$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:



We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the $24\,\Omega$ resistor:

$$-120\,\text{V} + v_3 = 0$$

Use Ohm's law to calculate the voltage across the $8\,\Omega$ resistor in terms of its current:

$$v_3 = 8i_3$$

Substitute the expression for v_3 into the first equation:

$$-120\,\text{V} + 8i_3 = 0 \qquad \text{so} \qquad i_3 = \frac{120}{8} = 15\,\text{A}$$

Also use Ohm's law to calculate the value of the current through the $24\,\Omega$ resistor:

$$i_2 = \frac{120\,\text{V}}{24\,\Omega} = 5\,\text{A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \qquad \text{so} \qquad i_1 = i_2 + i_3 = 5 + 15 = 20\,\text{A}$$

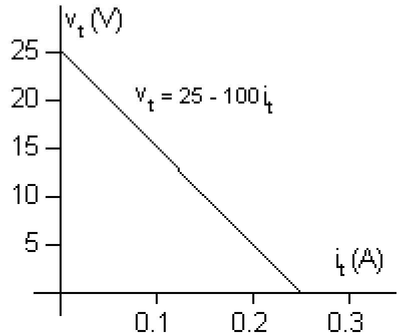
Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\,\text{V} + v_1 + 120\,\text{V} = 0 \qquad \text{so} \qquad v_1 = 200 - 120 = 80\,\text{V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$\mathrm{R} = \frac{v_1}{i_1} = \frac{80}{20} = 4\,\Omega$$

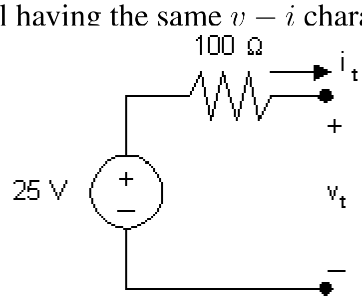
AP 2.7 **[a]** Plotting a graph



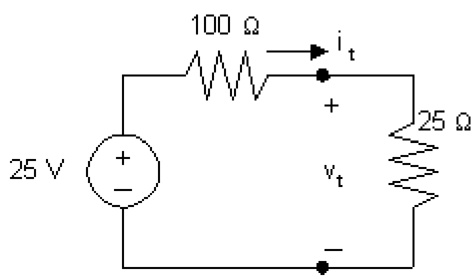
Note that when $i = 0$, $v = 25$ V; therefore the voltage source must be 25 V. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω re:



[b] Draw the circuit resistor:



To find the power delivered to the $25\,\Omega$ resistor we must calculate the current through the $25\,\Omega$ resistor. Do this by first using KCL to recognize that the current in each of the components is i, flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i flowing through the resistors:

$$-25\,\mathrm{V} + 100i + 25i = 0 \qquad \text{so} \qquad 125i = 25 \qquad \text{so} \qquad i = \frac{25}{125} = 0.2\,\mathrm{A}$$

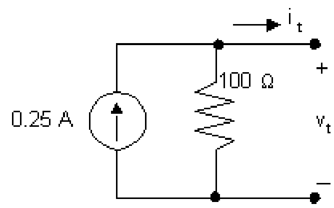
Thus, the power delivered to the $25\,\Omega$ resistor is

$$p_{25} = (25)i^2 = (25)(0.2)^2 = 1\,\mathrm{W}.$$

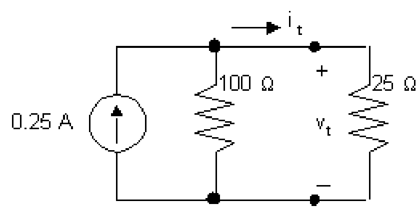
AP 2.8 **[a]** From the graph in Assessment Problem 2.7(a), we see that when $v = 0$, $i = 0.25$ A. Therefore the current source must be 0.25 A. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a $25\,\Omega$ resistor:



Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v . Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

$$-0.25 + \frac{v}{100} + \frac{v}{25} = 0, \qquad \text{so} \qquad 5v = 25, \qquad \text{thus} \qquad v = 5\,\text{V}$$

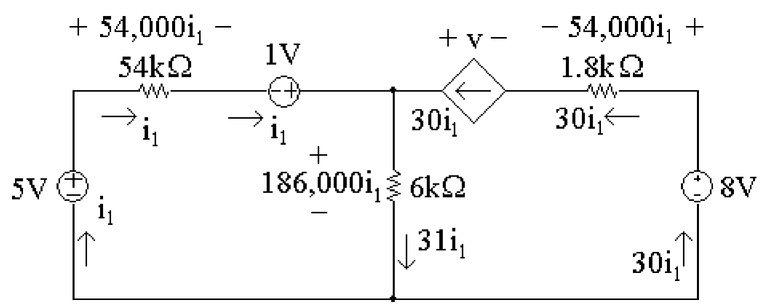
$$p_{25} = \frac{v^2}{25} = 1\,\text{W}.$$

AP 2.9 First note that we know the current through all elements in the circuit except the 6 kΩ resistor (the current in the three elements to the left of the 6 kΩ resistor is i_1; the current in the three elements to the right of the 6 kΩ resistor is $30i_1$). To find the current in the 6 kΩ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\text{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1.

The results are shown in the figure below:



[a] To find i_1, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5\,\text{V} + 54{,}000 i_1 - 1\,\text{V} + 186{,}000 i_1 = 0$$

Solving for i_1

$$54{,}000 i_1 + 189{,}000 i_1 = 6\,\text{V} \qquad \text{so} \qquad 240{,}000 i_1 = 6\,\text{V}$$

Thus,

$$i_1 = \frac{6}{240{,}000} = 25\,\mu\text{A}$$

[b] Now that we have the value of i_1, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54{,}000 i_1 + 8\,\text{V} - 186{,}000 i_1 = 0$$

Thus,

$$v = 240{,}000 i_1 - 8\,\text{V} = 240{,}000(25 \times 10^{-6}) - 8\,\text{V} = 6\,\text{V} - 8\,\text{V} = -2\,\text{V}$$

We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (μA)**	**Voltage (V)**	**Power Equation**	**Power (μW)**
5 V	25	5	$p = -vi$	-125
54 kΩ	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	-25
6 kΩ	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	-2	$p = -vi$	1500
1.8 kΩ	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	-6000

[c] The total power generated in the circuit is the sum of the negative power values in the power table:

$$-125\,\mu\text{W} + -25\,\mu\text{W} + -6000\,\mu\text{W} = -6150\,\mu\text{W}$$

Thus, the total power generated in the circuit is $6150\,\mu$W.

[d] The total power absorbed in the circuit is the sum of the positive power values in the power table:

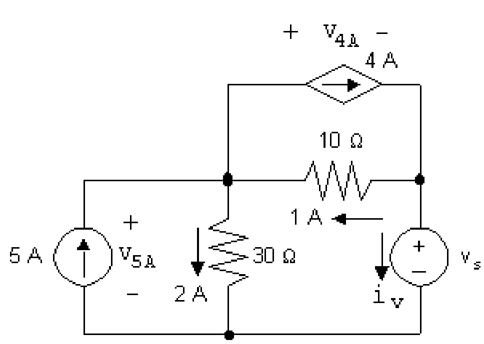
$$33.75\,\mu\text{W} + 3603.75\,\mu\text{W} + 1500\,\mu\text{W} + 1012.5\,\mu\text{W} = 6150\,\mu\text{W}$$

Thus, the total power absorbed in the circuit is $6150\,\mu$W.

AP 2.10 Given that $i \;\; = 2\,\text{A}$, we know the current in the dependent source is $2i \;\; = 4\,\text{A}$. We can write a KCL equation at the left node to find the current in the $10\,\Omega$ resistor. Summing the currents leaving the node,

$$-5\,\text{A} + 2\,\text{A} + 4\,\text{A} + i_{10\Omega} = 0 \qquad \text{so} \qquad i_{10\Omega} = 5\,\text{A} - 2\,\text{A} - 4\,\text{A} = -1\,\text{A}$$

Thus, the current in the $10\,\Omega$ resistor is $1\,\text{A}$, flowing right to left, as seen in the circuit below.



[a] To find v , write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v + (1\,\text{A})(10\,\Omega) + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v = 10\,\text{V} + 60\,\text{V} = 70\,\text{V}$$

[b] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4\,\text{A} + 1\,\text{A} + i = 0 \qquad \text{so} \qquad i = 4\,\text{A} - 1\,\text{A} = 3\,\text{A}$$

The current in the voltage source is $3\,\text{A}$, flowing top to bottom. The power associated with this source is

$$p = vi = (70\,\text{V})(3\,\text{A}) = 210\,\text{W}$$

Thus, $210\,\text{W}$ are absorbed by the voltage source.

[c] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_5 + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_5 = 60\,\text{V}$$

The power associated with this source is

$$p = -v_5\ i = -(60\,\text{V})(5\,\text{A}) = -300\,\text{W}$$

This source thus delivers $300\,\text{W}$ of power to the circuit.

[d] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_4 + (10\,\Omega)(1\,\text{A}) = 0 \qquad \text{so} \qquad v_4 = -10\,\text{V}$$

The power associated with this source is

$$p = v_4\ i = (-10\,\text{V})(4\,\text{A}) = -40\,\text{W}$$

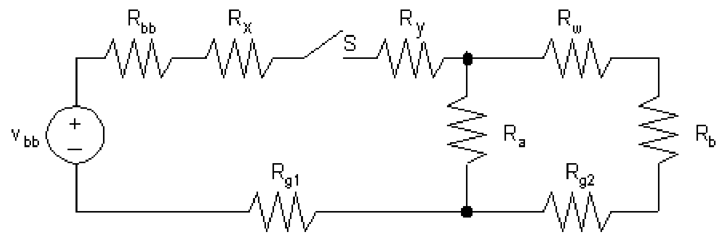
This source thus delivers $40\,\text{W}$ of power to the circuit.

[e] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30\,\Omega) + (i_{10\Omega})^2(10\,\Omega) = (2)^2(30\,\Omega) + (1)^2(10\,\Omega) = 120 + 10 = 130\,\text{W}$$

Problems

P 2.1



V	=	no-load voltage of battery
R	=	internal resistance of battery
R	=	resistance of wire between battery and switch
R	=	resistance of wire between switch and lamp A
R_{a}	=	resistance of lamp A
R_{b}	=	resistance of lamp B
R	=	resistance of wire between lamp A and lamp B
R_{1}	=	resistance of frame between battery and lamp A
R_{2}	=	resistance of frame between lamp A and lamp B
S	=	switch

P 2.2 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.2(a),

$$v = Ri \qquad \text{so} \qquad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.2(b),

$$R = \frac{-160}{-0.02} = \frac{-80}{-0.01} = \frac{80}{0.01} = \frac{160}{0.02} = \frac{240}{0.03} = 8\text{k}\Omega$$

P 2.3 The resistor value is the ratio of the power to the square of the current:

$$\frac{500}{1^2} = \frac{2000}{2^2} = \frac{4500}{3^2} = \frac{8000}{4^2} = \frac{12{,}500}{5^2} = \frac{18{,}000}{6^2} = 500\,\Omega$$

P 2.4 Since we know the device is a resistor, we can use the power equation. From Fig. P2.4(a),

$$p = vi = \frac{v^2}{R} \qquad \text{so} \qquad R = \frac{v^2}{p}$$

Using the values in the table of Fig. P2.4(b)

$$R = \frac{(-8)^2}{3.2} = \frac{(-4)^2}{0.8} = \frac{(4)^2}{0.8} = \frac{(8)^2}{3.2} = \frac{(12)^2}{7.2} = \frac{(16)^2}{12.8} = 20\,\Omega$$

P 2.5 **[a]** Yes, independent voltage sources can carry whatever current is required by the connection; independent current source can support any voltage required by the connection.

[b] 18 V source: absorbing

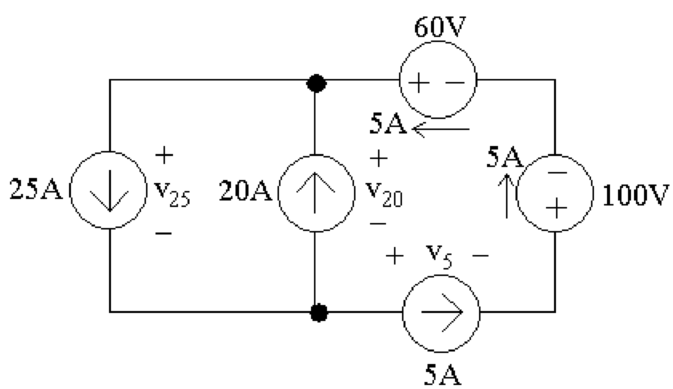
5 mA source: delivering

7 V source: absorbing

[c]
$$P_{18V} = (5\times10^{-3})(18) = 90\text{ mW} \quad \text{(abs)}$$
$$P_{5mA} = -(5\times10^{-3})(25) = -125\text{ mW} \quad \text{(del)}$$
$$P_{7V} = (5\times10^{-3})(7) = 35\text{ mW} \quad \text{(abs)}$$
$$\sum P_{abs} = \sum P_{del} = 125\text{ mW}$$

[d] Yes; 18 V source is delivering, the 5 mA source is absorbing, and the 7 V source is absorbing
$$P_{18V} = -(5\times10^{-3})(18) = -90\text{ mW} \quad \text{(del)}$$
$$P_{5mA} = (5\times10^{-3})(11) = 55\text{ mW} \quad \text{(abs)}$$
$$P_{7V} = (5\times10^{-3})(7) = 35\text{ mW} \quad \text{(abs)}$$
$$\sum P_{abs} = \sum P_{del} = 90\text{ mW}$$

P 2.6



Write the two KCL equations, summing the currents leaving the node:

KCL, top node: $25\text{A} - 20\text{A} - 5\text{A} = 0\text{A}$

KCL, bottom node: $-25\text{A} + 20\text{A} + 5\text{A} = 0\text{A}$

Write the three KVL equations, summing the voltages in a clockwise direction:

KVL, left loop: $-v_{25} + v_{20} = 0$

KVL, right loop: $60\text{V} - 100\text{V} - v_5 - v_{20} = 0$

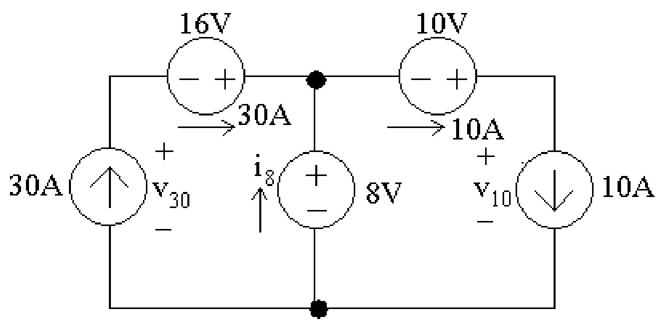
KVL, outer loop: $60\text{V} - 100\text{V} - v_5 - v_{25} = 0$

Note that since v_5, v_{20}, and v_{25} are not specified, we can choose values that satisfy the equations. For example, let $v_5 = -80\text{V}$, $v_{20} = 40\text{V}$, and $v_{25} = 40\text{V}$. There are many other voltage values that will satisfy the equations, too.
Thus, the interconnection is valid because it does not violate Kirchhoff's laws. We can now calculate the power developed by the two voltage sources:

$$p_{\text{v-sources}} = p_{60} + p_{100} = -(60)(5) + (100)(5) = 200\,\text{W}.$$

Since the power is positive, the sources are absorbing 200 W of power, or developing -200 W of power.

P 2.7



Write the two KCL equations, summing the currents leaving the node:

KCL, top node: $-30\text{A} - i_8 + 10\text{A} = 0\text{A}$

KCL, bottom node: $30\text{A} + i_8 - 10\text{A} = 0\text{A}$

Note that the value $i_8 = -20\text{A}$ satisfies these two equations.
Write the three KVL equations, summing the voltages in a clockwise direction:

KVL, left loop: $-v_{30} - 16\text{V} + 8\text{V} = 0$

KVL, right loop: $-10\text{V} + v_{10} - 8\text{V} = 0$

KVL, outer loop: $-16\text{V} - 10\text{V} + v_{10} - v_{30} = 0$

Note that $v_{30} = -8\text{V}$ and $v_{10} = 18\text{V}$ satisfy the three KVL equations.
The interconnection is valid, since neither of Kirchhoff's laws is violated. We use the values of i_8, v_{30} and v_{10} stated above to calculate the power associated with each source:

$p_{30\text{A}} = -(30)(-8) = 240\,\text{W}$ $\qquad p_{16\text{V}} = -(30)(16) = -480\,\text{W}$

$p_{8\text{V}} = -(-20)(8) = 160\,\text{W}$ $\qquad p_{10\text{V}} = -(10)(10) = -100\,\text{W}$

$p_{10\text{A}} = (10)(18) = 180\,\text{W}$

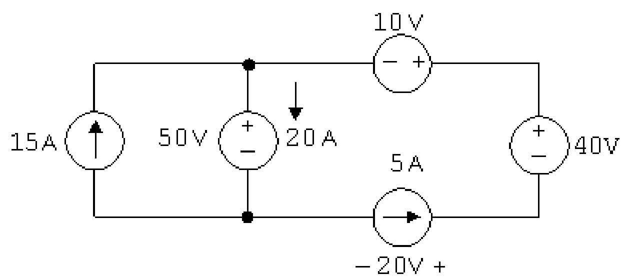
$\sum P_{\text{abs}} = \sum P_{\text{del}} = 580\,\text{W}$

Power developed by the current sources:

$p_{\text{ -sources}} = p_{30\text{A}} + p_{10\text{A}} = 240 + 180 = 420\,\text{W}$

Since power is positive, the sources are absorbing 420 W of power, or developing -420 W of power.

P 2.8 The interconnect is valid since it does not violate Kirchhoff's laws.



$-10 + 40 + v_{5\text{A}} - 50 = 0$ so $v_{5\text{A}} = 20\,\text{V}$ (KVL)

$15 + 5 + i_{50\text{V}} = 0$ so $i_{50\text{V}} = -20\,\text{A}$ (KCL)

$p_{15\text{A}} = -(15)(50) = -750\,\text{W}$ $\qquad p_{50\text{V}} = (20)(50) = 1000\,\text{W}$

$p_{5\text{A}} = -(5)(20) = -100\,\text{W}$ $\qquad p_{10\text{V}} = (5)(10) = 50\,\text{W}$

$p_{40\text{V}} = -(5)(40) = -200\,\text{W}$

$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 1050\,\text{W}$

P 2.9 First there is no violation of Kirchhoff's laws, hence the interconnection is valid. Kirchhoff's voltage law requires

$-20 + 60 + v_1 - v_2 = 0$ so $v_1 - v_2 = -40\,\text{V}$

The conservation of energy law requires

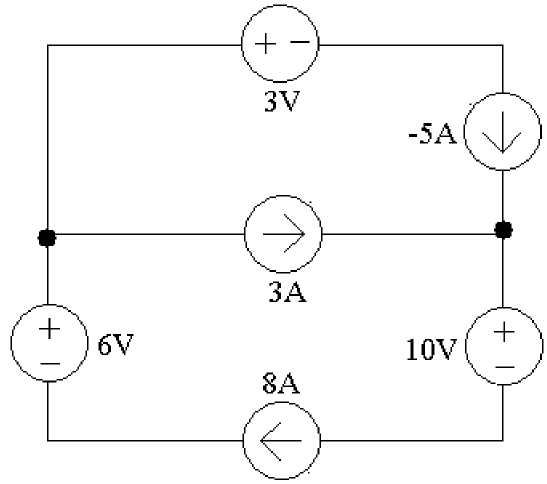
$$-(5\times10^{-3})v_2 - (15\times10^{-3})v_2 - (20\times10^{-3})(20) + (20\times10^{-3})(60) + (20\times10^{-3})v_1 = 0$$

or

$v_1 - v_2 = -40\,\text{V}$

Hence any combination of v_1 and v_2 such that $v_1 - v_2 = -40\,\text{V}$ is a valid solution.

P 2.10

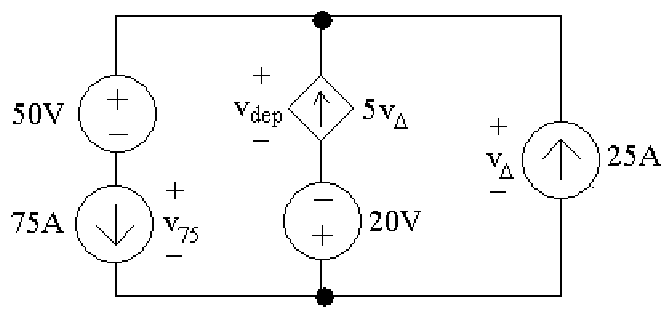


The interconnection is invalid because KCL is violated at the right-hand node. Summing the currents leaving,

$$-(-5\text{A}) - 3\text{A} + 8\text{A} = 10\text{A} \neq 0$$

Note that KCL is also violated at the left-hand node.

P 2.11



Write the two KCL equations, summing the currents leaving the node:

KCL, top node: $75\text{A} - 5v_\Delta - 25\text{A} = 0\text{A}$

KCL, bottom node: $-75\text{A} + 5v_\Delta + 25\text{A} = 0\text{A}$

To satisfy KCL, note that $v_\Delta = 10$ V.

Write the three KVL equations, summing the voltages in a clockwise direction:

KVL, left loop: $-v_{75} - 50\text{V} + v_{\text{dep}} - 20\text{V} = 0$

KVL, right loop: $20\text{V} - v_{\text{dep}} + v_\Delta = 0$

KVL, outer loop: $-v_{75} - 50\text{V} + v_\Delta = 0$

Substitute the value $v_\Delta = 10$ V into the second KVL equation and find $v_{\text{dep}} = 30$ V. Substitute the value $v_\Delta = 10$ V into the third equation and find $v_{75} = -40$ V. These values satisfy the first equation.
Thus, the interconnection is valid because it does not violate Kirchhoff's laws.
Use the values for v_Δ, v_{75}, and v_{dep} above to calculate the total power developed in the circuit:

$$p_{50\text{V}} = (75)(50) = 3750\,\text{W} \qquad p_{75\text{A}} = (75)(-40) = -3000\,\text{W}$$

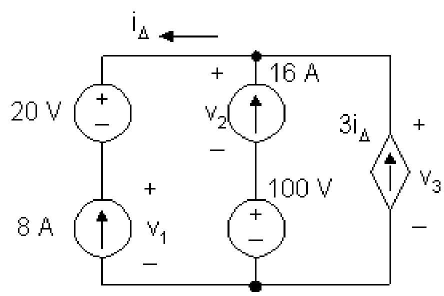
$$p_{20\text{V}} = [5(10)](20) = 1000\,\text{W} \qquad p_{\text{ds}} = -(50)(30) = -1500\,\text{W}$$

$$p_{25\text{A}} = -(25)(10) = -250\,\text{W}$$

$$\sum P_{\text{dev}} = 3750 + 1000 = 4750\,\text{W} = \sum P_{\text{abs}}$$

P 2.12 **[a]** Yes, Kirchhoff's laws are not violated. (Note that $i_\Delta = -8$ A.)

[b] No, because the voltages across the independent and dependent current sources are indeterminate. For example, define v_1, v_2, and v_3 as shown:



Kirchhoff's voltage law requires

$$v_1 + 20 = v_3$$

$$v_2 + 100 = v_3$$

Conservation of energy requires

$$-8(20) - 8v_1 - 16v_2 - 16(100) + 24v_3 = 0$$

or

$$v_1 + 2v_2 - 3v_3 = -220$$

Now arbitrarily select a value of v_3 and show the conservation of energy will be satisfied. Examples:
If $v_3 = 200$ V then $v_1 = 180$ V and $v_2 = 100$ V. Then

$$180 + 200 - 600 = -220 \text{ (CHECKS)}$$

If $v_3 = -100$ V, then $v_1 = -120$ V and $v_2 = -200$ V. Then

$$-120 - 400 + 300 = -220 \text{ (CHECKS)}$$

P 2.13 First, $10v\ \ = 5$ V, so $v\ \ = 0.5$ V

KVL for the outer loop: $5 - 20 + v_{9\text{A}} = 0$ so $v_{9\text{A}} = 15$ V

KVL for the right loop: $5 - 0.5 + v\ \ = 0$ so $v\ \ = -4.5$ V

KCL at the top node: $9 + 6 + i_{\text{ds}} = 9$ so $i_{\text{ds}} = -15$ A

Thus,

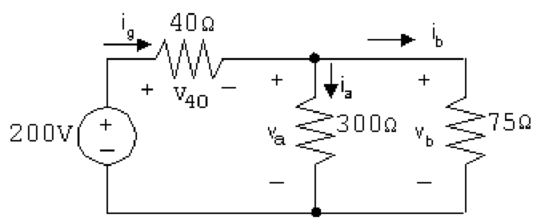
$p_{9\text{A}} = -(9)(15) = -135$ W $\qquad$ $p_{20\text{V}} = (9)(20) = 180$ W

$p\ _{g} = -(6)(-4.5) = 27$ W $\qquad$ $p_{6\text{A}} = (6)(0.5) = 3$ W

$p_{\text{ds}} = -(15)(5) = -75$ W

$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 210$ W

P 2.14



[a] Write a KVL equation clockwise aroud the right loop, starting below the 300 Ω resistor:

$-v\ \ + v\ \ = -0$ so $v\ \ = v$

Using Ohm's law,

$v\ \ = 300i$ and $v\ \ = 75i$

Substituting,

$300i\ \ = 75i$ so $i\ \ = 4i$

Write a KCL equation at the top middle node, summing the currents leaving:

$-i\ \ + i\ \ + i\ \ = 0$ so $i\ \ = i\ \ + i\ \ = i\ \ + 4i\ \ = 5i$

Write a KVL equation clockwise around the left loop, starting below the voltage source:

$-200\text{ V} + v_{40} + v\ \ = 0$

From Ohm's law,

$v_{40} = 40i$ and $v\ \ = 300i$

Substituting,

$-200\,\text{V} + 40i \;\; + 300i \;\; = 0$

Subsituting for i :

$-200\,\text{V} + 40(5i \;\;) + 300i \;\; = -200\,\text{V} + 200i \;\; + 300i \;\; = -200\,\text{V} + 500i \;\; = 0$

Thus,

$500i \;\; = 200\,\text{V} \qquad \text{so} \qquad i \;\; = \dfrac{200\,\text{V}}{500} = 0.4\,\text{A}$

[b] From part (a), $i \;\; = 4i \;\; = 4(0.4\,\text{A}) = 1.6\,\text{A}$.

[c] From the circuit, $v \;\; = 75\,\Omega(i \;\;) = 75\,\Omega(1.6\,\text{A}) = 120\,\text{V}$.

[d] Use the formula $p \;\; = Ri^2$ to calculate the power absorbed by each resistor:

$p_{40\Omega} = i^2(40\,\Omega) = (5i \;\;)^2(40\,\Omega) = [5(0.4)]^2(40\,\Omega) = (2)^2(40\,\Omega) = 160\,\text{W}$

$p_{300\Omega} = i_{\text{a}}^2(300\,\Omega) = (0.4)^2(300\,\Omega) = 48\,\text{W}$

$p_{75\Omega} = i_{\text{b}}^2(75\,\Omega) = (4i \;\;)^2(75\,\Omega) = [4(0.4)]^2(75\,\Omega) = (1.6)^2(75\,\Omega) = 192\,\text{W}$

[e] Using the passive sign convention,

$$p_{\text{source}} = -(200\,\text{V})i \;\; = -(200\,\text{V})(5i \;\;) = -(200\,\text{V})[5(0.4\,\text{A})]$$

$$= -(200\,\text{V})(2\,\text{A}) = -400\,\text{W}$$

Thus the voltage source delivers 400 W of power to the circuit. Check:

$\sum P_{\text{dis}} = 160 + 48 + 192 = 400\,\text{W}$

$\sum P_{\text{del}} = 400\,\text{W}$

P 2.15 **[a]**

$$v = 8i_{\text{a}} + 14i_{\text{a}} + 18i_{\text{a}} = 40(20) = 800\text{ V}$$

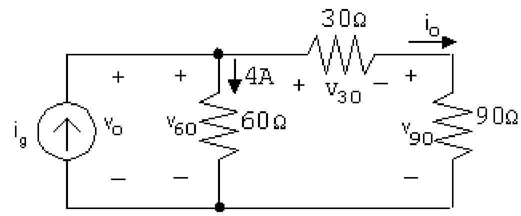
$$800 = 10i$$

$$i = 800/10 = 80\text{ A}$$

[b] $i \;\; = i_{\text{a}} + i \;\; = 20 + 80 = 100$ A

[c] p (delivered) $= (100)(800) = 80{,}000$ W $= 80$ kW

P 2.16



[a] Write a KVL equation clockwise around the right loop:

$$-v_{60} + v_{30} + v_{90} = 0$$

From Ohm's law,

$$v_{60} = (60\,\Omega)(4\,\text{A}) = 240\,\text{V}, \qquad v_{30} = 30i\ , \qquad v_{90} = 90i$$

Substituting,

$$-240\,\text{V} + 30i\ + 90i\ = 0 \qquad \text{so} \qquad 120i\ = 240\,\text{V}$$

$$\text{Thus} \qquad i\ = \frac{240\,\text{V}}{120} = 2\,\text{A}$$

Now write a KCL equatiohn at the top middle node, summing the currents leaving:

$$-i\ + 4\,\text{A} + i\ = 0 \qquad \text{so} \qquad i\ = 4\,\text{A} + 2\,\text{A} = 6\,\text{A}$$

[b] Write a KVL equation clockwise around the left loop:

$$-v\ + v_{60} = 0 \qquad \text{so} \qquad v\ = v_{60} = 240\,\text{V}$$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$$p_{\text{source}} = -v\ i\ = -(240\,\text{V})(6\,\text{A}) = -1440\,\text{W}$$

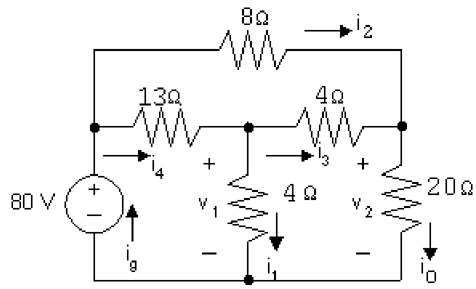
$$p_{60\Omega} = 4^2(60) = 960\,\text{W}$$

$$p_{30\Omega} = 30i^2 = (30)2^2 = 120\,\text{W}$$

$$p_{90\Omega} = 90i^2 = (90)2^2 = 360\,\text{W}$$

$$\sum P_{\text{dev}} = 1440\,\text{W} \qquad \sum P_{\text{abs}} = 960 + 120 + 360 = 1440\,\text{W}$$

P 2.17 **[a]**



$v_2 = 2(20) = 40$ V
$v_{8\Omega} = 80 - 40 = 40$ V
$i_2 = 40\,\text{V}/8\,\Omega = 5\,\text{A}$
$i_3 = i\ \ - i_2 = 2 - 5 = -3$ A
$v_{4\Omega} = (-3)(4) = -12$ V
$v_1 = 4i_3 + v_2 = -12 + 40 = 28$ V
$i_1 = 28\,\text{V}/4\,\Omega = 7$ A

[b] $i_4 = i_1 + i_3 = 7 - 3 = 4$ A

$$p_{13\Omega} = 4^2(13) = 208\text{ W}$$

$$p_{8\Omega} = (5)^2(8) = 200\text{ W}$$

$$p_{4\Omega} = 7^2(4) = 196\text{ W}$$

$$p_{4\Omega} = (-3)^2(4) = 36\text{ W}$$

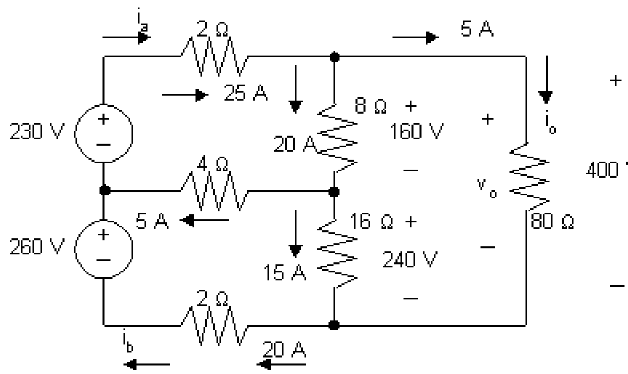
$$p_{20\Omega} = 2^2(20) = 80\text{ W}$$

[c] $\sum P_{\text{dis}} = 208 + 200 + 196 + 36 + 80 = 720\,\text{W}$

$i\ \ = i_4 + i_2 = 4 + 5 = 9\,\text{A}$

$P_{\text{dev}} = (9)(80) = 720\,\text{W}$

P 2.18 **[a]**



$$v = 20(8) + 16(15) = 400 \text{ V}$$

$$i = 400/80 = 5 \text{ A}$$

$$i_a = 25 \text{ A}$$

P_{230} (supplied) $= (230)(25) = 5750\,\text{W}$

$i_b = 5 + 15 = 20\,\text{A}$

P_{260} (supplied) $= (260)(20) = 5200\,\text{W}$

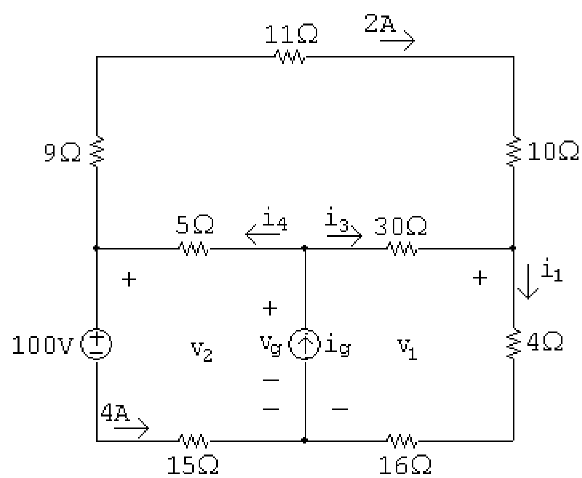
[b] $\sum P_{dis} = (25)^2(2) + (20)^2(8) + (5)^2(4) + (15)^2 16 + (20)^2 2 + (5)^2(80)$

$= 1250 + 3200 + 100 + 3600 + 800 + 2000 = 10{,}950 \text{ W}$

$\sum P_{sup} = 5750 + 5200 = 10{,}950 \text{ W}$

Therefore, $\sum P_{dis} = \sum P_{sup} = 10{,}950 \text{ W}$

P 2.19 **[a]**



$v_2 = 100 + 4(15) = 160\,\text{V}; \qquad v_1 = 160 - 30(2) = 100\,\text{V}$

$i_1 = \dfrac{v_1}{20} = \dfrac{100}{20} = 5\,\text{A}; \qquad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$

$$v = v_1 + 30i_3 = 100 + 30(3) = 190\,\text{V}$$

$$v - 5i_4 = v_2 \qquad \text{so} \qquad 5i_4 = v - v_2 = 190 - 160 = 30\,\text{V}$$

Thus $\quad i_4 = \dfrac{30}{5} = 6\,\text{A}$

$$i = i_3 + i_4 = 3 + 6 = 9\,\text{A}$$

[b] Calculate power using the formula $p = Ri^2$:

$p_{9\,\Omega} = (9)(2)^2 = 36\,\text{W}$; $\quad p_{11\,\Omega} = (11)(2)^2 = 44\,\text{W}$

$p_{10\,\Omega} = (10)(2)^2 = 40\,\text{W}$; $\quad p_{30\,\Omega} = (30)(3)^2 = 270\,\text{W}$

$p_{5\,\Omega} = (5)(6)^2 = 180\,\text{W}$; $\quad p_{4\,\Omega} = (4)(5)^2 = 100\,\text{W}$

$p_{16\,\Omega} = (16)(5)^2 = 400\,\text{W}$; $\quad p_{15\,\Omega} = (15)(4)^2 = 240\,\text{W}$

[c] $v = 190\,\text{V}$

[d] Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 270 + 180 + 100 + 400 + 240 = 1310\,\text{W}$$

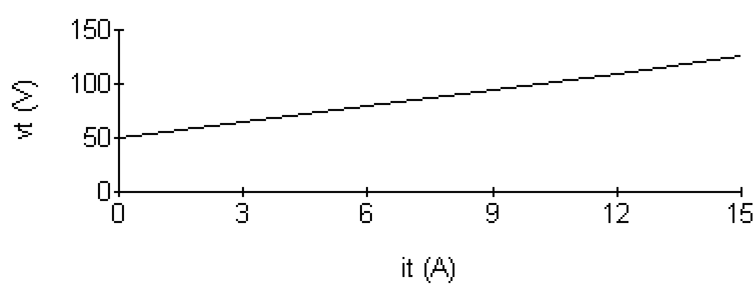
The power associated with the sources is

$$p_{\text{volt-source}} = (100\,\text{V})(4\,\text{A}) = 400\,\text{W}$$

$$p_{\text{curr-source}} = -v\,i = -(190\,\text{V})(9\,\text{A}) = -1710\,\text{W}$$

Thus the total power dissipated is $1310 + 400 = 1710$ W and the total power developed is 1710 W, so the power balances.

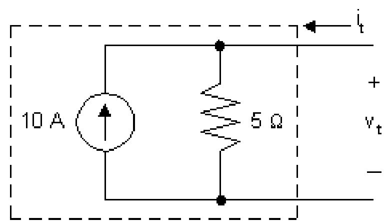
P 2.20 **[a]** Plot tl



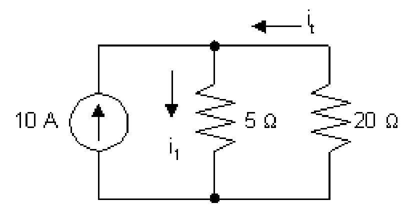
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(125 - 50)}{(15 - 0)} = 5\,\Omega$$

When $i = 0$, $v = 50$ V; therefore the ideal current source has a current of 10 A



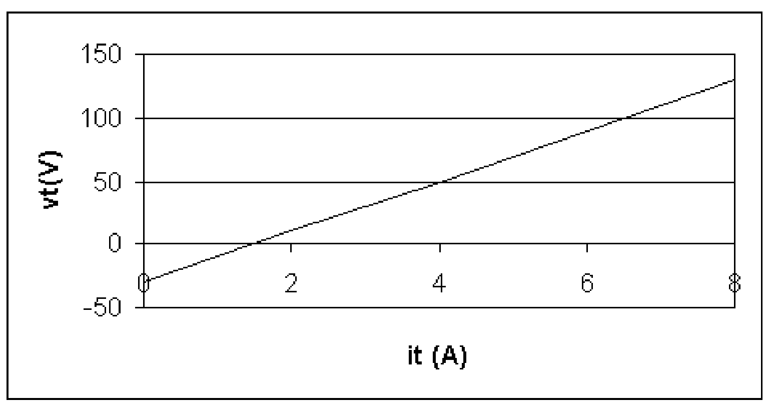
[b]



$10 + i = i_1$ and $5i_1 = -20i$

Therefore, $10 + i = -4i$ so $i = -2$ A

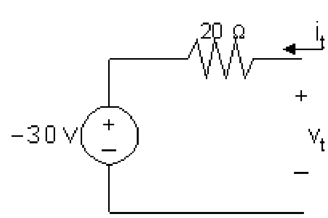
P 2.21 **[a]** Plot the v—i characteristic:



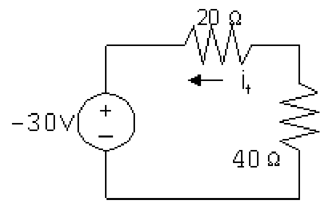
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{130 - (-30)}{8 - 0} = 20\,\Omega$$

When $i = 0$, $v = -30$ V; therefore the ideal voltage source has a voltage of -30 V. Thus the device can be modeled as a -30 V source in series with a $20\,\Omega$ resistor, as shown below:



[b] We attach a $40\,\Omega$ resistor to the device model developed in part (a):



Write a KVL equation clockwise around the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current i through the resistors:

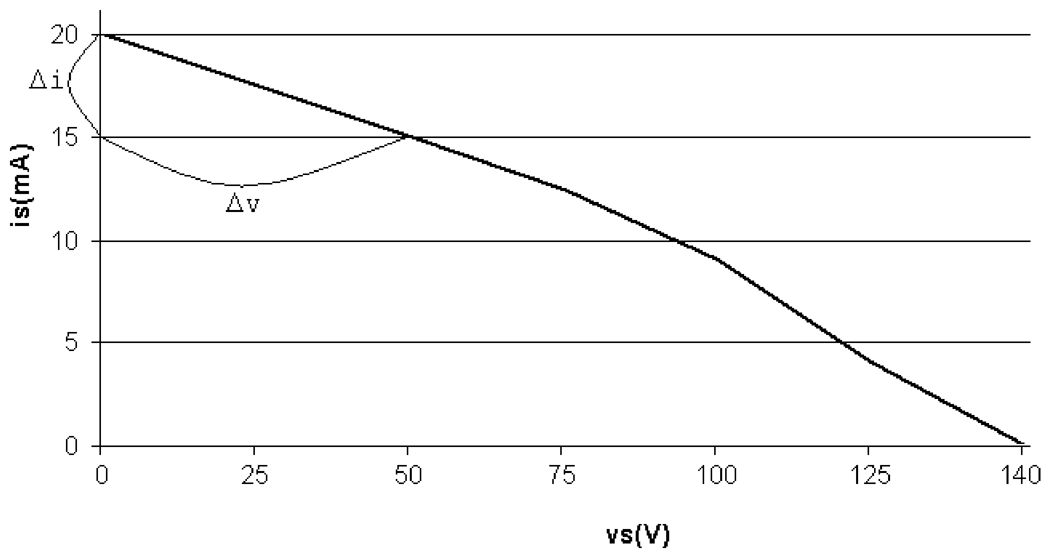
$$-(-30\,\mathrm{V}) - 20i - 40i = 0 \qquad \text{so} \qquad -60i = -30\,\mathrm{V}$$

$$\text{Thus} \qquad i = \frac{-30\,\mathrm{V}}{-60} = +0.5\,\mathrm{A}$$

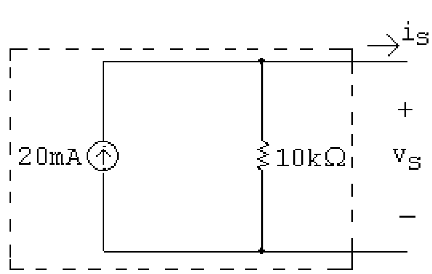
Now calculate the power dissipated by the resistor:

$$p_{40\,\Omega} = 40i^2 = (40)(0.5)^2 = 10\,\mathrm{W}$$

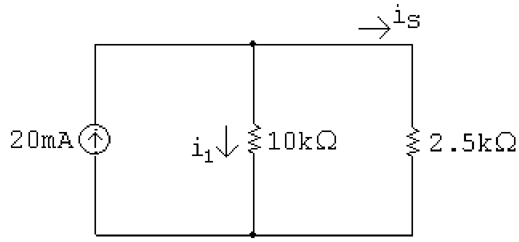
P 2.22 **[a]**



[b] $\Delta v = 50\,\mathrm{V}; \qquad \Delta i = 5\,\mathrm{mA}; \qquad R = \dfrac{\Delta v}{\Delta i} = \dfrac{50\,\mathrm{V}}{5\,\mathrm{mA}} = 10\,\mathrm{k}\Omega$



[c]



$10{,}000 i_1 = 2500 i \qquad \text{so} \qquad i_1 = 0.25 i$

$0.02 = i_1 + i = 0.25 i + i = 1.25 i$

$$\text{Thus,} \qquad i = \frac{0.02}{1.25} = 0.016\,\text{A} = 16\,\text{mA}$$

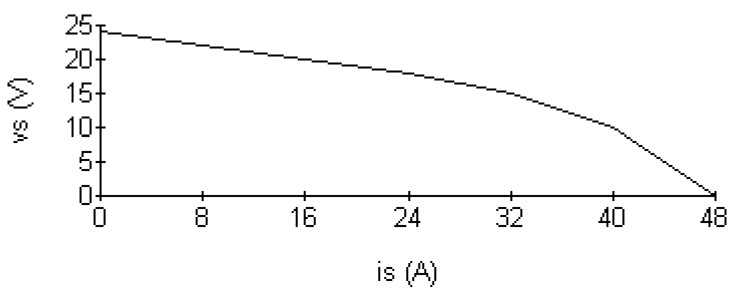
[d] Predicted open circuit voltage:

$v_{\text{oc}} = v = (0.02)(10{,}000) = 200\,\text{V}$

[e] From the table, the actual open circuit voltage is 140 V.

[f] This is a practical current source and is not a linear device.

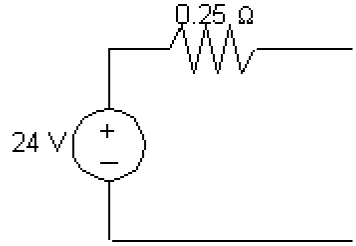
P 2.23 **[a]** Begin



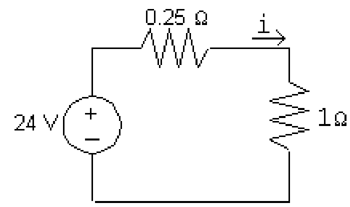
[b] Since the plot is linear for $0 \leq i \leq 24$ A amd since $R = \Delta v / \Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{24 - 18}{24 - 0} = \frac{6}{24} = 0.25\,\Omega$$

We can determine the value of the ideal voltage source by considering the value of v when $i = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 24 V. The model, valid for $0 \leq i \leq 24$



[c] The circuit is shown below:

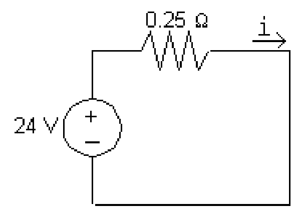


Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 0.25i + 1i = 0 \qquad \text{so} \qquad 1.25i = 24\,\text{V}$$

Thus, $\quad i = \dfrac{24\,\text{V}}{1.25\,\Omega} = 19.2\,\text{A}$

[d] The circuit is shown below:



Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 0.25i = 0 \qquad \text{so} \qquad 0.25i = 24\,\text{V}$$

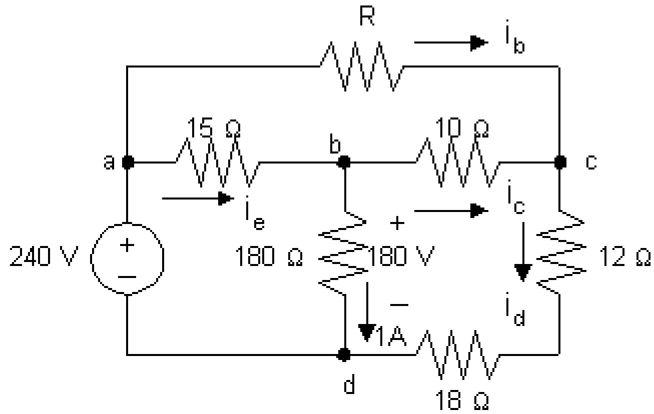
Thus, $\quad i = \dfrac{24\,\text{V}}{0.25\,\Omega} = 96\,\text{A}$

[e] The short circuit current can be found in the table of values (or from the plot) as the value of the current i when the voltage $v = 0$. Thus,

$i = 48\,\text{A}$ (from table)

[f] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

P 2.24



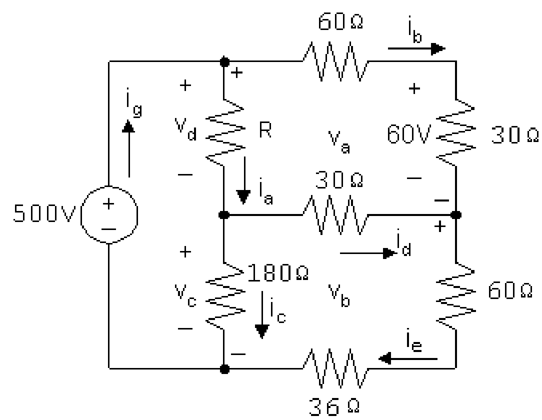
$v_{ab} = 240 - 180 = 60\,\text{V};$ therefore, $i_e = 60/15 = 4\,\text{A}$
$i_c = i_e - 1 = 4 - 1 = 3\,\text{A};$ therefore, $v_{bc} = 10i_c = 30\,\text{V}$
$v_{cd} = 180 - v_{bc} = 180 - 30 = 150\,\text{V};$
therefore, $i_d = v_{cd}/(12 + 18) = 150/30 = 5\,\text{A}$
$i_b = i_d - i_c = 5 - 3 = 2\,\text{A}$
$v_{ac} = v_{ab} + v_{bc} = 60 + 30 = 90\,\text{V}$
$R = v_{ac}/i_b = 90/2 = 45\,\Omega$

CHECK: $i \;= i_b + i_e = 2 + 4 = 6$ A

$p_{dev} = (240)(6) = 1440$ W

$\sum P_{dis} =$ $1(180) + 4(45) + 9(10) + 25(12) + 25(18) + 16(15) = 1440$ W (CHECKS)

P 2.25 **[a]**



$i \;= 60\,\text{V}/30\,\Omega = 2\,\text{A}$
$v \;= (30 + 60)(2) = 180\,\text{V}$
$-500 + v \;+ v \;= 0$ so $v \;= 500 - v \;= 500 - 180 = 320\,\text{V}$
$i \;= v \;/(60 + 36) = 320/96 = (10/3)\,\text{A}$
$i \;= i \;- i \;= (10/3) - 2 = (4/3)\,\text{A}$
$v \;= 30i \;+ v \;= 40 + 320 = 360\,\text{V}$
$i \;= v \;/180 = 360/180 = 2\,\text{A}$
$v \;= 500 - v \;= 500 - 360 = 140\,\text{V}$
$i \;= i \;+ i \;= 4/3 + 2 = (10/3)\,\text{A}$
$R = v \;/i \;= 140/(10/3) = 42\,\Omega$

[b] $i \;= i \;+ i \;= (10/3) + 2 = (16/3)$ A
p (supplied) $= (500)(16/3) = 2666.67$ W

P 2.26 **[a]** Start with the $22.5\,\Omega$ resistor. Since the voltage drop across this resistor is 90 V, we can use Ohm's law to calculate the current:

$$i_{22\ 5\,\Omega} = \frac{90\,\mathrm{V}}{22.5\,\Omega} = 4\,\mathrm{A}$$

Next we can calculate the voltage drop across the $15\,\Omega$ resistor by writing a KVL equation around the outer loop of the circuit:

$$-240\,\mathrm{V} + 90\,\mathrm{V} + v_{15\,\Omega} = 0 \qquad \text{so} \qquad v_{15\,\Omega} = 240 - 90 = 150\,\mathrm{V}$$

Now that we know the voltage drop across the $15\,\Omega$ resistor, we can use Ohm's law to find the current in this resistor:

$$i_{15\,\Omega} = \frac{150\,\mathrm{V}}{15\,\Omega} = 10\,\mathrm{A}$$

Write a KCL equation at the middle right node to find the current through the $5\,\Omega$ resistor. Sum the currents entering:

$$4\,\mathrm{A} - 10\,\mathrm{A} + i_{5\,\Omega} = 0 \qquad \text{so} \qquad i_{5\,\Omega} = 10\,\mathrm{A} - 4\,\mathrm{A} = 6\,\mathrm{A}$$

Write a KVL equation clockwise around the upper right loop, starting below the $4\,\Omega$ resistor. Use Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v_{4\,\Omega} + 90\,\mathrm{V} + (5\,\Omega)(-6\,\mathrm{A}) = 0 \qquad \text{so} \qquad v_{4\,\Omega} = 90\,\mathrm{V} - 30\,\mathrm{V} = 60\,\mathrm{V}$$

Using Ohm's law we can find the current through the $4\,\Omega$ resistor:

$$i_{4\,\Omega} = \frac{60\,\mathrm{V}}{4\,\Omega} = 15\,\mathrm{A}$$

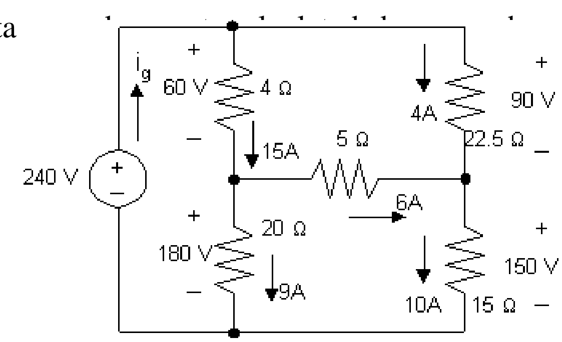
Write a KCL equation at the middle node. Sum the currents entering:

$$15\,\mathrm{A} - 6\,\mathrm{A} - i_{20\,\Omega} = 0 \qquad \text{so} \qquad i_{20\,\Omega} = 15\,\mathrm{A} - 6\,\mathrm{A} = 9\,\mathrm{A}$$

Use Ohm's law to calculate the voltage drop across the $20\,\Omega$ resistor:

$$v_{20\,\Omega} = (20\,\Omega)(9\,\mathrm{A}) = 180\,\mathrm{V}$$

All of the volta ı in the figure below:



Calculate the power dissipated by the resistors using the equation $p = Ri^2$:

$$p_{4\Omega} = (4)(15)^2 = 900\,\mathrm{W} \qquad p_{20\Omega} = (20)(9)^2 = 1620\,\mathrm{W}$$

$$p_{5\Omega} = (5)(6)^2 = 180\,\mathrm{W} \qquad p_{22\ 5\Omega} = (22.5)(4)^2 = 360\,\mathrm{W}$$

$$p_{15\Omega} = (15)(10)^2 = 1500\,\mathrm{W}$$

[b] We can calculate the current in the voltage source, i by writing a KCL equation at the top middle node:

$i \quad = 15\,\text{A} + 4\,\text{A} = 19\,\text{A}$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

$p \quad = -240(19) = -4560\,\text{W}$ thus p (supplied) $= 4560\,\text{W}$

[c] $\sum P_{\text{dis}} = 900 + 1620 + 180 + 360 + 1500 = 4560\,\text{W}$
Therefore,

$\sum P_{\text{supp}} = \sum P_{\text{dis}}$

P 2.27 $i \quad - i \quad - i \quad = 0$

$i \quad = \beta i$ therefore $i \quad = (1 + \beta)i$

$i_2 = -i \quad + i_1$

$V \quad + i \quad R \quad - (i_1 - i \quad)R_2 = 0$

$-i_1 R_1 + V \quad - (i_1 - i \quad)R_2 = 0$ or $i_1 = \dfrac{V \quad + i \quad R_2}{R_1 + R_2}$

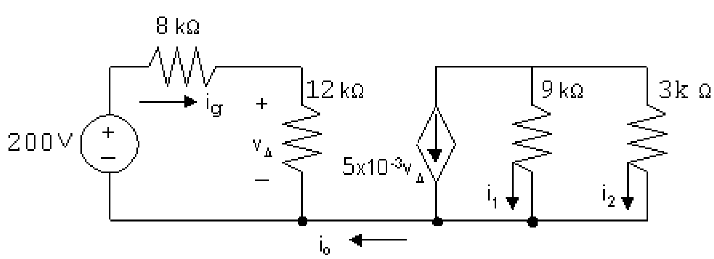
$V \quad + i \quad R \quad + i \quad R_2 - \dfrac{V \quad + i \quad R_2}{R_1 + R_2}R_2 = 0$

Now replace i by $(1 + \beta)i$ and solve for i. Thus

$i \quad = \dfrac{[V \quad R_2/(R_1 + R_2)] - V}{(1 + \beta)R \quad + R_1 R_2/(R_1 + R_2)}$

P 2.28 **[a]** $i \quad = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



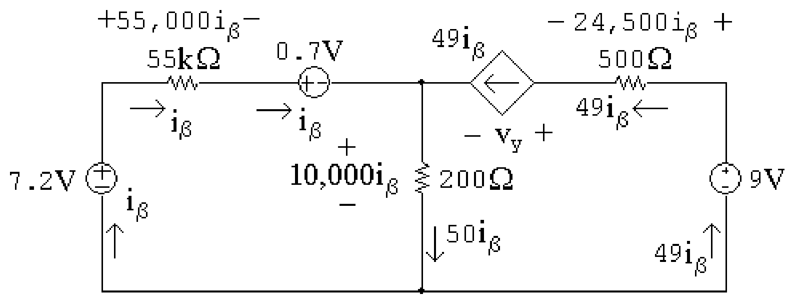
$-200 + 8000i \quad + 12{,}000i \quad = 0$ so $i \quad = 200/20{,}000 = 10$ mA
$v_\Delta = (12 \times 10^3)(10 \times 10^{-3}) = 120$ V
$5 \times 10^{-3}v_\Delta = 0.6\,\text{A}$
$9000i_1 = 3000i_2$ so $i_2 = 3i_1$
$0.6 + i_1 + i_2 = 0$ so $0.6 + i_1 + 3i_1 = 0$ thus $i_1 = -0.15$ A

[c] $i_2 = 3i_1 = -0.45$ A

P 2.29 First note that we know the current through all elements in the circuit except the $200\,\Omega$ resistor (the current in the three elements to the left of the $200\,\Omega$ resistor is i ; the current in the three elements to the right of the $200\,\Omega$ resistor is $49i$). To find the current in the $200\,\Omega$ resistor, write a KCL equation at the top node:

$$i \;+ 49i \;= i_{200\,\Omega} = 50i$$

We can then use Ohm's law to find the voltages across each resistor in terms of i . The results are shown in the figure below:



[a] To find i , write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 7.2V source:

$$-7.2\,\text{V} + 55{,}000i_1 + 0.7\,\text{V} + 10{,}000i \;= 0$$

Solving for i

$$55{,}000i \;+ 10{,}000i \;= 6.5\,\text{V} \qquad \text{so} \qquad 65{,}000i \;= 6.5\,\text{V}$$

Thus,

$$i \;= \frac{6.5}{65{,}000} = 100\,\mu\text{A}$$

Now that we have the value of i , we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$-v \;- 24{,}500i \;+ 9\,\text{V} - 10{,}000i \;= 0$$

Thus,

$$v \;= 9\,\text{V} - 34{,}500i \;= 9\,\text{V} - 34{,}500(100 \times 10^{-6}) = 9\,\text{V} - 3.45\,\text{V} = 5.55\,\text{V}$$

[b] We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (μA)**	**Voltage (V)**	**Power Equation**	**Power (μW)**
$7.2\,\text{V}$	100	7.2	$p = -vi$	-720
$55\,\text{k}\Omega$	100	5.5	$p = Ri^2$	550
$0.7\,\text{V}$	100	0.7	$p = vi$	70
$200\,\Omega$	5000	1	$p = Ri^2$	5000
Dep. source	4900	5.55	$p = vi$	27,195
$500\,\Omega$	4900	2.45	$p = Ri^2$	12,005
$9\,\text{V}$	4900	9	$p = -vi$	$-44{,}100$

The total power generated in the circuit is the sum of the negative power values in the power table:

$$-720\,\mu\text{W} + -44{,}100\,\mu\text{W} = -44{,}820\,\mu\text{W}$$

Thus, the total power generated in the circuit is $44{,}820\,\mu$W. The total power absorbed in the circuit is the sum of the positive power values in the power table:

$$550\,\mu\text{W} + 70\,\mu\text{W} + 5000\,\mu\text{W} + 27{,}195\,\mu\text{W} + 12{,}005\,\mu\text{W} = 44{,}820\,\mu\text{W}$$

Thus, the total power absorbed in the circuit is $44{,}820\,\mu$W and the power in the circuit balances.

P 2.30 **[a]** $12 - 2i \;\; = 5i_\Delta$

$5i_\Delta = 8i \;\; + 2i \;\; = 10i$

Therefore, $12 - 2i \;\; = 10i \;\;$, so $i \;\; = 1\,\text{A}$

$5i_\Delta = 10i \;\; = 10;$ so $i_\Delta = 2\,\text{A}$

$v \;\; = 2i \;\; = 2\,\text{V}$

[b] $i \;\;$ = current out of the positive terminal of the 12 V source
v_d = voltage drop across the $8i_\Delta$ source

$i \;\; = i_\Delta + i \;\; + 8i_\Delta = 9i_\Delta + i \;\; = 19\,\text{A}$

$v \;\; = 2 + 8 = 10\,\text{V}$

$$\sum P_{\text{gen}} = 12i \; + 8i_\Delta(8) = 12(19) + 8(2)(8) = 356 \text{ W}$$

$$\sum P_{\text{diss}} = 2i \; i \; + 5i_\Delta^2 + 8i \; (i \; + 8i_\Delta) + 2i^2 + 8i_\Delta v$$

$$= 2(1)(19) + 5(2)^2 + 8(1)(17) + 2(1)^2 + 8(2)(10)$$

$$= 356 \text{ W; Therefore,}$$

$$\sum P_{\text{gen}} = \sum P_{\text{diss}} = 356 \text{ W}$$

P 2.31 $40i_2 + \frac{5}{40} + \frac{5}{10} = 0$ so $i_2 = -15.625\,\text{mA}$

$v_1 = 80i_2 = -1.25\,\text{V}$

$25i_1 + \frac{-1.25}{20} + (-15.625 \times 10^{-3}) = 0$ so $i_1 = 3.125\,\text{mA}$

$v \; = 60i_1 + 260i_1 = 320i_1$

Therefore, $v \; = 1\,\text{V}$

P 2.32 $\frac{V \quad R_2}{R_1 + R_2} = \frac{(10)(60 \times 10^3)}{100 \times 10^3} = 6\text{V}$

$\frac{R_1 R_2}{R_1 + R_2} = \frac{(40 \times 10^3)(60 \times 10^3)}{100 \times 10^3} = 24\text{ k}\Omega$

$i \; = \frac{6 - 0.6}{24 \times 10^3 + 50(120)} = \frac{5.4}{(24 + 6) \times 10^3} = 0.18\text{ mA}$

$i \; = \beta i \; = (49)(0.18) = 8.82\text{ mA}$

$i \; = i \; + i \; = 8.82 + 0.18 = 9\text{ mA}$

$v_3 \; = (0.009)(120) = 1.08\,\text{V}$

$v \; = V \; + v_3 \; = 1.68\,\text{V}$

$i_2 = \frac{v}{R_2} = \frac{1.68}{60} \times 10^{-3} = 28\,\mu\text{A}$

$i_1 = i_2 + i \; = 28\,\mu\text{A} + 180\,\mu\text{A} = 208\,\mu\text{A}$

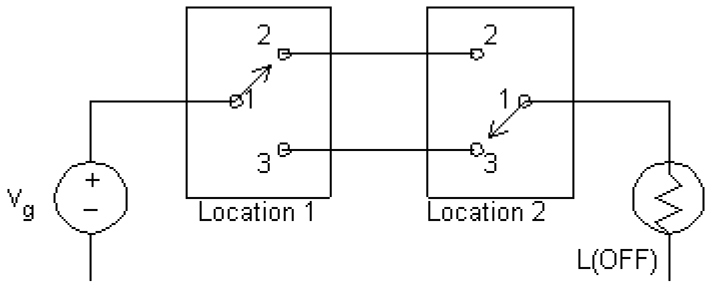
$v_{\text{ab}} = (40 \times 10^3)(208 \times 10^{-6}) = 8.32\text{V}$

$i \; = i \; + i_1 = 8.82\text{ mA} + 208\,\mu\text{A} = 9.028\text{ mA}$

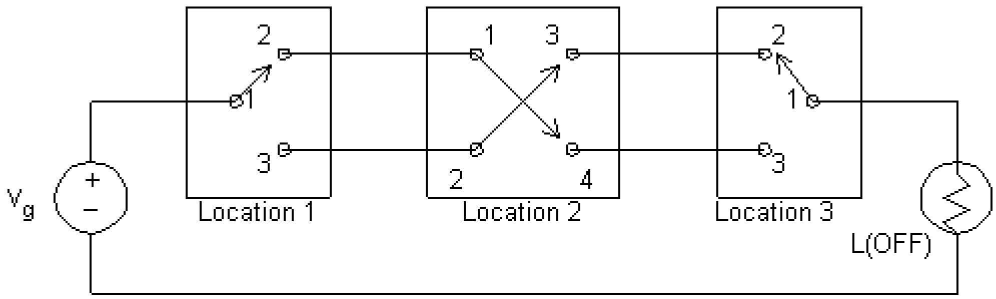
$v_{13} + (8.82 \times 10^{-3})(750) + 1.08 = 10$

Thus, $v_{13} = 2.305\text{V}$

P 2.33 **[a]**



[b]

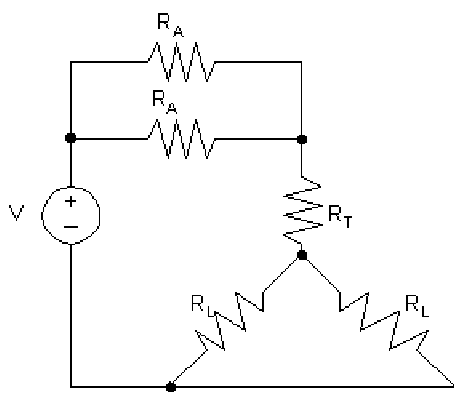


P 2.34 From the simplified circuit model, using Ohm's law and KVL:

$$400i + 50i + 200i - 250 = 0 \qquad \text{so} \qquad i = 250/650 = 385 \text{ mA}$$

This current is nearly enough to stop the heart, according to Table 2.1, so a warning sign should be posted at the 250 V source.

P 2.35



P 2.36 **[a]** $p = i^2R$

$$p_{\text{arm}} = \left(\frac{250}{650}\right)^2 (400) = 59.17\,\text{W}$$

$$p_{\text{leg}} = \left(\frac{250}{650}\right)^2 (200) = 29.59\,\text{W}$$

$$p_{\text{trunk}} = \left(\frac{250}{650}\right)^2 (50) = 7.40\,\text{W}$$

[b] $$\left(\frac{dT}{dt}\right)_{\text{arm}} = \frac{2.39 \times 10^{-4} p_{\text{arm}}}{4} = 35.36 \times 10^{-4}\,^\circ\text{ C/s}$$

$$t_{\text{arm}} = \frac{5}{35.36} \times 10^4 = 1414.23 \text{ s or } 23.57 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{leg}} = \frac{2.39 \times 10^{-4}}{10} P_{\text{leg}} = 7.07 \times 10^{-4\circ}\text{ C/s}$$

$$t_{\text{leg}} = \frac{5 \times 10^4}{7.07} = 7{,}071.13 \text{ s or } 117.85 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{trunk}} = \frac{2.39 \times 10^{-4}(7.4)}{25} = 0.71 \times 10^{-4}\,^\circ\text{ C/s}$$

$$t_{\text{trunk}} = \frac{5 \times 10^4}{0.71} = 70{,}677.37 \text{ s or } 1{,}177.96 \text{ min}$$

[c] They are all much greater than a few minutes.

P 2.37 **[a]** $R_{\text{arms}} = 400 + 400 = 800\Omega$

$$i_{\text{letgo}} = 50 \text{ mA (minimum)}$$

$$v_{\text{min}} = (800)(50) \times 10^{-3} = 40\,\text{V}$$

[b] No, $12/800 = 15$ mA. Note this current is sufficient to give a perceptible shock.

P 2.38 $R_{\text{space}} = 1\ \text{M}\Omega$

$$i_{\text{space}} = 3 \text{ mA}$$

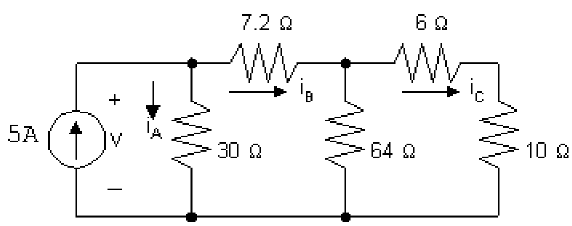
$$v = i_{\text{space}} R_{\text{space}} = 3000\,\text{V}.$$

3

Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the $6\,\Omega$ resistor and the $10\,\Omega$ resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this $16\,\Omega$ resistor in parallel with the $64\,\Omega$ resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

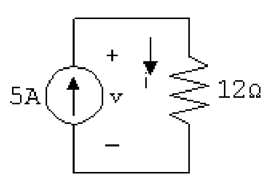
This equivalent $12.8\,\Omega$ resistor is in series with the $7.2\,\Omega$ resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent $20\,\Omega$ resistor is in parallel with the $30\,\Omega$ resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i \;\; = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i :

$-5\text{ A} + i \;\; + i \;\; = 0 \qquad \text{so} \qquad i \;\; = 5\text{ A} - i \;\; = 5\text{ A} - 2\text{ A} = 3\text{ A}$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$-v + 7.2i \;\; + 6i \;\; + 10i \;\; = 0$

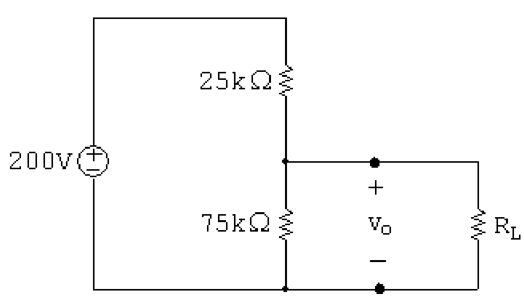
So $\qquad 16i \;\; = v - 7.2i \;\; = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$

Thus $\qquad i \;\; = \dfrac{38.4}{16} = 2.4\text{ A}$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$

AP 3.2



[a] We can use voltage division to calculate the voltage v across the $75\text{ k}\Omega$ resistor:

$$v\text{ (no load)} = \frac{75{,}000}{75{,}000 + 25{,}000}(200\text{ V}) = 150\text{ V}$$

[b] When we have a load resistance of $150\ \text{k}\Omega$ then the voltage v is across the parallel combination of the $75\ \text{k}\Omega$ resistor and the $150\ \text{k}\Omega$ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\ \text{k}\Omega \| 150\ \text{k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\ \text{k}\Omega$$

Now use voltage division to find v across this equivalent resistance:

$$v = \frac{50{,}000}{50{,}000 + 25{,}000}(200\ \text{V}) = 133.3\ \text{V}$$

[c] If the load terminals are short-circuited, the $75\ \text{k}\Omega$ resistor is effectively removed from the circuit, leaving only the voltage source and the $25\ \text{k}\Omega$ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\ \text{V}}{25\ \text{k}\Omega} = 8\ \text{mA}$$

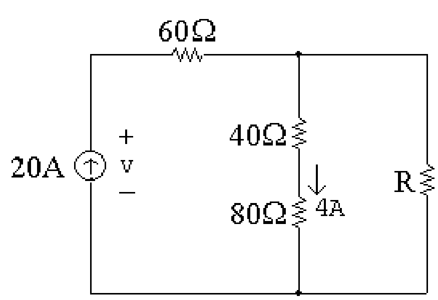
Now we can use the formula $p = Ri^2$ to find the power dissipated in the $25\ \text{k}\Omega$ resistor:

$$p_{25} = (25{,}000)(0.008)^2 = 1.6\ \text{W}$$

[d] The power dissipated in the $75\ \text{k}\Omega$ resistor will be maximum at no load since v is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75}\ (\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\ \text{W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\ \text{A}) = 4\ \text{A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i = \frac{40+80}{40+80+30}(20\text{ A}) = 16\text{ A} \qquad \text{so} \qquad p = (30)(16)^2 = 7680\text{ W}$$

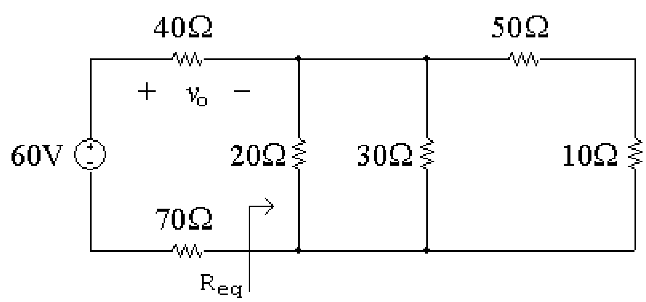
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\text{ A}) + (30\,\Omega)(16\text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\text{ V}$$

Thus, $p_{\text{source}} = -(1680\text{ V})(20\text{ A}) = -33{,}600\text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4



[a] First we need to determine the equivalent resistance to the right of the $40\,\Omega$ and $70\,\Omega$ resistors:

$$R_{\text{eq}} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v :

$$v = \frac{40}{40+10+70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the $40\,\Omega$ resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the $40\,\Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20\,\Omega$ resistor and the $50\,\Omega$ and $10\,\Omega$ resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20+60} = 15\,\Omega$$

Now we use current division to find the current in the $30\,\Omega$ branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

[c] We can find the power dissipated by the $50\,\Omega$ resistor if we can find the current in this resistor. We can use current division to find this current from the current in the $40\,\Omega$ resistor, but first we need to calculate the equivalent resistance of the $20\,\Omega$ branch and the $30\,\Omega$ branch:

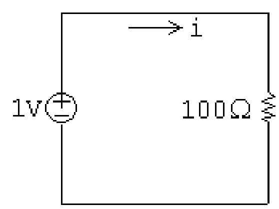
$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\text{ A}) = 0.08333\text{ A}$$

Thus, $\qquad p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$

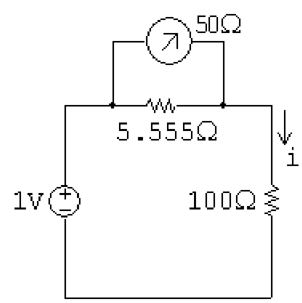
AP 3.5 **[a]**



We can find the current i using Ohm's law:

$$i = \frac{1\text{ V}}{100\,\Omega} = 0.01\text{ A} = 10\text{ mA}$$

[b]

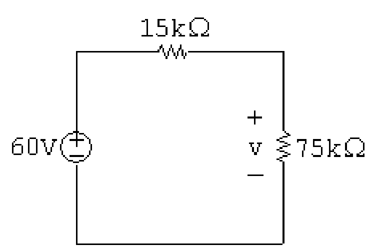


$$R \;= 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\text{ V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524\text{ mA}$$

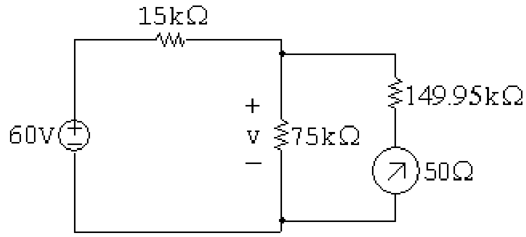
AP 3.6 **[a]**



Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000 + 15{,}000}(60\text{ V}) = 50\text{ V}$$

[b]



The meter resistance is a series combination of resistances:

$$R \;\; = 149{,}950 + 50 = 150{,}000\,\Omega$$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50\text{ k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60\text{ V}) = 46.15\text{ V}$$

AP 3.7 **[a]** Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R \;\; = (1000)(150) \qquad \text{so} \qquad R \;\; = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5\text{ k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R . We can find the current in the latter two branches using Ohm's law:

$$i_{1\;\;3} = \frac{5\text{ V}}{100\,\Omega + 150\,\Omega} = 20\text{ mA}; \qquad i_{2\;\;x} = \frac{5\text{ V}}{1000 + 1500} = 2\text{ mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02\text{ A})^2 = 40\text{ mW}$$

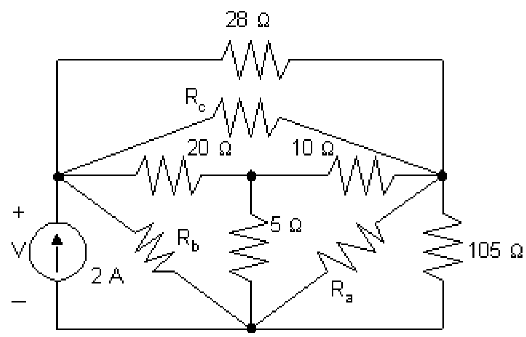
$$p_{150\Omega} = (150\,\Omega)(0.02\text{ A})^2 = 60\text{ mW}$$

$$p_{1000\Omega} = (1000\,\Omega)(0.002\text{ A})^2 = 4\text{ mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002\text{ A})^2 = 6\text{ mW}$$

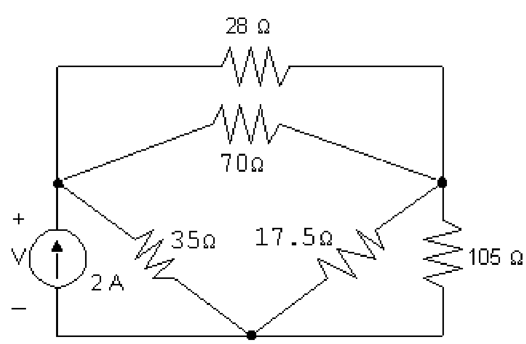
Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, $20\,\Omega$, $10\,\Omega$, and $5\,\Omega$ to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors



$$R_\mathrm{a} = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$
$$R_\mathrm{b} = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$
$$R_\mathrm{c} = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:



From this circuit we see that the $70\,\Omega$ resistor is parallel to the $28\,\Omega$ resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the $17.5\,\Omega$ resistor is parallel to the $105\,\Omega$ resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

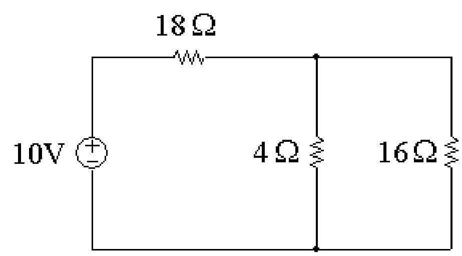
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

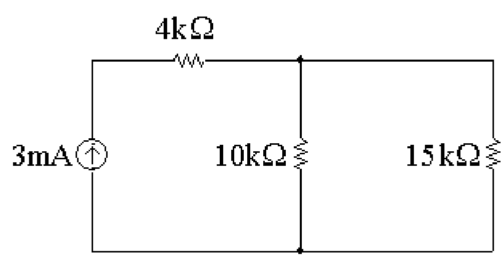
$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

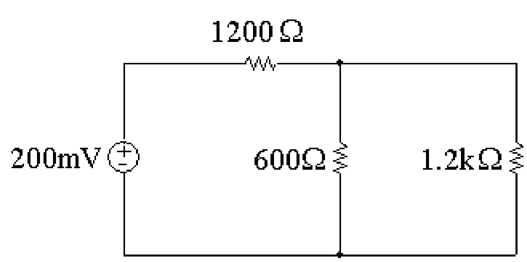
P 3.1 **[a]** The $6\,\Omega$ and $12\,\Omega$ resistors are in series, as are the $9\,\Omega$ and $7\,\Omega$ resistors. The simplified circuit is shown below:



[b] The 3 kΩ, 5 kΩ, and 7 kΩ resistors are in series. The simplified circuit is shown below:

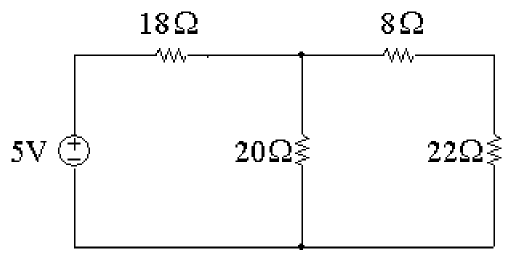


[c] The $300\,\Omega$, $400\,\Omega$, and $500\,\Omega$ resistors are in series. The simplified circuit is shown below:

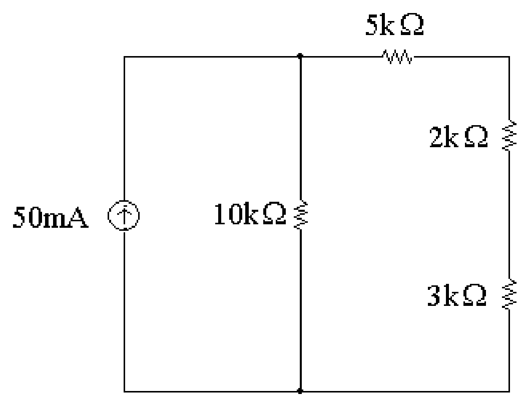


P 3.2 **[a]** The $10\,\Omega$ and $40\,\Omega$ resistors are in parallel, as are the $100\,\Omega$ and $25\,\Omega$ resistors.

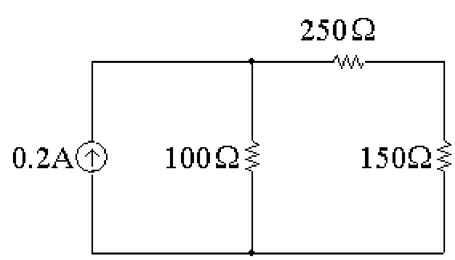
The simplified circuit is shown below:



[b] The 9 kΩ, 18 kΩ, and 6 kΩ resistors are in parallel. The simplified circuit is shown below:



[c] The 600 Ω, 200 Ω, and 300 Ω resistors are in series. The simplified circuit is shown below:



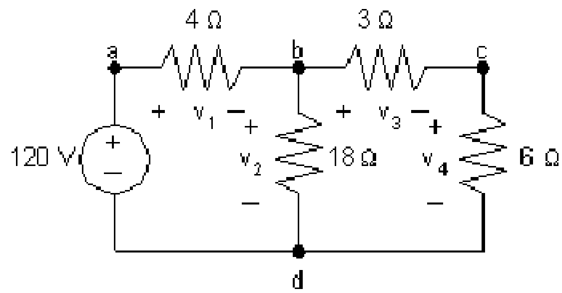
P 3.3 **[a]** $p_{4\Omega} = i^2 4 = (12)^2 4 = 576$ W $\quad p_{18\Omega} = (4)^2 18 = 288$ W

$p_{3\Omega} = (8)^2 3 = 192$ W $\quad p_{6\Omega} = (8)^2 6 = 384$ W

[b] $p_{120\text{V}}(\text{delivered}) = 120i = 120(12) = 1440$ W

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440$ W

P 3.4 **[a]** From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i = 12$ A
at node x: $-12 + 4 + 8 = 0$, $\quad$ at node y: $12 - 4 - 8 = 0$



[b] $v_1 = 4i = 48$ V $\qquad v_3 = 3i_2 = 24$ V

$v_2 = 18i_1 = 72$ V $\qquad v_4 = 6i_2 = 48$ V

loop abda: $-120 + 48 + 72 = 0,$

loop bcdb: $-72 + 24 + 48 = 0,$

loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{\text{eq}} = 6 + 12 + [4\|(9 + 7)] = 18 + (4\|16) = 18 + 3.2 = 21.2\,\Omega$

[b] $R_{\text{eq}} = 4\text{ k} + [10\text{ k}\|(3\text{ k} + 5\text{ k} + 7\text{ k})] = 4\text{ k} + (10\text{ k}\|15\text{ k}) = 4\text{ k} + 6\text{ k} = 10\text{ k}\Omega$

[c] $R_{\text{eq}} = (300 + 400 + 500) + (600\|1200) = 1200 + 400 = 1600\,\Omega$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{\text{eq}} = 18 + (100\|25\|(22 + (10\|40))) = 18 + (20\|(22 + 8) = 18 + 12 = 30\,\Omega$

[b] $R_{\text{eq}} = 10\text{ k}\|(5\text{ k} + 2\text{ k} + (9\text{ k}\|18\text{ k}\|6\text{ k})) = 10\text{ k}\|(7\text{ k} + 3\text{ k}) = 10\text{ k}\|10\text{ k} = 5\ \text{k}\Omega$

[c] $R_{\text{eq}} = 600\|200\|300\|(250 + 150) = 600\|200\|300\|400 = 80\,\Omega$

P 3.7 **[a]** $R_{\text{eq}} = 12 + (24\|(30 + 18)) + 10 = 12 + (24\|48) + 10 = 12 + 16 + 10 = 38\,\Omega$

[b] $R_{\text{eq}} = 4\text{ k}\|30\text{ k}\|60\text{ k}\|(1.2\text{ k} + (7.2\text{ k}\|2.4\text{ k}) + 2\text{ k}) = 4\text{ k}\|30\text{ k}\|60\text{ k}\|(3.2\text{ k} + 1.8\text{ k})$

$$= 4\text{ k}\|30\text{ k}\|60\text{ k}\|5\text{ k} = 2\ \text{k}\Omega$$

P 3.8 **[a]** $5\|20 = 100/25 = 4\,\Omega \qquad 5\|20 + 9\|18 + 10 = 20\,\Omega$

$9\|18 = 162/27 = 6\,\Omega \qquad 20\|30 = 600/50 = 12\,\Omega$

$R_{\text{ab}} = 5 + 12 + 3 = 20\,\Omega$

[b] $5 + 15 = 20\,\Omega$

$20\|60 = 1200/80 = 15\,\Omega$

$15 + 10 = 25\,\Omega$

$25\|75 = 1875/100 = 18.75\,\Omega$

$18.75 + 11.25 = 30\,\Omega$

$30\|20 = 600/50 = 12\,\Omega$

$3\|6 = 18/9 = 2\,\Omega$

$3\|6 + 30\|20 = 2 + 12 = 14\,\Omega$

$26\|14 = 364/40 = 9.1\,\Omega$

$R_{\text{ab}} = 2.5 + 9.1 + 3.4 = 15\,\Omega$

[c] $3 + 5 = 8\,\Omega$

$8\|12 = 96/20 = 4.8\,\Omega$

$4.8 + 5.2 = 10\,\Omega$

$45 + 15 = 60\,\Omega$

$60\|40 = 2400/100 = 24\,\Omega$

$24 + 6 = 30\,\Omega$

$30\|10 = 300/40 = 7.5\,\Omega$

$R_{\text{ab}} = 1.5 + 7.5 + 1.0 = 10\,\Omega$

P 3.9 **[a]** For circuit (a)

$$R_{\text{ab}} = 360\|(90 + 120\|(160 + 200)) = 360\|(90 + (120\|360)) = 360\|(90 + 90)$$
$$= 360\|180 = 120\,\Omega$$

For circuit (b)

$$\frac{1}{R} = \frac{1}{20} + \frac{1}{15} + \frac{1}{20} + \frac{1}{4} + \frac{1}{12} = \frac{30}{60} = \frac{1}{2}$$

$R = 2\,\Omega$

$R + 16 = 18\,\Omega$

$18\|18 = 9\,\Omega$

$R_{\text{ab}} = 10 + 8 + 9 = 27\,\Omega$

For circuit (c)

$15\|30 = 10\,\Omega$

$10 + 20 = 30\,\Omega$

$60\|30 = 20\,\Omega$

$20 + 10 = 30\,\Omega$

$30\|80\|(40 + 20) = 30\|80\|60 = 16\,\Omega$

$R_{\text{ab}} = 16 + 24 + 10 = 50\,\Omega$

[b] $P \ = (0.03^2)(120) = 108$ mW

$$P \ = \frac{144^2}{27} = 768 \text{ W}$$

$$P \ = \frac{0.08^2}{50} = 128\,\mu \text{ W}$$

P 3.10 The equivalent resistance to the right of the $10\,\Omega$ resistor is

$$(6 + 5\|(8 + 12)) = 6 + 5\|20 = 6 + 4 = 10\,\Omega.$$

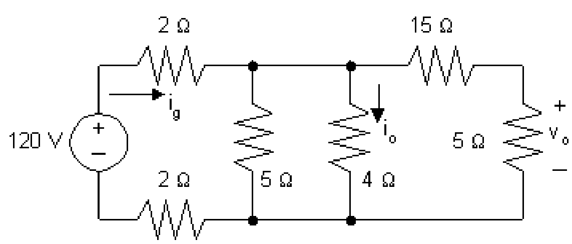
We can use current division to see that the current then splits equally between the two $10\,\Omega$ branches. Thus the current through the $6\,\Omega$ branch in the original circuit is 5 A. This 5 A current splits between the branch with the $5\,\Omega$ resistor and the branch with the $8 + 12 = 20\,\Omega$ resistor and we use current division to determine the current in the $5\,\Omega$ resistor:

$$i_{5\Omega} = \frac{20}{20+5}(5) = 4 \text{ A}$$

Thus the power in the $5\,\Omega$ resistor is

$$p_{5\Omega} = i_{5\Omega}^2(5) = 4^2(5) = 80 \text{ W}$$

P 3.11 **[a]**



$$R_{\text{eq}} = 2 + 2 + (1/4 + 1/5 + 1/20)^{-1} = 6\,\Omega$$

$$i \ = 120/6 = 20 \text{ A}$$

$$v_{4\Omega} = 120 - (2+2)20 = 40 \text{ V}$$

$$i \ = 40/4 = 10 \text{ A}$$

$$i_{(15+5)\Omega} = 40/(15+5) = 2 \text{ A}$$

$$v \ = (5)(2) = 10 \text{ V}$$

[b] $i_{15\Omega} = 2$ A; $\qquad P_{15\Omega} = (2)^2(15) = 60$ W

[c] $P_{120\text{V}} = (120)(20) = 2.4$ kW

P 3.12 **[a]** $R_{\text{eq}} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$R_{\text{eq}} = R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s})$$
$$= R\|\frac{R}{n-1}$$
$$= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n}$$

[c] One solution:

$$400 = \frac{2000}{n} \qquad \text{so} \qquad n = \frac{2000}{400} = 5$$

You can place 5 identical $2\ \text{k}\Omega$ resistors in parallel to get an equivalent resistance of $400\,\Omega$.

[d] One solution:

$$12{,}500 = \frac{100{,}000}{n} \qquad \text{so} \qquad n = \frac{100{,}000}{12{,}500} = 8$$

You can place 8 identical $100\ \text{k}\Omega$ resistors in parallel to get an equivalent resistance of $12.5\ \text{k}\Omega$.

P 3.13 **[a]** We can calculate the no-load voltage using voltage division to determine the voltage drop across the $500\,\Omega$ resistor:

$$v = \frac{500}{(2000 + 500)}(75\text{ V}) = 15\text{ V}$$

[b] We can calculate the power if we know the current in each of the resistors. Under no-load conditions, the resistors are in series, so we can use Ohm's law to calculate the current they share:

$$i = \frac{75\text{ V}}{2000\,\Omega + 500\,\Omega} = 0.03\text{ A} = 30\text{ mA}$$

Now use the formula $p = Ri^2$ to calculate the power dissipated by each resistor:

$$P_1 = (2000)(0.03)^2 = 1.8\text{ W} = 1800\text{ mW}$$

$$P_2 = (500)(0.03)^2 = 0.45\text{ W} = 450\text{ mW}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the no-load voltage requirement, first pick R_1 to meet the 1 W specification

$$i_1 = \frac{75 - 15}{R_1}, \qquad \text{Therefore, } \left(\frac{60}{R_1}\right)^2 R_1 \le 1$$

$$\text{Thus, } R_1 \ge \frac{60^2}{1} \qquad \text{or} \qquad R_1 \ge 3600\,\Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 3600}(75) = 15$$

Thus, $R_2 = 900\,\Omega$

$R_1 = 1600\,\Omega$ and $R_2 = 400\,\Omega$ are the smallest values of resistors that satisfy the 1 W specification.

P 3.14 Use voltage division to determine R_2 from the no-load voltage specification:

$$6\text{ V} = \frac{R_2}{(R_2 + 40)}(18\text{ V}); \qquad \text{so} \qquad 18R_2 = 6(R_2 + 40)$$

$$\text{Thus,} \qquad 12R_2 = 240 \qquad \text{so} \qquad R_2 = \frac{240}{12} = 20\,\Omega$$

Now use voltage division again, this time to determine the value of R_e, the parallel combination of R_2 and R . We use the loaded voltage specification:

$$4\text{ V} = \frac{R_e}{(40 + R_e)}(18\text{ V}) \qquad \text{so} \qquad 18R_e = 4(40 + R_e)$$

$$\text{Thus,} \qquad 14R_e = 160 \qquad \text{so} \qquad R_e = \frac{160}{14} = 11.43\,\Omega$$

Now use the definition R_e to calculate the value of R given that $R_2 = 20\,\Omega$:

$$R_e = \frac{20R}{20 + R} = 11.43 \qquad \text{so} \qquad 20R = 11.43(R + 20)$$

$$\text{Therefore,} \qquad 8.57R = 228.6 \qquad \text{and} \qquad R = \frac{226.8}{8.57} = 26.67\,\Omega$$

P 3.15 **[a]** From the constraint on the no-load voltage,

$$\frac{R_2}{R_1 + R_2}(40) = 8 \qquad \text{so} \qquad R_1 = 4R_2$$

From the constraint on the loaded voltage divider:

$$7.5 = \frac{\dfrac{3600R_2}{3600 + R_2}}{R_1 + \dfrac{3600R_2}{3600 + R_2}}(40)$$

$$= \frac{\dfrac{3600R_2}{3600 + R_2}}{4R_2 + \dfrac{3600R_2}{3600 + R_2}}(40)$$

$$= \frac{3600R_2}{4R_2(3600+R_2)+3600R_2}(40) = \frac{144{,}000R_2}{4R_2^2+18{,}000R_2}$$

So, $\dfrac{144{,}000}{4R_2+18{,}000} = 7.5 \qquad \therefore \quad R_2 = 300\,\Omega \quad$ and $\quad R_1 = 4R_2 = 1200\,\Omega$

[b] Power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur under load conditions.

$v_{1} = 40 - 7.5 = 32.5$ V; $\qquad P_{1} = \dfrac{(32.5)^2}{1200} = 880.2$ mW

So specify a 1 W power rating for the resistor R_1.
The power dissipated in R_2 will be maximum when the voltage drop across R_2 is maximum. This occurs under no-load conditions with $v \;= 8$ V.

$$P_{2} = \frac{(8)^2}{300} = 213.3 \text{ m W}$$

So specify a $1/4$ W power rating for the resistor R_2.

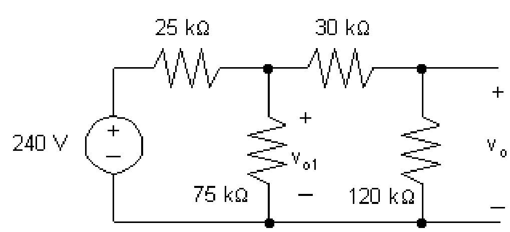
P 3.16 Refer to the solution of Problem 3.15. The divider will reach its dissipation limit when the power dissipated in R_1 equals 1 W

So $(v^2{}_{1}/1200) = 1; \quad v_{1} = 34.641$ V $\quad v \;= 40 - 34.641 = 5.359$ V

Therefore, $\dfrac{R_e}{1200+R_e}(40) = 5.359,\quad$ and $\quad R_e = 185.641\,\Omega$

$$\frac{1200R_L}{1200+R_L} = 185.641 \qquad \therefore \quad R_L = 219.62\,\Omega$$

P 3.17 **[a]**



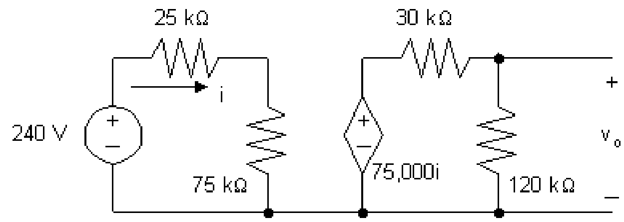
$120 \text{ k}\Omega + 30 \text{ k}\Omega = 150 \text{ k}\Omega$

$75 \text{ k}\Omega \| 150 \text{ k}\Omega = 50 \text{ k}\Omega$

$$v_{1} = \frac{240}{(25{,}000+50{,}000)}(50{,}000) = 160 \text{ V}$$

$$v \;= \frac{120{,}000}{(150{,}000)}(v_{1}) = 128 \text{ V}, \qquad v \;= 128 \text{ V}$$

[b]



$$i = \frac{240}{100{,}000} = 2.4\text{ mA}$$

$$75{,}000i = 180\text{ V}$$

$$v\ = \frac{120{,}000}{150{,}000}(180) = 144\text{ V}; \qquad v\ = 144\text{ V}$$

[c] It removes loading effect of second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_1 = \frac{75{,}000}{(100{,}000)}(240) = 180\text{ V}$$

Now note this is the input voltage to the second voltage divider when the current controlled voltage source is used.

P 3.18 $\frac{(24)^2}{R_1 + R_2 + R_3} = 36, \qquad$ Therefore, $R_1 + R_2 + R_3 = 16\,\Omega$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

Thus, $R_1 + R_2 = R_3; \qquad 2R_3 = 16; \qquad R_3 = 8\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 6$$

$4R_2 = R_1 + R_2 + R_3 \qquad$ so $R_2 = R_3/2 = 4\,\Omega$

$R_2 = 4\,\Omega; \qquad R_1 = 16 - 8 - 4 = 4\,\Omega$

P 3.19 Note – in the problem description, the first equation defines R_1 not R_L.

[a] At no load: $\quad v\ = kv\ = \frac{R_2}{R_1 + R_2}v\ .$

At full load: $\quad v\ = \alpha v\ = \frac{R_e}{R_1 + R_e}v\ , \qquad$ where $R_e = \frac{R\ R_2}{R\ + R_2}$

Therefore $\quad k = \frac{R_2}{R_1 + R_2} \quad$ and $\quad R_1 = \frac{(1-k)}{k}R_2$

$\alpha = \frac{R_e}{R_1 + R_e} \quad$ and $\quad R_1 = \frac{(1-\alpha)}{\alpha}R_e$

Thus $\left(\frac{1-\alpha}{\alpha}\right)\left[\frac{R_2 R}{R + R_2}\right] = \frac{(1-k)}{k} R_2$

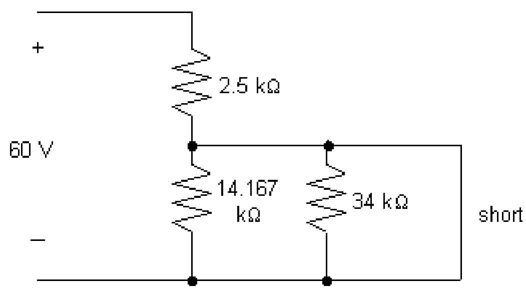
Solving for R_2 yields $R_2 = \frac{(k-\alpha)}{\alpha(1-k)} R$

Also, $R_1 = \frac{(1-k)}{k} R_2 \quad \therefore \quad R_1 = \frac{(k-\alpha)}{\alpha k} R$

[b] $R_1 = \left(\frac{0.05}{0.68}\right) R = 2.5\ \text{k}\Omega$

$R_2 = \left(\frac{0.05}{0.12}\right) R = 14.167\ \text{k}\Omega$

[c]



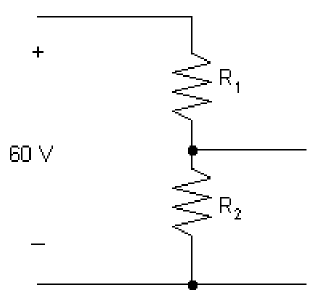
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{2(\text{max})} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6\ \text{mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{1(\text{max})} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60\ \text{mW}$$

[d]



$$P_1 = \frac{(60)^2}{2500} = 1.44\ \text{W} = 1440\ \text{mW}$$

$$P_2 = \frac{(0)^2}{14{,}167} = 0\ \text{W}$$

P 3.20 **[a]** Let v be the voltage across the parallel branches, positive at the upper terminal, then

$$i = v G_1 + v G_2 + \cdots + v G = v (G_1 + G_2 + \cdots + G)$$

It follows that $v = \frac{i}{(G_1 + G_2 + \cdots + G)}$

The current in the k^{th} branch is $\quad i \;\; = v \; G \;\; ;$ Thus,

$$i \;\; = \frac{i \; G}{[G_1 + G_2 + \cdot + G \;\;]}$$

[b] $i \;\; = \dfrac{120(0.00125)}{[0.0025 + 0.0004167 + 0.00125 + 0.000625 + 0.0002083]} = 30 \text{ mA}$

P 3.21 Begin by using the relationships among the branch currents to express all branch currents in terms of i_4:

$$i_1 = 2i_2 = 2(10i_3) = 20i_4$$

$$i_2 = 10i_3 = 10i_4$$

$$i_3 = i_4$$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.

$$i_1 + i_2 + i_3 + i_4 = 8 \text{ mA}$$

Express the branch currents in terms of i_4 and solve for i_4:

$$8 \text{ mA} = 20i_4 + 10i_4 + i_4 + i_4 = 32i_4 \qquad \text{so} \qquad i_4 = \frac{0.008}{32} = 0.00025 = 0.25 \text{ mA}$$

Since the resistors are in parallel, the same voltage, 4 V appears across each of them. We know the current and the voltage for R_4 so we can use Ohm's law to calculate R_4:

$$R_4 = \frac{v}{i_4} = \frac{4 \text{ V}}{0.25 \text{ mA}} = 16 \text{ k}\Omega$$

Calculate i_3 from i_4 and use Ohm's law as above to find R_3:

$$i_3 = i_4 = 0.25 \text{ mA} \qquad \therefore \quad R_3 = \frac{v}{i_3} = \frac{4 \text{ V}}{0.25 \text{ mA}} = 16 \text{ k}\Omega$$

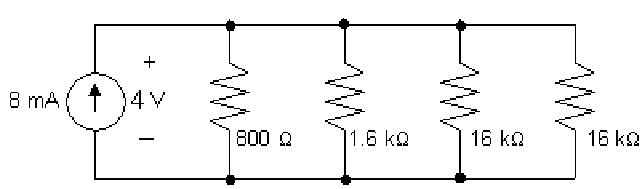
Calculate i_2 from i_4 and use Ohm's law as above to find R_2:

$$i_2 = 10i_4 = 10(0.25 \text{ mA}) = 2.5 \text{ mA} \qquad \therefore \quad R_2 = \frac{v}{i_2} = \frac{4 \text{ V}}{2.5 \text{ mA}} = 1.6 \text{ k}\Omega$$

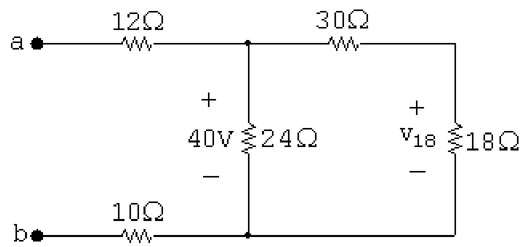
Calculate i_1 from i_4 and use Ohm's law as above to find R_1:

$$i_1 = 20i_4 = 20(0.25 \text{ mA}) = 5 \text{ mA} \qquad \therefore \quad R_1 = \frac{v}{i_1} = \frac{4 \text{ V}}{5 \text{ mA}} = 800\,\Omega$$

The resulting circuit is shown below:



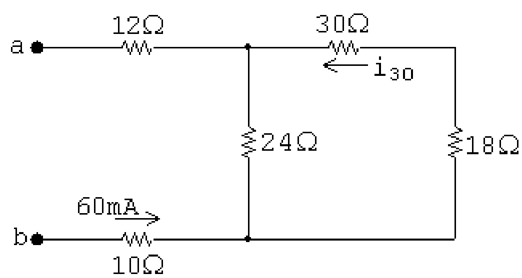
P 3.22 **[a]**



Using voltage division,

$$v_{18\Omega} = \frac{18}{18+30}(40) = 15 \text{ V positive at the top}$$

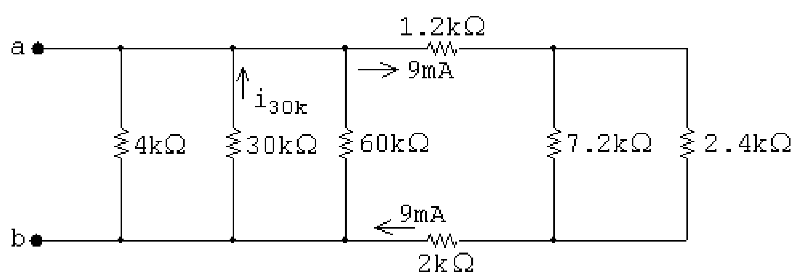
[b]



Using current division,

$$i_{30\Omega} = \frac{24}{24+30+18}(60 \times 10^{-3}) = 20 \text{ mA flowing from right to left}$$

[c]



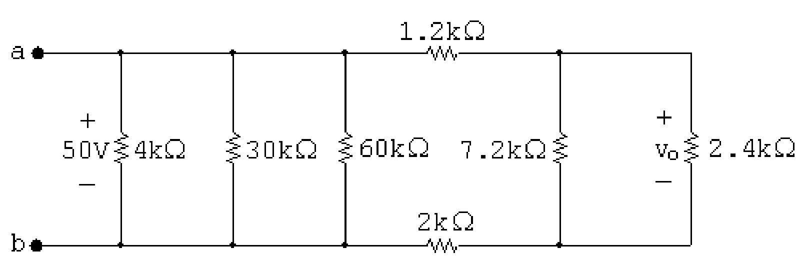
The 9 mA current in the $1.2\ \text{k}\Omega$ resistor is also the current in the $2\ \text{k}\Omega$ resistor. It then divides among the $4\ \text{k}\Omega$, $30\ \text{k}\Omega$, and $60\ \text{k}\Omega$ resistors.

$$4\ \text{k}\Omega \| 60\ \text{k}\Omega = 3.75\ \text{k}\Omega$$

Using current division,

$$i_{30\ \text{k}\Omega} = \frac{3.75\ \text{k}}{30\ \text{k} + 3.75\ \text{k}}(9 \times 10^{-3}) = 1\ \text{m A}, \quad \text{flowing bottom to top}$$

[d]



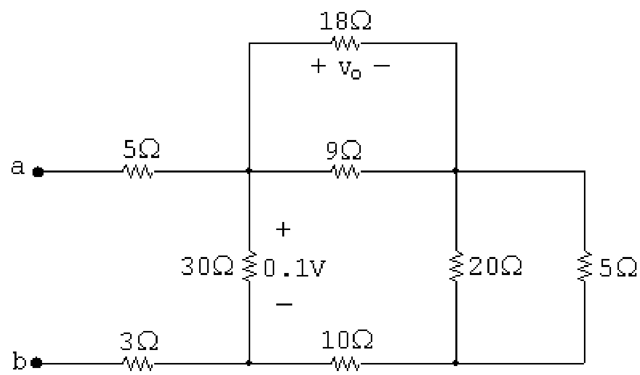
The voltage drop across the $4\ \mathrm{k}\Omega$ resistor is the same as the voltage drop across the series combination of the $1.2\ \mathrm{k}\Omega$, the $(7.2\ \mathrm{k}\|2.4\ \mathrm{k})\Omega$ combined resistor, and the $2\ \mathrm{k}\Omega$ resistor. Note that

$$7.2\ \mathrm{k}\|2.4\ \mathrm{k} = \frac{(7200)(2400)}{9600} = 1.8\ \mathrm{k}\Omega$$

Using voltage division,

$$v = \frac{1800}{1200 + 1800 + 2000}(50) = 18 \text{ V positive at the top}$$

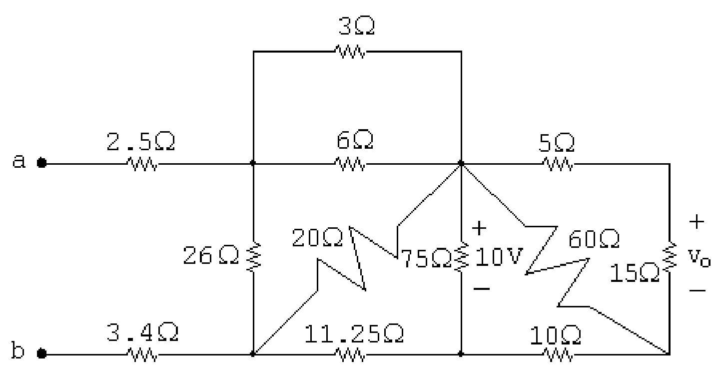
P 3.23 **[a]**



First, note the following: $18\|9 = 6\,\Omega$; $20\|5 = 4\,\Omega$; and the voltage drop across the $18\,\Omega$ resistor is the same as the voltage drop across the parallel combination of the $18\,\Omega$ and $9\,\Omega$ resistors. Using voltage division,

$$v = \frac{6}{6 + 4 + 10}(0.1 \text{ V}) = 30 \text{ mV positive at the left}$$

[b]



The equivalent resistance of the $5\,\Omega$, $15\,\Omega$, and $60\,\Omega$ resistors is

$$R = (5 + 15)\|60 = 15\,\Omega$$

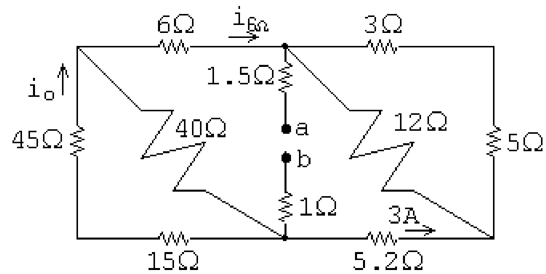
Using voltage division to find the voltage across the equivalent resistance,

$$v_e = \frac{15}{15 + 10}(10) = 6 \text{ V}$$

Using voltage division again,

$$v = \frac{15}{5 + 15}(6) = 4.5 \text{ V positive at the top}$$

[c]



Find equivalent resistance on the right side

$$R = 5.2 + \frac{(12)(5+3)}{(12+3+5)} = 10\ \Omega$$

Find voltage bottom to top across R

$$(10)(3) = 30\text{ V}$$

Find the equivalent resistance on the left side

$$R = 6 + \frac{(40)(45+15)}{(40+45+15)} = 30\ \Omega$$

The current in the $6\ \Omega$ is

$$i_{6\ \Omega} = \frac{30}{30} = 1\text{ A} \quad \text{left to right}$$

Use current division to find i

$$i = (1)\left(\frac{40}{40+15+45}\right) = 0.4\text{ A} \quad \text{bottom to top}$$

P 3.24 **[a]** $v_{20\text{k}} = \dfrac{20}{20+5}(45) = 36\text{ V}$

$$v_{90\text{k}} = \frac{90}{90+60}(45) = 27\text{ V}$$

$$v = v_{20\text{k}} - v_{90\text{k}} = 36 - 27 = 9\text{ V}$$

[b] $v_{20\text{k}} = \dfrac{20}{25}(V) = 0.8V$

$$v_{90\text{k}} = \frac{90}{150}(V) = 0.6V$$

$$v = 0.8V - 0.6V = 0.2V$$

P 3.25 $150\|75 = 50\,\Omega$

The equivalent resistance to the right of the $90\,\Omega$ resistor is

$$(50+40)\|(60+30) = 45\,\Omega$$

The voltage drop across this equivalent resistance is

$$\frac{45}{90+45}(3) = 1 \text{ V}$$

Use voltage division to find v_1, which is the voltage drop across the parallel combination whose equivalent resistance is $50\,\Omega$:

$$v_1 = \frac{50}{50+40}(1) = 5/9 \text{ V}$$

Use voltage division to find v_2:

$$v_2 = \frac{30}{30+60}(1) = 1/3 \text{ V}$$

P 3.26 $$i_{300\Omega} = \frac{1000+200}{1000+200+300+300}(15\times10^{-3}) = 10 \text{ mA}$$

$$v_{300\Omega} = (300)(10\times10^{-3}) = 3 \text{ V}$$

$$i_{200\Omega} = i_{1\text{ k}\Omega} = 15\times10^{-3} - i_{300\Omega} = 5 \text{ mA}$$

$$v_{1\text{k}} = (1000)(5\times10^{-3}) = 5 \text{ V}$$

$$v \ = 3-5 = -2 \text{ V}$$

P 3.27 $5\,\Omega\|20\,\Omega = 4\,\Omega$; $\quad 4\,\Omega + 6\,\Omega = 10\,\Omega$; $\quad 10\|40 = 8\,\Omega$;

Therefore, $i \ = \dfrac{125}{8+2} = 12.5$ A

$$i_{6\Omega} = \frac{(40)(12.5)}{50} = 10 \text{ A}; \quad i \ = \frac{(5)(10)}{25} = 2 \text{ A}$$

P 3.28 **[a]** Combine resistors in series and parallel to find the equivalent resistance seen by the source. Use this equivalent resistance to find the current through the source, and use current division to find i :

$80+70 = 150\,\Omega$ $\qquad 100\|150\|90 = 36\,\Omega$ $\qquad 36+24 = 60\,\Omega$

$$i_{24\Omega} = \frac{60 \text{ V}}{60\Omega} = 1 \text{ A}$$

$$i \ = \frac{100\|90\|150}{150}(1) = \frac{36}{150} = 0.24 \text{ A}$$

[b] Use current division to find the current through the $90\,\Omega$ resistor from the source current found in part (a), and use the calculated current to find the power in the $90\,\Omega$ resistor:

$$i_{90\Omega} = \frac{100\|90\|150}{90}(1) = \frac{36}{90} = 0.4 \text{ A}$$

$$p_{90\Omega} = i_{90\Omega}^2(90) = (0.4)^2(90) = 14.4 \text{ W}$$

P 3.29 **[a]** $v_{9\Omega} = (1)(9) = 9 \text{ V}$

$i_{2\Omega} = 9/(2+1) = 3 \text{ A}$

$i_{4\Omega} = 1 + 3 = 4 \text{ A};$

$v_{25\Omega} = (4)(4) + 9 = 25 \text{ V}$

$i_{25\Omega} = 25/25 = 1 \text{ A};$

$i_{3\Omega} = i_{25\Omega} + i_{9\Omega} + i_{2\Omega} = 1 + 1 + 3 = 5 \text{ A};$

$v_{40\Omega} = v_{25\Omega} + v_{3\Omega} = 25 + (5)(3) = 40 \text{ V}$

$i_{40\Omega} = 40/40 = 1 \text{ A}$

$i_{5\|20\Omega} = i_{40\Omega} + i_{25\Omega} + i_{4\Omega} = 1 + 1 + 4 = 6 \text{ A}$

$v_{5\|20\Omega} = (4)(6) = 24 \text{ V}$

$v_{32\Omega} = v_{40\Omega} + v_{5\|20\Omega} = 40 + 24 = 64 \text{ V}$

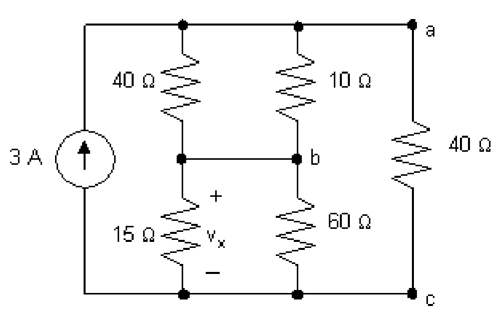
$i_{32\Omega} = 64/32 = 2 \text{ A};$

$i_{10\Omega} = i_{32\Omega} + i_{5\|20\Omega} = 2 + 6 = 8 \text{ A}$

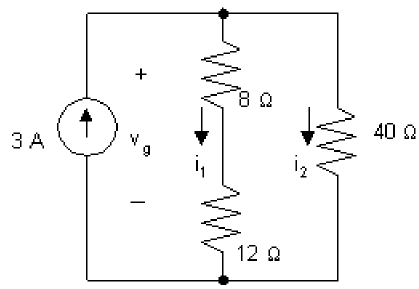
$v \;= 10(8) + v_{32\Omega} = 80 + 64 = 144 \text{ V}.$

[b] $P_{20\Omega} = \dfrac{(v_{5\|20\Omega})^2}{20} = \dfrac{24^2}{20} = 28.8 \text{ W}$

P 3.30



$40\|10 = 8\,\Omega \qquad 15\|60 = 12\,\Omega$



$$i_1 = \frac{(3)(40)}{(60)} = 2 \text{ A}; \qquad v \; = 8i_1 = 16 \text{ V}$$

$$v \; = 20i_1 = 40 \text{ V}$$

$$v_{60} = v \; - v \; = 24 \text{ V}$$

$$P_{\text{device}} = \frac{24^2}{60} + \frac{16^2}{10} + \frac{40^2}{40} = 75.2 \text{ W}$$

P 3.31 **[a]** The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R \; = \frac{100\,\mu\text{V}}{10\,\mu\text{A}} = 10\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i \;\; = \frac{(10/99)}{10 + (10/99)}(i_{\text{meas}}) = \frac{10}{990 + 10}(i_{\text{meas}}) = \frac{1}{100} i_{\text{meas}}$$

[b] $R \; = \dfrac{100\,\mu\text{V}}{10\,\mu\text{A}} = 10\,\Omega.$

$$i \;\; = \frac{(100/999{,}990)}{10 + (100/999{,}990)}(i_{\text{meas}}) = \frac{1}{100{,}000}(i_{\text{meas}})$$

[c] Yes

P 3.32 Measured value: $60 \| 20.1 = 15.056\,\Omega$

$$i \;\; = \frac{50}{(15.056 + 10)} = 1.9955 \text{ A}; \qquad i_{\text{meas}} = (1.9955)\frac{60}{80.1} = 1.495 \text{ A}$$

True value: $60 \| 20 = 15\,\Omega$

$$i \;\; = \frac{50}{(15 + 10)} = \frac{50}{25} = 2.0 \text{ A}; \qquad i_{\text{true}} = (2)\left(\frac{60}{80}\right) = 1.5 \text{ A}$$

$$\% \text{ error} \; = \left[\frac{1.495}{1.5} - 1\right] \times 100 = -0.3488\%$$

P 3.33 Begin by using current division to find the actual value of the current i :

$$i_{\text{true}} = \frac{15}{15+45}(50 \text{ mA}) = 12.5 \text{ mA}$$

$$i_{\text{meas}} = \frac{15}{15+45+0.1}(50 \text{ mA}) = 12.48 \text{ mA}$$

$$\% \text{ error } = \left[\frac{12.48}{12.5} - 1\right] 100 = -0.1664\%$$

P 3.34 For all full-scale readings the total resistance is

$$R \;\; + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\text{movement}} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20\,\Omega$$

Therefore, $R \;\; = 1000 \text{ (full-scale reading) } - 20$

[a] $R \;\; = 1000(50) - 20 = 49,980\,\Omega$

[b] $R \;\; = 1000(5) - 20 = 4980\,\Omega$

[c] $R \;\; = 1000(0.25) - 20 = 230\,\Omega$

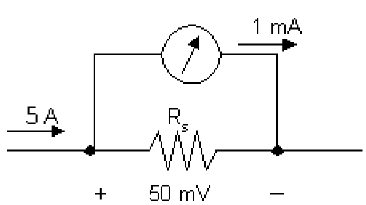
[d] $R \;\; = 1000(0.025) - 20 = 5\,\Omega$

P 3.35 **[a]** $v_{\text{meas}} = (50 \times 10^{-3})[15\|45\|(4980+20)] = 0.5612 \text{ V}$

[b] $v_{\text{true}} = (50 \times 10^{-3})(15\|45) = 0.5625 \text{ V}$

$$\% \text{ error } = \left(\frac{0.5612}{0.5625} - 1\right) \times 100 = -0.23\%$$

P 3.36



Original meter: $R_{\text{e}} = \dfrac{50 \times 10^{-3}}{5} = 0.01\,\Omega$

Modified meter: $R_{\text{e}} = \dfrac{(0.02)(0.01)}{0.03} = 0.00667\,\Omega$

$\therefore \quad (I_{\text{fs}})(0.00667) = 50 \times 10^{-3}$

$\therefore \quad I_{\text{fs}} = 7.5 \text{ A}$

P 3.37 At full scale the voltage across the shunt resistor will be 100 mV; therefore the power dissipated will be

$$P_\mathrm{A} = \frac{(100 \times 10^{-3})^2}{R_\mathrm{A}}$$

Therefore $R_\mathrm{A} \geq \dfrac{(100 \times 10^{-3})^2}{0.25} = 40\ \mathrm{m}\Omega$

Otherwise the power dissipated in R_A will exceed its power rating of 0.25 W
When $R_\mathrm{A} = 40\ \mathrm{m}\Omega$, the shunt current will be

$$i_\mathrm{A} = \frac{100 \times 10^{-3}}{40 \times 10^{-3}} = 2.5\ \mathrm{A}$$

The measured current will be $i_\mathrm{meas} = 2.5 + 0.001 = 2.501$ A
$\therefore$ Full-scale reading is for practical purposes is 2.5 A

P 3.38 The current in the shunt resistor at full-scale deflection is

$$i_\mathrm{A} = i_\mathrm{fullscale} - 20 \times 10^{-6}$$

The voltage across R_A at full-scale deflection is always 10 mV, therefore

$$R_\mathrm{A} = \frac{10 \times 10^{-3}}{i_\mathrm{fullscale} - 2 \times 10^{-3}} = \frac{10}{1000 i_\mathrm{fs} - 0.02}$$

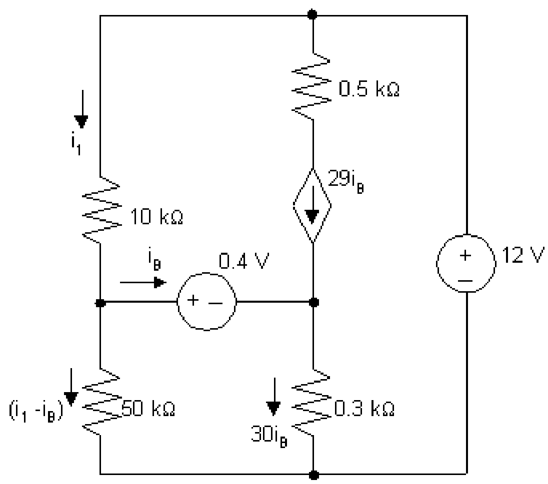
[a] $R_\mathrm{A} = \dfrac{10}{10{,}000 - 0.02} = 1\ \mathrm{m}\Omega$

[b] $R_\mathrm{A} = \dfrac{10}{1000 - 0.02} = 10\ \mathrm{m}\Omega$

[c] $R_\mathrm{A} = \dfrac{10}{100 - 0.02} = 1\,\Omega$

[d] $R_\mathrm{A} = \dfrac{10}{0.1 - 0.02} = 125\,\Omega$

P 3.39 **[a]**



$$10 \times 10^3 i_1 + 50 \times 10^3 (i_1 - i_B) = 12$$

$$50 \times 10^3 (i_1 - i_B) = 0.4 + 30 i_B (0.3 \times 10^3)$$

$$\therefore \quad 60 i_1 - 50 i_B = 12 \times 10^{-3}$$

$$50 i_1 - 59 i_B = 0.4 \times 10^{-3}$$

Calculator solution yields $i_B = 553.85\,\mu$A

[b] With the insertion of the ammeter the equations become

$$60 i_1 - 50 i_B = 12 \times 10^{-3} \quad \text{(no change)}$$

$$50 \times 10^3 (i_1 - i_B) = 2 \times 10^3 i_B + 0.4 + 30 i_B (300)$$

$$50 i_1 - 61 i_B = 0.4 \times 10^{-3}$$

Calculator solution yields $i_B = 496.6\,\mu$A

[c] $\%$ error $= \left(\frac{496.6}{553.85} - 1\right) 100 = -10.34\%$

P 3.40 **[a]** $v_{\text{meter}} = 100$ V

[b] $R_{\text{meter}} = (100\,\Omega/\text{V})(100\text{ V}) = 10$ kΩ

$$10\text{ k}\|60\text{ k} = 8.57\text{ k}\Omega$$

$$v_{\text{meter}} = \frac{8.57\text{ k}}{23.57\text{ k}}(100) = 36.36\text{ V}$$

[c] $10\text{ k}\|1\text{ k} = 6\text{ k}\Omega$

$$v_{\text{meter}} = \frac{6}{66}(100) = 9.09\text{ V}$$

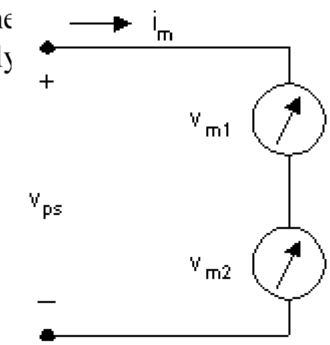
[d] $v_{\text{meter a}} = 100$ V

$v_{\text{meter b}} + v_{\text{meter c}} = 45.45$ V

No, because of the loading effect of the meter.

P 3.41 **[a]** Since the ater than either voltmeter's maximum reading, the only voltmeters would be to connect them in series.

[b]



$R_1 = (300)(1000) = 300$ kΩ; $\quad R_2 = (150)(800) = 120$ kΩ

$\therefore \quad R_1 + R_2 = 420$ kΩ

$i_{1\,\text{max}} = \dfrac{300}{300} \times 10^{-3} = 1$ mA; $\quad i_{2\,\text{max}} = \dfrac{150}{120} \times 10^{-3} = 1.25$ mA

$\therefore \quad i_{\text{max}} = 1$ mA since meters are in series

$v_{\text{max}} = 10^{-3}(300 + 120)10^3 = 420$ V

Thus the meters can be used to measure the voltage

[c] $i = \dfrac{399}{420 \times 10^3} = 0.95$ mA

$v_1 = (0.95)(300) = 285$ V $\quad v_2 = (0.95)(120) = 114$ V

P 3.42 The current in the series-connected voltmeters is

$$i = \frac{288}{300} = 0.96 \text{ mA}$$

$$v_{80\text{ k}\Omega} = (0.96)(80) = 76.8 \text{ V}$$

$$V_{\text{power supply}} = 288 + 115.2 + 76.8 = 480 \text{ V}$$

P 3.43 $R_{\text{meter}} = R + R_{\text{movement}} = \dfrac{750 \text{ V}}{1.5 \text{ mA}} = 500$ kΩ

$$v_{\text{meas}} = (25\text{ k}\Omega \| 125\text{ k}\Omega \| 500\text{ k}\Omega)(30\text{ mA}) = (20\text{ k}\Omega)(30\text{ mA}) = 600\text{ V}$$

$$v_{\text{true}} = (25\text{ k}\Omega \| 125\text{ k}\Omega)(30\text{ mA}) = (20.833\text{ k}\Omega)(30\text{ mA}) = 625\text{ V}$$

$$\%\text{ error} = \left(\frac{600}{625} - 1\right)100 = -4\%$$

P 3.44 Note – the upper terminal of the voltmeter should be labeled 820 V, not 300 V.

[a] $R_{\text{meter}} = 360\text{ k}\Omega + 200\text{ k}\Omega \| 50\text{ k}\Omega = 400\text{ k}\Omega$

$$400\|600 = 240\text{ k}\Omega$$

$$V_{\text{meter}} = \frac{240}{300}(300) = 240\text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value } = \frac{600}{660}(300) = 272.73\text{ V}$$

$$\%\text{ error } = \left(\frac{240}{272.73} - 1\right)100 = -12\%$$

P 3.45 **[a]**

$$R_1 = \frac{100\text{ V}}{2\text{ mA}} = 50\text{ k}\Omega$$

$$R_2 = \frac{10\text{ V}}{2\text{ mA}} = 5\text{ k}\Omega$$

$$R_3 = \frac{1\text{ V}}{2\text{ mA}} = 500\,\Omega$$

[b] Let i_{a} = actual current in the movement

i_{d} = design current in the movement

$$\text{Then \% error } = \left(\frac{i_{\text{a}}}{i_{\text{d}}} - 1\right)100$$

For the 100 V scale:

$$i_{\text{a}} = \frac{100}{50{,}000 + 25} = \frac{100}{50{,}025}, \qquad i_{\text{d}} = \frac{100}{50{,}000}$$

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{50{,}000}{50{,}025} = 0.9995 \qquad \%\text{ error } = (0.9995 - 1)100 = -0.05\%$$

For the 10 V scale:

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{5000}{5025} = 0.995 \qquad \%\text{ error } = (0.995 - 1.0)100 = -0.5\%$$

For the 1 V scale:

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{500}{525} = 0.9524 \qquad \%\text{ error } = (0.9524 - 1.0)100 = -4.76\%$$

P 3.46 **[a]** $R_{\text{movement}} = 50\,\Omega$

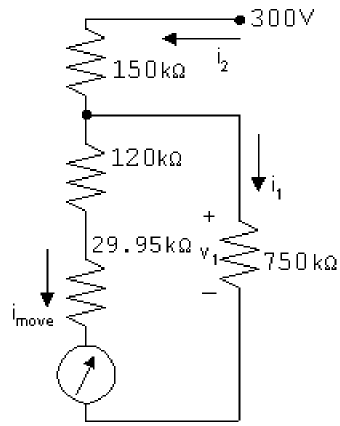
$$R_1 + R_{\text{movement}} = \frac{30}{1 \times 10^{-3}} = 30\text{ k}\Omega \quad \therefore \quad R_1 = 29{,}950\,\Omega$$

$$R_2 + R_1 + R_{\text{movement}} = \frac{150}{1 \times 10^{-3}} = 150\text{ k}\Omega \quad \therefore \quad R_2 = 120\text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{300}{1 \times 10^{-3}} = 300\ \text{k}\Omega$$

$$\therefore \quad R_3 = 150\ \text{k}\Omega$$

[b]



$$i_{\text{move}} = \frac{288}{300}(1) = 0.96 \text{ mA}$$

$$v_1 = (0.96 \text{ m})(150 \text{ k}) = 144 \text{ V}$$

$$i_1 = \frac{144}{750 \text{ k}} = 0.192 \text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 0.96 \text{ m} + 0.192 \text{ m} = 1.152 \text{ mA}$$

$$v_{\text{meas}} = v = 144 + 150 i_2 = 316.8 \text{ V}$$

[c] $v_1 = 150$ V; $\qquad i_2 = 1 \text{ m} + 0.20 \text{ m} = 1.20 \text{ mA}$

$$i_1 = 150/750{,}000 = 0.20 \text{ mA}$$

$$\therefore \quad v_{\text{meas}} = v = 150 + (150 \text{ k})(1.20 \text{ m}) = 330 \text{ V}$$

P 3.47 From the problem statement we have

$$50 = \frac{V(10)}{10 + R} \qquad (1) \quad V \text{ in mV; } R \text{ in M}\Omega$$

$$48.75 = \frac{V(6)}{6 + R} \qquad (2)$$

[a] From Eq (1) $\quad 10 + R = 0.2V$

$$\therefore \quad R = 0.2V - 10$$

Substituting into Eq (2) yields

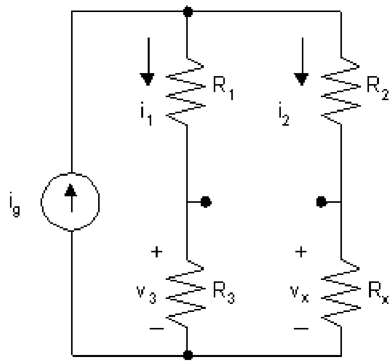
$$48.75 = \frac{6V}{0.2V - 6} \quad \text{or} \quad V = 52 \text{ mV}$$

[b] From Eq (1)

$$50 = \frac{520}{10 + R} \quad \text{or} \quad 50R = 20$$

So $R = 400$ kΩ

P 3.48 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i\ (R_2 + R\ )}{R_1 + R_2 + R_3 + R\ } = \frac{i\ (R_2 + R\ )}{\sum R}$$

$$i_2 = \frac{i\ (R_1 + R_3)}{R_1 + R_2 + R_3 + R\ } = \frac{i\ (R_1 + R_3)}{\sum R}$$

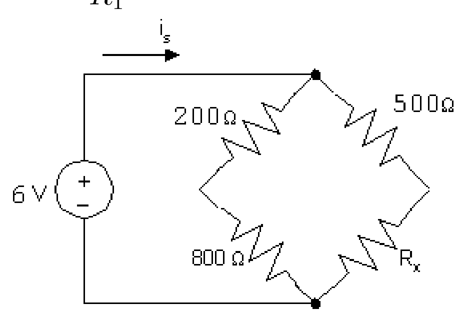
$$v_3 = R_3 i_1 = v\ \ = i_2 R$$

$$\therefore \quad \frac{R_3 i\ (R_2 + R\ )}{\sum R} = \frac{R\ i\ (R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R\ ) = R\ (R_1 + R_3)$$

From which $R\ \ = \dfrac{R_2 R_3}{R_1}$

P 3.49 **[a]**



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(200)(R\ ) = (500)(800) \qquad \text{so} \qquad R\ \ = \frac{(500)(800)}{200} = 2000\,\Omega$$

[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 6 V:

$$i = \frac{6\text{ V}}{200\,\Omega + 800\,\Omega} + \frac{6\text{ V}}{500\,\Omega + 2000\,\Omega} = 8.4\text{ mA}$$

[c] We can use current division to find the current in each branch:

$$i_{\text{left}} = \frac{500 + 2000}{500 + 2000 + 200 + 800}(8.4\text{ mA}) = 6\text{ mA}$$

$$i_{\text{right}} = 8.4\text{ mA} - 6\text{ mA} = 2.4\text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

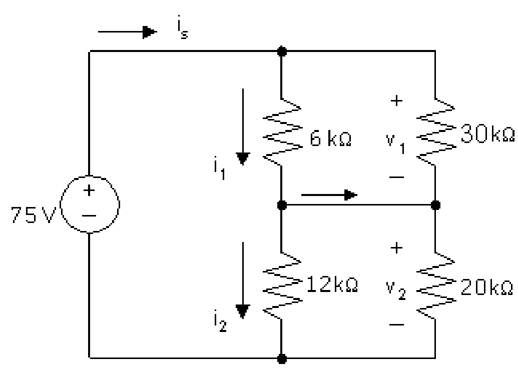
$$p_{200} = (200)(0.006)^2 = 7.2\text{ mW} \qquad p_{800} = (800)(0.006)^2 = 28.8\text{ mW}$$

$$p_{500} = (500)(0.0024)^2 = 2.88\text{ mW} \qquad p_{2000} = (2000)(0.0024)^2 = 11.52\text{ mW}$$

Thus, the $800\,\Omega$ resistor absorbs the most power; it absorbs 28.8 mW of power.

[d] From the analysis in part (c), the $500\,\Omega$ resistor absorbs the least power; it absorbs 2.88 mW of power.

P 3.50 Redraw the circuit, replacing the detector branch with a short circuit.



$6\text{ k}\Omega \| 30\text{ k}\Omega = 5\text{ k}\Omega$

$12\text{ k}\Omega \| 20\text{ k}\Omega = 7.5\text{ k}\Omega$

$$i = \frac{75}{5000 + 7500} = 6\text{ mA}$$

$v_1 = 6\text{ mA}(5000) = 30\text{ V}$

$v_2 = 6\text{ mA}(7500) = 45\text{ V}$

$$i_1 = \frac{30\text{ V}}{6000\,\Omega} = 5\text{ mA}$$

$$i_2 = \frac{45\text{ V}}{12{,}000\,\Omega} = 3.75\text{ mA}$$

$$i_\text{d} = i_1 - i_2 = 5\text{ mA} - 3.75\text{ mA} = 1.25\text{ mA}$$

P 3.51 Note the bridge structure is balanced, that is $15 \times 5 = 25 \times 3$, hence there is no current in the 5 kΩ resistor. It follows that the equivalent resistance of the circuit is

$$R_\text{eq} = 0.750 + 11.25 = 12\text{ k}\Omega$$

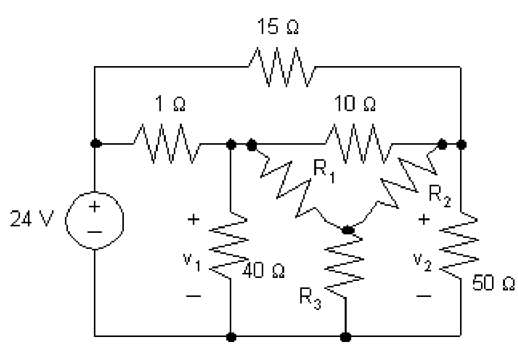
The source current is $192/12{,}000 = 16$ mA.
The current down through the 3 kΩ resistor is

$$i_3 \;= 16\frac{30}{48} = 10\text{ mA}$$

$$\therefore\;\; p_3 \;= (10 \times 10^{-3})^2(3 \times 10^3) = 300\text{ mW}$$

P 3.52 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

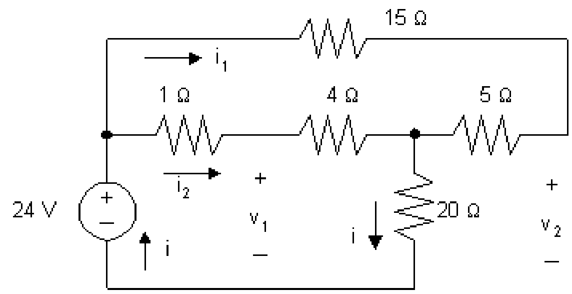
P 3.53 Begin by transforming the Y-connected resistors ($10\,\Omega, 40\,\Omega, 50\,\Omega$) to Δ-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Δ-connected resistors:

$$R_1 = \frac{(40)(10)}{10+40+50} = 4\,\Omega; \quad R_2 = \frac{(50)(10)}{10+40+50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10+40+50} = 20\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\rm eq} = (15+5)\|(4+1) + 20 = 20\|5 + 20 = 4 + 20 = 24\,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24\text{ V}}{24\,\Omega} = 1\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $15\,\Omega$ and $5\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $1\,\Omega$ and $4\,\Omega$ resistors:

$$i_1 = \frac{1+4}{1+4+15+5}(i) = \frac{5}{25}(1\text{ A}) = 0.2\text{ A}, \qquad \text{and} \qquad i_2 = 1\text{ A} - 0.2\text{ A} = 0.8\text{ A}$$

Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $4\,\Omega$ resistor, $4i_2$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8\text{ A}) + 20(1\text{ A}) = 3.2 + 20 = 23.2\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $5\,\Omega$ resistor, $5i_1$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

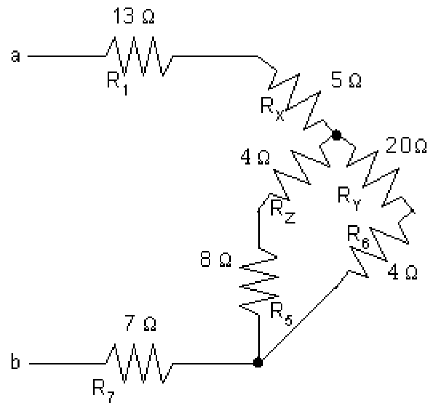
$$v_2 = 5i_1 + 20i = 5(0.2\text{ A}) + 20(1\text{ A}) = 1 + 20 = 21\text{ V}$$

P 3.54 **[a]** Calculate the values of the Y-connected resistors that are equivalent to the $10\,\Omega$, $40\,\Omega$, and 50Ω Δ-connected resistors:

$$R = \frac{(10)(50)}{10+40+50} = 5\,\Omega; \qquad R = \frac{(40)(50)}{10+40+50} = 20\,\Omega;$$

$$R = \frac{(10)(40)}{10+40+50} = 4\,\Omega$$

Replacing the R_2—R_3—R_4 delta with its equivalent Y gives



Now calculate the equivalent resistance R_{ab} by making series and parallel combinations of the resistors:

$$R_{ab} = 13 + 5 + [(4 + 8)\|(20 + 4)] + 7 = 33\,\Omega$$

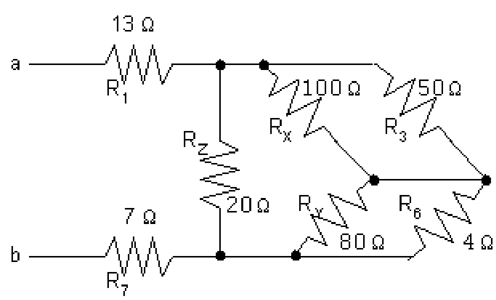
[b] Calculate the values of the Δ-connected resistors that are equivalent to the $8\,\Omega$, $10\,\Omega$, and $40\,\Omega$ Y-connected resistors:

$$R = \frac{(10)(40) + (40)(8) + (8)(10)}{8} = \frac{800}{8} = 100\,\Omega$$

$$R = \frac{(10)(40) + (40)(8) + (8)(10)}{10} = \frac{800}{10} = 80\,\Omega$$

$$R = \frac{(10)(40) + (40)(8) + (8)(10)}{40} = \frac{800}{40} = 20\,\Omega$$

Replacing the R_2, R_4, R_5 wye with its equivalent Δ gives



Make series and parallel combinations of the resistors to find the equivalent resistance R_{ab}:

$$100\,\Omega\|50\,\Omega = 33.33\,\Omega; \qquad 80\,\Omega\|4\,\Omega = 3.81\,\Omega$$

$$\therefore \quad 100\|50 + 80\|4 = 33.33 + 3.81 = 37.14\,\Omega$$

$$\therefore \quad 37.14\|20 = \frac{(37.14)(20)}{57.14} = 13\,\Omega$$

$$\therefore \quad R_{ab} = 13 + 13 + 7 = 33\,\Omega$$

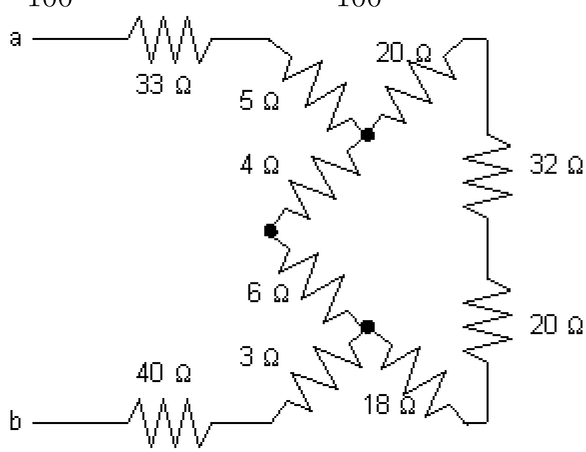
[c] Convert the delta connection R_4—R_5—R_6 to its equivalent wye.
Convert the wye connection R_3—R_4—R_6 to its equivalent delta.

P 3.55 Replace the upper and lower deltas with the equivalent wyes:

$$R_{1U} = \frac{(50)(10)}{100} = 5\,\Omega;\ R_{2U} = \frac{(50)(40)}{100} = 20\,\Omega;\ R_{3U} = \frac{(40)(10)}{100} = 4\,\Omega$$

$$R_{1L} = \frac{(60)(10)}{100} = 6\,\Omega;\ R_{2L} = \frac{(60)(30)}{100} = 18\,\Omega;\ R_{3L} = \frac{(30)(10)}{100} = 3\,\Omega$$

The res



Now make series and parallel combinations of the resistors:

$$(4+6)\|(20+32+20+18) = 10\|90 = 9\,\Omega$$

$$R_{ab} = 33 + 5 + 9 + 3 + 40 = 90\,\Omega$$

P 3.56 $18 + 2 = 20\,\Omega$

$20\|80 = 16\,\Omega$

$16 + 4 = 20\,\Omega$

$20\|30 = 12\,\Omega$

$12 + 8 = 20\,\Omega$

$20\|60 = 15\,\Omega$

$15 + 5 = 20\,\Omega$

$$i = \frac{240\text{ V}}{20\,\Omega} = 12\text{ A}$$

$$i = \frac{60}{60+20}(12\text{ A}) = 9\text{ A}$$

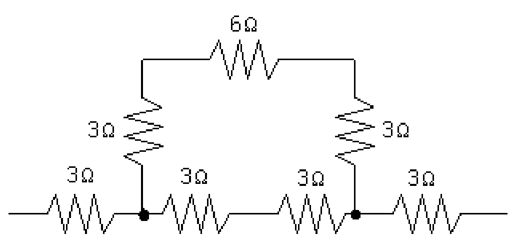
$$i_{30\Omega} = \frac{20}{20+30}(9\text{ A}) = 3.6\text{ A}$$

$$p_{30\Omega} = (30)(3.6)^2 = 388.8\text{ W}$$

P 3.57 The top of the pyramid can be replaced by a resistor equal to

$$R_1 = \frac{(18)(9)}{27} = 6\,\Omega$$

The lower left and right deltas can be replaced by wyes. Each resistance in the wye equals $3\,\Omega$. Thus our circuit can be reduced to



Now the $12\,\Omega$ in parallel with $6\,\Omega$ reduces to $4\,\Omega$.

$$\therefore \quad R_{\text{ab}} = 3 + 4 = 3 = 10\,\Omega$$

P 3.58 Note – the top resistor to the right of the $1.5\,\Omega$ resistor is $20\,\Omega$.

[a] Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(50)(100)}{200} = 25\,\Omega$$

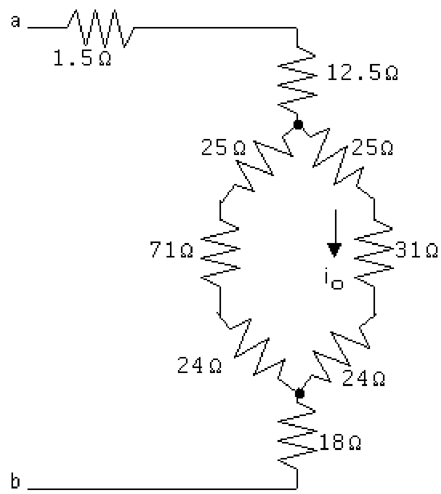
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(60)(80)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 1.5 + 12.5 + (25 + 71 + 24)\|(25 + 31 + 24) + 18$$

$$= 1.5 + 12.5 + (120\|85) + 18 = 1.5 + 12.5 + 48 + 18 = 80\,\Omega$$

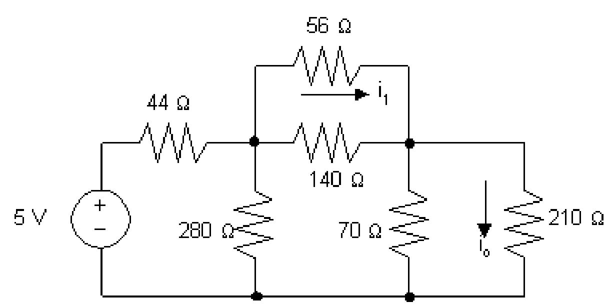
[b] When $v_{ab} = 400$ V

$$i = \frac{400}{80} = 5 \text{ A}$$

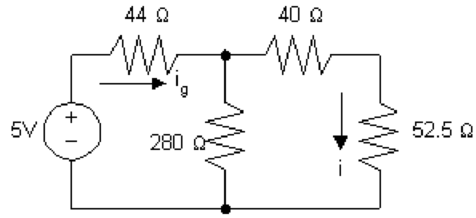
$$i = \frac{120}{120 + 80}(5) = 3 \text{ A}$$

$$p_{31\Omega} = (31)(3)^2 = 279 \text{ W}$$

P 3.59 **[a]** After the $20\,\Omega$—$80\,\Omega$—$40\,\Omega$ wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to



$R_{\text{eq}} = 44 + 280\|92.5 = 113.53\,\Omega$

$i \ = 5/113.53 = 44.04$ mA

$i = (280/372.5)(44) = 33.11$ mA

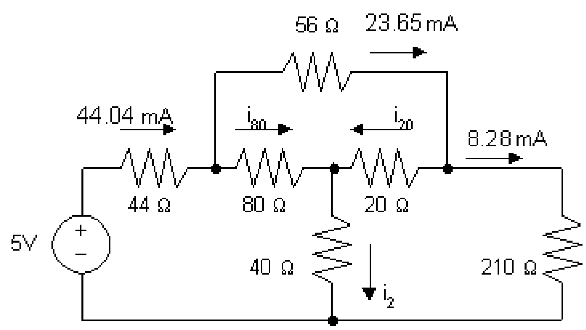
$v_{52\ 5\Omega} = (52.5)(33.11\text{ m}) = 1.74$ V

$i \ = 1.74/210 = 8.28$ mA

[b] $v_{40\Omega} = (40)(33.11\text{ m}) = 1.32$ V

$i_1 = 1.32/56 = 23.65$ mA

[c] Now that i and i_1 are known return to the original circuit



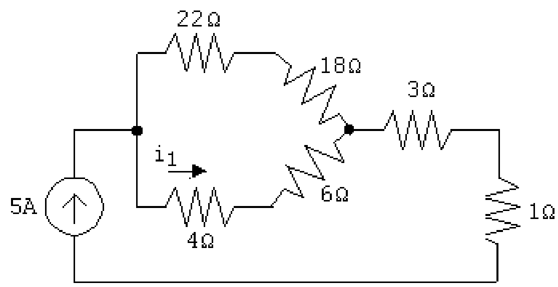
$i_{80\Omega} = 44.04\text{ m} - 23.65\text{ m} = 20.39$ mA

$i_{20\Omega} = 23.65\text{ m} - 8.28\text{ m} = 15.37$ mA

$i_2 = i_{80\Omega} + i_{20\Omega} = 35.76$ mA

[d] $p_{\text{del}} = (5)(44.04\text{ m}) = 220.2$ mW

P 3.60 **[a]** After the $30\,\Omega$—$60\,\Omega$—$10\,\Omega$ delta is replaced by its equivalent wye, the circuit reduces to



Use current division to calculate i_1:

$$i_1 = \frac{22+18}{22+18+4+6}(5\text{ A}) = \frac{40}{50}(5\text{ A}) = 4\text{ A}$$

[b] Return to the original circuit and write a KVL equation around the upper left loop:

$$(22\,\Omega)i_{22\Omega} + v - (4\,\Omega)(i_1) = 0$$

so $\quad v = (4\,\Omega)(4\text{ A}) - (22\,\Omega)(5\text{ A} - 4\text{ A}) = -6\text{ V}$

[c] Write a KCL equation at the lower center node of the original circuit:

$$i_2 = i_1 + \frac{v}{60} = 4 + \frac{-6}{60} = 3.9\text{ A}$$

[d] Write a KVL equation around the bottom loop of the original circuit:

$$-v_{5\text{A}} + (4\,\Omega)(4\text{ A}) + (10\,\Omega)(3.9\text{ A}) + (1\,\Omega)(5\text{ A}) = 0$$

So, $\quad v_{5\text{A}} = (4)(4) + (10)(3.9) + (1)(5) = 60\text{ V}$

Thus, $\quad p_{5\text{A}} = (5\text{ A})(60\text{ V}) = 300\text{ W}$

P 3.61 Subtracting Eq. 3.42 from Eq. 3.43 gives

$$R_1 - R_2 = (R_cR_b - R_cR_a)/(R_a + R_b + R_c).$$

Adding this expression to Eq. 3.41 and solving for R_1 gives

$$R_1 = R_cR_b/(R_a + R_b + R_c).$$

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43. Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1+R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_\text{b} = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_\text{c}/R_\text{a})$ or $R_\text{c} = (R_1/R_3)R_\text{a}$ and $(R_1/R_2) = (R_\text{b}/R_\text{a})$ or $R_\text{b} = (R_1/R_2)R_\text{a}$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_\text{b} = (R_3/R_2)R_\text{c}$ and $R_\text{a} = (R_3/R_1)R_\text{c}$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.62

$$G_\text{a} = \frac{1}{R_\text{a}} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)}$$

$$= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.63 **[a]** $R_\text{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_\text{L})}{2R_1 + R_2 + R_\text{L}} = R_\text{L}$

Therefore $\quad 2R_1 - R_\text{L} + \dfrac{R_2(2R_1 + R_\text{L})}{2R_1 + R_2 + R_\text{L}} = 0$

Thus $\quad R_\text{L}^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_\text{ab} = R_\text{L}$, the current into terminal a of the attenuator will be $v\,/R_\text{L}$ Using current division, the current in the R_L branch will be

$$\frac{v}{R_\text{L}} \cdot \frac{R_2}{2R_1 + R_2 + R_\text{L}}$$

Therefore $\quad v = \dfrac{v}{R_\text{L}} \cdot \dfrac{R_2}{2R_1 + R_2 + R_\text{L}} R_\text{L}$

and $\quad \dfrac{v}{v} = \dfrac{R_2}{2R_1 + R_2 + R_\text{L}}$

[b] $(600)^2 = 4(R_1 + R_2)R_1$

$$9 \times 10^4 = R_1^2 + R_1R_2$$

$$\frac{v}{v} = 0.6 = \frac{R_2}{2R_1 + R_2 + 600}$$

$$\therefore \quad 1.2R_1 + 0.6R_2 + 360 = R_2$$

$$0.4R_2 = 1.2R_1 + 360$$

$$R_2 = 3R_1 + 900$$

$$\therefore \quad 9 \times 10^4 = R_1^2 + R_1(3R_1 + 900) = 4R_1^2 + 900R_1$$

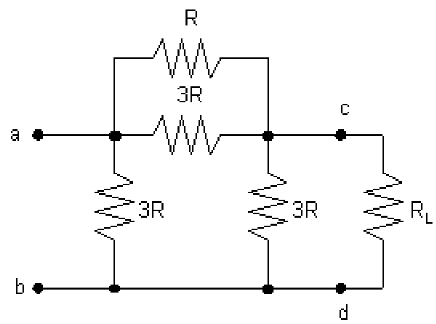
$$\therefore \quad R_1^2 + 225R_1 - 22{,}500 = 0$$

$$R_1 = -112.5 \pm \sqrt{(112.5)^2 + 22{,}500} = -112.5 \pm 187.5$$

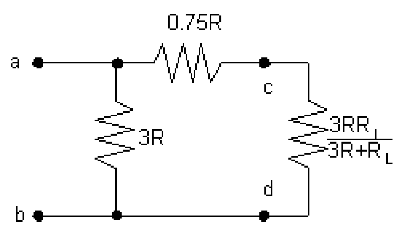
$$\therefore \quad R_1 = 75\,\Omega$$

$$\therefore \quad R_2 = 3(75) + 900 = 1125\,\Omega$$

P 3.64 **[a]** After making the Y-to-Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



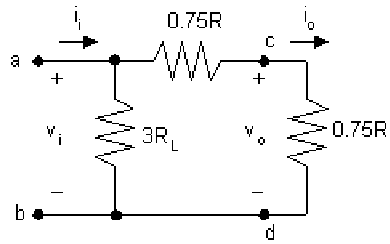
Now note: $$0.75R + \frac{3RR_\mathrm{L}}{3R + R_\mathrm{L}} = \frac{2.25R_\mathrm{L}^2 + 3.75RR_\mathrm{L}}{3R + R_\mathrm{L}}$$

Therefore $$R_\mathrm{ab} = \frac{3R\left(\dfrac{2.25R_\mathrm{L}^2 + 3.75RR_\mathrm{L}}{3R + R_\mathrm{L}}\right)}{3R + \left(\dfrac{2.25R_\mathrm{L}^2 + 3.75RR_\mathrm{L}}{3R + R_\mathrm{L}}\right)} = \frac{3R(3R + 5R_\mathrm{L})}{15R + 9R_\mathrm{L}}$$

When $R_\mathrm{ab} = R_\mathrm{L}$, we have $$15RR_\mathrm{L} + 9R_\mathrm{L}^2 = 9R^2 + 15RR_\mathrm{L}$$

Therefore $R_\mathrm{L}^2 = R^2$ or $R_\mathrm{L} = R$

[b] When $R = R_L$, the circuit reduces to



$$i = \frac{i\,(3R_L)}{4.5R_L} = \frac{1}{1.5}i = \frac{1}{1.5}\frac{v}{R_L}, \qquad v = 0.75R_L i = \frac{1}{2}v\,,$$

Therefore $\quad \dfrac{v}{v} = 0.5$

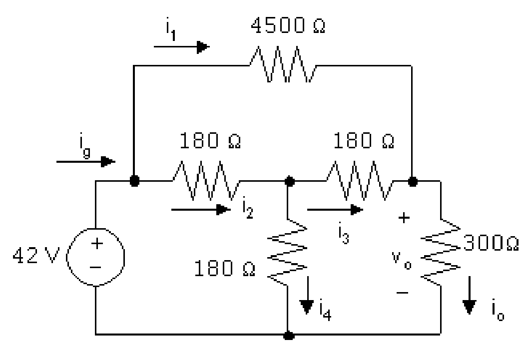
P 3.65 **[a]** $3.5(3R - R_L) = 3R + R_L$

$$10.5R - 1050 = 3R + 300$$

$$7.5R = 1350, \qquad R = 180\,\Omega$$

$$R_2 = \frac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500\,\Omega$$

[b]



$$v = \frac{v}{3.5} = \frac{42}{3.5} = 12\text{ V}$$

$$i = \frac{12}{300} = 40\text{ mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67\text{ mA}$$

$$i = \frac{42}{300} = 140\text{ mA}$$

$$i_2 = 140\text{ m} - 6.67\text{ m} = 133.33\text{ mA}$$

$$i_3 = 40\text{ m} - 6.67\text{ m} = 33.33\text{ mA}$$

$$i_4 = 133.33\text{ m} - 33.33\text{ m} = 100\text{ mA}$$

$$p_{4500\text{ top}} = (6.67 \times 10^{-3})^2(4500) = 0.2 \text{ W}$$

$$p_{180\text{ left}} = (133.33 \times 10^{-3})^2(180) = 3.2 \text{ W}$$

$$p_{180\text{ right}} = (33.33 \times 10^{-3})^2(180) = 0.2 \text{ W}$$

$$p_{180\text{ vertical}} = (100 \times 10^{-3})^2(180) = 1.8 \text{ W}$$

$$p_{300\text{ load}} = (40 \times 10^{-3})^2(300) = 0.48 \text{ W}$$

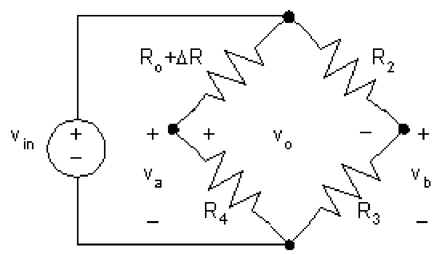
The 180 Ω resistor carrying i_2 dissipates the most power.

[c] $p_{180\text{ left}} = 3.2$ W

[d] Two resistors dissipate minimum power – the $4500\,\Omega$ and the $180\,\Omega$ carrying i_3.

[e] Both resistors dissipate 0.2 W or 200 mW.

P 3.66 **[a]**



$$v_a = \frac{v_{in}R_4}{R + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3}v_{in}$$

$$v = v_a - v_b = \frac{R_4 v_{in}}{R + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3}v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R + R_4}v_{in} = \frac{R_3}{R_2 + R_3}v_{in}$$

$$\therefore \quad \frac{R_4}{R + R_4} = \frac{R_3}{R_2 + R_3}$$

Thus,

$$\begin{aligned} v &= \frac{R_4 v_{in}}{R + R_4 + \Delta R} - \frac{R_4 v_{in}}{R + R_4} \\ &= R_4 v_{in}\left[\frac{1}{R + R_4 + \Delta R} - \frac{1}{R + R_4}\right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R + R_4 + \Delta R)(R + R_4)} \\ &\approx \frac{-(\Delta R)R_4 v_{in}}{(R + R_4)^2} \end{aligned}$$

[b] $\Delta R = 0.03R$

$$R = \frac{R_2R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[c]
$$v = \frac{-(\Delta R)R_4 v_{\text{in}}}{(R + R_4 + \Delta R)(R + R_4)} = \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} = -39.2157 \text{ mV}$$

P 3.67 **[a]**
$$\text{approx value} = \frac{-(\Delta R)R_4 v_{\text{in}}}{(R + R_4)^2}$$

$$\text{true value} = \frac{-(\Delta R)R_4 v_{\text{in}}}{(R + R_4 + \Delta R)(R + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R + R_4 + \Delta R)}{(R + R_4)}$$

$$\therefore \quad \%\text{ error} = \left[\frac{R + R_4 + \Delta R}{R + R_4} - 1\right] \times 100 = \frac{\Delta R}{R + R_4} \times 100$$

$$\text{But } R = \frac{R_2R_4}{R_3}$$

$$\therefore \quad \%\text{ error} = \frac{R_3\Delta R}{R_4(R_2 + R_3)}$$

[b]
$$\%\text{ error} = \frac{(500)(300)}{(5000)(1500)} \times 100 = 2\%$$

P 3.68
$$\frac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\text{ change} = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.69 **[a]** From Eq 3.64 we have

$$\left(\frac{i_1}{i_2}\right)^2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}$$

Substituting into Eq 3.63 yields

$$R_2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}R_1$$

Solving for R_2 yields

$$R_2 = (1+2\sigma)^2 R_1$$

[b] From Eq 3.63 we have

$$\frac{i_1}{i_\mathrm{b}} = \frac{R_2}{R_1 + R_2 + 2R_\mathrm{a}}$$

But $R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{a} = \sigma R_1$ therefore

$$\frac{i_1}{l_\mathrm{b}} = \frac{(1+2\sigma)^2 R_1}{R_1 + (1+2\sigma)^2 R_1 + 2\sigma R_1} = \frac{(1+2\sigma)^2}{(1+2\sigma)+(1+2\sigma)^2}$$

$$= \frac{1+2\sigma}{2(1+\sigma)}$$

It follows that

$$\left(\frac{i_1}{i_\mathrm{b}}\right)^2 = \frac{(1+2\sigma)^2}{4(1+\sigma)^2}$$

Substituting into Eq 3.66 gives

$$R_\mathrm{b} = \frac{(1+2\sigma)^2 R_\mathrm{a}}{4(1+\sigma)^2} = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$$

P 3.70 From Eq 3.69

$$\frac{i_1}{i_3} = \frac{R_2 R_3}{D}$$

But $D = (R_1 + 2R_\mathrm{a})(R_2 + 2R_\mathrm{b}) + 2R_\mathrm{b}R_2$

where $R_\mathrm{a} = \sigma R_1$; $R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{b} = \dfrac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$

Therefore D can be written as

$$\begin{aligned}
D &= (R_1 + 2\sigma R_1)\left[(1+2\sigma)^2 R_1 + \frac{2(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] + \\
&\quad 2(1+2\sigma)^2 R_1 \left[\frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] \\
&= (1+2\sigma)^3 R_1^2 \left[1 + \frac{\sigma}{2(1+\sigma)^2} + \frac{(1+2\sigma)\sigma}{2(1+\sigma)^2}\right] \\
&= \frac{(1+2\sigma)^3 R_1^2}{2(1+\sigma)^2}\{2(1+\sigma)^2 + \sigma + (1+2\sigma)\sigma\} \\
&= \frac{(1+2\sigma)^3 R_1^2}{(1+\sigma)^2}\{1 + 3\sigma + 2\sigma^2\}
\end{aligned}$$

$$D = \frac{(1+2\sigma)^4 R_1^2}{(1+\sigma)}$$

$$\begin{aligned}
\therefore\ \frac{i_1}{i_3} &= \frac{R_2 R_3 (1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+2\sigma)^2 R_1 R_3 (1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+\sigma) R_3}{(1+2\sigma)^2 R_1}
\end{aligned}$$

When this result is substituted into Eq 3.69 we get

$$R_3 = \frac{(1+\sigma)^2 R_3^2 R_1}{(1+2\sigma)^4 R_1^2}$$

Solving for R_3 gives

$$R_3 = \frac{(1+2\sigma)^4 R_1}{(1+\sigma)^2}$$

P 3.71 From the dimensional specifications, calculate σ and R_3:

$$\sigma = \frac{y}{x} = \frac{0.025}{1} = 0.025; \qquad R_3 = \frac{V_{\mathrm{dc}}^2}{p} = \frac{12^2}{120} = 1.2\,\Omega$$

Calculate R_1 from R_3 and σ:

$$R_1 = \frac{(1+\sigma)^2}{(1+2\sigma)^4} R_3 = 1.0372\,\Omega$$

Calculate R , R , and R_2:

$$R \;\; = \sigma R_1 = 0.0259\,\Omega \qquad R \;\; = \frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2} = 0.0068\,\Omega$$

$$R_2 = (1 + 2\sigma)^2 R_1 = 1.1435\,\Omega$$

Using symmetry,

$$R_4 = R_2 = 1.1435\,\Omega \qquad R_5 = R_1 = 1.0372\,\Omega$$

$$R\ \ = R\ \ = 0.0068\,\Omega \qquad R\ \ = R\ \ = 0.0259\,\Omega$$

Test the calculations by checking the power dissipated, which should be 120 W/m. Calculate D, then use Eqs. (3.58)-(3.60) to calculate $i\ $, i_1, and i_2:

$$D = (R_1 + 2R\ \ )(R_2 + 2R\ \ ) + 2R_2R\ \ = 1.2758$$

$$i\ \ = \frac{V_{\rm dc}(R_1 + R_2 + 2R\ \ )}{D} = 21\text{ A}$$

$$i_1 = \frac{V_{\rm dc}R_2}{D} = 10.7561\text{ A} \qquad i_2 = \frac{V_{\rm dc}(R_1 + 2R\ \ )}{D} = 10.2439\text{ A}$$

It follows that $i^2R\ \ = 3$ W and the power dissipation per meter is $3/0.025 = 120$ W/m. The value of $i_1^2R_1 = 120$ W/m. The value of $i_2^2R_2 = 120$ W/m. Finally, $i_1^2R\ \ = 3$ W/m.

P 3.72 From the solution to Problem 3.71 we have $i_{\rm b} = 21$ A and $i_3 = 10$ A. By symmetry $i_{\rm c} = 21$ A thus the total current supplied by the 12 V source is $21 + 21 + 10$ or 52 A. Therefore the total power delivered by the source is $p_{12\,\rm V}$ (del) $= (12)(52) = 624$ W. We also have from the solution that $p_{\rm a} = p_{\rm b} = p_{\rm c} = p_{\rm d} = 3$ W. Therefore the total power delivered to the vertical resistors is $p_{\rm V} = (8)(3) = 24$ W. The total power delivered to the five horizontal resistors is $p_{\rm H} = 5(120) = 600$ W.

$$\therefore\ \ \sum p_{\rm diss} = p_{\rm H} + p_{\rm V} = 624\text{ W} = \sum p_{\rm del}$$

P 3.73 **[a]** $\sigma = 0.03/1.5 = 0.02$

Since the power dissipation is 150 W/m the power dissipated in R_3 must be $200(1.5)$ or 300 W. Therefore

$$R_3 = \frac{12^2}{300} = 0.48\,\Omega$$

From Table 3.1 we have

$$R_1 = \frac{(1+\sigma)^2R_3}{(1+2\sigma)^4} = 0.4269\,\Omega$$

$$R_{\rm a} = \sigma R_1 = 0.0085\,\Omega$$

$$R_2 = (1 + 2\sigma)^2 R_1 = 0.4617\,\Omega$$

$$R_{\text{b}} = \frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2} = 0.0022\,\Omega$$

Therefore

$R_4 = R_2 = 0.4617\,\Omega$ $\qquad$ $R_5 = R_1 = 0.4269\,\Omega$

$R_{\text{c}} = R_{\text{b}} = 0.0022\,\Omega$ $\qquad$ $R_{\text{d}} = R_{\text{a}} = 0.0085\,\Omega$

[b] $D = [0.4269 + 2(0.0085)][0.4617 + 2(0.0022)] + 2(0.4617)(0.0022) = 0.2090$

$$i_1 = \frac{V_{\text{dc}}R_2}{D} = 26.51 \text{ A}$$

$i_1^2 R_1 = 300$ W or 200 W/m

$$i_2 = \frac{R_1 + 2R_{\text{a}}}{D}V_{\text{dc}} = 25.49 \text{ A}$$

$i_2^2 R_2 = 300$ W or 200 W/m

$i_1^2 R_{\text{a}} = 6$ W or 200 W/m

$$i_{\text{b}} = \frac{R_1 + R_2 + 2R_{\text{a}}}{D}V_{\text{dc}} = 52 \text{ A}$$

$i_{\text{b}}^2 R_{\text{b}} = 6$ W or 200 W/m

$$i_{\text{source}} = 52 + 52 + \frac{12}{0.48} = 129 \text{ A}$$

$p_{\text{del}} = 12(129) = 1548$ W

$p \quad = 5(300) = 1500$ W

$p_{\text{V}} = 8(6) = 48$ W

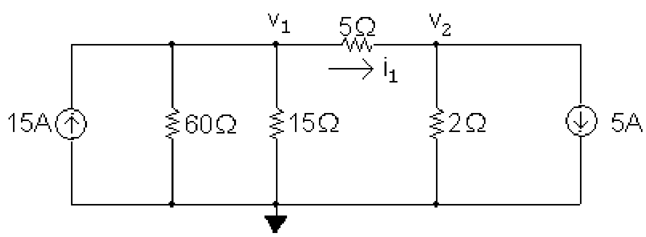
$\sum p_{\text{del}} = \sum p_{\text{diss}} = 1548$ W

4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 **[a]** Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

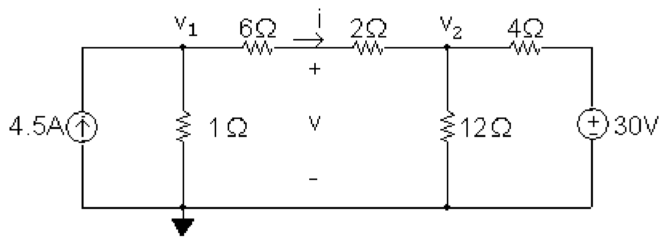
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900$ W(delivered)

[c] $p_{5\text{A}} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50$ W$= -50$ W(delivered)

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6 + 2} = 0$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{6 + 2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$
$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\quad v_2 = 18$ V

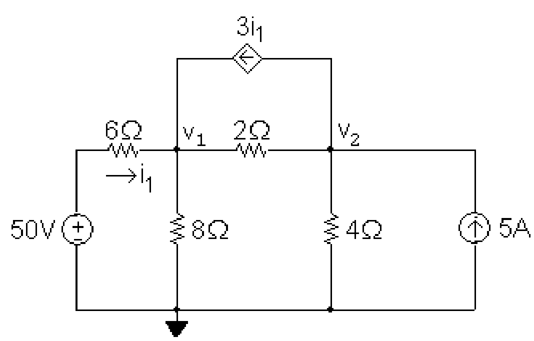
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6 + 2} = \frac{6 - 18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$
$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) &+ v_2\left(-\frac{1}{2}\right) &+ i_1(-3) &= \frac{50}{6} \\
v_1\left(-\frac{1}{2}\right) &+ v_2\left(\frac{1}{4}+\frac{1}{2}\right) &+ i_1(3) &= 5 \\
v_1\left(\frac{1}{6}\right) &+ v_2(0) &+ i_1(1) &= \frac{50}{6}
\end{aligned}$$

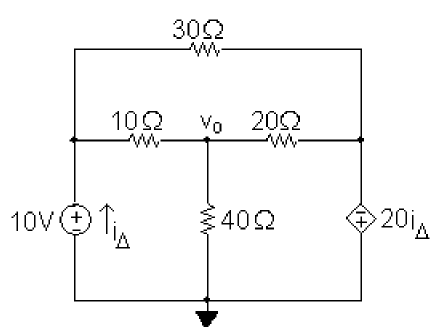
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50 i_1 &= -150 \text{ W} \\
p_{5\text{A}} = -5(v_2) &= -80 \text{ W} \\
p_{3\ 1} = 3 i_1 (v_2 - v_1) &= -144 \text{ W}
\end{aligned}$$

[b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v}{40} + \frac{v\ \ - 10}{10} + \frac{v\ \ + 20 i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v}{10} + \frac{10 + 20 i_\Delta}{30}$$

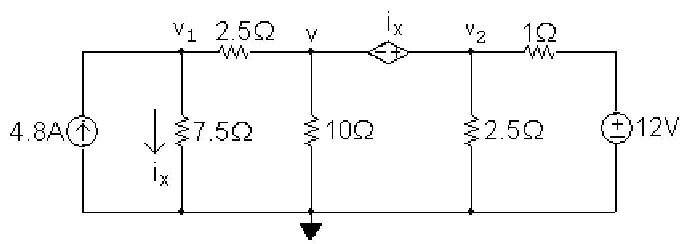
Place these equations in standard form:

$$v\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) + i_\Delta(1) = 1$$

$$v\left(\frac{1}{10}\right) + i_\Delta\left(1-\frac{20}{30}\right) = 1+\frac{10}{30}$$

Solving, $v = 24$ V $\quad i_\Delta = -3.2$ A

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5}+\frac{v_1-v}{2.5}-4.8=0$$

The supernode equation is

$$\frac{v-v_1}{2.5}+\frac{v}{10}+\frac{v_2}{2.5}+\frac{v_2-12}{1}=0$$

The constraint equation due to the dependent source is

$$i = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v+i = v_2$$

Place this set of equations in standard form:

$$v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) + v\left(-\frac{1}{2.5}\right) + v_2(0) + i\ (0) = 4.8$$

$$v_1\left(-\frac{1}{2.5}\right) + v\left(\frac{1}{2.5}+\frac{1}{10}\right) + v_2\left(\frac{1}{2.5}+1\right) + i\ (0) = 12$$

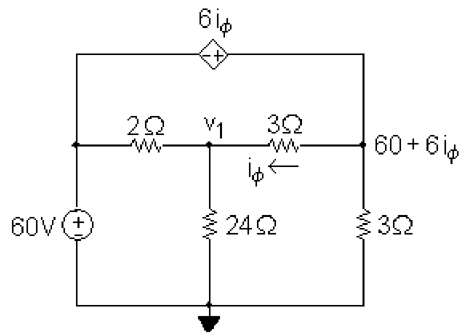
$$v_1\left(-\frac{1}{7.5}\right) + v(0) + v_2(0) + i\ (1) = 0$$

$$v_1(0) + v(1) + v_2(-1) + i\ (1) = 0$$

Solving this set of equations for v gives $v = 8$ V

$v_1 = 15$ V, $\quad v_2 = 10$ V, $\quad i = 2$ A

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i\ \ )}{3} = 0$$

The constraint equation due to the dependent source is

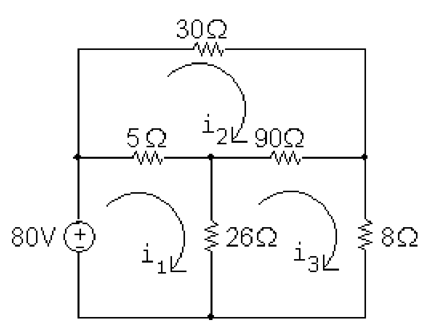
$$i\ \ = \frac{60 + 6i\ \ - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) \quad &+ \quad i\ \ (-2) &= \quad 30 + 20 \\ v_1\left(\frac{1}{3}\right) \quad &+ \quad i\ \ (1 - 2) &= \quad 20 \end{aligned}$$

Solving, $\quad v_1 = 48$ V $\quad i\ \ = -4$ A

AP 4.7 **[a]** Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$i_1 = 5$ A; $\quad i_2 = 2$ A; $\quad i_3 = 2.5$ A

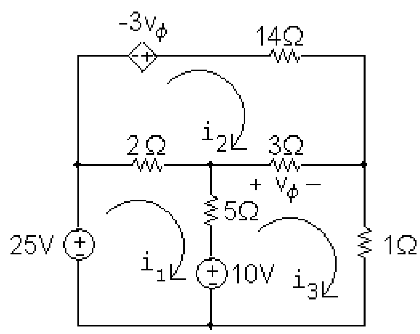
$p_{80} = -(80)i_1 = -(80)(5) = -400$ W

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v\ \ ) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$v\ \ = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v &= 0 \end{aligned}$$

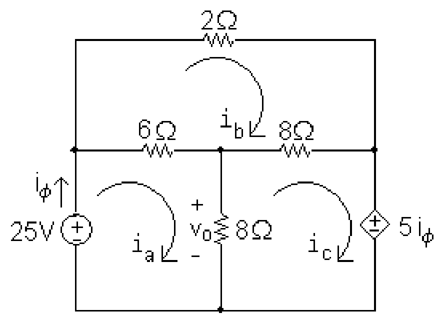
Solving

$i_1 = 4$ A; $\quad i_2 = -1$ A; $\quad i_3 = 3$ A; $\quad v\ \ = 12$ V

$$p_{\rm ds} = -(-3v\ \ )i_2 = 3(12)(-1) = -36\ \text{W}$$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -25 + 6(i_{\rm a} - i_{\rm b}) + 8(i_{\rm a} - i_{\rm c}) &= 0 \\ 2i_{\rm b} + 8(i_{\rm b} - i_{\rm c}) + 6(i_{\rm b} - i_{\rm a}) &= 0 \\ 5i\ \ + 8(i_{\rm c} - i_{\rm a}) + 8(i_{\rm c} - i_{\rm b}) &= 0 \end{aligned}$$

The dependent source constraint equation is $i\ \ = i_{\rm a}$. We can substitute this simple expression for i into the third mesh equation and place the equations in standard form:

$$\begin{aligned} 14i_{\rm a} - 6i_{\rm b} - 8i_{\rm c} &= 25 \\ -6i_{\rm a} + 16i_{\rm b} - 8i_{\rm c} &= 0 \\ -3i_{\rm a} - 8i_{\rm b} + 16i_{\rm c} &= 0 \end{aligned}$$

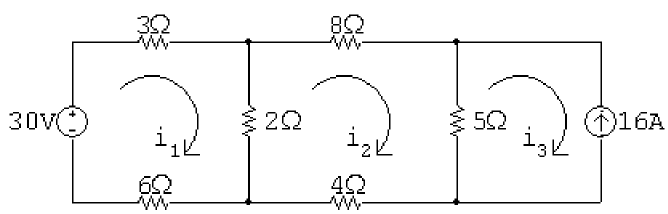
Solving,

$i_{\rm a} = 4$ A; $\quad i_{\rm b} = 2.5$ A; $\quad i_{\rm c} = 2$ A

Thus,

$$v\ \ = 8(i_{\rm a} - i_{\rm c}) = 8(4 - 2) = 16\ \text{V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

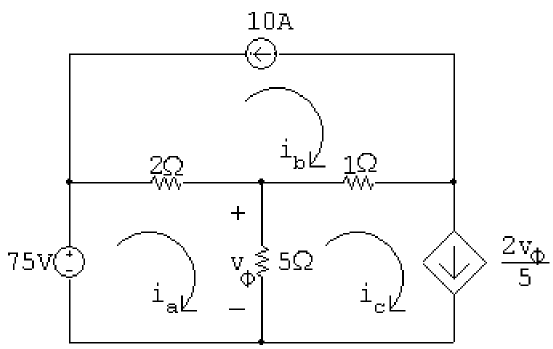
$$-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 = 0$$
$$8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) = 0$$

Place these equations in standard form:

$$11i_1 - 2i_2 = 30$$
$$-2i_1 + 19i_2 = -80$$

Solving: $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72$ W
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_b = -10 \text{ A}; \qquad i_c = \frac{2v}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_a + 10) + 5(i_a - 0.4v\ ) = 0$$

The dependent source requires the following constraint equation:

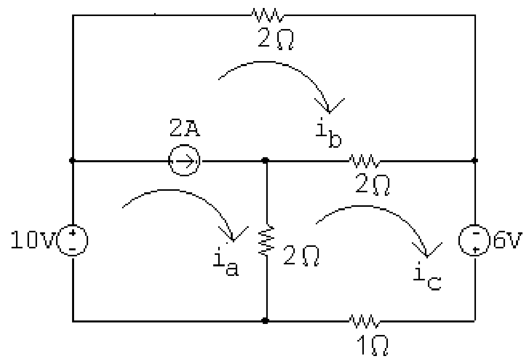
$$v\ = 5(i_a - i_c) = 5(i_a - 0.4v\ )$$

Place the mesh current equation and the dependent source equation is standard form:

$$7i_a - 2v = 55$$
$$5i_a - 3v = 0$$

Solving: $i_a = 15$ A; $i_b = -10$ A; $i_c = 10$ A; $v\ = 25$ V
Thus, $i_a = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_\mathrm{b} + 2(i_\mathrm{b} - i_\mathrm{c}) + 2(i_\mathrm{a} - i_\mathrm{c}) = 0$$

The other mesh current equation is

$$-6 + 1i_\mathrm{c} + 2(i_\mathrm{c} - i_\mathrm{a}) + 2(i_\mathrm{c} - i_\mathrm{b}) = 0$$

The supermesh constraint equation is

$$i_\mathrm{a} - i_\mathrm{b} = 2$$

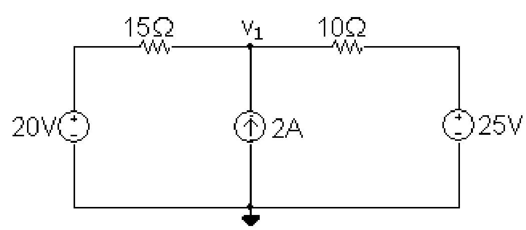
Place these three equations in standard form:

$$\begin{aligned} 2i_\mathrm{a} + 4i_\mathrm{b} - 4i_\mathrm{c} &= 10 \\ -2i_\mathrm{a} - 2i_\mathrm{b} + 5i_\mathrm{c} &= 6 \\ i_\mathrm{a} - i_\mathrm{b} + 0i_\mathrm{c} &= 2 \end{aligned}$$

Solving, $i_\mathrm{a} = 7$ A; $i_\mathrm{b} = 5$ A; $i_\mathrm{c} = 6$ A

Thus, $p_{1\,\Omega} = i_\mathrm{c}^2(1) = (6)^2(1) = 36$ W

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

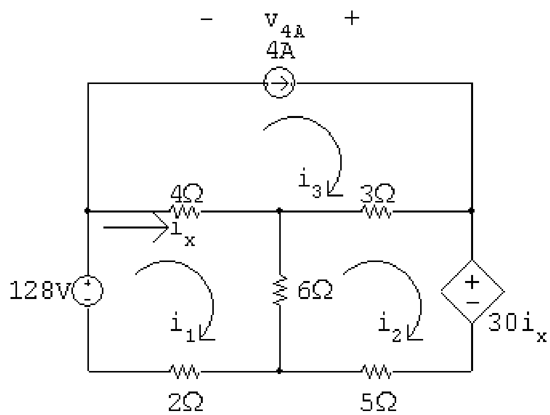
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right)=2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1 = 35\text{ V}$$

$$p_2 \ = -35(2) = -70\text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$\begin{aligned} -128 + 4(i_1 - 4) + 6(i_1 - i_2) + 2i_1 &= 0 \\ 30i \ \ + 5i_2 + 6(i_2 - i_1) + 3(i_2 - 4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i \ \ = i_1 - i_3 = i_1 - 4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1 - 6i_2 &= 144 \\ 24i_1 + 14i_2 &= 132 \end{aligned}$$

Solving,

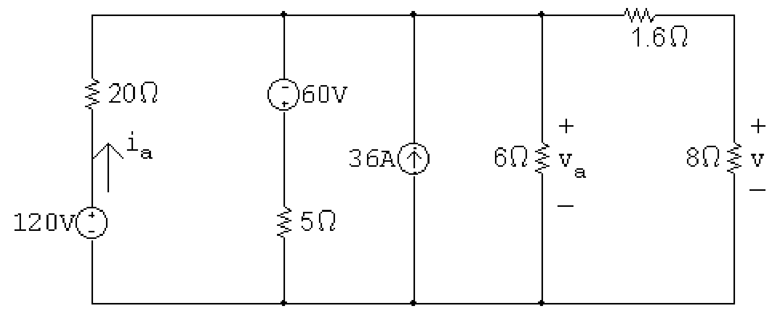
$$i_1 = 9\text{ A}; \qquad i_2 = -6\text{ A}; \qquad i_3 = 4\text{ A}; \qquad i \ \ = 9 - 4 = 5\text{ A}$$

$$\therefore \quad v_{4\text{A}} = 3(i_3 - i_2) - 4i \ \ = 10\text{ V}$$

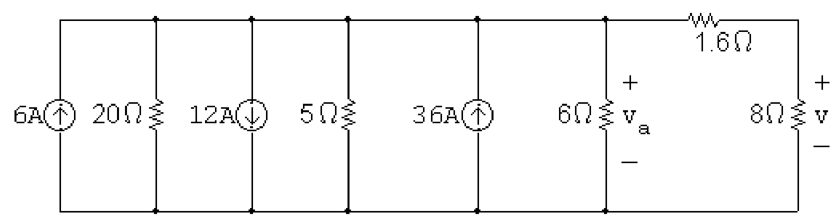
$$p_4 \ = -v_{4\text{A}}(4) = -(10)(4) = -40\text{ W}$$

Thus, the 2 A current source delivers 40 W.

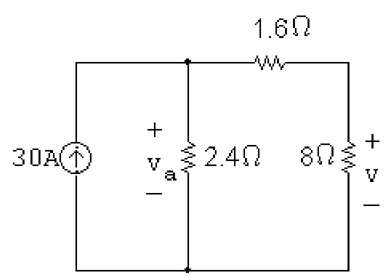
AP 4.15 **[a]** Redraw the circuit with a helpful voltage and current labeled:

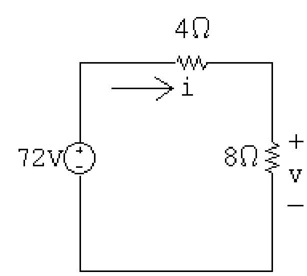


Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.





Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A}$$

Now use i to calculate v_{a} in the circuit on the left:

$$v_{\mathrm{a}} = 6(1.6 + 8) = 57.6 \text{ V}$$

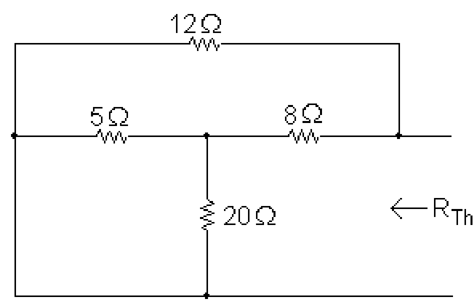
Returning back to the original circuit, note that the voltage v_{a} is also the voltage drop across the series combination of the 120 V source and $20\,\Omega$ resistor. Use this fact to calculate the current in the 120 V source, i_{a}:

$$i_{\mathrm{a}} = \frac{120 - v_{\mathrm{a}}}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120} \quad = -(120)i_{\mathrm{a}} = -(120)(3.12) = -374.40 \text{ W}$$

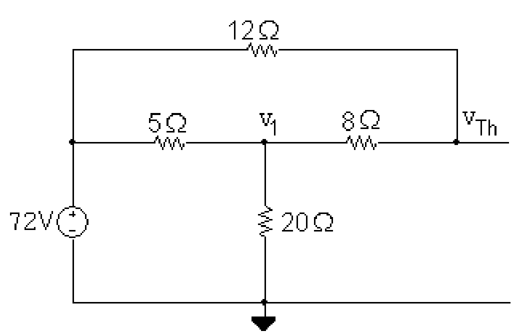
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{\mathrm{Th}} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

$$\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{\mathrm{Th}}}{8} = 0$$

$$\frac{v_{\mathrm{Th}} - v_1}{8} + \frac{v_{\mathrm{Th}} - 72}{12} = 0$$

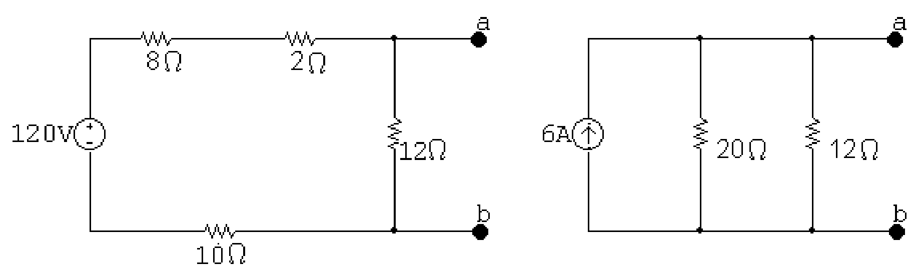
Place these equations in standard form:

$$v_1\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{8}\right) + v_{\text{Th}}\left(-\frac{1}{8}\right) = \frac{72}{5}$$

$$v_1\left(-\frac{1}{8}\right) + v_{\text{Th}}\left(\frac{1}{8}+\frac{1}{12}\right) = 6$$

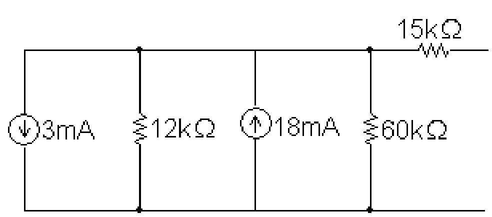
Solving, $v_1 = 60$ V and $v_{\text{Th}} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and $8\,\Omega$ resistor into a series combination of a 120 V source and an $8\,\Omega$ resistor, as shown in the figure on the left. Next, combine the $2\,\Omega$, $8\,\Omega$ and $10\,\Omega$ resistors in series to give an equivalent $20\,\Omega$ resistance. Then transform the series combination of the 120 V source and the $20\,\Omega$ equivalent resistance into a parallel combination of a 6 A source and a $20\,\Omega$ resistor, as shown in the figure on the right.



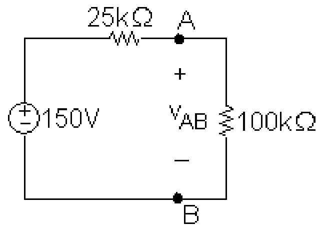
Finally, combine the $20\,\Omega$ and $12\,\Omega$ parallel resistors to give $R_{\text{N}} = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a $7.5\,\Omega$ resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and 12 kΩ resistor into a parallel combination of a -3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k$\|60$ k$= 10$ kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ

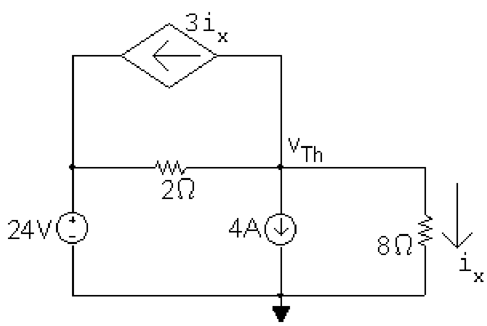
resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a $100\ \text{k}\Omega$ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{\text{AB}} = \frac{100{,}000}{125{,}000}(150) = 120\ \text{V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th}, from the circuit below:



Summing the currents away from the node labeled v_{Th} We have

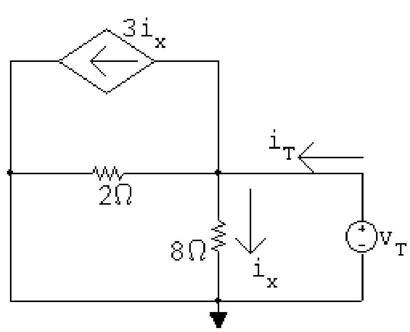
$$\frac{v_{\text{Th}}}{8} + 4 + 3i \;\; + \frac{v_{\text{Th}} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i \;\; = \frac{v_{\text{Th}}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{\text{Th}} = 8$ V.

Now calculate R_{Th}. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_{T}, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_{\mathrm{T}} = i \;\; + 3i \;\; + v_{\mathrm{T}}/2 = 4i \;\; + v_{\mathrm{T}}/2$$

Use Ohm's law to determine i as a function of v_{T}:

$$i \;\; = v_{\mathrm{T}}/8$$

Substitute the second equation into the first equation:

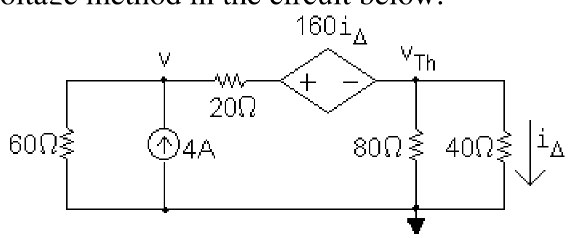
$$i_{\mathrm{T}} = 4(v_{\mathrm{T}}/8) + v_{\mathrm{T}}/2 = v_{\mathrm{T}}$$

Thus,

$$R_{\mathrm{Th}} = v_{\mathrm{T}}/i_{\mathrm{T}} = 1\,\Omega$$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\frac{v}{60} + \frac{v - (v_{\mathrm{Th}} + 160 i_\Delta)}{20} - 4 = 0,$$

$$\frac{v_{\mathrm{Th}}}{40} + \frac{v_{\mathrm{Th}}}{80} + \frac{v_{\mathrm{Th}} + 160 i_\Delta - v}{20} = 0$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{\mathrm{Th}}}{40}$$

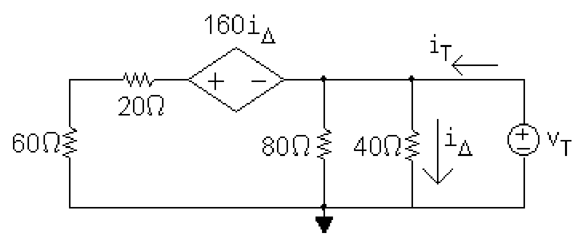
Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$v\left(\frac{1}{60}+\frac{1}{20}\right) + v_{\rm Th}\left(-\frac{5}{20}\right) = 4$$

$$v\left(-\frac{1}{20}\right) + v_{\rm Th}\left(\frac{1}{40}+\frac{1}{80}+\frac{5}{20}\right) = 0$$

Solving, $v = 172.5$ V and $v_{\rm Th} = 30$ V.

Now use the test source method to calculate the test current and thus $R_{\rm Th}$. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\rm T} = \frac{v_{\rm T}}{80} + \frac{v_{\rm T}}{40} + \frac{v_{\rm T}+160i_\Delta}{80}$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{\rm T}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

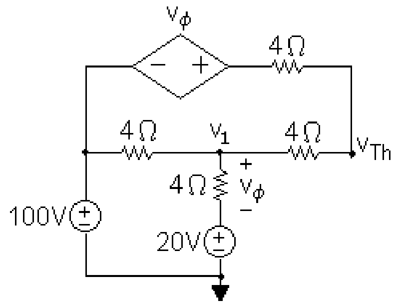
$$i_{\rm T} = \frac{v_{\rm T}}{10}$$

Therefore,

$$R_{\rm Th} = \frac{v_{\rm T}}{i_{\rm T}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find $v_{\rm Th}$, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\frac{v_{\rm Th} - (100 + v\ \ )}{4} + \frac{v_{\rm Th} - v_1}{4} = 0$$

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{\rm Th}}{4} = 0$$

The dependent source constraint equation is

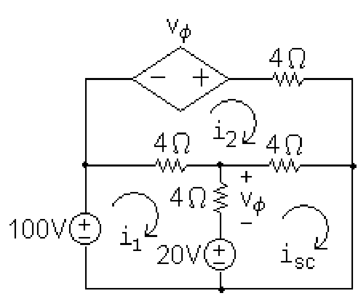
$$v\ \ = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}
v_{\rm Th}\left(\frac{1}{4} + \frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v\ \ \left(-\frac{1}{4}\right) &= 25\\
v_{\rm Th}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) &+ v\ \ (0) &= 30\\
v_{\rm Th}\,(0) &+ v_1\,(1) &+ v\ \ (-1) &= 20
\end{aligned}$$

Solving, $v_{\rm Th} = 120$ V, $v_1 = 80$ V, and $v\ \ = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned}
-100 + 4(i_1 - i_2) + v\ \ + 20 &= 0\\
-v\ \ + 4i_2 + 4(i_2 - i_{\rm sc}) + 4(i_2 - i_1) &= 0\\
-20 - v\ \ + 4(i_{\rm sc} - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is

$$v \;\; = 4(i_1 - i_{sc})$$

Place these four equations in standard form:

$$\begin{aligned}
4i_1 - 4i_2 + 0i_{sc} + v \;\; &= 80 \\
-4i_1 + 12i_2 - 4i_{sc} - v \;\; &= 0 \\
0i_1 - 4i_2 + 4i_{sc} - v \;\; &= 20 \\
4i_1 + 0i_2 - 4i_{sc} - v \;\; &= 0
\end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{sc} = 40$ A, and $v \;\; = 20$ V. Thus,

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{Th} = 3\,\Omega$

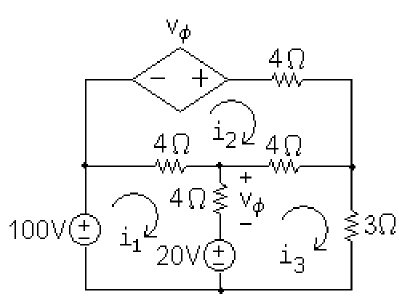
[b] The Thévenin voltage, $v_{Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{load} = \frac{120}{2} = 60 \text{ V}$$

Therefore,

$$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200 \text{ W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned}
-100 + 4(i_1 - i_2) + v \;\; + 20 &= 0 \\
-v \;\; + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\
-20 - v \;\; + 4(i_3 - i_2) + 3i_3 &= 0
\end{aligned}$$

The dependent source constraint equation is

$$v_\Delta = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned}
4i_1 - 4i_2 + 0i_3 + v_\Delta &= 80 \\
-4i_1 + 12i_2 - 4i_3 - v_\Delta &= 0 \\
0i_1 - 4i_2 + 7i_3 - v_\Delta &= 20 \\
4i_1 + 0i_2 - 4i_3 - v_\Delta &= 0
\end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\Delta = 40$ V.

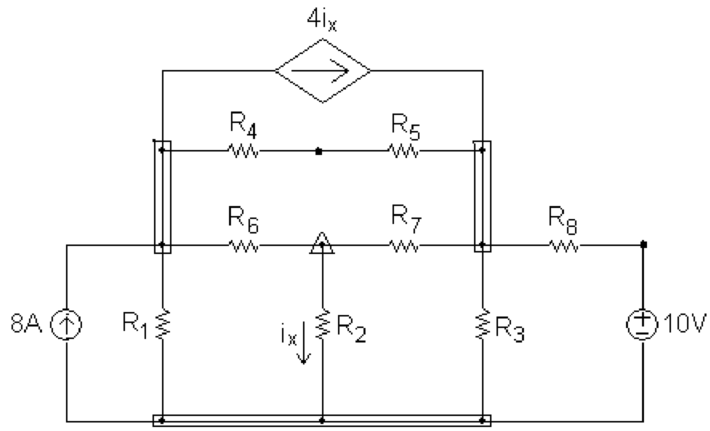
[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\Delta i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1



[a] 11 branches, 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source

[b] The current is unknown in every branch except the one containing the 8 A current source, so the current is unknown in 10 branches.

[c] 9 essential branches – $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches are essential branches that contain a single element.

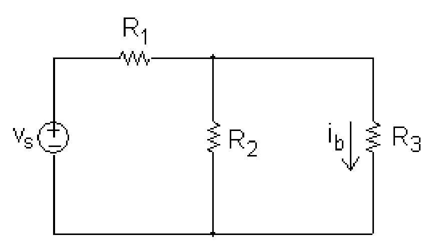
[d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 essential branches

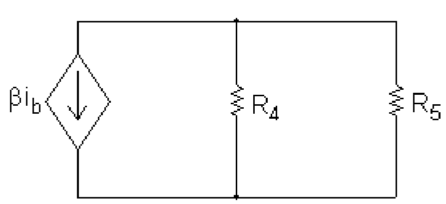
[e] From the figure there are 6 nodes – three identified by rectangular boxes, two identified with single black dots, and one identified by a triangle.

[f] There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle

[g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

P 4.2





[a] As can be seen from the figure, the circuit has 2 separate parts.

[b] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_1.

[c] There are 7 branches, each containing one of the seven circuit components.

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.3 **[a]** From Problem 4.1(d) there are 8 essential branches were the current is unknown, so we need 8 simultaneous equations to describe the circuit.

[b] From Problem 4.1(f), there are 4 essential nodes, so we can apply KCL at $(4-1) = 3$ of these essential nodes. These would also be a dependent source constraint equation.

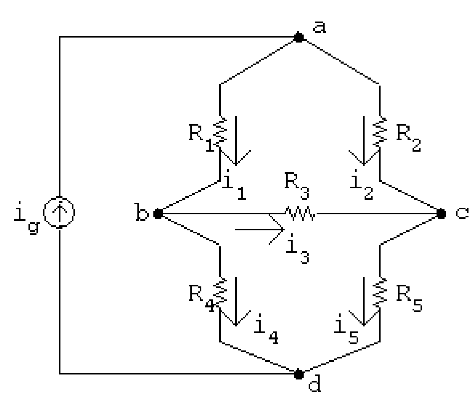
[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the topmost mesh and the leftmost mesh. Each of these meshes contains a current source, and we have no way of determining the voltage drop across a current source.

P 4.4 **[a]** There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$-i\ \ + i_1 + i_2 = 0$

$-i_1 + i_4 + i_3 = 0$

$i_5 - i_2 - i_3 = 0$

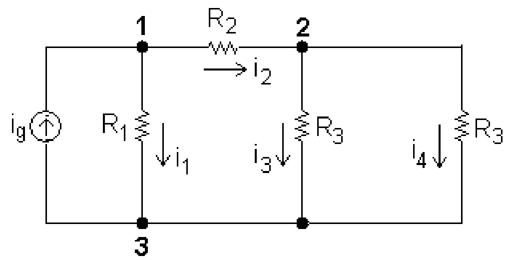
[d] There are three meshes in this circuit: one on the left with the components i , R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$

$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$

P 4.5



[a] At node 1: $\quad - i \ + i_1 + i_2 = 0$

At node 2: $\quad - i_2 + i_3 + i_4 = 0$

At node 3: $\quad i \ - i_1 - i_3 - i_4 = 0$

[b] There are many possible solutions. For example, solve the equation at node 1 for i :

$i \ = i_1 + i_2$

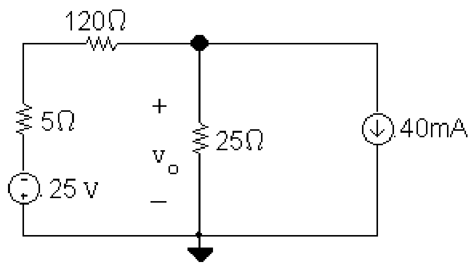
Substitute this expression for i into the equation at node 3:

$(i_1 + i_2) - i_1 - i_3 - i_4 = 0 \qquad \text{so} \qquad i_2 - i_3 - i_4 = 0$

Multiply this last equation by -1 to get the equation at node 2:

$-(i_2 - i_3 - i_4) = -0 \qquad \text{so} \qquad - i_2 + i_3 + i_4 = 0$

P 4.6



Note that we have chosen the lower node as the reference node, and that the voltage at the upper node with respect to the reference node is v . Write a KCL equation (node voltage equation)by summing the currents leaving the upper node:

$$\frac{v\ \ + 25}{120 + 5} + \frac{v}{25} + 0.04 = 0$$

Solve by multiplying both sides of the KCL equation by 125 and collecting the terms involving v on one side of the equation and the constants on the other side of the equation:

$v\ \ + 25 + 5v\ \ + 5 = 0 \qquad \therefore \qquad 6v\ \ = -30 \qquad \text{so} \qquad v\ \ = -30/6 = -5$ V

P 4.7 **[a]** From the solution to Problem 4.6 we know $v\ \ = -5$ V; therefore

$p_{40\text{mA}} = (-5)(0.04) = -0.2$ W

The power developed by the 40 mA source is 200 mW

[b] The current into the negative terminal of the 25 V source in the figure of Problem 4.6 is

$i\ \ = (-5 + 25)/125 = 160$ mA

The power in the 25 V source is

$p_{25\text{V}} = -(25)(0.16) = -4$ W

The power developed by the 25 V source is 4 W

[c]

$$p_{5\Omega} = (0.16)^2(5) = 128 \text{ mW}$$

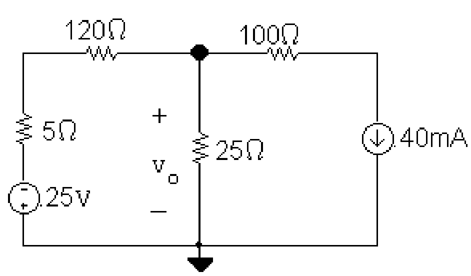
$$p_{120\Omega} = (0.16)^2(120) = 3.072 \text{ W}$$

$$p_{25\Omega} = (-5)^2/25 = 1 \text{ W}$$

$$\sum p_{\text{dis}} = 0.128 + 3.072 + 1 = 4.2 \text{ W}$$

$$\sum p_{\text{dev}} = 0.2 + 4 = 4.2 \text{ W (checks!)}$$

P 4.8



[a] The node voltage equation is:

$$\frac{v\ \ + 25}{125} + \frac{v}{25} + 0.04 = 0$$

Solving,

$v \ + 25 + 5v \ + 5 = 0 \qquad \therefore \qquad 6v \ = -30 \qquad \text{so} \qquad v \ = -5$ V

[b] Let v = voltage drop across 40 mA source:

$v \ = v \ - (100)(0.04) = -5 - 4 = -9$ V

$p_{40\text{mA}} = (-9)(0.04) = -360$ mW

The power developed by the 40 mA source is 360 mW

[c] Let i = current into negative terminal of 25 V source:

$i \ = (-5 + 25)/125 = 160$ mA

$p_{25\text{V}} = -(25)(0.16) = -4$ W

The power developed by the 25 V source is 4 W

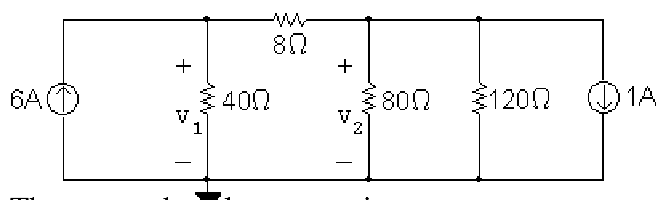
[d]
$$p_{5\Omega} = (0.16)^2(5) = 128 \text{ mW}$$
$$p_{120\Omega} = (0.16)^2(120) = 3.072 \text{ W}$$
$$p_{25\Omega} = (-5)^2/25 = 1 \text{ W}$$
$$p_{100\Omega} = (0.04)^2(100) = 160 \text{ mW}$$

$\sum p_{\text{dis}} = 0.128 + 3.072 + 1 + 0.160 = 4.36$ W

$\sum p_{\text{dev}} = 0.360 + 4 = 4.36$ W (checks!)

[e] v is independent of any finite resistance connected in series with the 40 mA current source

P 4.9



The two node voltage equations are:

$$-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$$
$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Place these equations in standard form:

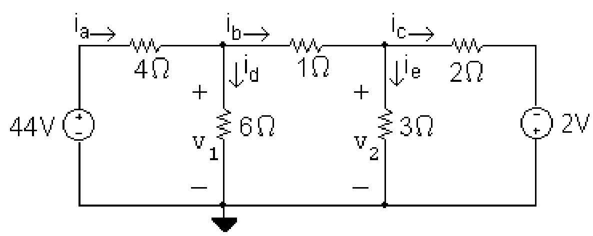
$$v_1\left(\frac{1}{40} + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 6$$
$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{8} + \frac{1}{80} + \frac{1}{120}\right) = -1$$

Solving, $v_1 = 120$ V and $v_2 = 96$ V.

Check this result by calculating the power associated with each component:

Component	Power Delivered (W)	Power Absorbed (W)
6A	$-(6\text{ A})(120\text{ V}) = -720$	
$40\,\Omega$		$\frac{120^2}{40} = 360$
$8\,\Omega$		$\frac{(120-96)^2}{8} = 72$
$80\,\Omega$		$\frac{96^2}{80} = 115.2$
$120\,\Omega$		$\frac{96^2}{120} = 76.8$
1 A		$(96\text{ V})(1\text{ A}) = 96$
Total	-720	720

P 4.10 **[a]**



The two node voltage equations are:

$$\frac{v_1}{6} + \frac{v_1 - 44}{4} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_2}{3} + \frac{v_2 - v_1}{1} + \frac{v_2 + 2}{2} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{6} + \frac{1}{4} + 1\right) + v_2(-1) = \frac{44}{4}$$

$$v_1(-1) + v_2\left(\frac{1}{3} + 1 + \frac{1}{2}\right) = -\frac{2}{2}$$

Solving, $v_1 = 12$ V; $\quad v_2 = 6$ V

Now calculate the branch currents from the node voltage values:

$$i_a = \frac{44 - 12}{4} = 8\text{ A}$$

$$i_b = \frac{12}{6} = 2\text{ A}$$

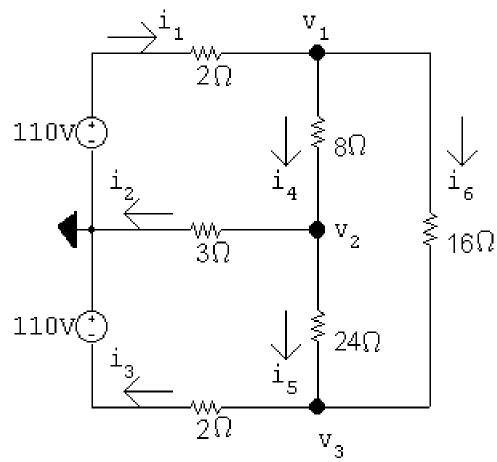
$$i_c = \frac{12 - 6}{1} = 6\text{ A}$$

$$i_d = \frac{6}{3} = 2\text{ A}$$

$$i_e = \frac{6 + 2}{2} = 4\text{ A}$$

[b] $p_{\rm sources} = p_{44\rm V} + p_{2\rm V} = -(44)i_{\rm a} - (2)i_{\rm e} = -(44)(8) - (2)(4) = -352 - 8 = -360$ W
Thus, the power developed in the circuit is 360 W. Note that the resistors cannot develop power!

P 4.11 **[a]**



$$\frac{v_1-110}{2}+\frac{v_1-v_2}{8}+\frac{v_1-v_3}{16}=0 \qquad \text{so} \qquad 11v_1-2v_2-v_3=880$$

$$\frac{v_2-v_1}{8}+\frac{v_2}{3}+\frac{v_2-v_3}{24}=0 \qquad \text{so} \qquad -3v_1+12v_2-v_3=0$$

$$\frac{v_3+110}{2}+\frac{v_3-v_2}{24}+\frac{v_3-v_1}{16}=0 \qquad \text{so} \qquad -3v_1-2v_2+29v_3=-2640$$

Solving, $v_1 = 74.64$ V; $v_2 = 11.79$ V; $v_3 = -82.5$ V

Thus, $i_1 = \frac{110-v_1}{2} = 17.68$ A $\quad i_4 = \frac{v_1-v_2}{8} = 7.86$ A

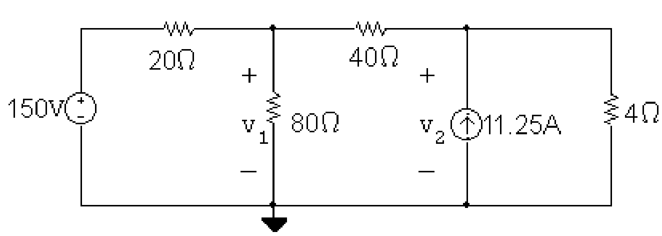
$i_2 = \frac{v_2}{3} = 3.93$ A $\quad i_5 = \frac{v_2-v_3}{24} = 3.93$ A

$i_3 = \frac{v_3+110}{2} = 13.75$ A $\quad i_6 = \frac{v_1-v_3}{16} = 9.82$ A

[b] $\sum P_{\rm dev} = 110i_1 + 110i_3 = 3457.14$ W

$\sum P_{\rm dis} = i_1^2(2) + i_2^2(3) + i_3^2(2) + i_4^2(8) + i_5^2(24) + i_6^2(16) = 3457.14$ W

P 4.12



The two node voltage equations are:

$$\frac{v_1-150}{20}+\frac{v_1}{80}+\frac{v_1-v_2}{40} = 0$$

$$\frac{v_2-v_1}{40}-11.25+\frac{v_2}{4} = 0$$

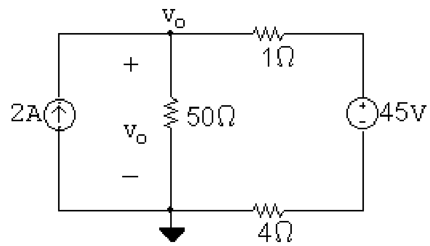
Place these equations in standard form:

$$v_1\left(\frac{1}{20}+\frac{1}{80}+\frac{1}{40}\right) + v_2(-\frac{1}{40}) = \frac{150}{20}$$

$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40}+\frac{1}{4}\right) = 11.25$$

Solving, $v_1 = 100$ V; $v_2 = 50$ V

P 4.13



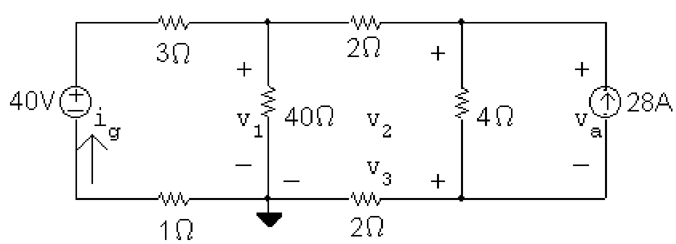
At v : $-2+\frac{v}{50}+\frac{v-45}{4+1}=0$

Solving, $v = 50$ V

$p_{2\text{A}} = -(50)(2) = -100$ W

Thus, the 2 A current source delivers 100 W, or the current source extracts -100 W from the circuit.

P 4.14



The three node voltage equations are:

$$\frac{v_1-40}{4}+\frac{v_1}{40}+\frac{v_1-v_2}{2} = 0$$

$$\frac{v_2-v_1}{2}+\frac{v_2-v_3}{4}-28 = 0$$

$$\frac{v_3}{2}+\frac{v_3-v_2}{4}+28 = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{4}+\frac{1}{40}+\frac{1}{2}\right) + v_2\left(-\frac{1}{2}\right) + v_3(0) = \frac{40}{4}$$

$$v_1\left(-\frac{1}{2}\right) + v_2\left(\frac{1}{2}+\frac{1}{4}\right) + v_3\left(-\frac{1}{4}\right) = 28$$

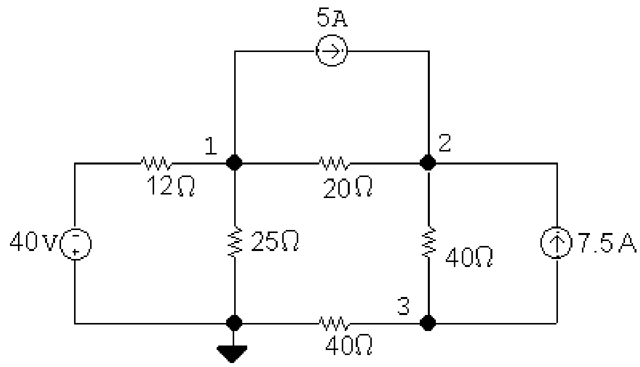
$$v_1(0) + v_2\left(-\frac{1}{4}\right) + v_3\left(\frac{1}{2}+\frac{1}{4}\right) = -28$$

Solving, $v_1 = 60$ V; $v_2 = 73$ V; $v_3 = -13$ V.

$$p_{28\text{A}} = -v_\text{a}(28\text{ A}) = -(v_2 - v_3)(28\text{ A}) = -(73 + 13)(28) = -2408\text{ W}$$

The 28 A source delivers 2408 W.

P 4.15



The node voltage equations are:

$$\begin{aligned}
\frac{v_1 + 40}{12} + \frac{v_1}{25} + 5 + \frac{v_1 - v_2}{20} &= 0 \\
\frac{v_2 - v_1}{20} + \frac{v_2 - v_3}{40} - 7.5 - 5 &= 0 \\
\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{12} + \frac{1}{25} + \frac{1}{20}\right) + v_2\left(-\frac{1}{20}\right) + v_3(0) &= -\frac{40}{12} - 5 \\
v_1\left(-\frac{1}{20}\right) + v_2\left(\frac{1}{20} + \frac{1}{40}\right) + v_3\left(-\frac{1}{40}\right) &= 12.5 \\
v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3\left(\frac{1}{40} + \frac{1}{40}\right) &= -7.5
\end{aligned}$$

Solving, $v_1 = -10$ V; $v_2 = 132$ V; $v_3 = -84$ V.

Find the power:

$$\begin{aligned}
i_{40\text{V}} &= (-10 + 40)/12 = 2.5\text{ A} \\
p_{40\text{V}} &= -(2.5)(40) = -100\text{ W} \quad \text{(del)} \\
p_{5\text{A}} &= (5)(-10 - 132) = -710\text{ W} \quad \text{(del)} \\
p_{7\,5\text{A}} &= (7.5)(-84 - 132) = -1620\text{ W} \quad \text{(del)} \\
p_{12\Omega} &= (-10 + 40)^2/12 = 75\text{ W} \quad \text{(abs)} \\
p_{25\Omega} &= (-10)^2/25 = 4\text{ W} \quad \text{(abs)}
\end{aligned}$$

$$p_{20\Omega} = (132+10)^2/20 = 1008.2\text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = (132+84)^2/40 = 1166.4\text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = (-84)^2/40 = 176.4\text{ W} \quad \text{(abs)}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 1166.4 + 176.4 = 2430\text{ W}$$

$$\sum p_{\text{dev}} = 100 + 710 + 1620\text{ W} = 2430\text{ W} \qquad \text{(CHECKS)}$$

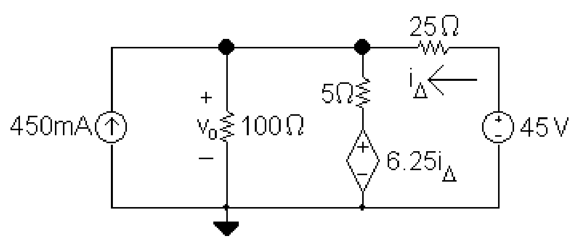
P 4.16 **[a]** $\frac{v \;\; - v_1}{R} + \frac{v \;\; - v_2}{R} + \frac{v \;\; - v_3}{R} + \cdots + \frac{v \;\; - v}{R} = 0$

$\therefore \quad nv \;\; = v_1 + v_2 + v_3 + \cdots + v$

$\therefore \quad v \;\; = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v \;\;] = \frac{1}{n}\sum_{\;\;=1} v$

[b] $v \;\; = \frac{1}{3}(120 + 60 - 30) = 50\text{ V}$

P 4.17 **[a]**



The node voltage equation is:

$$-0.45 + \frac{v}{100} + \frac{v \;\; - 6.25i_\Delta}{5} + \frac{v \;\; - 45}{25} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{45 - v}{25}$$

Place these equations in standard form:

$$v \left(\frac{1}{100} + \frac{1}{5} + \frac{1}{25}\right) + i_\Delta\left(-\frac{6.25}{5}\right) = \frac{45}{25} + 0.45$$

$$v \left(\frac{1}{25}\right) + i_\Delta(1) = \frac{45}{25}$$

Solving, $\quad v \;\; = 15\text{ V}; \qquad i_\Delta = 1.2\text{ A}$

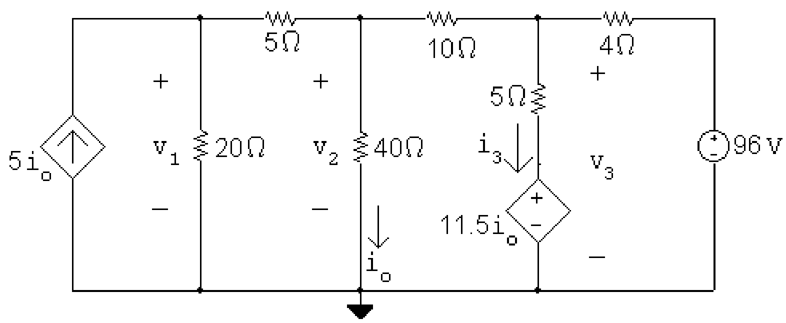
[b] $i_{\text{ds}} = \frac{v \;\; - 6.25i_\Delta}{5} = \frac{15 - 7.5}{5} = 1.5\text{ A}$

$p_{\text{ds}} = [6.25(1.2)](1.5) = 11.25\text{ W}$

Thus, the dependent source absorbs 11.25 W

[c] $p_{450\text{mA}} = -(0.45)(15) = -6.75$ W
$p_{45\text{V}} = -(1.2)(45) = -54$ W
$\sum p_{\text{dev}} = 6.75 + 54 = 60.75$ W
Thus the independent sources develop 60.75 W
Also,
$\sum p_{\text{dis}} = p_{\text{ds}} + p_{100\Omega} + p_{5\Omega} + p_{25\Omega}$
$= 11.25 + (15)^2/100 + (1.5)^2(5) + (1.2)^2(25)$
$= 11.25 + 2.25 + 11.25 + 36 = 60.75$ W (checks!)

P 4.18 **[a]**



The node voltage equations are:

$$-5i + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i}{5} + \frac{v_3 - 96}{4} = 0$$

The dependent source constraint equation is:
$i = v_2/40$
Place these equations in standard form:

$$v_1\left(\frac{1}{20} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) + i\ (-5) = 0$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) + i\ (0) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right) + i\ \left(-\frac{11.5}{5}\right) = \frac{96}{4}$$

$$v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3(0) + i\ (1) = 0$$

Solving, $v_1 = 156$ V; $v_2 = 120$ V; $v_3 = 78$ V; $i = 3$ A

[b] Calculate the power:

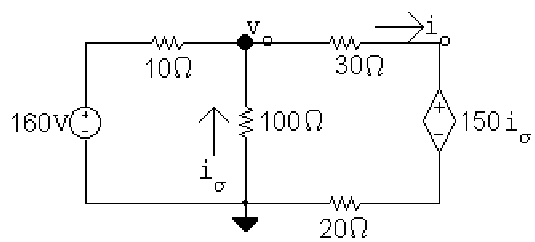
$$
\begin{aligned}
p_{\text{cccs}} &= -[5(3)](156) = -2340 \text{ W} \\
p_{20\Omega} &= (156)^2/20 = 1216.8 \text{ W} \\
p_{5\Omega} &= (156-120)^2/5 = 259.2 \text{ W} \\
p_{40\Omega} &= (120)^2/40 = 360 \text{ W} \\
p_{10\Omega} &= (120-78)^2/10 = 176.4 \text{ W} \\
p_{5\Omega} &= (78-11.5\cdot 3)^2/5 = 378.45 \text{ W} \\
p_{4\Omega} &= (78-96)^2/4 = 81 \text{ W} \\
p_{96\text{V}} &= [(78-96)/4](96) = -432 \text{ W} \\
p_{\text{ccvs}} &= [(78-3\cdot 11.5)/5](11.5\cdot 3) = 300.15 \text{ W}
\end{aligned}
$$

$\sum p_{\text{dev}} = 2340 + 432 = 2772$ W

$\sum p_{\text{dis}} = 1216.8 + 259.2 + 360 + 176.4 + 378.45 + 81 + 300.15 = 2772$ W (checks)

Thus, the circuit dissipates 2772 W

P 4.19



The node voltage equation is

$$\frac{v\ \ - 160}{10} + \frac{v}{100} + \frac{v\ \ - 150i}{30+20} = 0$$

The dependent source constraint equation is:

$$i\ \ = -\frac{v}{100}$$

Place these equations in standard form:

$$
\begin{aligned}
v\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{50}\right) &+ i\left(-\frac{150}{50}\right) &= \frac{160}{10} \\
v\left(\frac{1}{100}\right) &+ i\ (1) &= 0
\end{aligned}
$$

Solving, $v\ = 100$ V; $i\ = -1$ A

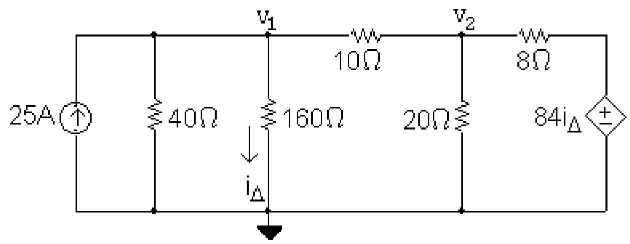
Now find the power:

$$i\ = \frac{160-100}{10} - 1 = 5 \text{ A}$$

$p_{\text{ds}} = [150(-1)](5) = -750$ W.

Thus, the dependent source delivers 750 W

P 4.20 **[a]**



The node voltage equations are:

$$-25 + \frac{v_1}{40} + \frac{v_1}{160} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} + \frac{v_2}{20} + \frac{v_2 - 84i_\Delta}{8} = 0$$

The dependent source constraint equation is:

$i_\Delta = v_1/160$

Place these three equations in standard form:

$$v_1\left(\frac{1}{40} + \frac{1}{160} + \frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) + i_\Delta(0) = 25$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{8}\right) + i_\Delta\left(-\frac{84}{8}\right) = 0$$

$$v_1\left(-\frac{1}{160}\right) + v_2(0) + i_\Delta(1) = 0$$

Solving, $v_1 = 352$ V; $v_2 = 212$ V; $i_\Delta = 2.2$ A

Now calculate the power. Only the two sources can develop power, so focus on the sources:

$$p_{25\text{A}} = -(352)(25) = -8800 \text{ W}$$

$$i_{\text{dep source}} = (v_2 - 84i_\Delta)/8 = (212 - 84 \cdot 2.2)/8 = 3.4 \text{ A}$$

$$p_{\text{dep source}} = (84 \cdot 2.2)(3.4) = 628.32 \text{ W}$$

Thus, only the current source develops power, so the total power developed in the circuit is 8800 W

[b] The dependent source and all of the resistors dissipate the power developed by the current source. Check that the power developed equals the power dissipated:

$$p_{40\Omega} = (352)^2/40 = 3097.6 \text{ W}$$

$$p_{160\Omega} = (352)^2/160 = 774.4 \text{ W}$$

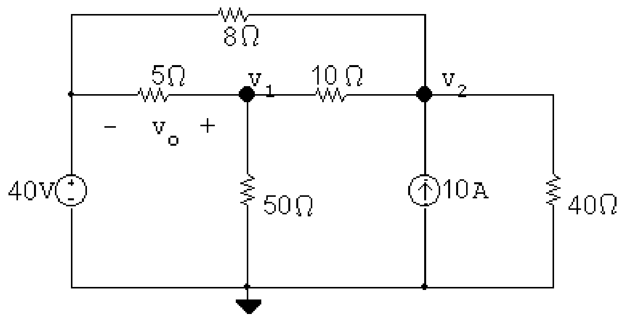
$$p_{10\Omega} = (352 - 212)^2/10 = 1960 \text{ W}$$

$$p_{20\Omega} = (212)^2/20 = 2247.2 \text{ W}$$

$$p_{8\Omega} = (212 - 84 \cdot 2.2)^2/8 = 92.48 \text{ W}$$

$\sum p_{\text{diss}} = 628.32 + 3097.6 + 774.4 + 1960 + 2247.2 + 92.48 = 8800$ W so the power balances.

P 4.21



The two node voltage equations are:

$$\frac{v_1 - 40}{5} + \frac{v_1}{50} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} - 10 + \frac{v_2}{40} + \frac{v_2 - 40}{8} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{5} + \frac{1}{50} + \frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) = \frac{40}{5}$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10} + \frac{1}{40} + \frac{1}{8}\right) = 10 + \frac{40}{8}$$

Solving, $v_1 = 50$ V; $v_2 = 80$ V.
Thus, $v\ = v_1 - 40 = 50 - 40 = 10$ V.
POWER CHECK:

$$i = (50 - 40)/5 + (80 - 40)/8 = 7 \text{ A}$$

$$p_{40\text{V}} = (40)(7) = 280 \text{ W} \quad \text{(abs)}$$

$$p_{5\Omega} = (50 - 40)^2/5 = 20 \text{ W} \quad \text{(abs)}$$

$$p_{8\Omega} = (80 - 40)^2/8 = 200 \text{ W} \quad \text{(abs)}$$

$$p_{10\Omega} = (80 - 50)^2/10 = 90 \text{ W} \quad \text{(abs)}$$

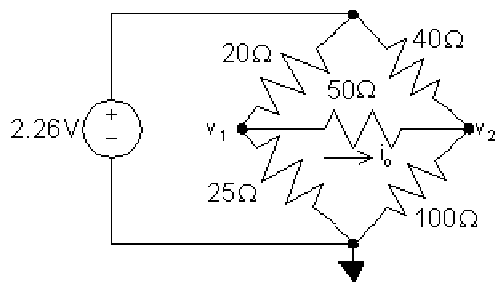
$$p_{50\Omega} = 50^2/50 = 50 \text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = 80^2/40 = 160 \text{ W} \quad \text{(abs)}$$

$$p_{10\text{A}} = -(80)(10) = -800 \text{ W} \quad \text{(del)}$$

$$\sum p_{\text{abs}} = 280 + 20 + 200 + 90 + 50 + 160 = 800 \text{ W} = \sum p_{\text{del}}$$

P 4.22



The node voltage equations are:

$$\frac{v_1 - 2.26}{20} + \frac{v_1 - v_2}{50} + \frac{v_1}{25} = 0$$

$$\frac{v_2 - 2.26}{40} + \frac{v_2 - v_1}{50} + \frac{v_2}{100} = 0$$

Place these equations in standard form:

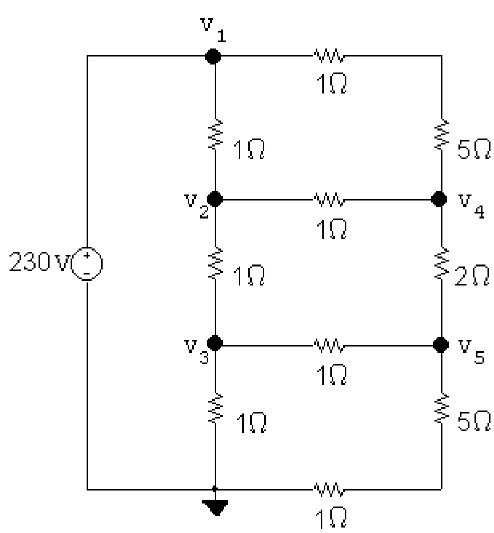
$$v_1\left(\frac{1}{20} + \frac{1}{50} + \frac{1}{25}\right) + v_2\left(-\frac{1}{50}\right) = \frac{2.26}{20}$$

$$v_1\left(-\frac{1}{50}\right) + v_2\left(\frac{1}{40} + \frac{1}{50} + \frac{1}{100}\right) = \frac{2.26}{40}$$

Solving, $v_1 = 1.3$ V; $v_2 = 1.5$ V.

Thus, $i = \dfrac{v_1 - v_2}{50} = \dfrac{1.3 - 1.5}{50} = -4$ mA

P 4.23 **[a]**



The node voltage equations are:

$$\frac{v_2-230}{1}+\frac{v_2-v_4}{1}+\frac{v_2-v_3}{1} = 0$$
$$\frac{v_3-v_2}{1}+\frac{v_3-v_5}{1}+\frac{v_3}{1} = 0$$
$$\frac{v_4-230}{5+1}+\frac{v_4-v_2}{1}+\frac{v_4-v_5}{2} = 0$$
$$\frac{v_5-v_4}{2}+\frac{v_5-v_3}{1}+\frac{v_5}{5+1} = 0$$

Place these equations in standard form:

$$v_2(1+1+1)+v_3(-1)+v_4(-1)+v_5(0) = 230$$
$$v_2(-1)+v_3(1+1+1)+v_4(0)+v_5(-1) = 0$$
$$v_2(-1)+v_3(0)+v_4\left(\frac{1}{6}+1+\frac{1}{2}\right)+v_5\left(-\frac{1}{2}\right) = \frac{230}{6}$$
$$v_2(0)+v_3(-1)+v_4\left(-\frac{1}{2}\right)+v_5\left(\frac{1}{2}+1+\frac{1}{6}\right) = 0$$

Solving, $v_2 = 150$ V; $v_3 = 80$ V; $v_4 = 140$ V; $v_5 = 90$ V

Find the power dissipated by the $2\,\Omega$ resistor:

$i_{2\Omega} = \frac{v_4-v_5}{2} = \frac{140-90}{2} = 25$ A

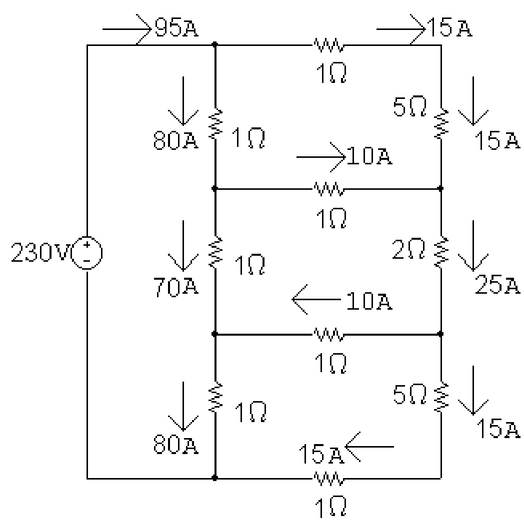
$p_{2\Omega} = (25)^2(2) = 1250$ W

[b] Find the power developed by the 230 V source:

$i_{230\text{V}} = \frac{v_2-230}{1}+\frac{v_4-230}{6} = -80-15 = -95$ A

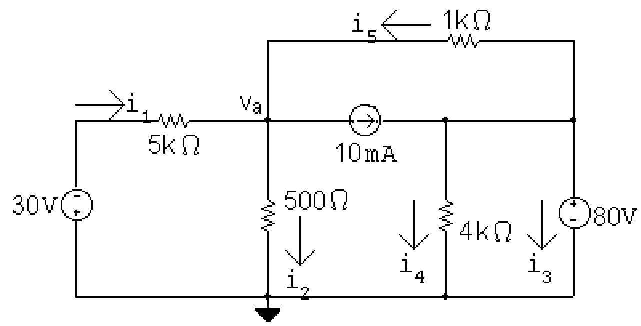
$p_{230\text{V}} = (230)(-95) = -21{,}850$ W, so the source supplies $21{,}850$ W

Check:



$$\begin{aligned}\sum P_{\text{dis}} &= (80)^2(1)+(15)^2(1)+(15)^2(5)+(70)^2(1)+(10)^2(1)\\ &\quad +(25)^2(2)+(10)^2(1)+(80)^2(1)+(15)^2(5)+(15)^2(1)\\ &= 21{,}850 \text{ W(checks)}\end{aligned}$$

P 4.24 **[a]**



There is only one node voltage equation:

$$\frac{v_a + 30}{5000} + \frac{v_a}{500} + \frac{v_a - 80}{1000} + 0.01 = 0$$

Solving,

$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0$ so $16v_a = 320$

$\therefore \quad v_a = 20$ V

Calculate the currents:

$$i_1 = (-30 - 20)/5000 = -10 \text{ mA}$$

$$i_2 = 20/500 = 40 \text{ mA}$$

$$i_4 = 80/4000 = 20 \text{ mA}$$

$$i_5 = (80 - 20)/1000 = 60 \text{ mA}$$

$i_3 + i_4 + i_5 - 10 \text{ mA} = 0$ so $i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70$ mA

[b]

$$p_{30\text{V}} = (30)(-0.01) = -0.3 \text{ W}$$

$$p_{10\text{mA}} = (20 - 80)(0.01) = -0.6 \text{ W}$$

$$p_{80\text{V}} = (80)(-0.07) = -5.6 \text{ W}$$

$$p_{5\text{k}} = (-0.01)^2(5000) = 0.5 \text{ W}$$

$$p_{500\Omega} = (0.04)^2(500) = 0.8 \text{ W}$$

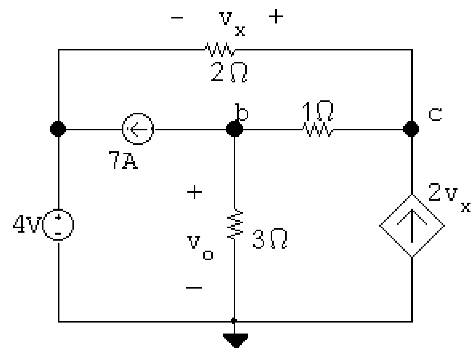
$$p_{1\text{k}} = (80 - 20)^2/(1000) = 3.6 \text{ W}$$

$$p_{4\text{k}} = (80)^2/(4000) = 1.6 \text{ W}$$

$\sum p_{\text{abs}} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5$ W

$\sum p_{\text{del}} = 0.3 + 0.6 + 5.6 = 6.5$ W (checks!)

P 4.25



The two node voltage equations are:

$$7 + \frac{v_\mathrm{b}}{3} + \frac{v_\mathrm{b} - v_\mathrm{c}}{1} = 0$$

$$-2v\ \ + \frac{v_\mathrm{c} - v_\mathrm{b}}{1} + \frac{v_\mathrm{c} - 4}{2} = 0$$

The constraint equation for the dependent source is:
$v\ \ = v_\mathrm{c} - 4$

Place these equations in standard form:

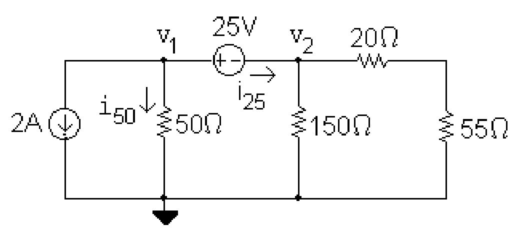
$$v_\mathrm{b}\left(\frac{1}{3} + 1\right) + v_\mathrm{c}(-1) + v\ \ (0) = -7$$

$$v_\mathrm{b}(-1) + v_\mathrm{c}\left(1 + \frac{1}{2}\right) + v\ \ (-2) = \frac{4}{2}$$

$$v_\mathrm{b}(0) + v_\mathrm{c}(1) + v\ \ (-1) = 4$$

Solving, $v\ \ = v_\mathrm{b} = 1.5$ V Also, $v\ \ = 9$ V and $v\ \ = 5$ V.

P 4.26



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{20 + 55} = 0$$

The supernode constraint equation is

$$v_2 + 25 = v_1$$

Place these two equations in standard form:

$$v_1\left(\frac{1}{50}\right) \quad + \quad v_2\left(\frac{1}{150}+\frac{1}{75}\right) \quad = \quad -2$$

$$v_1(1) \quad + \quad v_2(-1) \quad = \quad 25$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V.

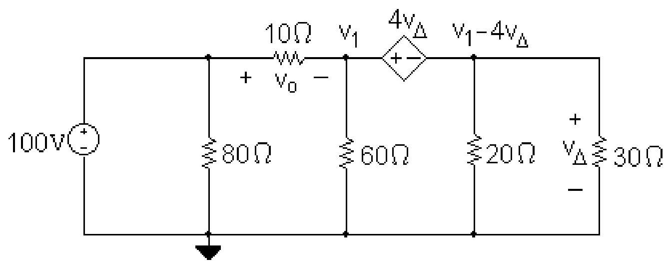
$p_{25\text{V}} = (25)i_{25}$

$$i_{25} = -2\text{ A} - i_{50} = -2\text{ A} - \frac{v_1}{50} = 2\text{ A} - \frac{-37.5}{50} = -2\text{ A} + 0.75\text{ A} = -1.25\text{ A}$$

Thus, $\quad p_{25\text{V}} = (25)(-1.25) = -31.25$ W

The 25 V source delivers 31.25 W.

P 4.27



The supernode equation is:

$$\frac{v_1-100}{10}+\frac{v_1}{60}+\frac{v_1-4v_\Delta}{20}+\frac{v_1-4v_\Delta}{30}=0$$

The constraint equation for the dependent source is:

$4v_\Delta = v_1 - v_\Delta$

Place these equations in standard form:

$$v_1\left(\frac{1}{10}+\frac{1}{60}+\frac{1}{20}+\frac{1}{30}\right) \quad + \quad v_\Delta\left(-\frac{4}{20}-\frac{4}{30}\right) \quad = \quad \frac{100}{10}$$

$$v_1(1) \quad + \quad v_\Delta(-5) \quad = \quad 0$$

Solving, $\quad v_1 = 75$ V; $\quad v_\Delta = 15$ V

Thus, $v = 100 - v_1 = 25$ V

P 4.28 Calculate currents and voltages needed to calculate the power for the various components:

$$i \quad = \quad \frac{v_4-v_3}{8} = \frac{81.6-108}{8} = -3.3\text{ A}$$

$$\frac{40}{3}i \quad = \quad \frac{40}{3}(-3.3) = -44\text{ V}$$

$$v_1 \quad = \quad v_4 + \frac{40}{3}i \quad = 81.6 - 44 = 37.6\text{ V}$$

$$v_3 + v_\Delta = 120 \quad \therefore \quad v_\Delta = 120 - 108 = 12 \text{ V}$$

$$1.75v_\Delta = (1.75)(12) = 21 \text{ A}$$

$$i_{120\text{V}} = \frac{v_1 - 120}{4} + \frac{v_3 - 120}{2} = \frac{37.6 - 120}{4} + \frac{108 - 120}{2} = -26.6 \text{ A}$$

$$i_{\text{ccvs}} = \frac{0 - v_1}{20} + \frac{v_2 - v_1}{4} = \frac{-37.6}{20} + \frac{120 - 37.6}{4} = 18.72 \text{ A}$$

Now calculate the power associated with each circuit element:

$$p_{20\Omega} = (37.6)^2/20 = 70.688 \text{ W}$$

$$p_{4\Omega} = (37.6 - 120)^2/4 = 1697.44 \text{ W}$$

$$p_{120\text{V}} = (120)(-26.6) = -3192 \text{ W}$$

$$p_{2\Omega} = (12)^2/2 = 72 \text{ W}$$

$$p_{40\Omega} = (108)^2/40 = 291.6 \text{ W}$$

$$p_{8\Omega} = (108 - 81.6)^2/8 = 87.12 \text{ W}$$

$$p_{80\Omega} = (81.6)^2/80 = 83.232 \text{ W}$$

$$p_{\text{vccs}} = (81.6)[1.75(12)] = 1713.6 \text{ W} \qquad \sum p_{\text{abs}} = \sum p_{\text{del}} = 4015.6 \text{ W}$$

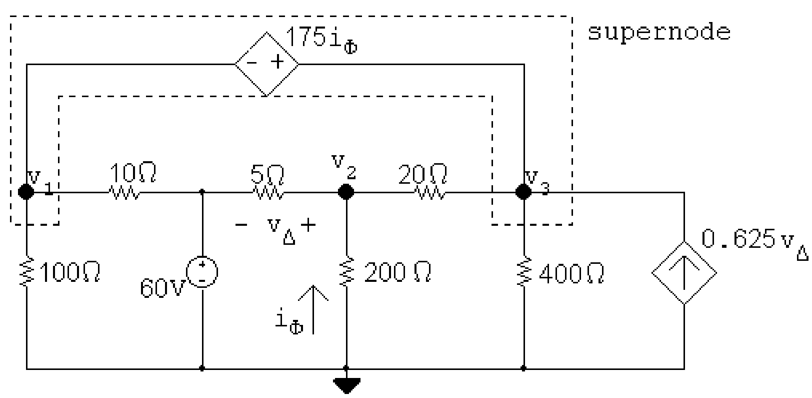
$$p_{\text{ccvs}} = (18.72)(-44) = -823.68 \text{ W}$$

Now sum the powers:

$$\sum p_{\text{total}} = 70.688 + 1697.44 - 3192 + 72 + 291.6 + 87.12 + 83.232 + 1712.6 - 823.68 = 0 \text{ W}$$

Thus, the power balances and the staff analyst has correctly calculated the voltage values

P 4.29



The supernode equation is:

$$\frac{v_1}{100} + \frac{v_1 - 60}{10} + \frac{v_3 - v_2}{20} + \frac{v_3}{400} - 0.625v_\Delta = 0$$

The node voltage equation at v_2 is:

$$\frac{v_2 - 60}{5} + \frac{v_2}{200} + \frac{v_2 - v_3}{20} = 0$$

The supernode constraint equation is:

$v_3 - v_1 = 175i$

The two dependent source constraint equations are:

$v_\Delta = v_2 - 60$

$i \ = -v_2/200$

Place the four equations above in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{100}+\frac{1}{10}\right) + v_2\left(-\frac{1}{20}\right) + v_3\left(\frac{1}{400}+\frac{1}{20}\right) + i\ (0) + v_\Delta(-0.625) &= \frac{60}{10} \\
v_1(0) + v_2\left(\frac{1}{5}+\frac{1}{200}+\frac{1}{20}\right) + v_3\left(-\frac{1}{20}\right) + i\ (0) + v_\Delta(0) &= \frac{60}{5} \\
v_1(1) + v_2(0) + v_3(-1) + i\ (175) + v_\Delta(0) &= 0 \\
v_1(0) + v_2(1) + v_3(0) + i\ (0) + v_\Delta(-1) &= 60 \\
v_1(0) + v_2\left(\frac{1}{200}\right) + v_3(0) + i\ (1) + v_\Delta(0) &= 0
\end{aligned}$$

Solving,

$v_1 = -60.75$ V $\quad v_2 = 30$ V; $\quad v_3 = -87$ V; $\quad i \ = -0.15$ A; $\quad v_\Delta = -30$ V

Calculate the power for the 60 V source:

$$\begin{aligned}
i_{60\text{V}} &= \frac{v_1 - 60}{10} + \frac{v_2 - 60}{5} \\
&= \frac{-60.75 - 60}{10} + \frac{30 - 60}{5} = -18.075 \text{ A} \\
p_{60\text{V}} &= (60)(-18.075) = -1084.5 \text{ W}
\end{aligned}$$

Thus, the 60 V source delivers 1084.5 W

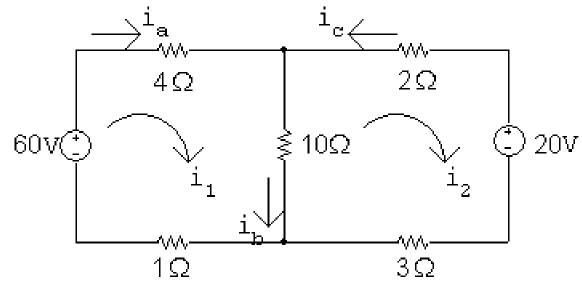
P 4.30 From Eq. 4.16, $\quad i \ = v \ /(1+\beta)R$

From Eq. 4.17, $\quad i \ = (v \ - V \ )/(1+\beta)R$

From Eq. 4.19,

$$\begin{aligned}
i \ &= \frac{1}{(1+\beta)R} \left[\frac{V \ \ (1+\beta)R \ \ R_2 + V\, R_1R_2}{R_1R_2 + (1+\beta)R \ \ (R_1+R_2)} - V\right] \\
&= \frac{V \ \ R_2 - V\,(R_1+R_2)}{R_1R_2 + (1+\beta)R \ \ (R_1+R_2)} = \frac{[V \ \ R_2/(R_1+R_2)] - V}{[R_1R_2/(R_1+R_2)] + (1+\beta)R}
\end{aligned}$$

P 4.31 **[a]**



The mesh current equations are:

$$-60 + 4i_1 + 10(i_1 - i_2) + 1i_1 = 0$$
$$20 + 3i_2 + 10(i_2 - i_1) + 2i_2 = 0$$

Place the equations in standard form:

$$i_1(4 + 10 + 1) + i_2(-10) = 60$$
$$i_1(-10) + i_2(3 + 10 + 2) = -20$$

Solving, $i_1 = 5.6$ A; $i_2 = 2.4$ A

Now solve for the requested currents:

$i_{\rm a} = i_1 = 5.6$ A; $i_{\rm b} = i_1 - i_2 = 3.2$ A; $i_{\rm c} = -i_2 = -2.4$ A

[b] If the polarity of the 60 V source is reversed, we have the following mesh current equations in standard form:

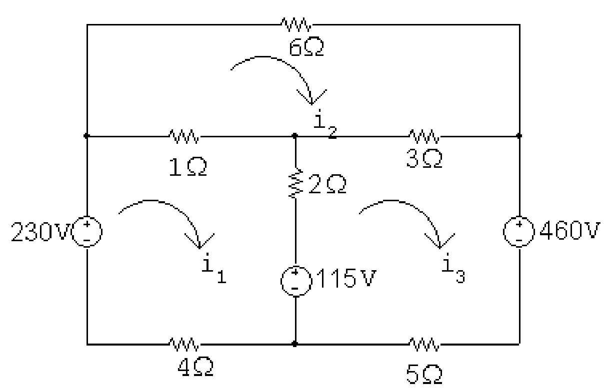
$$i_1(4 + 10 + 1) + i_2(-10) = -60$$
$$i_1(-10) + i_2(3 + 10 + 2) = -20$$

Solving, $i_1 = -8.8$ A; $i_2 = -7.2$ A

Now solve for the requested currents:

$i_{\rm a} = i_1 = -8.8$ A; $i_{\rm b} = i_1 - i_2 = -1.6$ A; $i_{\rm c} = -i_2 = 7.2$ A

P 4.32 **[a]**



The mesh current equations are:

$$-230 + 1(i_1 - i_2) + 2(i_1 - i_3) + 115 + 4i_1 = 0$$
$$6i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) = 0$$
$$460 + 5i_3 - 115 + 2(i_3 - i_1) + 3(i_3 - i_2) = 0$$

Place these equations in standard form:

$$i_1(1 + 2 + 4) + i_2(-1) + i_3(-2) = 115$$
$$i_1(-1) + i_2(6 + 3 + 1) + i_3(-3) = 0$$
$$i_1(-2) + i_2(-3) + i_3(5 + 2 + 3) = -345$$

Solving, $i_1 = 4.4$ A; $i_2 = -10.6$ A; $i_3 = -36.8$ A

The only components that can develop power in the circuit are the sources:

$$p_{230\text{V}} = -(230)(4.4) = -1012 \text{ W}$$
$$p_{115\text{V}} = -(115)(-36.8 - 4.4) = 4738 \text{ W}$$
$$p_{460\text{V}} = (460)(-36.8) = -16{,}928 \text{ W}$$

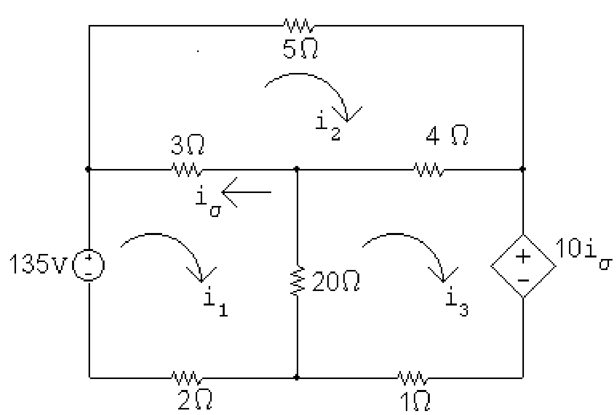
$\therefore \sum p_{\text{dev}} = 1012 + 16{,}928 = 17940$ W

[b] From part (a) we know that the 115 V source is dissipating power; compute the power dissipated by the resistors:

$$p_{1\Omega} = (1)(4.4 + 10.6)^2 = 225 \text{ W}$$
$$p_{4\Omega} = (4)(4.4)^2 = 77.44 \text{ W}$$
$$p_{6\Omega} = (6)(-10.6)^2 = 674.16 \text{ W}$$
$$p_{2\Omega} = (2)(4.4 + 36.8)^2 = 3394.88 \text{ W}$$
$$p_{3\Omega} = (3)(-10.6 + 36.8)^2 = 2059.32 \text{ W}$$
$$p_{5\Omega} = (5)(-36.8)^2 = 6771.2 \text{ W}$$

$\therefore \sum p_{\text{dis}} = 4738 + 225 + 77.44 + 674.16 + 3394.88 + 2059.32 + 6771.2$
$= 17940$ W (checks!)

P 4.33



The mesh current equations are:

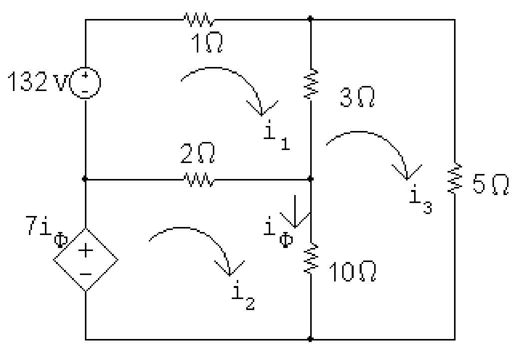
$$-135 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 = 0$$
$$5i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i \;\; + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:
$i \;\; = i_2 - i_1$
Place these equations in standard form:

$$i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i \;\; (0) = 135$$
$$i_1(-3) + i_2(5 + 4 + 3) + i_3(-4) + i \;\; (0) = 0$$
$$i_1(-20) + i_2(-4) + i_3(1 + 20 + 4) + i \;\; (10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i \;\; (1) = 0$$

Solving, $i_1 = 64.8$ A, $i_2 = 39$ A; $i_3 = 68.4$ A; $i \;\; = -25.8$ A
Calculate the power:
$p_{20\Omega} = 20(68.4 - 64.8)^2 = 259.2$ W
Thus the $20\,\Omega$ resistor dissipates 259.2 W.

P 4.34



The mesh current equations:

$$-132 + 1i_1 + 3(i_1 - i_3) + 2(i_1 - i_2) = 0$$
$$-7i \;\; + 2(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$5i_3 + 10(i_3 - i_2) + 3(i_3 - i_1) = 0$$

The dependent source constraint equation:
$i \;\; = i_2 - i_3$

Place these equations in standard form:

$$i_1(1 + 3 + 2) + i_2(-2) + i_3(-3) + i \;\; (0) = 132$$
$$i_1(-2) + i_2(10 + 2) + i_3(-10) + i \;\; (-7) = 0$$
$$i_1(-3) + i_2(-10) + i_3(5 + 10 + 3) + i \;\; (0) = 0$$
$$i_1(0) + i_2(-1) + i_3(1) + i \;\; (1) = 0$$

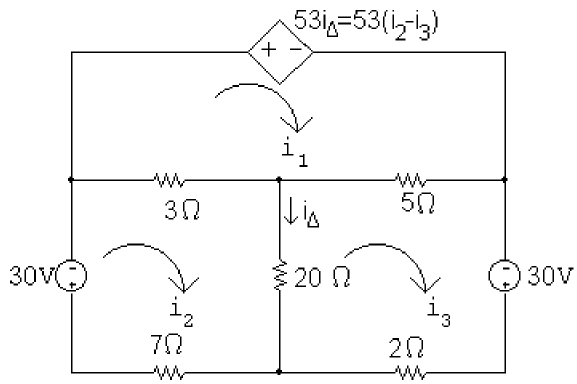
Solving, $i_1 = 48$ A; $i_2 = 36$ A; $i_3 = 28$ A; $i \;\; = 8$ A

Solve for the power:

$p_{\text{dep source}} = -7(i \;\;)i_2 = -7(8)(36) = -2016$ W

Thus, the dependent source is developing 2016 W.

P 4.35



The mesh current equations:

$$53(i_2 - i_3) + 5(i_1 - i_3) + 3(i_1 - i_2) = 0$$

$$30 + 3(i_2 - i_1) + 20(i_2 - i_3) + 7i_2 = 0$$

$$-30 + 2i_3 + 20(i_3 - i_2) + 5(i_3 - i_1) = 0$$

Place these equations in standard form:

$$i_1(5+3) + i_2(53-3) + i_3(-53-5) = 0$$

$$i_1(-3) + i_2(3+20+7) + i_3(-20) = -30$$

$$i_1(-5) + i_2(-20) + i_3(2+20+5) = 30$$

Solving, $i_1 = 186$ A; $i_2 = 81.6$ A; $i_3 = 96$ A

Calculate the power:

$$p_{30\text{V(left)}} = (30)(81.6) = 2448 \text{ W}$$

$$p_{30\text{V(right)}} = -(30)(96) = -2880 \text{ W}$$

$$p_{\text{dep source}} = 53(81.6 - 96)(186) = -141{,}955.2 \text{ W}$$

$$p_{3\Omega} = (3)(186 - 81.6)^2 = 32{,}698.08 \text{ W}$$

$$p_{5\Omega} = (5)(186 - 96)^2 = 40{,}500 \text{ W}$$

$$p_{20\Omega} = (20)(81.6 - 96)^2 = 4147.2 \text{ W}$$

$$p_{7\Omega} = (7)(81.6)^2 = 46{,}609.92 \text{ W}$$

$$p_{2\Omega} = (2)(96)^2 = 18{,}432 \text{ W}$$

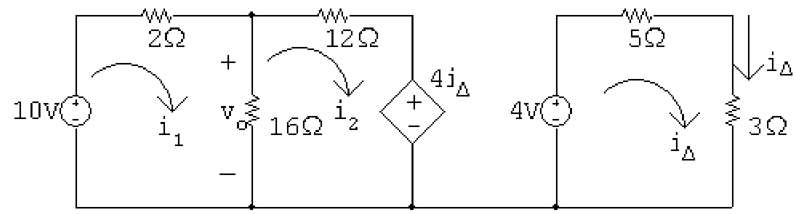
$$\sum p_{\text{dev}} = 2880 + 141{,}955.2 = 144{,}835.2 \text{ W}$$

$\sum p_{\text{dis}} = 2448 + 32{,}698.08 + 40{,}500 + 4147.2 + 46{,}609.92 + 18{,}432$

$= 144{,}835.2$ W(checks)

Thus the dependent source develops $141{,}955.2$ W.

P 4.36 **[a]**



$10 = 18i_1 - 16i_2$

$0 = -16i_1 + 28i_2 + 4i_\Delta$

$4 = 8i_\Delta$

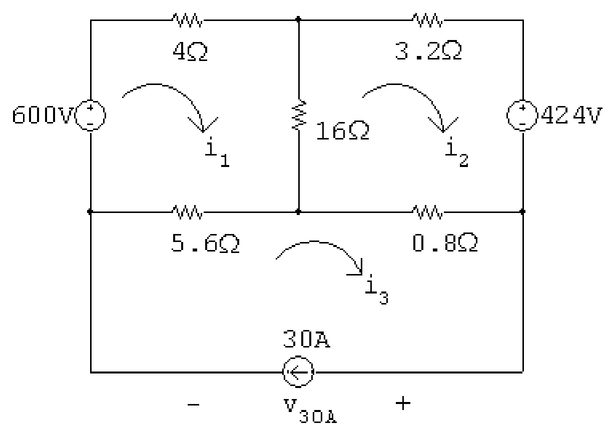
Solving, $i_1 = 1$ A; $\quad i_2 = 0.5$ A; $\quad i_\Delta = 0.5$ A

$v_0 = 16(i_1 - i_2) = 16(0.5) = 8$ V

[b] $p_{4\,\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1$ W (abs)

$\therefore\ p_{4\,\Delta}$ (deliver) $= -1$ W

P 4.37



$600 = 25.6i_1 - 16i_2 - 5.6i_3$

$-424 = -16i_1 + 20i_2 - 0.8i_3$

$30 = i_3$

Solving, $i_1 = 35$ A; $\quad i_2 = 8$ A; $\quad i_3 = 30$ A

[a] $v_{30A} = 0.8(i_2 - i_3) + 5.6(i_1 - i_3)$

$= 0.8(8 - 30) + 5.6(35 - 30) = 10.4$ V

$p_{30A} = 30v_{30A} = 30(10.4) = 312$ W (abs)

Therefore, the 30 A source delivers -312 W.

[b] $p_{600V} = -600(35) = -21{,}000$ W(del)

$p_{424V} = 424(8) = 3392$ W(abs)

Therefore, the total power delivered is $21{,}000$ W

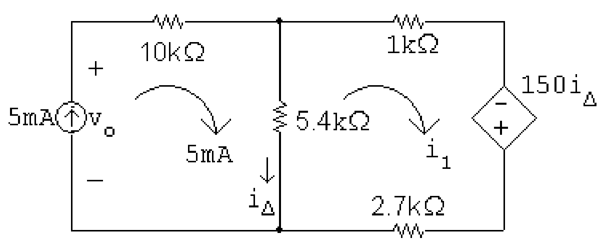
[c]

$$p_{4\Omega} = (35)^2(4) = 4900 \text{ W}$$
$$p_{3\,2\Omega} = (8)^2(3.2) = 204.8 \text{ W}$$
$$p_{16\Omega} = (35-8)^2(16) = 11{,}664 \text{ W}$$
$$p_{5\,6\Omega} = (35-30)^2(5.6) = 140 \text{ W}$$
$$p_{0\,8\Omega} = (-30+8)^2(0.8) = 387.2 \text{ W}$$

$\sum p_{\text{resistors}} = 17{,}296$ W

$\sum p_{\text{abs}} = 17{,}296 + 312 + 3392 = 21{,}000$ W (CHECKS)

P 4.38 **[a]**



The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $9250i_1 = 27.75 \quad \therefore \quad i_1 = 3$ mA
Then, $i_\Delta = 0.005 - i_1 = 0.005 - 0.003 = 0.002 = 2$ mA

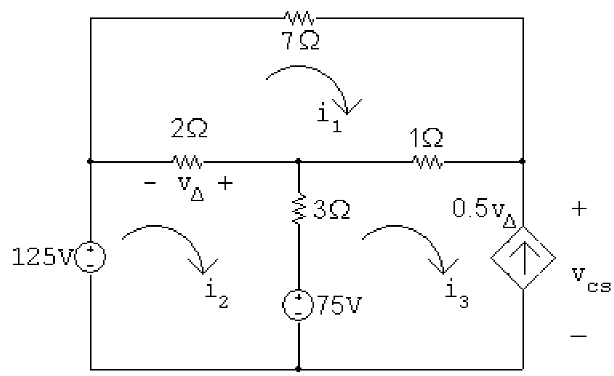
[b] $v = (0.005)(10{,}000) + (0.002)(5400) = 60.8$ V
$p_{5mA} = -(60.8)(0.005) = -304$ mW
Thus, the 5 mA source delivers 304 mW

[c] $150i_\Delta = 150(0.002) = 0.3$ V

$p_{\text{dep source}} = 150i_\Delta i_1 = -(0.3)(0.003) = -0.9$ mW

The dependent source delivers 0.9 mW.

P 4.39



Mesh equations:

$$7i_1 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0$$

$$-125 + 2(i_2 - i_1) + 3(i_2 - i_3) + 75 = 0$$

Constraint equations:

$i_3 = -0.5v_\Delta;$ $\qquad v_\Delta = 2(i_1 - i_2)$

Place these equations in standard form:

$$i_1(7+1+2) + i_2(-2) + i_3(-1) + v_\Delta(0) = 0$$

$$i_1(-2) + i_2(2+3) + i_3(-3) + v_\Delta(0) = 50$$

$$i_1(0) + i_2(0) + i_3(1) + v_\Delta(0.5) = 0$$

$$i_1(2) + i_2(-2) + i_3(0) + v_\Delta(-1) = 0$$

Solving, $i_1 = 6$ A; $\quad i_2 = 22$ A; $\quad i_3 = 16$ A; $\quad v_\Delta = -32$ V

Solve the outer loop KVL equation to find v_{cs}:

$-125 + 7i_1 + v_{\text{cs}} = 0;$ $\qquad \therefore \qquad v_{\text{cs}} = 125 - 7(6) = 83$ V

Calculate the power:

$$p_{125\text{V}} = -(125)(22) = -2750 \text{ W}$$

$$p_{75\text{V}} = (75)(22-16) = 450 \text{ W}$$

$$p_{\text{dep source}} = -(83)[0.5(-32)] = 1328 \text{ W}$$

Thus, the total power developed is 2750 W.

CHECK:

$$p_{7\Omega} = (6)^2(7) = 252 \text{ W}$$

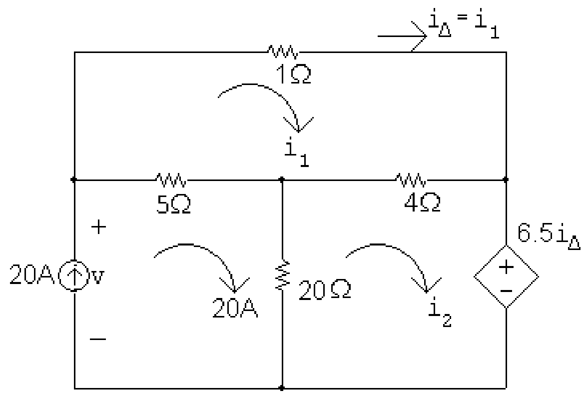
$$p_{2\Omega} = (22-6)^2(2) = 512 \text{ W}$$

$$p_{3\Omega} = (22-16)^2(3) = 108 \text{ W}$$

$$p_{1\Omega} = (16-6)^2(1) = 100 \text{ W}$$

$\therefore \quad \sum p_{\text{abs}} = 450 + 1328 + 252 + 512 + 108 + 100 = 2750$ W (checks!)

P 4.40



Since the bottom left mesh current value is known, we need only two mesh current equations:

$$1i_1 + 4(i_1 - i_2) + 5(i_1 - 20) = 0$$

$$6.5i_1 + 20(i_2 - 20) + 4(i_2 - i_1) = 0$$

Place these equations in standard form:

$$i_1(1 + 4 + 5) + i_2(-4) = 100$$

$$i_1(6.5 - 4) + i_2(20 + 4) = 400$$

Solving, $\quad i_1 = 16$ A; $\quad i_2 = 15$ A

Find v:

$-v + 5(20 - i_1) + 20(20 - i_2) = 0 \quad \therefore \quad v = 5(4) + 20(5) = 120$ V

Calculate the power:

$$p_{20\text{A}} = -(120)(20) = -2400 \text{ W}$$

$$p_{\text{dep source}} = [6.5(16)](15) = 1560 \text{ W}$$

$$p_{1\Omega} = 1(16)^2 = 256 \text{ W}$$

$$p_{5\Omega} = 5(20 - 16)^2 = 80 \text{ W}$$

$$p_{4\Omega} = 4(16 - 15)^2 = 4 \text{ W}$$

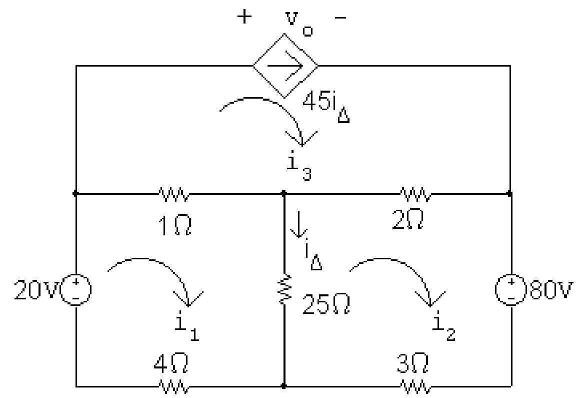
$$p_{20\Omega} = 20(20 - 15)^2 = 500 \text{ W}$$

$\sum p_{\text{dev}} = 2400$ W

$\sum p_{\text{dis}} = 1560 + 256 + 80 + 4 + 500 = 2400$ W (checks)

The power developed by the 20 A source is 2400 W

P 4.41 **[a]**



The mesh current equations are:

$$-20 + 1(i_1 - i_3) + 25(i_1 - i_2) + 4i_1 = 0$$

$$80 + 3i_2 + 25(i_2 - i_1) + 2(i_2 - i_3) = 0$$

The constraint equation is:
$i_3 = 45i_\Delta = 45(i_1 - i_2)$
Place these equations in standard form:

$$i_1(1 + 25 + 4) + i_2(-25) + i_3(-1) = 20$$

$$i_1(-25) + i_2(3 + 25 + 2) + i_3(-2) = -80$$

$$i_1(-45) + i_2(45) + i_3(1) = 0$$

Solving, $i_1 = 8$ A; $i_2 = 7$ A; $i_3 = 45$ A
Find the power in the $2\,\Omega$ resistor:
$p_{2\Omega} = 2(i_2 - i_3)^2 = 2(-38)^2 = 2888$ W
The $2\,\Omega$ resistor dissipates 2888 W.

[b] Find the power developed by the sources:
$v\ +80+3(7)+4(8)-20 = 0$ $\therefore$ $v\ = 20-80-21-32 = -113$ V

$$p_{\text{dep source}} = (-113)[45(8 - 7)] = -5085 \text{ W}$$

$$p_{80\text{V}} = (80)(7) = 560 \text{ W}$$

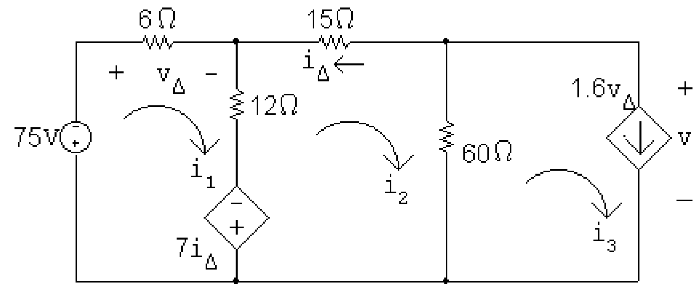
$$p_{20\text{V}} = -(20)(8) = -160 \text{ W}$$

$$\sum p_{\text{dev}} = 5085 + 160 = 5245 \text{ W}$$

The percent of the power developed that is deliverd to the $2\,\Omega$ resistor is:

$$\frac{2888}{5245} \times 100 = 55.06\%$$

P 4.42 **[a]**



The mesh current equations are:

$$75 + 6i_1 + 12(i_1 - i_2) - 7i_\Delta = 0$$

$$15i_2 + 60(i_2 - i_3) + 7i_\Delta + 12(i_2 - i_1) = 0$$

The two constraint equations are:

$$i_\Delta = -i_2$$

$$i_3 = 1.6v_\Delta = 1.6(6i_1) = 9.6i_1$$

Place these equations in standard form:

$$i_1(6+12) + i_2(-12) + i_3(0) + i_\Delta(-7) = -75$$

$$i_1(-12) + i_2(15+60+12) + i_3(-60) + i_\Delta(7) = 0$$

$$i_1(0) + i_2(1) + i_3(0) + i_\Delta(1) = 0$$

$$i_1(9.6) + i_2(0) + i_3(-1) + i_\Delta(0) = 0$$

Solving, $i_1 = 4$ A; $i_2 = 29.4$ A; $i_3 = 38.4$ A; $i_\Delta = -29.4$ A

Calculate the power associated with the three sources:

$$v = 60(i_2 - i_3) = -540 \text{ V}$$

$$v_\Delta = 6i_1 = 6(4) = 24 \text{ V}$$

$$p_{75\text{V}} = (75)(4) = 300 \text{ W}$$

$$p_{\text{CCVS}} = -7(-29.4)(4 - 29.4) = -5227.32 \text{ W}$$

$$p_{\text{VCCS}} = (-540)[1.6(24)] = -20{,}736 \text{ W}$$

The two dependent sources are generating a total of $5227.32 + 20{,}736 = 25{,}963.32$ W.

[b] Find the power dissipated. Remember that the 75 V source is generating 300 W, as calculated in part (a):

$$p_{6\Omega} = (6)(4)^2 = 96 \text{ W}$$

$$p_{12\Omega} = (12)(4 - 29.4)^2 = 7741.92 \text{ W}$$

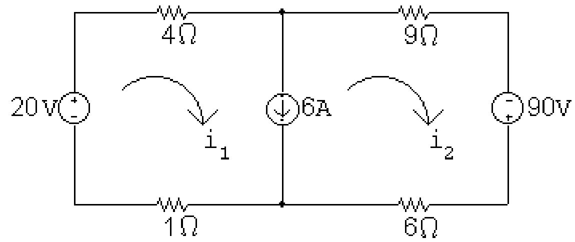
$$p_{15\Omega} = (15)(29.4)^2 = 12{,}965.4 \text{ W}$$

$$p_{60\Omega} = (60)(29.4 - 38.4)^2 = 4860 \text{ W}$$

$$\sum p_{\text{dis}} = 300 + 96 + 7741.92 + 12{,}965.4 + 4860 = 25{,}963.32 \text{ W(checks)}$$

Thus the power dissipated in the circuit is 25,963.32 W.

P 4.43



The supermesh equation is:
$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

$$i_1(4+1) + i_2(9+6) = 20 + 90$$

$$i_1(1) + i_2(-1) = 6$$

Solving, $i_1 = 10$ A; $i_2 = 4$ A
Now find the power:

$$p_{4\Omega} = 10^2(4) = 400 \text{ W}$$

$$p_{1\Omega} = 10^2(1) = 100 \text{ W}$$

$$p_{9\Omega} = 4^2(9) = 144 \text{ W}$$

$$p_{6\Omega} = 4^2(6) = 96 \text{ W}$$

$$p_{20\text{V}} = -(20)(10) = -200 \text{ W}$$

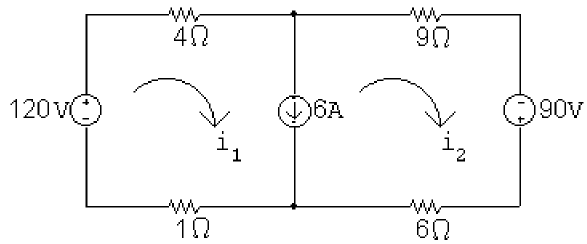
$$v_{6\text{A}} = 9i_2 - 90 + 6i_2 = (9)(4) - 90 + (6)(4) = -30 \text{ V}$$

$$p_{6\text{A}} = (-30)(6) = -180 \text{ W}$$

$$p_{90\text{V}} = -(90)(4) = -360 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 200 + 180 + 360 = 740$ W
$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740$ W
Thus the power dissipated in the circuit is 740 W

P 4.44



The supermesh equation is:
$-120 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

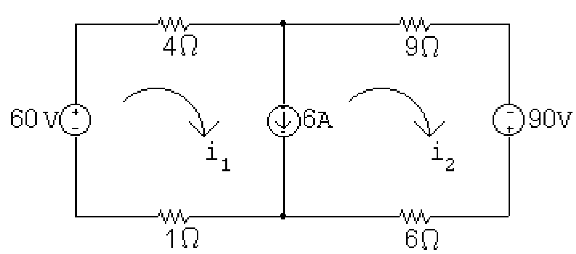
$$i_1(4+1) + i_2(9+6) = 120 + 90$$
$$i_1(1) + i_2(-1) = 6$$

Solving, $i_1 = 15$ A; $i_2 = 9$ A
Now find the power:

$$p_{4\Omega} = 15^2(4) = 900 \text{ W}$$
$$p_{1\Omega} = 15^2(1) = 225 \text{ W}$$
$$p_{9\Omega} = 9^2(9) = 729 \text{ W}$$
$$p_{6\Omega} = 9^2(6) = 486 \text{ W}$$
$$p_{120\text{V}} = -(120)(15) = -1800 \text{ W}$$
$$v = 9i_2 - 90 + 6i_2 = 9(9) - 90 + 6(9) = 45 \text{ V}$$
$$p_{6\text{A}} = (45)(6) = 270 \text{ W}$$
$$p_{90\text{V}} = -(90)(9) = -810 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 900 + 225 + 729 + 486 + 270 = 2610$ W (note that the 6 A source is now dissipating power!)
$\sum p_{\text{diss}} = 1800 + 810 = 2610$ W
Thus the power dissipated in the circuit is 2610 W

P 4.45 **[a]**



The supermesh equation is:
$-60 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

$$i_1(4+1) + i_2(9+6) = 60 + 90$$
$$i_1(1) + i_2(-1) = 6$$

Solving, $i_1 = 12$ A; $i_2 = 6$ A
Now find the power:

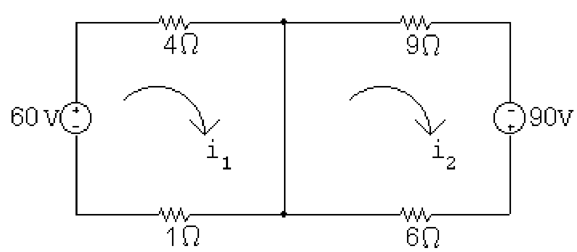
$$p_{4\Omega} = 12^2(4) = 576 \text{ W}$$
$$p_{1\Omega} = 12^2(1) = 144 \text{ W}$$
$$p_{9\Omega} = 6^2(9) = 324 \text{ W}$$
$$p_{6\Omega} = 6^2(6) = 216 \text{ W}$$
$$p_{60\text{V}} = -(60)(20) = -720 \text{ W}$$
$$v = 9i_2 - 90 + 6i_2 = 9(6) - 90 + 6(6) = 0 \text{ V}$$

(the 6 A source acts like a short circuit carrying 6 A of current)

$$p_{6\text{A}} = (0)(6) = 0 \text{ W}$$
$$p_{90\text{V}} = -(90)(6) = -540 \text{ W}$$

In summary:
$\sum p_{\text{dev}} = 576 + 144 + 324 + 216 = 1260$ W (note that the power of the 6 A source is zero)
$\sum p_{\text{diss}} = 720 + 540 = 1260$ W
Thus the power dissipated in the circuit is 1260 W

[b]



Now there is no longer a supermesh. The two simple mesh current equations are:

$$-60 + 4i_1 + 1i_1 = 0$$
$$-90 + 6i_2 + 9i_2 = 0$$

Since these equations are uncoupled, each can be solved separately:

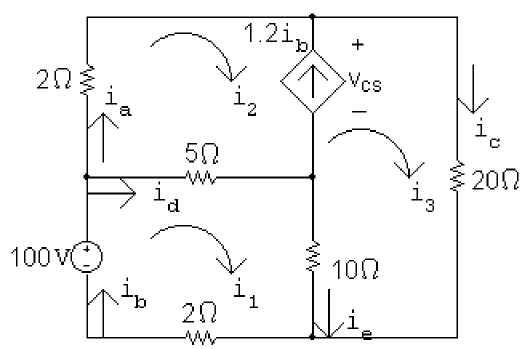
$5i_1 = 60 \quad \therefore \quad i_1 = 60/5 = 12$ A

$15i_2 = 90 \quad \therefore \quad i_2 = 90/15 = 6$ A

Since the currents are the same as in part (a), the power will be the same as calculated in part (a). Thus, the power dissipated in the circuit is again 1260 W.

[c] As noted in part (a), the 6 A source has zero voltage drop, so is equivalent to a short circuit (which has no voltage drop by definition) carrying 6 A of current, as in the circuit of part (b).

P 4.46 **[a]**



The i_1 mesh current equation:
$-100 + 5(i_1 - i_2) + 10(i_1 - i_3) + 2i_1 = 0$
The i_2 — i_3 supermesh equationa:
$2i_2 + 20i_3 + 10(i_3 - i_1) + 5(i_2 - i_1) = 0$
The supermesh constraint:
$i_3 - i_2 = 1.2i_{\rm b} = 1.2i_1$
Place these equations in standard form:

$$i_1(5 + 10 + 2) + i_2(-5) + i_3(-10) = 100$$
$$i_1(-10 - 5) + i_2(2 + 5) + i_3(20 + 10) = 0$$
$$i_1(1.2) + i_2(1) + i_3(-1) = 0$$

Solving, $\quad i_1 = 7.4$ A; $\quad i_2 = -4.2$ A; $\quad i_3 = 4.68$ A
Solve for the requested currents:

$$i_a = i_2 = -4.2 \text{ A}$$
$$i_b = i_1 = 7.4 \text{ A}$$
$$i_c = i_3 = 4.68 \text{ A}$$
$$i_d = i_1 - i_2 = 11.6 \text{ A}$$
$$i_e = i_1 - i_3 = 2.72 \text{ A}$$

[b] Find $v_{\rm cs}$:
$2i_2 + v_{\rm cs} + 5(i_2 - i_1) = 0 \quad \therefore \quad v_{\rm cs} = -2(-4.2) - 5(-4.2 - 7.4) = 66.4$ V

Calculate the power:

$$p_{100\text{V}} = -(100)(7.4) = -740 \text{ W}$$

$$p_{\text{dep source}} = -(66.4)[1.2(7.4)] = -589.632 \text{ W}$$

$$p_{2\Omega} = 2(-4.2)^2 = 35.28 \text{ W}$$

$$p_{5\Omega} = 5(7.4 + 4.2)^2 = 672.8 \text{ W}$$

$$p_{2\Omega} = 2(7.4)^2 = 109.52 \text{ W}$$

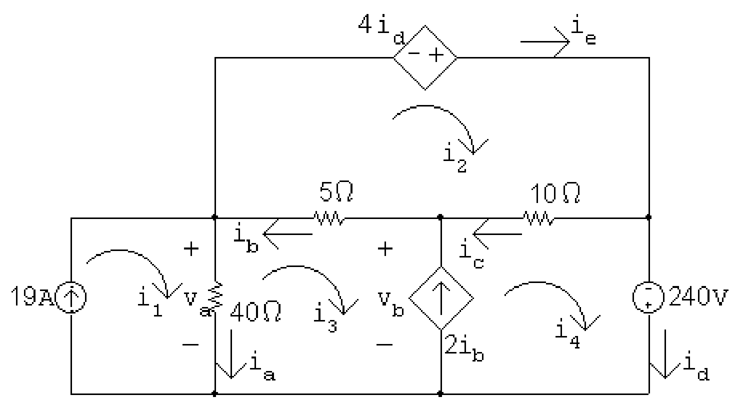
$$p_{10\Omega} = 10(7.4 - 4.68)^2 = 73.984 \text{ W}$$

$$p_{20\Omega} = 20(4.68)^2 = 438.048 \text{ W}$$

$$\sum p_{\text{dev}} = 740 + 589.632 = 1329.632 \text{ W}$$

$$\sum p_{\text{dis}} = 35.28 + 672.8 + 109.52 + 73.984 + 438.048 = 1329.632 \text{ W}$$

P 4.47 **[a]**



The i_2 mesh current equation:
$-4i_\text{d} + 10(i_2 - i_4) + 5(i_2 - i_3) = 0$
The i_3 — i_4 supermesh equation:
$40(i_3 - 19) + 5(i_3 - i_2) + 10(i_4 - i_2) - 240 = 0$
The supermesh constraint equation:
$i_4 - i_3 = 2i_\text{b} = 2(i_2 - i_3)$
Place the equations in standard form:

$$i_2(10 + 5) + i_3(-5) + i_4(-10 - 4) = 0$$

$$i_2(-5 - 10) + i_3(40 + 5) + i_4(10) = 240 + (40)(19)$$

$$i_2(2) + i_3(-1) + i_4(-1) = 0$$

Solving, $i_2 = 18$ A; $i_3 = 26$ A; $i_4 = 10$ A
Solve for the requested currents:

$i_a = 19 - i_3 = 19 - 26 = -7$ A

$i_b = i_2 - i_3 = 18 - 26 = -8$ A

$i_c = i_2 - i_4 = 18 - 10 = 8$ A

$i_d = i_4 = 10$ A

$i_e = i_2 = 18$ A

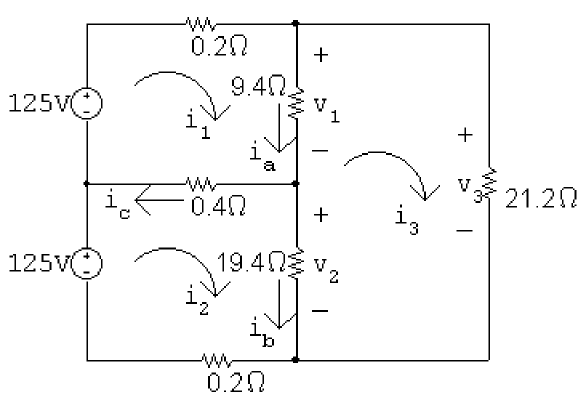
[b] Find the power in the circuit:

$$v_a = 40i_a = 40(-7) = -280 \text{ V}$$
$$v_b = -10i_c - 240 = -10(8) - 240 = -320 \text{ V}$$
$$p_{19A} = -(-280)(19) = 5320 \text{ W}$$
$$p_{CCCS} = -(-320)(2)(-8) = -5120 \text{ W}$$
$$p_{CCVS} = -(4)(10)(18) = -720 \text{ W}$$
$$p_{240V} = -(240)(10) = -2400 \text{ W}$$
$$p_{40\Omega} = (40)(-7)^2 = 1960 \text{ W}$$
$$p_{5\Omega} = (5)(-8)^2 = 320 \text{ W}$$
$$p_{10\Omega} = (10)(8)^2 = 640 \text{ W}$$

$$\sum p_{dev} = 5120 + 720 + 2400 = 8240 \text{ W}$$

$$\sum p_{dis} = 5320 + 1960 + 320 + 640 = 8240 \text{ W(checks)}$$

P 4.48 **[a]**



$$125 = 10i_1 - 0.4i_2 - 9.4i_3$$
$$125 = -0.4i_1 + 20i_2 - 19.4i_3$$
$$0 = -9.4i_1 - 19.4i_2 + 50i_3$$

Solving, $i_1 = 23.93$ A; $\quad i_2 = 17.79$ A; $\quad i_3 = 11.40$ A

$$v_1 = 9.4(i_1 - i_3) = 117.76 \text{ V}$$
$$v_2 = 19.4(i_2 - i_3) = 123.90 \text{ V}$$
$$v_3 = 21.2i_3 = 241.66 \text{ V}$$

[b] $p_1 = (i_1 - i_3)^2(9.4) = 1475.22$ W

$p_2 = (i_2 - i_3)^2(19.4) = 791.29$ W

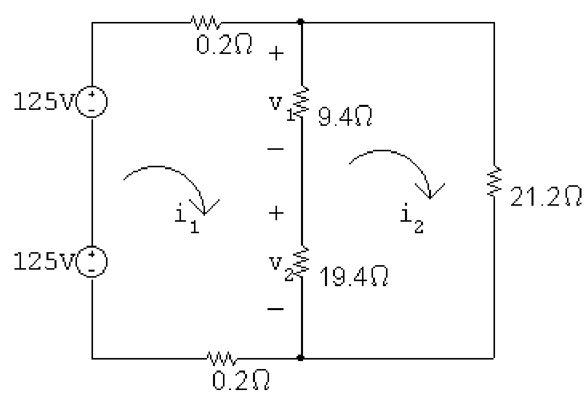
$p_3 = i_3^2(21.2) = 2754.64$ W

[c] $\sum p_{\text{dev}} = 125(i_1 + i_2) = 5213.99$ W

$\sum p_{\text{load}} = 5021.15$ W

$$\%\text{ delivered } = \frac{5021.15}{5213.99} \times 100 = 96.3\%$$

[d]



$250 = 29.2i_1 - 28.8i_2$

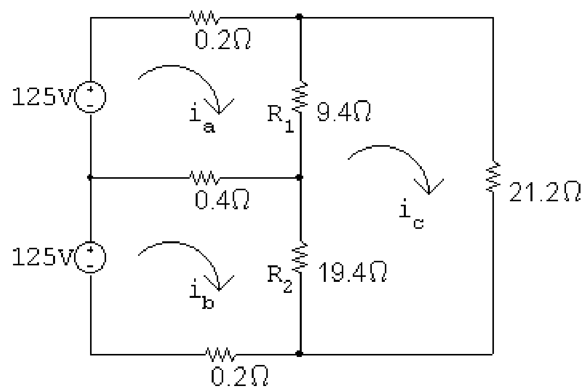
$0 = -28.8i_1 + 50i_2$

Solving, $i_1 = 19.82$ A; $\quad i_2 = 11.42$ A

$$i_1 - i_2 = 8.41 \text{ A}$$
$$v_1 = (8.41)(9.4) = 79.01 \text{ V}$$
$$v_2 = 8.41(19.4) = 163.06 \text{ V}$$

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 4.49



$$125 = (R_1 + 0.6)i_a - 0.4i_b - R_1 i_c$$

$$125 = -0.4i_a + (R_2 + 0.6)i_b - R_2 i_c$$

$$0 = -R_1 i_a - R_2 i_b + (R_1 + R_2 + 21.2)i_c$$

$$\Delta = \begin{vmatrix} (R_1+0.6) & -0.4 & -R_1 \\ -0.4 & (R_2+0.6) & -R_2 \\ -R_1 & -R_2 & (R_1+R_2+21.2) \end{vmatrix}$$

When $R_1 = R_2$, Δ reduces to

$$\Delta = 21.6R_1^2 + 25.84R_1 + 4.24.$$

$$\begin{aligned} N_a &= \begin{vmatrix} 125 & -0.4 & -R_1 \\ 125 & (R_2+0.6) & -R_2 \\ 0 & -R_2 & (R_1+R_2+21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + R_1 + 22.2R_2 + 21.2] \end{aligned}$$

$$\begin{aligned} N_b &= \begin{vmatrix} (R_1+0.6) & 125 & -R_1 \\ -0.4 & 125 & -R_2 \\ -R_1 & 0 & (R_1+R_2+21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + 22.2R_1 + R_2 + 21.2] \end{aligned}$$

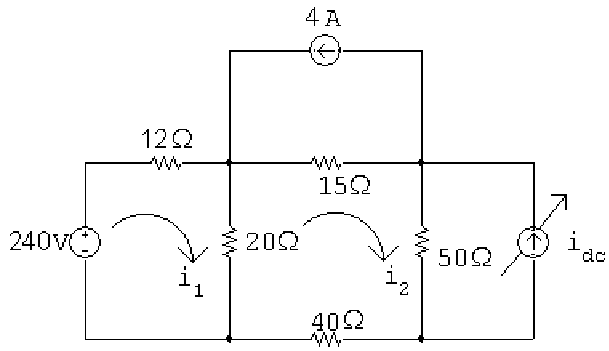
$$i_a = \frac{N_a}{\Delta}, \qquad i_b = \frac{N_b}{\Delta}$$

$$i_{neutral} = i_a - i_b = \frac{N_a - N_b}{\Delta} = \frac{125[(R_1 - R_2) + 22.2(R_2 - R_1)]}{\Delta}$$

Now note that when $R_1 = R_2$, i_{neutral} reduces to

$$i_{\text{neutral}} = \frac{0}{\Delta} = 0$$

P 4.50



The mesh current equations:

$$-240 + 12i_1 + 20(i_1 - i_2) = 0$$

$$20(i_2 - i_1) + 15(i_2 + 4) + 50(i_2 + i_{\text{dc}}) + 40i_2 = 0$$

Place these equations in standard form:

$$i_1(12 + 20) + i_2(-20) + i_{\text{dc}}(0) = 240$$

$$i_1(-20) + i_2(20 + 15 + 50 + 40) + i_{\text{dc}}(50) = -60$$

But if the power associated with the 4 A source is zero, the voltage drop across the source must be zero. This means that the voltage drop across the $15\,\Omega$ resistor is also zero, so the $15\,\Omega$ resistor is effectively removed from the circuit. Once this happens, $i_2 = -4$ A. Substitute this value into the first equation and solve for i_1:

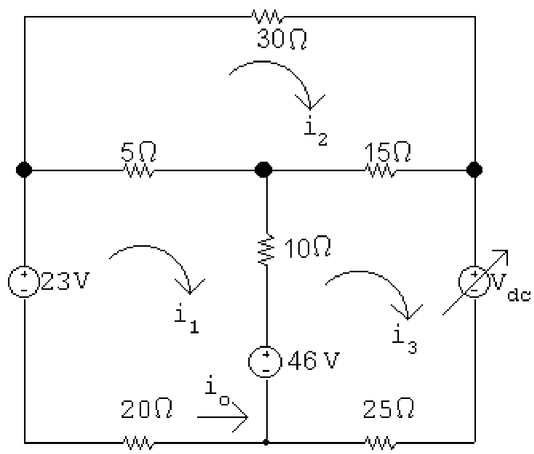
$32i_1 - 20(-4) = 240 \quad \therefore \quad 32i_1 = 160 \quad$ so $\quad i_1 = 5$ A

Now substitute this value for i_1 into the second equation and solve for i_{dc}:

$-20(5) + 125(-4) + 50i_{\text{dc}} = -60 \quad$ so $\quad 50i_{\text{dc}} = -60 + 100 + 500 = 540$

$\therefore \quad i_{\text{dc}} = 540/50 = 10.8$ A

P 4.51 **[a]**



Write the mesh current equations. Note that if $i \;= 0$, then $i_1 = 0$:

$$
\begin{aligned}
-23 + 5(-i_2) + 10(-i_3) + 46 &= 0 \\
30i_2 + 15(i_2 - i_3) + 5i_2 &= 0 \\
V_{\rm dc} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) &= 0
\end{aligned}
$$

Place the equations in standard form:

$$
\begin{aligned}
i_2(-5) + i_3(-10) + V_{\rm dc}(0) &= -23 \\
i_2(30 + 15 + 5) + i_3(-15) + V_{\rm dc}(0) &= 0 \\
i_2(-15) + i_3(25 + 10 + 15) + V_{\rm dc}(1) &= 46
\end{aligned}
$$

Solving, $\quad i_2 = 0.6$ A; $\quad i_3 = 2$ A; $\quad V_{\rm dc} = -45$ V

Thus, the value of $V_{\rm dc}$ required to make $i \;= 0$ is -45 V.

[b] Calculate the power:

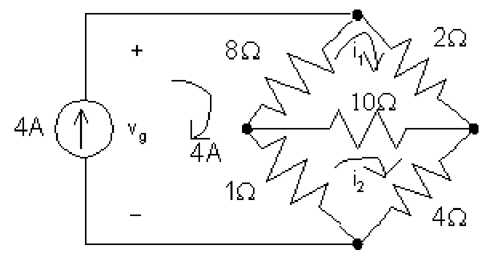
$$
\begin{aligned}
p_{\rm 23V} &= -(23)(0) = 0 \text{ W} \\
p_{\rm 46V} &= -(46)(2) = -92 \text{ W} \\
p_{\rm Vdc} &= (-45)(2) = -90 \text{ W} \\
p_{30\Omega} &= (30)(0.6)^2 = 10.8 \text{ W} \\
p_{5\Omega} &= (5)(0.6)^2 = 1.8 \text{ W} \\
p_{15\Omega} &= (15)(2 - 0.6)^2 = 29.4 \text{ W} \\
p_{10\Omega} &= (10)(2)^2 = 40 \text{ W} \\
p_{20\Omega} &= (20)(0)^2 = 0 \text{ W} \\
p_{25\Omega} &= (25)(2)^2 = 100 \text{ W}
\end{aligned}
$$

$$\sum p_{\rm dev} = 92 + 90 = 182 \text{ W}$$

$$\sum p_{\rm dis} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W(checks)}$$

P 4.52 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2i_1 + 10(i_1 - i_2) + 8(i_1 - 4) = 0$$

$$4i_2 + 1(i_2 - 4) + 10(i_2 - i_1) = 0$$

Place the equations in standard form:

$$i_1(2 + 10 + 8) + i_2(-10) = 32$$

$$i_1(-10) + i_2(4 + 1 + 10) = 4$$

Solving, $i_1 = 2.6$ A; $\qquad i_2 = 2$ A

Find the power in the $10\,\Omega$ resistor:

$i_{10\Omega} = i_1 - i_2 = 0.6$ A

$p_{10\Omega} = (0.6)^2(10) = 3.6$ W

[c] No, the voltage across the 4 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

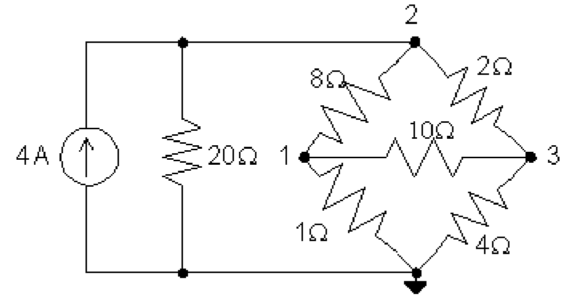
[d] $v \;= 2i_1 + 4i_2 = 2(2.6) + 4(2) = 13.2$ V

$p_{4\text{A}} = -(13.2)(4) = -52.8$ W

Thus the 4 A source develops 52.8 W.

P 4.53 **[a]** There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\frac{v_1}{1} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{10} = 0$$

$$-4 + \frac{v_2}{20} + \frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{2} = 0$$

$$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{2} + \frac{v_3}{4} = 0$$

Put the equations in standard form:

$$v_1\left(1 + \frac{1}{8} + \frac{1}{10}\right) + v_2\left(-\frac{1}{8}\right) + v_3\left(-\frac{1}{10}\right) = 0$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{20} + \frac{1}{8} + \frac{1}{2}\right) + v_3\left(-\frac{1}{2}\right) = 4$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(-\frac{1}{2}\right) + v_3\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{4}\right) = 0$$

Solving, $v_1 = 1.72$ V; $v_2 = 11.33$ V; $v_3 = 6.87$ V

$p_{4\text{A}} = -(11.33)(4) = -45.32$ W

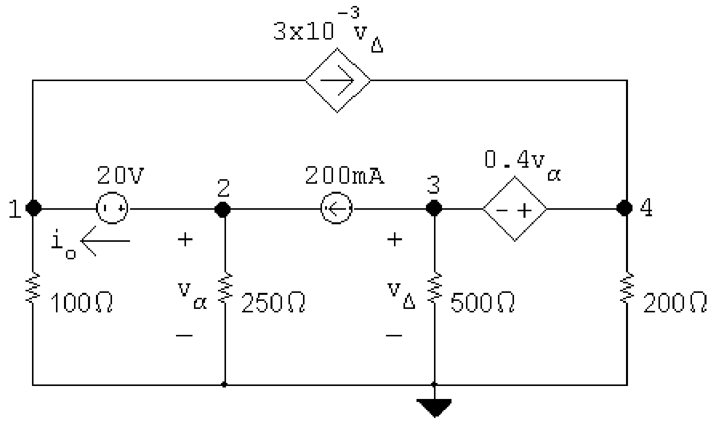
Therefore, the 4 A source is developing 45.32 W

P 4.54 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3} v_3 = 0$$

$$\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3} v_3 + 0.2 = 0$$

Constraints:

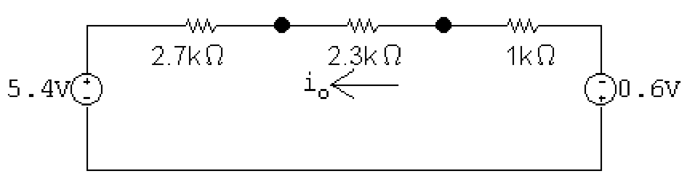
$v_2 - v_1 = 20$; $\qquad v_4 - v_3 = 0.4v \;\; ; v \quad = v_2$

Solving, $v_2 = 44$ V

$i \quad = 0.2 - 44/250 = 24$ mA

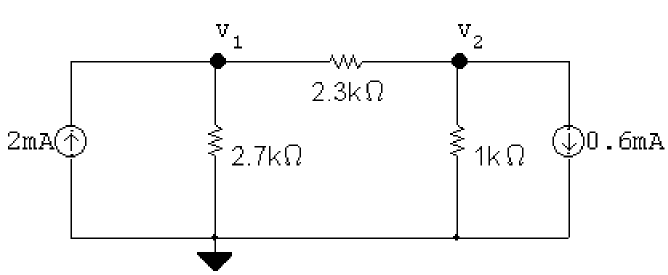
$p_{20\text{V}} = 20i \quad = 480$ mW (abs)

P 4.55 **[a]** Apply source transformations to both current sources to get



$$i \quad = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2 \times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 0.6 \times 10^{-3} = 0$$

Place these equations in standard form:

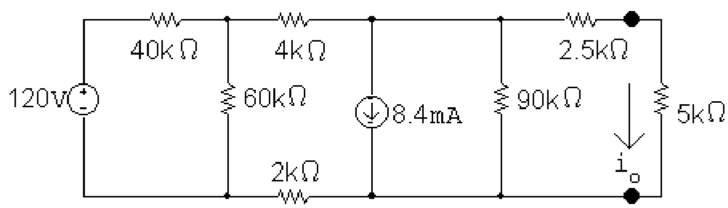
$$v_1\left(\frac{1}{2700} + \frac{1}{2300}\right) + v_2\left(-\frac{1}{2300}\right) = 2 \times 10^{-3}$$

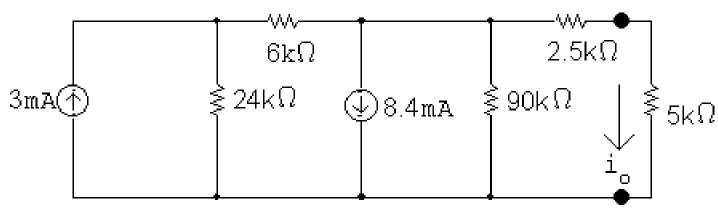
$$v_1\left(-\frac{1}{2300}\right) + v_2\left(\frac{1}{1000} + \frac{1}{2300}\right) = -0.6 \times 10^{-3}$$

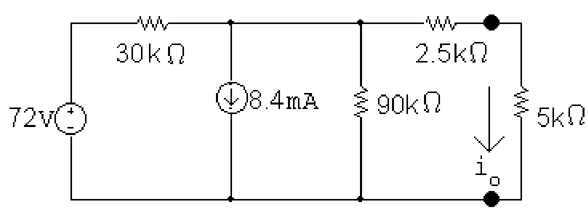
Solving, $v_1 = 2.7$ V; $\qquad v_2 = 0.4$ V

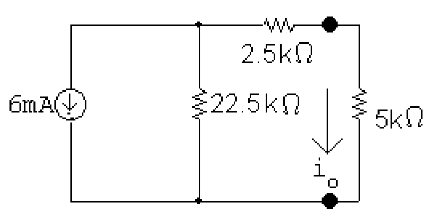
$$\therefore \quad i = \frac{v_2 - v_1}{2300} = -1 \text{ mA}$$

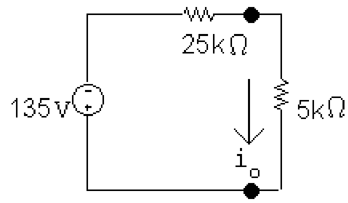
P 4.56 **[a]**





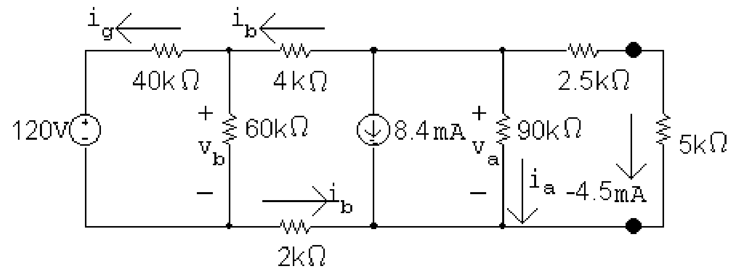






$i \;= -135/30{,}000 = -4.5$ mA

[b]



$$v_{\rm a} = (7500)(-0.0045) = -33.75 \text{ V}$$

$$i_{\rm a} = \frac{v_{\rm a}}{90{,}000} = \frac{-33.75}{90{,}000} = -0.375 \text{ mA}$$

$$i_{\rm b} = -8.4\times10^{-3} + 0.375\times10^{-3} + 4.5\times10^{-3} = -3.525 \text{ mA}$$

$$v_{\rm b} = (6000)(3.525\times10^{-3}) - 33.75 = -12.6 \text{ V}$$

$$i = \frac{-12.6-120}{40{,}000} = -3.315 \text{ mA}$$

$$p_{120\rm V} = (120)(-3.315\times10^{-3}) = -397.8 \text{ mW}$$

Check:

$$p_{8\ 4\rm mA} = (-33.75)(8.4\times10^{-3}) = -283.5 \text{ mW}$$

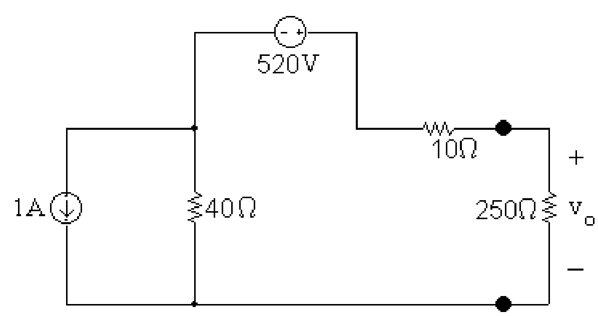
$$\sum P_{\rm dev} = 397.8 + 283.5 = 681.3 \text{ mW}$$

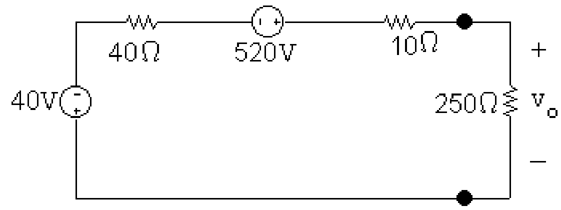
$$\sum P_{\rm dis} = (40{,}000)(-3.315\times10^{-3})^2 + \frac{(-12.6)^2}{60{,}000} + \frac{(-33.75)^2}{90{,}000}$$

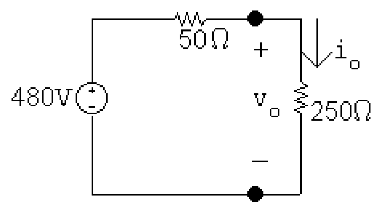
$$+(6000)(-3.525\times10^{-3})^2 + (7500)(-4.5\times10^{-3})^2$$

$$= 681.3 \text{ mW}$$

P 4.57 **[a]**

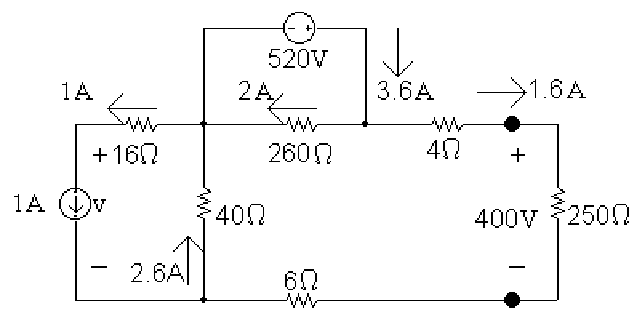






$$\therefore \quad v \;=\; \frac{250}{300}(480) = 400 \text{ V}; \qquad i \;=\; \frac{400}{250} = 1.6 \text{ A}$$

[b]



$p_{520\text{V}} = -(520)(3.6) = -1872$ W
Therefore, the 520 V source is developing 1872 kW.

[c] $v = -(16)(1) - 40(2.6) = -120$ V
$p_{1\text{A}} = (-120)(1) = -120$ W
Therefore the 1 A source is developing 120 W.

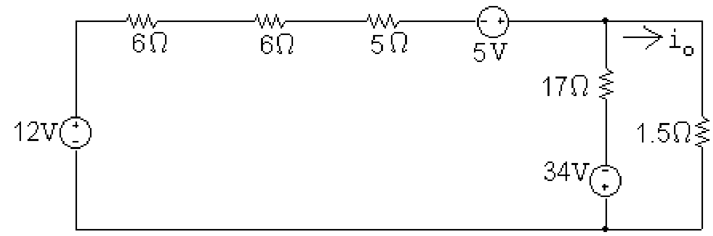
[d] Calculate the power dissipated by the resistors:

$$\begin{aligned}
p_{16\Omega} &= (16)(1)^2 = 16 \text{ W} \\
p_{260\Omega} &= (260)(2)^2 = 1040 \text{ W} \\
p_{40\Omega} &= (40)(2.6)^2 = 270.4 \text{ W} \\
p_{4\Omega} &= (4)(1.6)^2 = 10.24 \text{ W} \\
p_{250\Omega} &= (250)(1.6)^2 = 640 \text{ W} \\
p_{6\Omega} &= (6)(1.6)^2 = 15.36 \text{ W}
\end{aligned}$$

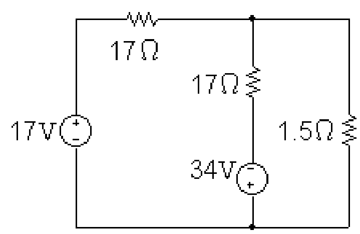
$$\sum p_{\text{dev}} = 120 + 1872 = 1992 \text{ W}$$

$$\sum p_{\text{dev}} = 16 + 1040 + 270.4 + 10.24 + 640 + 15.36 = 1992 \text{ W (CHECKS)}$$

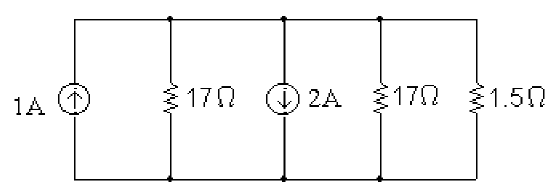
P 4.58 **[a]** Applying a source transformation to each current source yields



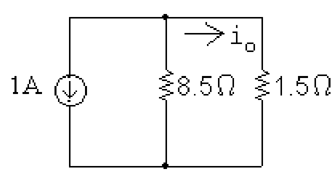
Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

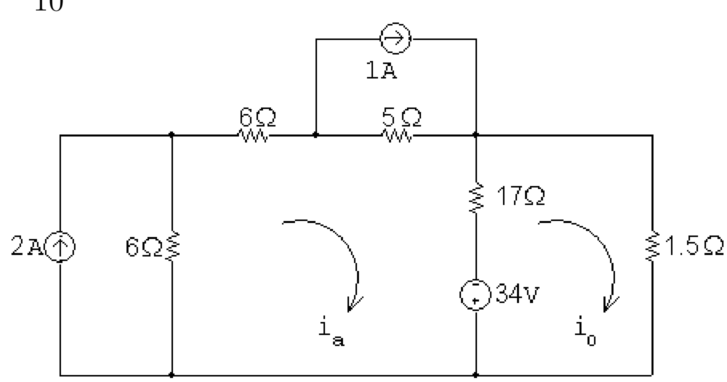


which can be reduced to



$$\therefore \quad i = \frac{8.5}{10}(-1) = -0.85 \text{ A}$$

[b]



The mesh current equations are:

$$6(i_a - 2) + 6i_a + 5(i_a - 1) + 17(i_a - i\ ) - 34 = 0$$

$$1.5i\ + 34 + 17(i\ - i_a) = 0$$

Put these equations in standard form:

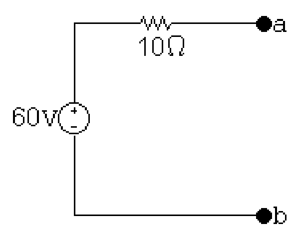
$$i_a(6 + 6 + 5 + 17) + i\ (-17) = 12 + 5 + 34$$

$$i_a(-17) + i\ (1.5 + 17) = -34$$

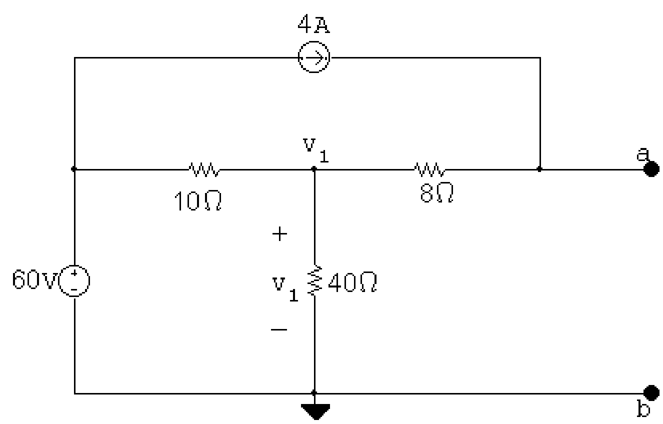
Solving, $i_a = 1.075$ A; $i\ = -0.85$ A

P 4.59 $V_{Th} = \dfrac{30}{30 + 10}(80) = 60$ V

$R_{Th} = 10\|30 + 2.5 = 10\,\Omega$



P 4.60



Write and solve the node voltage equation at v_1:

$$\frac{v_1 - 60}{10} + \frac{v_1}{40} - 4 = 0$$

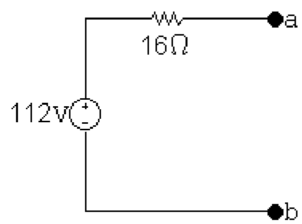
$4v_1 - 240 + v_1 - 160 = 0 \quad \therefore \quad v_1 = 400/5 = 80$ V

Calculate V_{Th}:

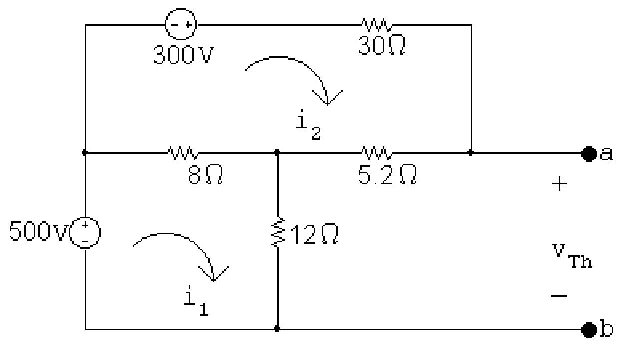
$V_{Th} = v_1 + (8)(4) = 80 + 32 = 112$ V

Calculate R_{Th} by removing the independent sources and making series and parallel combinations of the resistors:

$R_{Th} = 8 + 40\|10 = 8 + 8 = 16\,\Omega$



P 4.61 After making a source transformation the circuit becomes



The mesh current equations are:

$$-500 + 8(i_1 - i_2) + 12i_1 = 0$$

$$-300 + 30i_2 + 5.2i_2 + 8(i_2 - i_1) = 0$$

Put the equations in standard form:

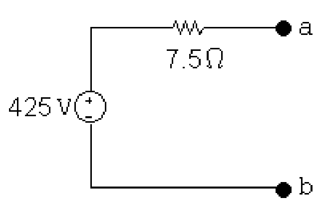
$$i_1(8 + 12) + i_2(-8) = 500$$

$$i_1(-8) + i_2(30 + 5.2 + 8) = 300$$

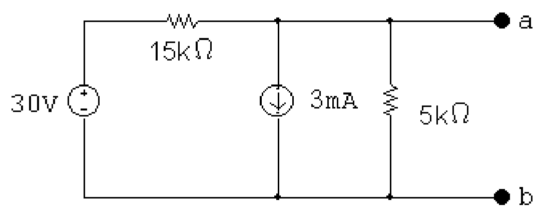
Solving, $i_1 = 30$ A; $i_2 = 12.5$ A

$V_{\rm Th} = 5.2i_2 + 12i_1 = 425$ V

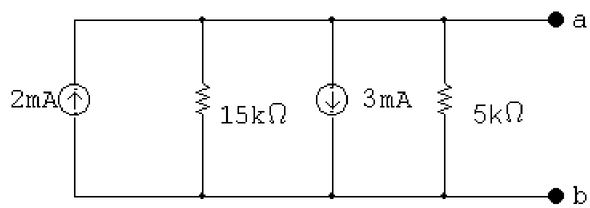
$R_{\rm Th} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$



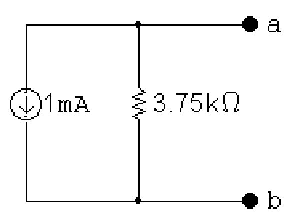
P 4.62 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



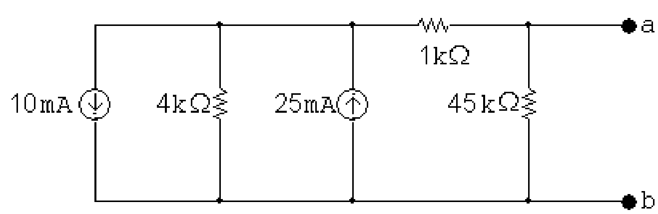
or

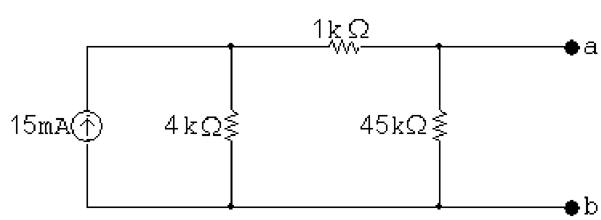


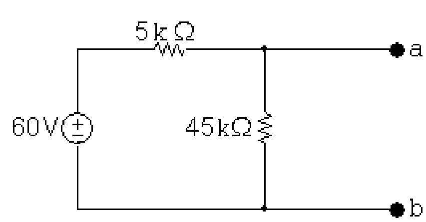
Therefore the Norton equivalent is determined by adding the current sources and combining the resistors in parallel:

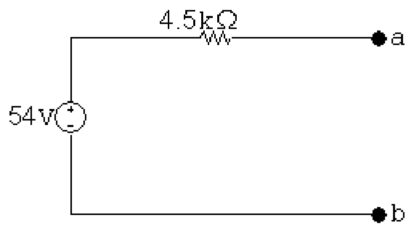


P 4.63 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.

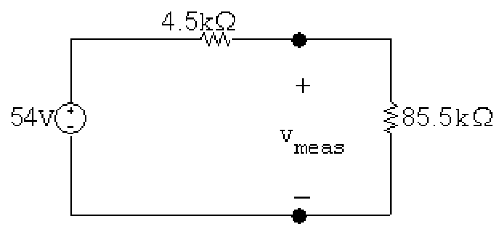








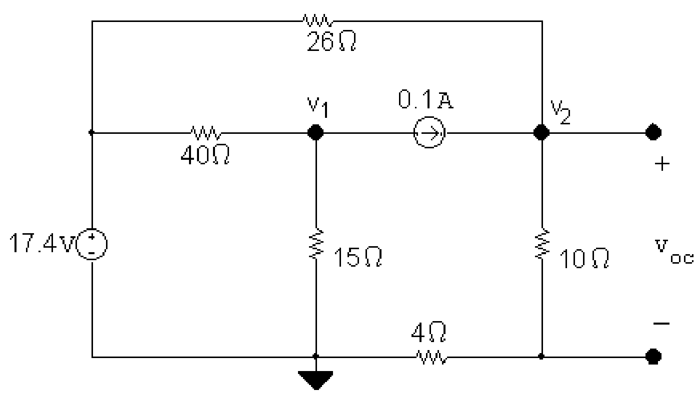
$\therefore \quad V_{\text{Th}} = 54\text{ V} \qquad R_{\text{Th}} = 4.5\text{ k}\Omega$



$$v_{\text{meas}} = \frac{85.5}{90}(54) = 51.3\text{ V}$$

[b] $\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

P 4.64 **[a]** Open circuit:



The node voltage equations are:

$$\frac{v_1 - 17.4}{40} + \frac{v_1}{15} + 0.1 = 0$$

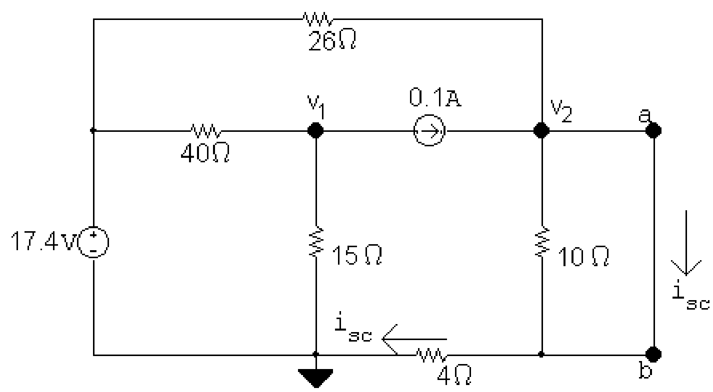
$$-0.1 + \frac{v_2}{14} + \frac{v_2 - 17.4}{26} = 0$$

The above equations are decoupled, so just solve the second equation for v_2 and use v_2 to solve for v_{oc}:

$-36.4 + 26v_2 + 14v_2 - 243.6 = 0 \quad \therefore \quad v_2 = 280/40 = 7\text{ V}$

$$v_{\text{oc}} = \frac{10}{10+4}(7) = 5\text{ V}$$

Short circuit:



Write a node voltage equation at v_2:

$$-0.1 + \frac{v_2 - 17.4}{26} + \frac{v_2}{4} = 0$$

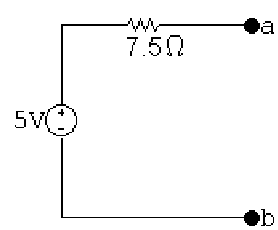
Solving,

$$-5.2 + 2v_2 - 34.8 + 13v_2 = 0 \qquad \therefore \qquad v_2 = 40/15 \text{ V}$$

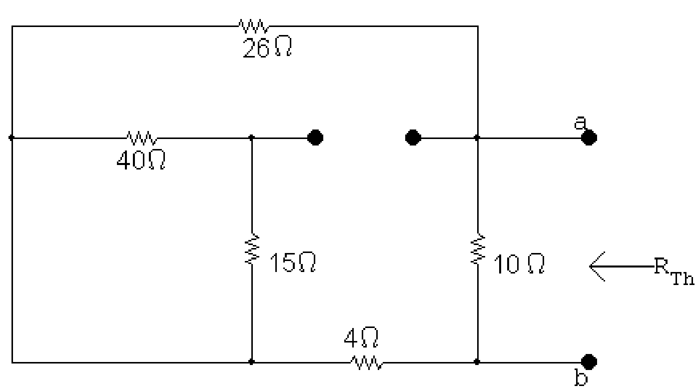
Calculate the short circuit current:

$i_{\rm sc} = (40/15)/4 = 2/3$ A

Therefore, $R_{\rm Th} = 5/(2/3) = 7.5\,\Omega$

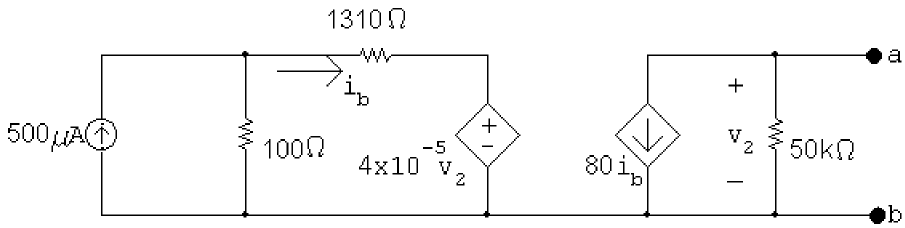


[b]



$R_{\rm Th} = 10\|(26+4) = 7.5\,\Omega$ (CHECKS)

P 4.65



OPEN CIRCUIT

Use Ohm's law to solve for v_2 on the right hand side of the circuit:

$v_2 = -80i_{\rm b}(50{,}000) = -40 \times 10^5 i_{\rm b}$

Use this value of v_2 to express the value of the dependent voltage source in terms of $i_{\rm b}$:

$4 \times 10^{-5}v_2 = 4 \times 10^{-5}(-40 \times 10^5 i_{\rm b}) = -160i_{\rm b}$

Write the mesh current equation for the $i_{\rm b}$ mesh:

$1310i_{\rm b} - 160i_{\rm b} + 100(i_{\rm b} - 500 \times 10^{-6}) = 0$

Solving,

$1250i_{\rm b} = 0.05 \quad \therefore \quad i_{\rm b} = 0.05/1250 = 40\,\mu$ A

Thus,

$V_{\rm Th} = v_2 = -40 \times 10^5 i_{\rm b} = -40 \times 10^5(40 \times 10^{-6}) = -160$ V

SHORT CIRCUIT

$v_2 = 0; \qquad i_{\rm sc} = -80i$

Calculate $i_{\rm b}$ using current division on the left hand side of the circuit:

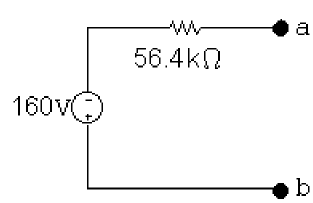
$$i \;= \frac{100}{100 + 1310}500 \times 10^{-6} = 35.461\,\mu\text{ A}$$

Calculate the short circuit current from the right hand side of the circuit:

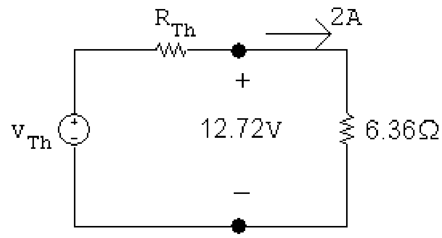
$i_{\rm sc} = -80(35.461 \times 10^{-6}) = -2.8369 \times 10^{-3}\,\text{mA}$

Calculate $R_{\rm Th}$ from the short circuit current and open circuit voltage:

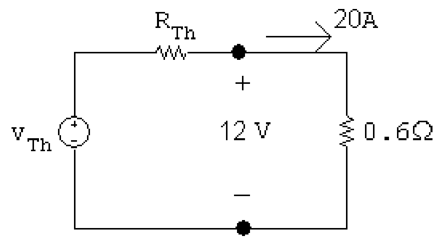
$$R_{\rm Th} = \frac{-160}{-2.8369 \times 10^{-3}} = 56.4\text{ k}\Omega$$



P 4.66



$$12.72 = V_{\text{Th}} - 2R_{\text{Th}}$$



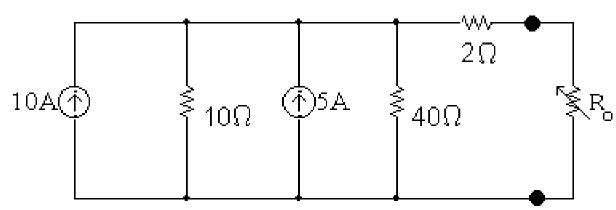
$$12 = V_{\text{Th}} - 20R_{\text{Th}}$$

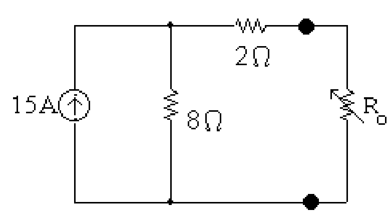
Solving the above equations for V_{Th} and R_{Th} yields

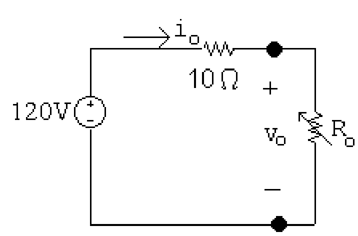
$V_{\text{Th}} = 12.8$ V, $\quad R_{\text{Th}} = 40$ mΩ

$\therefore \quad I \quad = 320$ A, $\quad R \quad = 40$ mΩ

P 4.67 First, find the Thévenin equivalent with respect to R .

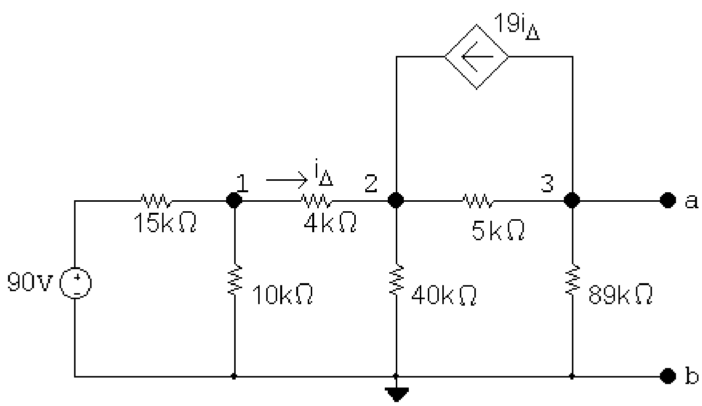






R	i	v	R	i	v
0	12	0	20	4	80
2	10	20	30	3	90
6	7.5	45	40	2.4	96
10	6	60	50	2	100
15	4.8	72	70	1.5	105

P 4.68



The node voltage equations are:

$$\frac{v_1 - 90}{15{,}000} + \frac{v_1}{10{,}000} + \frac{v_1 - v_2}{4000} = 0$$

$$\frac{v_2 - v_1}{4000} + \frac{v_2}{40{,}000} + \frac{v_2 - v_3}{5000} - 19i_\Delta = 0$$

$$\frac{v_3 - v_2}{5000} + \frac{v_3}{89{,}000} + 19i_\Delta = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1 - v_2}{4000}$$

Substitute the constraint equation into the node voltage equations and put the three remaining equations in standard form:

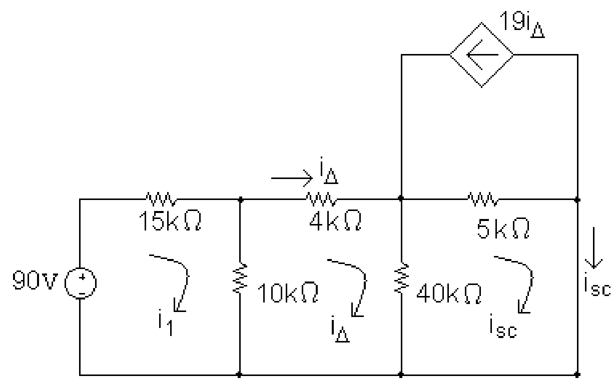
$$v_1\left(\frac{1}{15{,}000} + \frac{1}{10{,}000} + \frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) = \frac{90}{15{,}000}$$

$$v_1\left(-\frac{1}{4000} - \frac{19}{4000}\right) + v_2\left(\frac{1}{4000} + \frac{1}{40{,}000} + \frac{1}{5000} + \frac{19}{4000}\right) + v_3\left(-\frac{1}{5000}\right) = 0$$

$$v_1\left(\frac{19}{4000}\right) + v_2\left(-\frac{1}{5000} - \frac{19}{4000}\right) + v_3\left(\frac{1}{5000} + \frac{1}{89{,}000}\right) = 0$$

Solving, $v_1 = 32.75$ V; $v_2 = 30.58$ V; $v_3 = -19.8$ V

$V_{\text{Th}} = v_3 = -19.8$ V



The mesh current equations are:

$$-90 + 15{,}000 i_1 + 10{,}000(i_1 - i_\Delta) = 0$$

$$4000 i_\Delta + 40{,}000(i_\Delta - i_{\rm sc}) + 10{,}000(i_\Delta - i_1) = 0$$

$$40{,}000(i_{\rm sc} - i_\Delta) + 5000(i_{\rm sc} + 19 i_\Delta) = 0$$

Put these equations in standard form:

$$i_1(25{,}000) + i_\Delta(-10{,}000) + i_{\rm sc}(0) = +90$$

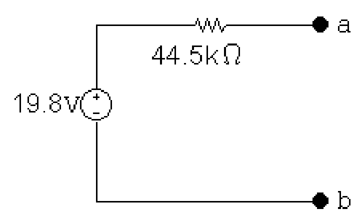
$$i_1(-10{,}000) + i_\Delta(54{,}000) + i_{\rm sc}(-40{,}000) = 0$$

$$i_1(0) + i_\Delta(55{,}000) + i_{\rm sc}(45{,}000) = 0$$

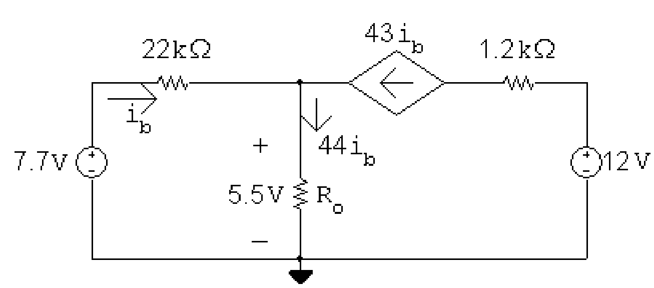
Solving, $i_1 = 3745.62\,\mu$A; $i_\Delta = 364.04\,\mu$ A; $i_{\rm sc} = -444.94\,\mu$ A

$i_{\rm sc} = -444.94\,\mu$ A

$R_{\rm Th} = -19.8/-444.94 \times 10^{-6} = 44.5$ kΩ



P 4.69 **[a]** Use source transformations to simplify the left side of the circuit.



$$i = \frac{7.7 - 5.5}{22{,}000} = 0.1 \text{ mA}$$

Let $R = R_{\text{meter}} \| 1.3\text{ k}\Omega = 5.5/4.4 \times 10^{-3} = 1250\,\Omega$

$$\therefore \quad \frac{(R_{\text{meter}})(1300)}{R_{\text{meter}} + 1300} = 1250; \qquad R_{\text{meter}} = \frac{(1250)(1300)}{50} = 32.5\text{ k}\Omega$$

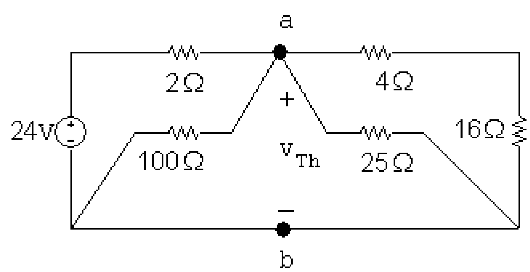
[b] Actual value of v :

$$i = \frac{7.7}{22{,}000 + 44(1300)} = 97.22\,\mu\text{ A}$$

$$v = 44i\ (1300) = 5.56\text{ V}$$

$$\%\text{ error} = \left(\frac{5.5 - 5.56}{5.56}\right) \times 100 = -1.10\%$$

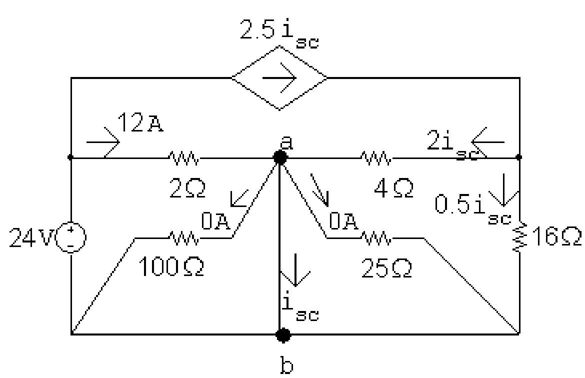
P 4.70 **[a]** Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $4.8\,\Omega$ resistor.
Thévenin voltage: note i is zero.



$$\frac{V_{\text{Th}} - 24}{2} + \frac{V_{\text{Th}}}{100} + \frac{V_{\text{Th}}}{25} + \frac{V_{\text{Th}}}{20} = 0$$

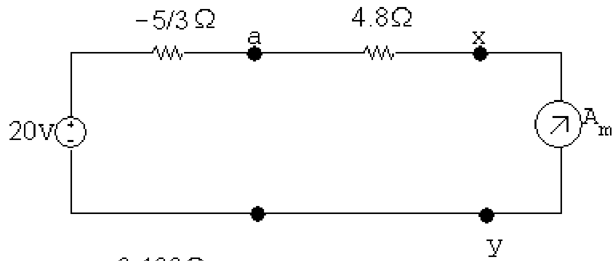
$$50V_{\text{Th}} + V_{\text{Th}} + 4V_{\text{Th}} + 5V_{\text{Th}} = 50(24) \qquad \therefore \qquad V_{\text{Th}} = 50(24)/60 = 20\text{ V}$$

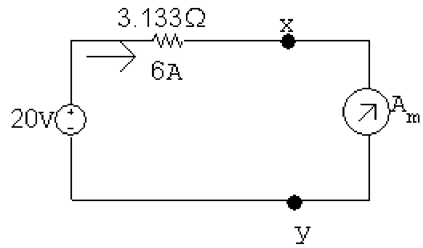
Short-circuit current:



$$i_{\text{sc}} = 12 + 2i_{\text{sc}}, \qquad \therefore \quad i_{\text{sc}} = -12\text{ A}$$

$$R_{\text{Th}} = \frac{20}{-12} = -1.67\,\Omega$$

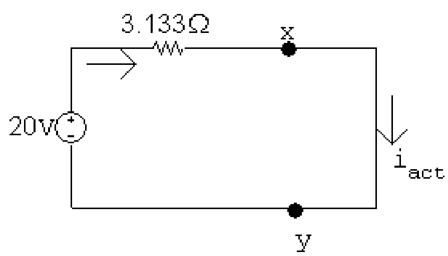




$$R_{\text{total}} = \frac{20}{6} = 3.333\,\Omega$$

$$R_{\text{meter}} = 3.333 - 3.133 = 0.20\,\Omega$$

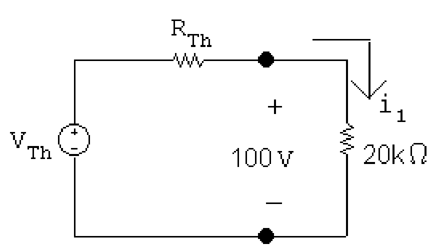
[b] Actual current:



$$i_{\text{actual}} = \frac{20}{3.133} = 6.383 \text{ A}$$

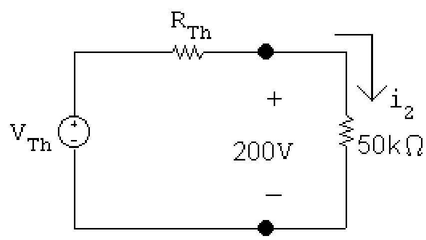
$$\%\text{ error } = \frac{6 - 6.383}{6.383} \times 100 = -6\%$$

P 4.71



$i_1 = 100/20{,}000 = 5$ mA

$$100 = V_{\text{Th}} - 0.005R_{\text{Th}}, \qquad V_{\text{Th}} = 100 + 0.005R_{\text{Th}}$$

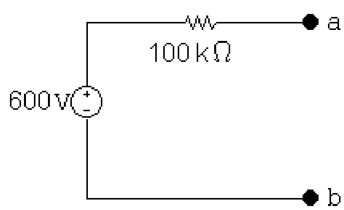


$i_2 = 200/50{,}000 = 4$ mA

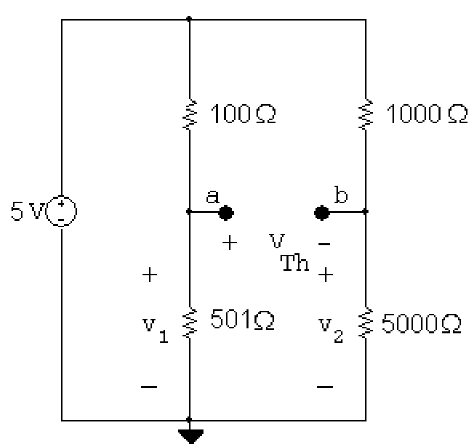
$200 = V_{\rm Th} - 0.004R_{\rm Th}, \qquad V_{\rm Th} = 200 + 0.004R_{\rm Th}$

$\therefore\quad 100 + 0.005R_{\rm Th} = 200 + 0.004R_{\rm Th}$ so $R_{\rm Th} = 100$ kΩ

$V_{\rm Th} = 100 + 500 = 600$ V



P 4.72



Use voltage division to calculate v_1 and v_2:

$$v_1 = \frac{501}{501 + 100}(5) = 4.168053 \text{ V}$$

$$v_2 = \frac{5000}{5000 + 1000}(5) = 4.1666667 \text{ V}$$

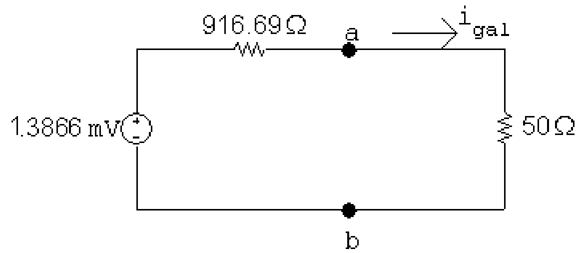
Now calculate $V_{\rm Th}$:

$V_{\rm Th} = v_1 - v_2 = 4.168053 - 4.1666667 = 1.3866$ mV

Calculate $R_{\rm Th}$ by removing the voltage source and creating series and parallel combinations of the resisitors:

$$R_{\rm Th} = 100\|501 + 1000\|5000 = \frac{(100)(501)}{601} + \frac{(1000)(5000)}{6000} = 916.69\,\Omega$$

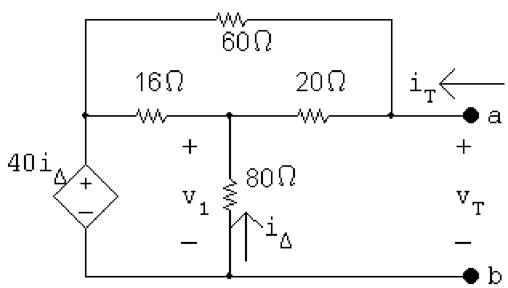
The resulting Thévenin equivalent circuit is shown below:



Use KVL to calculate $i_{\rm gal}$:

$$i_{\rm gal} = \frac{1.3866 \times 10^{-3}}{916.69 + 50} = 1.43\,\mu\text{A}$$

P 4.73 $V_{\rm Th} = 0$, since circuit contains no independent sources.



$$i_{\rm T} = \frac{v_{\rm T} - v_1}{20} + \frac{v_{\rm T} - 40i_\Delta}{60}$$

$$\frac{v_1 - 40i_\Delta}{16} + \frac{v_1}{80} + \frac{v_1 - v_{\rm T}}{20} = 0$$

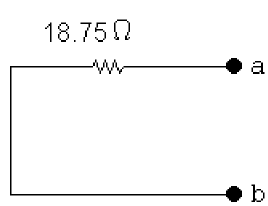
$$\therefore \quad 10v_1 - 200i_\Delta = 4v_{\rm T} \qquad i_\Delta = \frac{-v_1}{80}, \qquad 200i_\Delta = -2.5v_1$$

$$\therefore \quad 12.5v_1 = 4v_{\rm T}; \qquad v_1 = 0.32v_{\rm T}$$

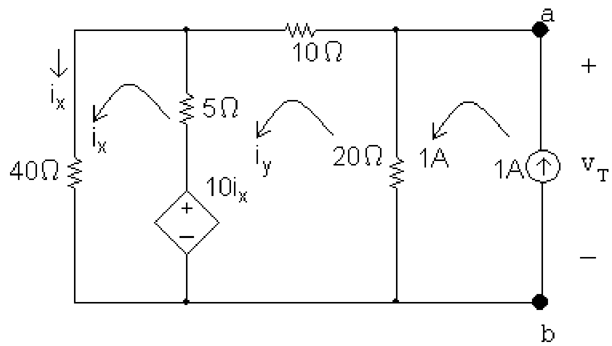
$$60i_{\rm T} = 4v_{\rm T} - 2.5v_1 = 3.2v_{\rm T}$$

$$\therefore \quad \frac{v_{\rm T}}{i_{\rm T}} = \frac{60}{3.2} = 18.75\,\Omega$$

$$R_{\rm Th} = 18.75\,\Omega$$



P 4.74 $V_{\mathrm{Th}} = 0$ since there are no independent sources in the circuit. To find R_{Th}, apply a 1 A test source and calculate the voltage drop across the test source. Use the mesh current method.



The mesh current equations for the two meshes on the left:

$$-10i \;\; + 5(i \;\; - i \;\;) + 40i \qquad = \quad 0$$

$$10i \;\; + 20(i \;\; - 1) + 10i \;\; + 5(i \;\; - i \;\;) \quad = \quad 0$$

Place these equations in standard form:

$$i \;\; (-10 + 5 + 40) + i \;\; (-5) \quad = \quad 0$$

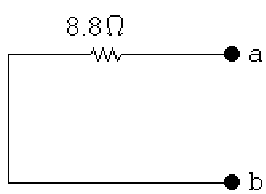
$$i \;\; (10 - 5) + i \;\; (20 + 10 + 5) \quad = \quad 20$$

Solving, $\qquad i \;\; = 80$ mA; $\qquad i \;\; = 560$ mA

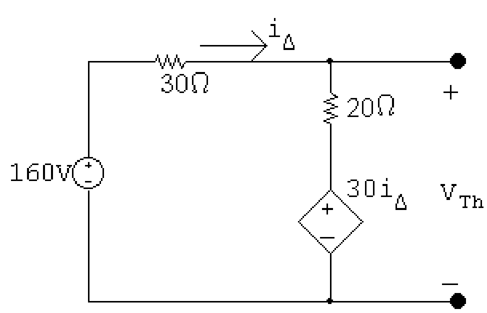
Find the voltage drop across the 1 A source:

$v_{\mathrm{T}} = 20(1 - 0.56) = 8.8$ V

$\therefore \qquad R_{\mathrm{Th}} = v_{\mathrm{T}}/1\text{ A} = 8.8/1 = 8.8\,\Omega$



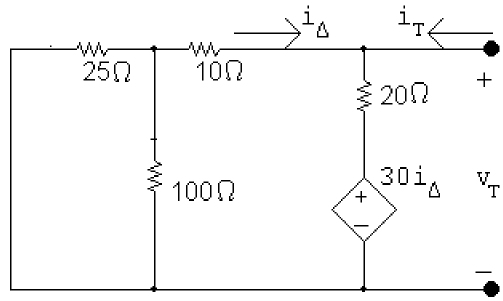
P 4.75 We begin by finding the Thévenin equivalent with respect to R . After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\text{ A}$$

$V_{\mathrm{Th}} = 20i_{\Delta} + 30i_{\Delta} = 50i_{\Delta} = 100$ V

Using the test-source method to find the Thévenin resistance gives

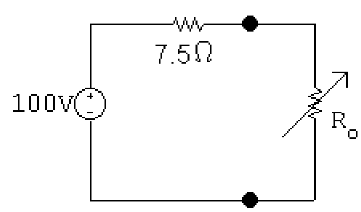


$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{30} + \frac{v_{\mathrm{T}} - 30(-v_{\mathrm{T}}/30)}{20}$$

$$\frac{i_{\mathrm{T}}}{v_{\mathrm{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R}\right)^2 R \;= 250$$

$$\frac{10^4}{R^2 + 15R \; + 56.25} R \;= 250$$

$$\frac{10^4 R}{250} = R^2 + 15R \; + 56.25$$

$$40R \;= R^2 + 15R \; + 56.25$$

$$R^2 - 25R \; + 56.25 = 0$$

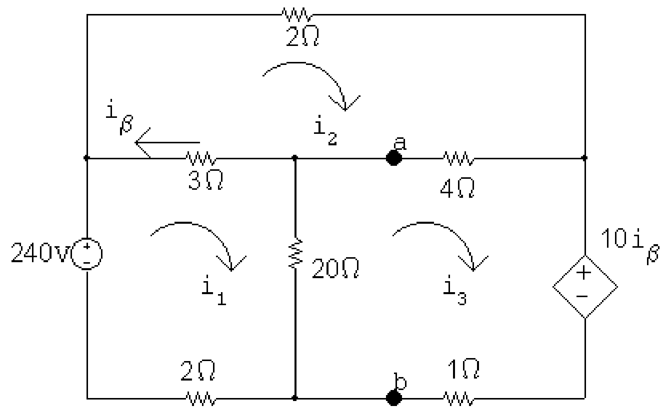
$$R \;= 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R \;= 22.5\,\Omega$$

$$R \;= 2.5\,\Omega$$

P 4.76 **[a]** Find the Thévenin equivalent with respect to the terminals of $R_{\rm L}$.

Open circuit voltage:



The mesh current equations are:

$$-240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 = 0$$
$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i\quad + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:

$i\quad = i_2 - i_1$

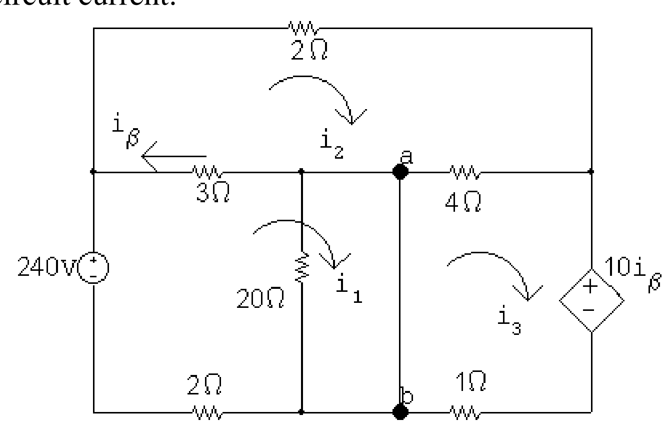
Place these equations in standard form:

$$i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i\ \ (0) = 240$$
$$i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i\ \ (0) = 0$$
$$i_1(-20) + i_2(-4) + i_3(4 + 1 + 20) + i\ \ (10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i\ \ (1) = 0$$

Solving, $i_1 = 99.6$ A; $i_2 = 78$ A; $i_3 = 100.8$ A; $i\quad = -21.6$ A

$V_{\rm Th} = 20(i_1 - i_3) = -24$ V

Short-circuit current:



The mesh current equations are:

$$-240 + 3(i_1 - i_2) + 2i_1 = 0$$
$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i\ \ + 1i_3 + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:

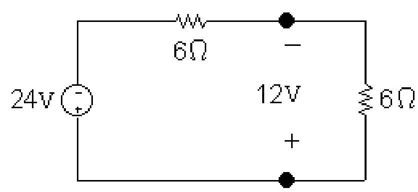
$i\ \ = i_2 - i_1$

Place these equations in standard form:

$$i_1(3+2) + i_2(-3) + i_3(0) + i\ \ (0) = 240$$
$$i_1(-3) + i_2(2+4+3) + i_3(-4) + i\ \ (0) = 0$$
$$i_1(0) + i_2(-4) + i_3(4+1) + i\ \ (10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i\ \ (1) = 0$$

Solving, $\quad i_1 = 92$ A; $\quad i_2 = 73.33$ A; $\quad i_3 = 96$ A; $\quad i\ \ = -18.67$ A

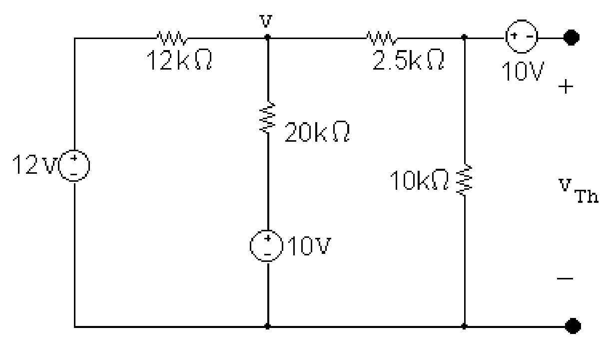
$i_{\rm sc} = i_1 - i_3 = -4$ A; $\qquad R_{\rm Th} = \dfrac{V_{\rm Th}}{i_{\rm sc}} = \dfrac{-24}{-4} = 6\,\Omega$



$R_{\rm L} = R_{\rm Th} = 6\,\Omega$

[b] $p_{\max} = \dfrac{12^2}{6} = 24$ W

P 4.77 **[a]**

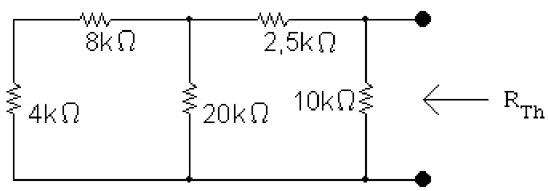


$$\frac{v-12}{12{,}000} + \frac{v-10}{20{,}000} + \frac{v}{12{,}500} = 0$$

Solving, $\qquad v = 7.03125$ V

$$v_{10\rm k} = \frac{10{,}000}{12{,}500}(7.03125) = 5.625\text{ V}$$

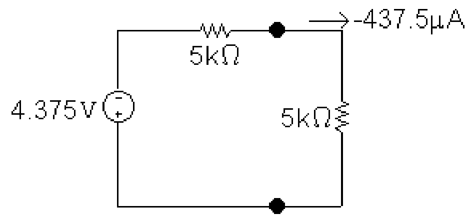
$\therefore\quad V_{\rm Th} = v - 10 = -4.375$ V



$$R_{\mathrm{Th}} = [(12{,}000\|20{,}000) + 2500] = 5\text{ k}\Omega$$

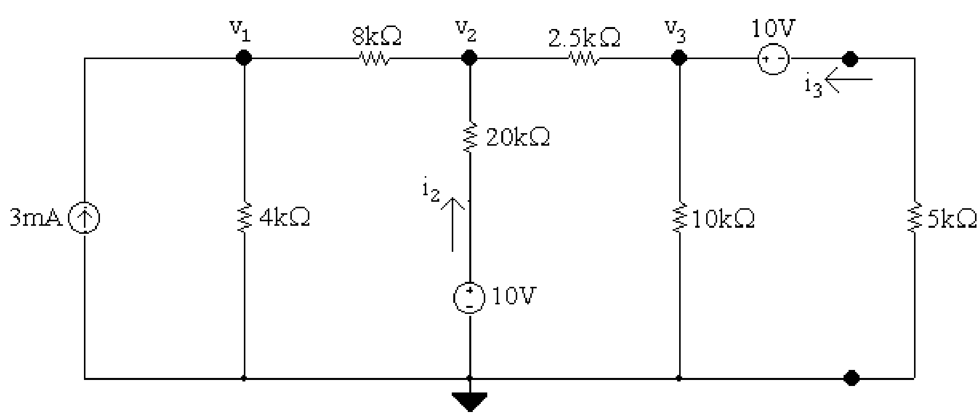
$$R \ = R_{\mathrm{Th}} = 5\text{ k}\Omega$$

[b]



$$p_{\max} = (-437.5 \times 10^{-6})^2(5000) = 957.03\,\mu\text{ W}$$

P 4.78 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-3 \times 10^{-3} + \frac{v_1}{4000} + \frac{v_1 - v_2}{8000} = 0$$

At v_2: $$\frac{v_2 - v_1}{8000} + \frac{v_2 - 10}{20{,}000} + \frac{v_2 - v_3}{2500} = 0$$

At v_3: $$\frac{v_3 - v_2}{2500} + \frac{v_3}{10{,}000} + \frac{v_3 - 10}{5000} = 0$$

Standard form:

$$v_1\left(\frac{1}{4000}+\frac{1}{8000}\right)+v_2\left(-\frac{1}{8000}\right)+v_3(0)=0.003$$

$$v_1\left(-\frac{1}{8000}\right)+v_2\left(\frac{1}{8000}+\frac{1}{20{,}000}+\frac{1}{2500}\right)+v_3\left(-\frac{1}{2500}\right)=\frac{10}{20{,}000}$$

$$v_1(0)+v_2\left(-\frac{1}{2500}\right)+v_3\left(\frac{1}{2500}+\frac{1}{10{,}000}+\frac{1}{5000}\right)=\frac{10}{5000}$$

Calculator solution:

$$v_1 = 10.890625 \text{ V} \qquad v_2 = 8.671875 \text{ V} \qquad v_3 = 7.8125 \text{ V}$$

Calculate currents:

$$i_2 = \frac{10 - v_2}{20{,}000} = 66.40625\,\mu\text{ A} \qquad i_3 = \frac{10 - v_3}{5000} = 437.5\,\mu\text{ A}$$

Calculate power delivered by the sources:

$$p_{3\text{mA}} = (3\times10^{-3})v_1 = (3\times10^{-3})(10.890625) = 32.671875 \text{ mW}$$

$$p_{10\text{Vmiddle}} = i_2(10) = (66.40625\times10^{-6})(10) = 0.6640625 \text{ mW}$$

$$p_{10\text{Vtop}} = i_3(10) = (437.5\times10^{-6})(10) = 4.375 \text{ mW}$$

$$p_{\text{deliveredtotal}} = 32.671875 + 0.6640625 + 4.375 = 37.7109375 \text{ mW}$$

Calculate power absorbed by the $5\text{ k}\Omega$ resistor and the percentage power:

$$p_{5\text{k}} = i_3^2(5000) = (437.5\times10^{-6})^2(5000) = 0.95703125 \text{ mW}$$

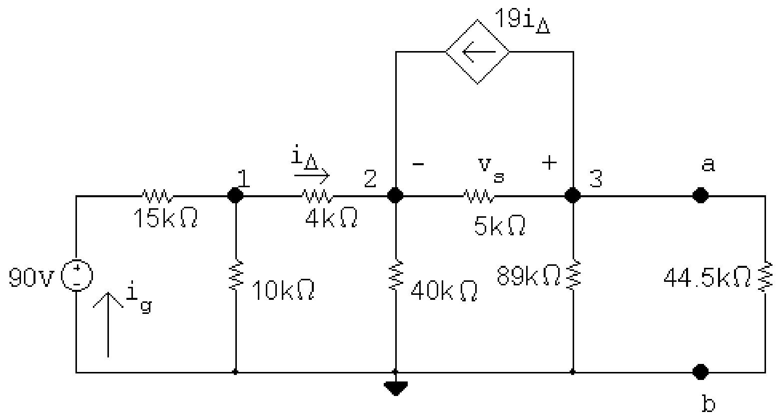
$$\%\text{ delivered to } R: \qquad \frac{0.95793125}{37.7109375}(100) = 2.54\%$$

P 4.79 **[a]** From the solution of Problem 4.68 we have $R_{\text{Th}} = 44.5\text{ k}\Omega$ and $V_{\text{Th}} = -19.8$ V. Therefore

$$R = R_{\text{Th}} = 44.5\text{ k}\Omega$$

[b] $p = \dfrac{(-9.9)^2}{44{,}500} = 2.2\text{ mW}$

[c]



The node voltage equations are:

$$\frac{v_1 - 90}{15{,}000} + \frac{v_1}{10{,}000} + \frac{v_1 - v_2}{4000} = 0$$

$$\frac{v_2 - v_1}{4000} + \frac{v_2}{40{,}000} + \frac{v_2 - v_3}{5000} - 19i_\Delta = 0$$

$$\frac{v_3 - v_2}{5000} + \frac{v_3}{89{,}000} + 19i_\Delta + \frac{v_3}{44{,}500} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1 - v_2}{4000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{15{,}000} + \frac{1}{10{,}000} + \frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) + i_\Delta(0) = \frac{90}{15{,}000}$$

$$v_1\left(-\frac{1}{4000}\right) + v_2\left(\frac{1}{4000} + \frac{1}{40{,}000} + \frac{1}{5000}\right) + v_3\left(-\frac{1}{5000}\right) + i_\Delta(-19) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{5000}\right) + v_3\left(\frac{1}{5000} + \frac{1}{89{,}000} + \frac{1}{44{,}500}\right) + i_\Delta(19) = 0$$

$$v_1\left(\frac{1}{4000}\right) + v_2\left(-\frac{1}{4000}\right) + v_3(0) + i_\Delta(-1) = 0$$

Solving,

$v_1 = 33.2818$ V; $\quad v_2 = 31.4697$ V; $\quad v_3 = -9.9$ V; $\quad i_\Delta = 453\,\mu$A

Calculate the power:

$$i = \frac{90 + 33.2818}{15{,}000} = 3.78 \text{ mA}$$

$$p_{90\text{V}} = -(90)(3.78 \times 10^{-3}) = -340.31 \text{ mW}$$

$$p_{\text{dep source}} = (v_3 - v_2)(19i_\Delta) = -356.07 \text{ mW}$$

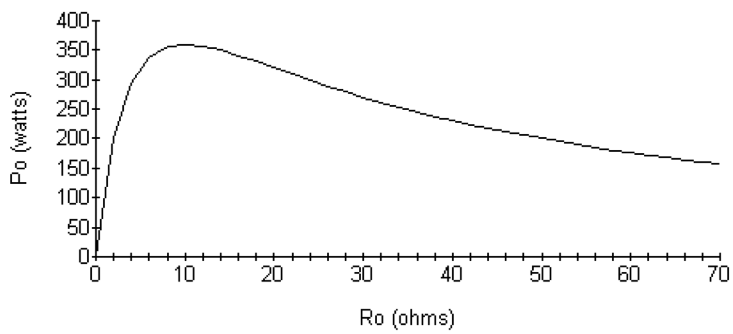
$$\sum p_{\text{dev}} = 340.31 + 356.07 = 696.38 \text{ mW}$$

$$\% \text{ delivered } = \frac{2.2 \times 10^{-3}}{696.38 \times 10^{-3}} \times 100 = 0.316\%$$

P 4.80 **[a]** From the solution to Problem 4.67 we have

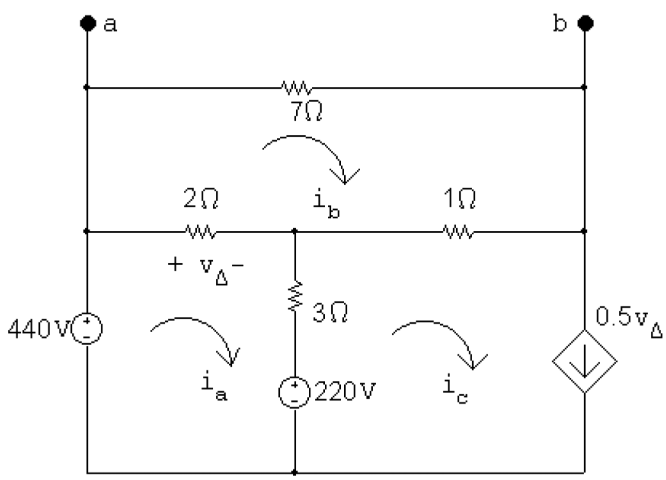
R (Ω)	P (W)	R (Ω)	P (W)
0	0	20	320.00
2	200.00	30	270.00
6	337.50	40	230.40
10	360.00	50	200.00
15	345.60	70	157.50

[b]



[c] $R \;= 10\,\Omega,\qquad P\;(\max) = 360$ W

P 4.81 Find the Thévenin equivalent with respect to the terminals of R .
Open circuit voltage:



$$(440 - 220) = 5i_{\mathrm{a}} - 2i_{\mathrm{b}} - 3i_{\mathrm{c}}$$

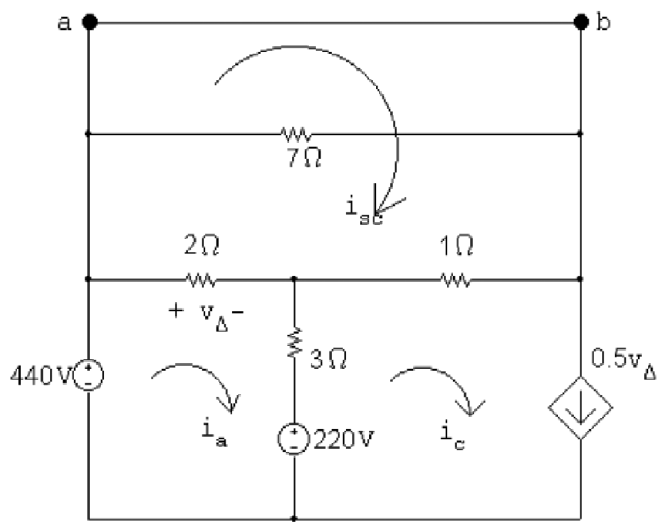
$$0 = -2i_{\mathrm{a}} + 10i_{\mathrm{b}} - i_{\mathrm{c}}$$

$$i_{\mathrm{c}} = 0.5v_{\Delta};\qquad v_{\Delta} = 2(i_{\mathrm{a}} - i_{\mathrm{b}});\qquad i_{\mathrm{c}} = i_{\mathrm{a}} - i_{\mathrm{b}}$$

Solving, $i_\mathrm{a} = 96.8$ A; $i_\mathrm{b} = 26.4$ A; $i_\mathrm{c} = 70.4$ A; $v_\Delta = 140.8$ V

$\therefore \quad V_\mathrm{Th} = 7i_\mathrm{b} = 184.8$ V

Short circuit current:



$$440 - 220 = 5i_\mathrm{a} - 2i_\mathrm{sc} - 3i_\mathrm{c}$$

$$0 = -2i_\mathrm{a} + 3i_\mathrm{sc} - 1i_\mathrm{c}$$

$$i_\mathrm{c} = 0.5v_\Delta; \qquad v_\Delta = 2(i_\mathrm{a} - i_\mathrm{sc}) \quad \therefore \quad i_\mathrm{c} = i_\mathrm{a} - i_\mathrm{sc}$$

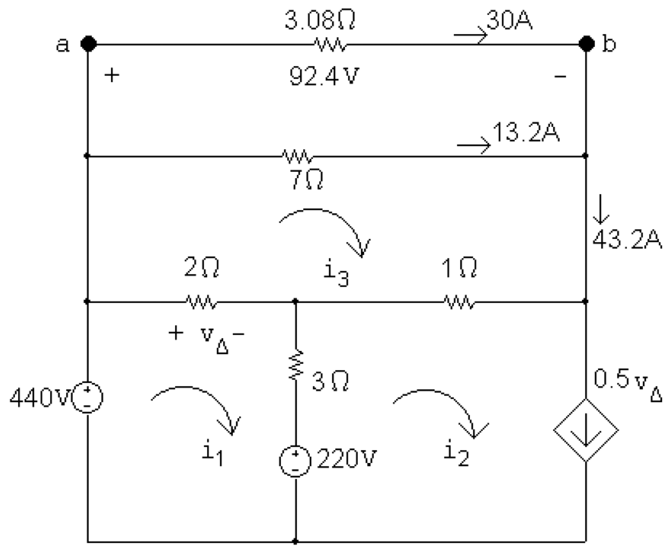
Solving, $i_\mathrm{sc} = 60$ A; $\quad i_\mathrm{a} = 80$ A; $\quad i_\mathrm{c} = 20$ A; $\quad v_\Delta = 40$ V

$$R_\mathrm{Th} = V_\mathrm{Th}/i_\mathrm{sc} = 184.8/60 = 3.08\,\Omega$$

$$R = 3.08\,\Omega$$

$$p_o = \frac{(92.4)^2}{3.08} = 2772 \text{ W}$$

With R equal to $3.08\,\Omega$ the circuit becomes



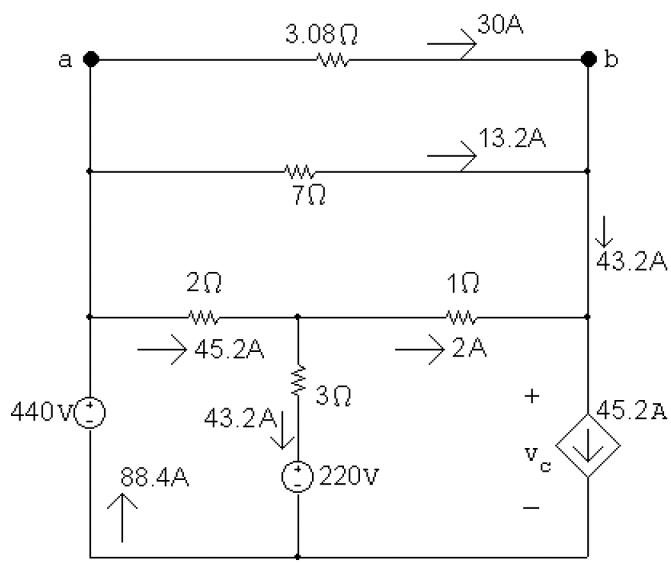
$$220 = 5i_1 - 3(0.5)(2)(i_1 - i_3) - 2i_3 = 2i_1 + i_3$$

$$\therefore \quad 2i_1 = 220 - i_3 = 220 - 43.2 = 176.8 \quad \therefore \; i_1 = 88.4 \text{ A}$$

$$v_\Delta = 2(i_1 - i_3) = 90.4 \text{ V}$$

$$i_2 = 0.5v_\Delta = 45.2 \text{ A}$$

Thus we have



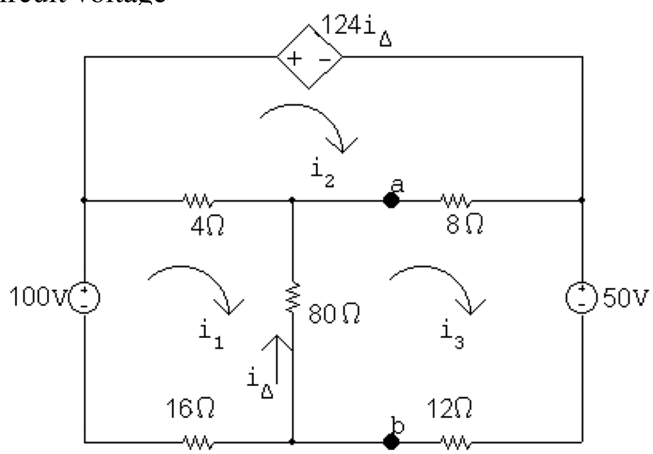
$$v \; = 220 + 3(43.2) - 2 = 347.6 \text{ V}$$

Therefore, the only source developing power is the 440 V source.

$p_{440\text{V}} = -(440)(88.4) = -38{,}896$ W   Power delivered is 38,896 W

$$\%\ \text{delivered} = \frac{2772}{38{,}896}(100) = 7.13\%$$

P 4.82 **[a]** We begin by finding the Thévenin equivalent with respect to the terminals of R .
Open circuit voltage



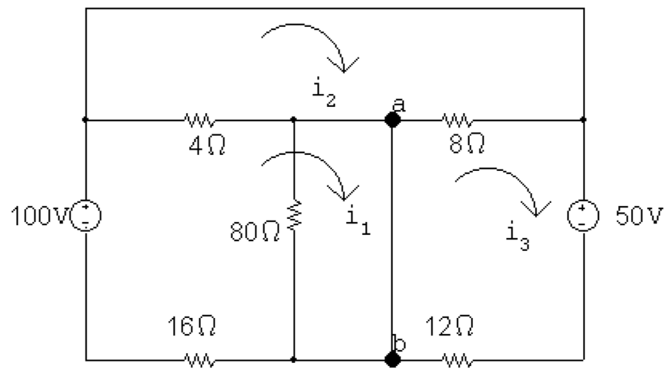
The mesh current equations are:

$$-100 + 4(i_1 - i_2) + 80(i_1 - i_3) + 16i_1 = 0$$
$$124i_\Delta + 8(i_2 - i_3) + 4(i_2 - i_1) = 0$$
$$50 + 12i_3 + 80(i_3 - i_1) + 8(i_3 - i_2) = 0$$

The constraint equation is:
$i_\Delta = i_3 - i_1$
Place these equations in standard form:

$$i_1(4 + 80 + 16) + i_2(-4) + i_3(-80) + i_\Delta(0) = 100$$
$$i_1(-4) + i_2(8 + 4) + i_3(-8) + i_\Delta(124) = 0$$
$$i_1(-80) + i_2(-8) + i_3(12 + 80 + 8) + i_\Delta(0) = -50$$
$$i_1(1) + i_2(0) + i_3(-1) + i_\Delta(1) = 0$$

Solving, $i_1 = 4.7$ A; $i_2 = 10.5$ A; $i_3 = 4.1$ A; $i_\Delta = -0.6$ A
Also, $V_{\text{Th}} = v_{\text{ab}} = -80i_\Delta = 48$ V
Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $124i_\Delta$ is also zero.

The mesh currents are:

$$-100 + 4(i_1 - i_2) + 16i_1 = 0$$
$$8(i_2 - i_3) + 4(i_2 - i_1) = 0$$
$$50 + 12i_3 + 8(i_3 - i_2) = 0$$

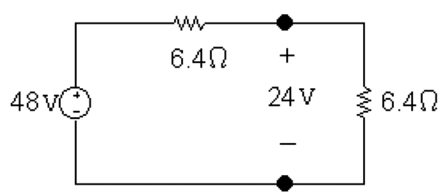
Place these equations in standard form:

$$i_1(4 + 16) + i_2(-4) + i_3(0) = 100$$
$$i_1(-4) + i_2(8 + 4) + i_3(-8) = 0$$
$$i_1(0) + i_2(-8) + i_3(12 + 8) = -50$$

Solving, $i_1 = 5$ A; $i_2 = 0$ A; $i_3 = -2.5$ A

Then, $i_{sc} = i_1 - i_3 = 7.5$ A

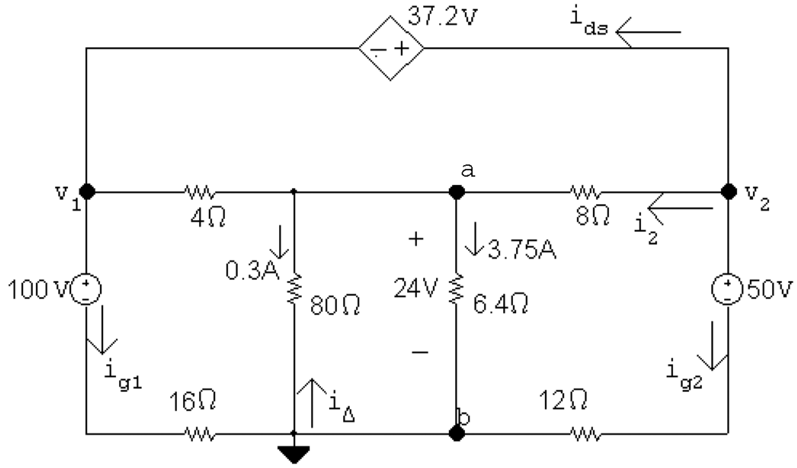
$R_{Th} = 48/7.5 = 6.4\,\Omega$



For maximum power transfer $R = R_{Th} = 6.4\,\Omega$

[b] $p_{max} = \dfrac{24^2}{6.4} = 90$ W

P 4.83 From the solution of Problem 4.82 we know that when R is $6.4\,\Omega$, the voltage across R is 24 V, positive at the upper terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is -0.3 A, and hence $124i_\Delta$ is -37.2 V.



Using the node voltage method to find v_1 and v_2 yields

$$4.05 + \frac{24 - v_1}{4} + \frac{24 - v_2}{8} = 0$$

$$2v_1 + v_2 = 104.4; \qquad v_1 + 37.2 = v_2$$

Solving, $v_1 = 22.4$ V; $\qquad v_2 = 59.6$ V.
It follows that

$$i_1 = \frac{22.4 - 100}{16} = -4.85 \text{ A}$$

$$i_2 = \frac{59.6 - 50}{12} = 0.8 \text{ A}$$

$$i_2 = \frac{59.6 - 24}{8} = 4.45 \text{ A}$$

$$i_{\text{ds}} = -4.45 - 0.8 = -5.25 \text{ A}$$

$$p_{100\text{V}} = 100i_1 = -485 \text{ W}$$

$$p_{50\text{V}} = 50i_2 = 40 \text{ W}$$

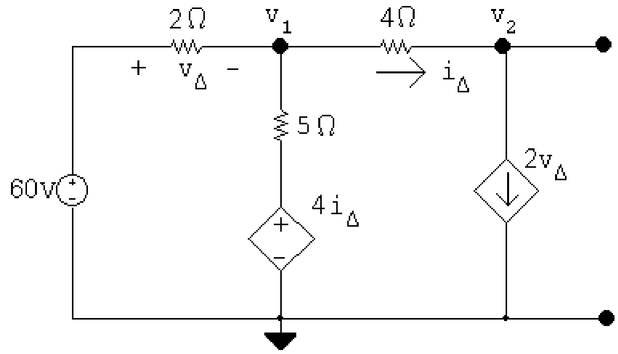
$$p_{\text{ds}} = 37.2i_{\text{ds}} = -195.3 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = 485 + 195.3 = 680.3 \text{ W}$$

$$\therefore \quad \% \text{ delivered } = \frac{90}{680.3}(100) = 13.23\%$$

$\therefore \quad 13.23\%$ of developed power is delivered to load

P 4.84 **[a]** Open circuit voltage



Node voltage equations:

$$\frac{v_1-60}{2}+\frac{v_1-4i_\Delta}{5}+\frac{v_1-v_2}{4} = 0$$

$$\frac{v_2-v_1}{4}+2v_\Delta = 0$$

Constraint equations:

$$v_\Delta = 60-v_1$$

$$i_\Delta = \frac{v_1-v_2}{4}$$

Place the equations in standard form:

$$v_1\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)+v_2\left(-\frac{1}{4}\right)+i_\Delta\left(-\frac{4}{5}\right)+v_\Delta(0) = 30$$

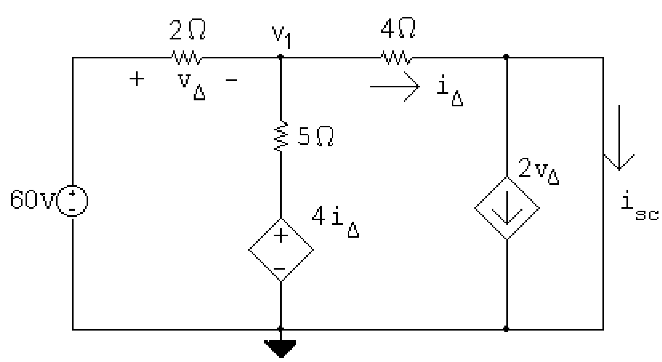
$$v_1\left(-\frac{1}{4}\right)+v_2\left(\frac{1}{4}\right)+i_\Delta(0)+v_\Delta(2) = 0$$

$$v_1(1)+v_2(0)+i_\Delta(0)+v_\Delta(1) = 60$$

$$v_1(1)+v_2(-1)+i_\Delta(-4)+v_\Delta(0) = 0$$

Solving, $v_1 = 20$ V; $v_2 = -300$ V; $i_\Delta = 80$ A; $v_\Delta = 40$ V

Short circuit current:



The node voltage equation:

$$\frac{v_1-60}{2}+\frac{v_1-4i_\Delta}{5}+\frac{v_1}{4}=0$$

The constraint equation:

$i_\Delta = v_1/4$

Place these equations in standard form:

$$v_1\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)+i_\Delta\left(-\frac{4}{5}\right) = 30$$

$$v_1\left(\frac{1}{4}\right)+i_\Delta(-1) = 0$$

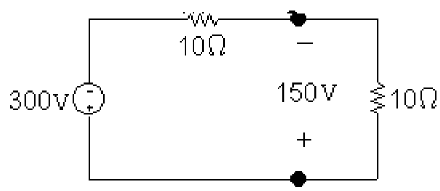
Solving, $v_1 = 40$ V; $i_\Delta = 10$ A

Then, $v_\Delta = 60 - 40 = 20$ V

and $i_{sc} = i_\Delta - 2v_\Delta = 10 - 40 = -30$ A

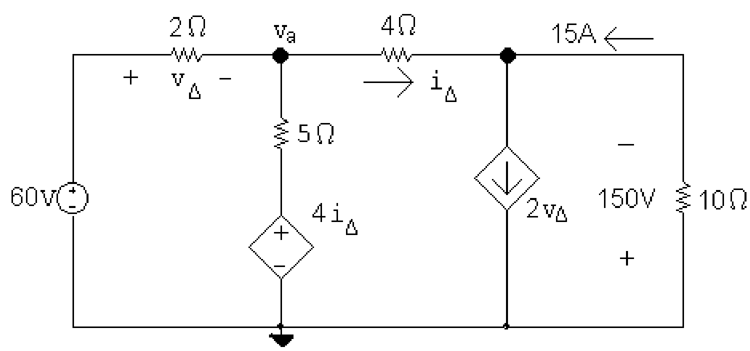
Thus, $R_{Th} = -300/-30 = 10\,\Omega$

[b]



$$p_{max} = \frac{(150)^2}{10} = 2250 \text{ W}$$

[c]



The node voltage equation:

$$\frac{v_a - 60}{2}+\frac{v_a - 4i_\Delta}{5}+\frac{v_a+150}{4} = 0$$

The constraint equation is:

$$i_\Delta = \frac{v_a+150}{4}$$

Place the equations in standard form:

$$v_a\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)+i_\Delta\left(-\frac{4}{5}\right) = 30-\frac{150}{4}$$

$$v_a\left(-\frac{1}{4}\right)+i_\Delta(1) = \frac{150}{4}$$

Solving, $v_a = 30$ V; $i_\Delta = 45$ A

Calculate the power:

$$i_{60\text{V}} = \frac{v_\text{a} - 60}{2} = -15 \text{ A}$$

$$p_{60\text{V}} = (60)(-15) = -900 \text{ W}$$

$$i_\text{ccvs} = \frac{v_\text{a} - 4i_\Delta}{5} = -30 \text{ A}$$

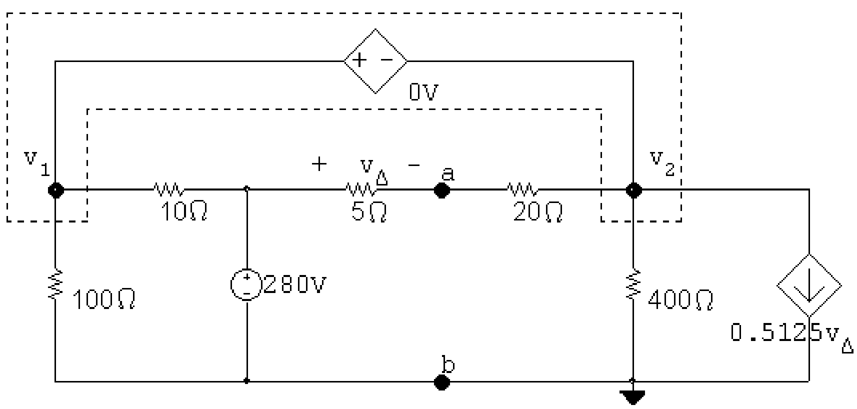
$$p_\text{ccvs} = 4(45)(-30) = -5400 \text{ W}$$

$$p_\text{vccs} = (-150)[2(30)] = -9000 \text{ W}$$

$$\sum p_\text{dev} = 900 + 5400 + 9000 = 15{,}300 \text{ W}$$

$$\% \text{ delivered} = \frac{2250}{15{,}3000} \times 100 = 14.7\%$$

P 4.85 **[a]** First find the Thévenin equivalent with respect to R .
Open circuit voltage: $i = 0$; $50i = 0$



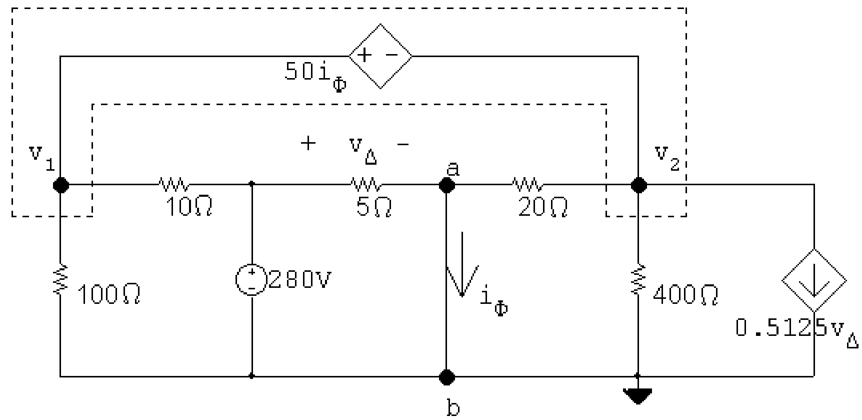
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_\text{Th} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$v_\Delta = 280$ V

$v_2 + 50i \ \ = v_1$

$$i \ \ = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

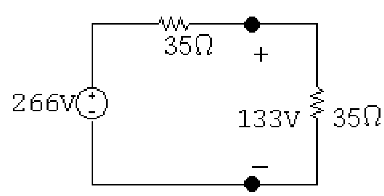
$v_2 = -968$ V; $v_1 = -588$ V

$i \ \ = i_{sc} = 56 + 0.05(-968) = 7.6$ A

$R_{Th} = V_{Th}/i_{sc} = 266/7.6 = 35\,\Omega$

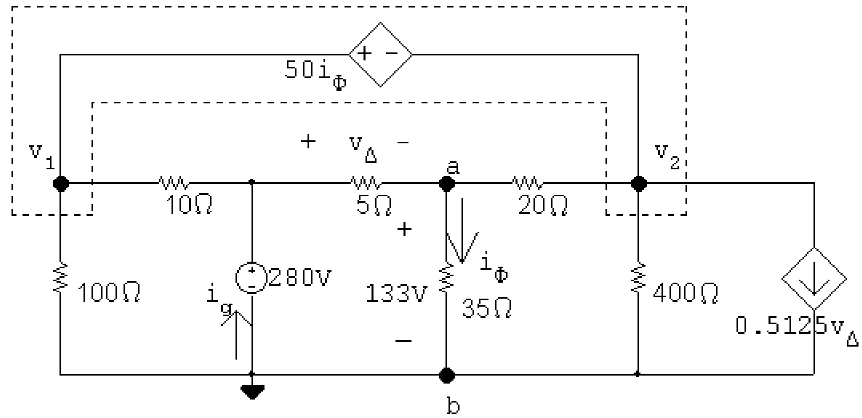
$\therefore \ \ R \ \ = 35\,\Omega$

[b]



$p_{max} = (133)^2/35 = 505.4$ W

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$$v_2 + 50i \;\; = v_1; \qquad i \;\; = 133/35 = 3.8 \text{ A}$$

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

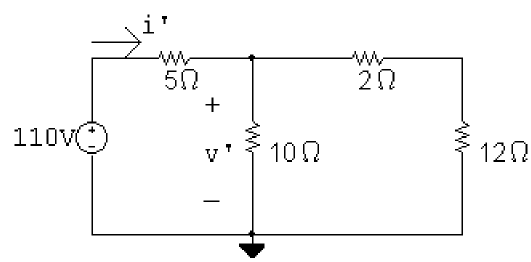
$$i \;\; = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$$p_{280\text{V}} \text{ (dev) } = (280)(76.3) = 21{,}364 \text{ W}$$

P 4.86 **[a]** Since $0 \leq R \;\; < \infty$ maximum power will be delivered to the 8 Ω resistor when $R \;\; = 0$.

[b] $P = \dfrac{24^2}{8} = 72$ W

P 4.87 **[a]** 110 V source acting alone:

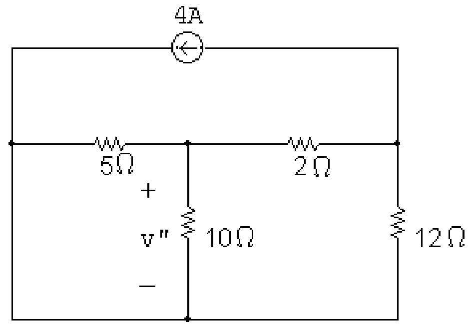


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\ \Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13} \text{ A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13} \text{ V}$$

4 A source acting alone:

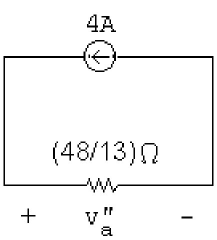


$5\,\Omega\|10\,\Omega = 50/15 = 10/3\,\Omega$

$10/3 + 2 = 16/3\,\Omega$

$16/3\|12 = 48/13\,\Omega$

Hence our circuit reduces to:



It follows that

$v'' = 4(48/13) = (192/13)$ V

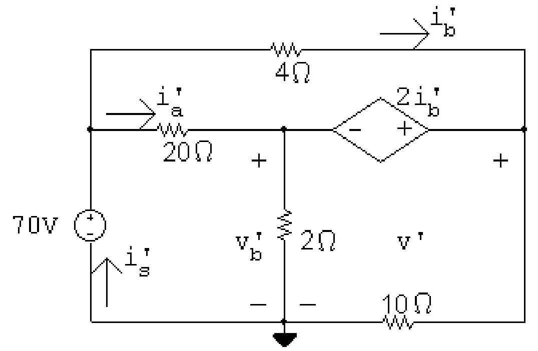
and

$$v'' = \frac{-v''}{(16/3)}(10/3) = -\frac{5}{8}v'' = -(120/13) \text{ V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50 \text{ V}$$

[b] $p = \dfrac{v^2}{10} = 250$ W

P 4.88 70-V source acting alone:



$v' = 70 - 4i'$

$i' = \dfrac{v'}{2} + \dfrac{v'}{10} = i' + i'$

$70 = 20i' + v'$

$i' = \dfrac{70 - v'}{20}$

$\therefore \qquad i' = \dfrac{v'}{2} + \dfrac{v'}{10} - \dfrac{70 - v'}{20} = \dfrac{11}{20}v' + \dfrac{v'}{10} - 3.5$

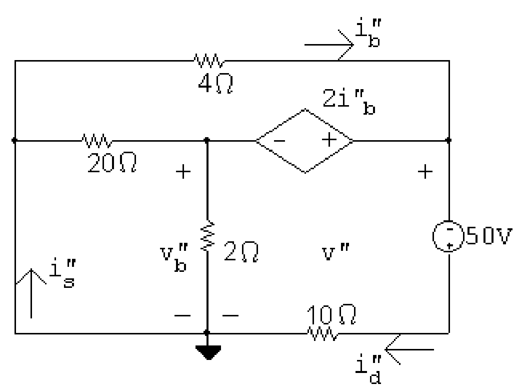
$v' = v' + 2i'$

$\therefore \quad v' = v' - 2i'$

$\therefore \quad i' = \dfrac{11}{20}(v' - 2i') + \dfrac{v'}{10} - 3.5 \quad$ or $\quad i' = \dfrac{13}{42}v' - \dfrac{70}{42}$

$\therefore \quad v' = 70 - 4\left(\dfrac{13}{42}v' - \dfrac{70}{42}\right) \quad$ or $\quad v' = \dfrac{3220}{94} = \dfrac{1610}{47}$ V

50-V source acting alone:



$v'' = -4i''$

$$v'' = v'' + 2i''$$

$$v'' = -50 + 10i''$$

$$\therefore \quad i'' = \frac{v'' + 50}{10}$$

$$i'' = \frac{v''}{2} + \frac{v'' + 50}{10}$$

$$i'' = \frac{v''}{20} + i'' = \frac{v''}{20} + \frac{v''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v'' + \frac{v'' + 50}{10}$$

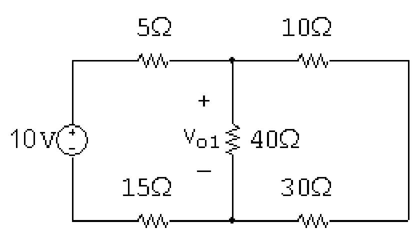
$$v'' = v'' - 2i''$$

$$\therefore \quad i'' = \frac{11}{20}(v'' - 2i'') + \frac{v'' + 50}{10} \quad \text{or} \quad i'' = \frac{13}{42}v'' + \frac{100}{42}$$

Thus, $v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right)$ or $v'' = -\frac{200}{47}$ V

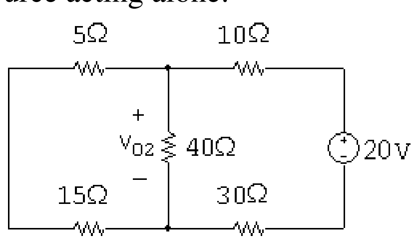
Hence, $v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30$ V

P 4.89 10 V source acting alone:



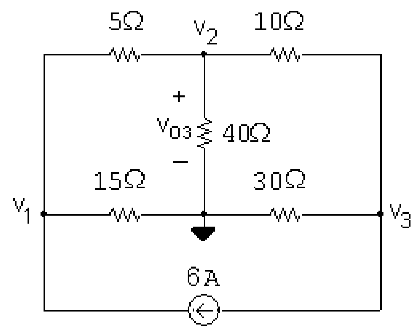
$$v_1 = \frac{20}{20 + 5 + 15}(10) = 5 \text{ V}$$

20 V source acting alone:



$$v_2 = \frac{13.333}{13.333 + 10 + 30}(20) = 5 \text{ V}$$

6 A current source acting alone:



Node voltage equations:

$$\frac{v_1}{15} + \frac{v_1 - v_2}{5} - 6 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3}{30} + 6 = 0$$

In standard form:

$$v_1\left(\frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) = 6$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) = 0$$

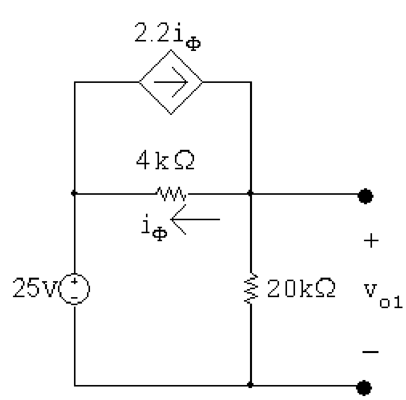
$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{30}\right) = -6$$

Solving, $v_1 = 22.5$ V; $v_2 = 0$ V; $v_3 = -45$ V

Note that $v_{\ 3} = v_2 = 0$ V

Finally, $v \ = v_{\ 1} + v_{\ 2} + v_{\ 3} = 5 + 5 + 0 = 10$ V

P 4.90 Voltage source acting alone:

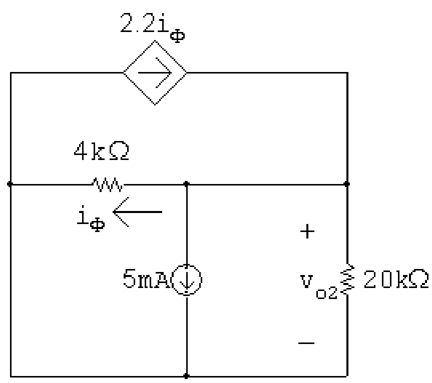


$$\frac{v_{\ 1} - 25}{4000} + \frac{v_{\ 1}}{20{,}000} - 2.2\left(\frac{v_{\ 1} - 25}{4000}\right) = 0$$

Simplifying $\quad 5v_1 - 125 + v_1 - 11v_1 + 275 = 0$

$\therefore \quad v_1 = 30$ V

Current source acting alone:



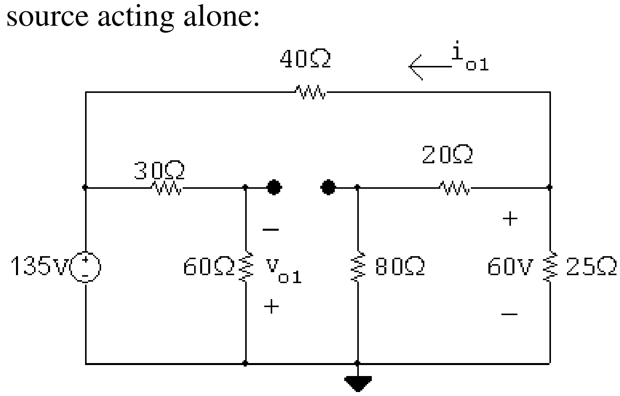
$$\frac{v_2}{4000} + \frac{v_2}{20{,}000} + 0.005 - 2.2\left(\frac{v_2}{4000}\right) = 0$$

Simplifying $\quad 5v_2 + v_2 + 100 - 11v_2 = 0$

$\therefore \quad v_2 = 20$ V

$v = v_1 + v_2 = 30 + 20 = 50$ V

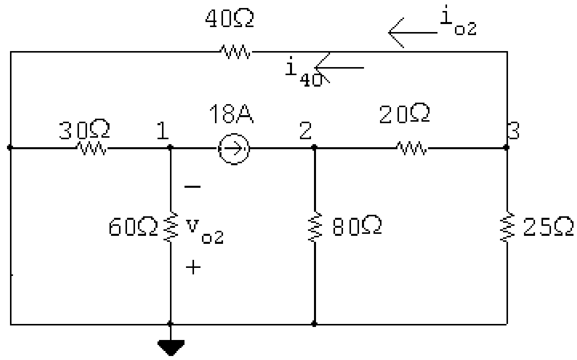
P 4.91 Voltage source acting alone:



$$i_1 = \frac{-135}{40 + 100\|25} = -2.25 \text{ A}$$

$$v_1 = \frac{60}{90}(-135) = -90 \text{ V}$$

Current source acting alone:



$$\frac{v_1}{30} + \frac{v_1}{60} + 18 = 0 \quad \therefore \quad v_1 = -360\text{ V}; \quad v_{\ 2} = 360\text{ V}$$

$$-18 + \frac{v_2}{80} + \frac{v_2 - v_3}{20} = 0$$

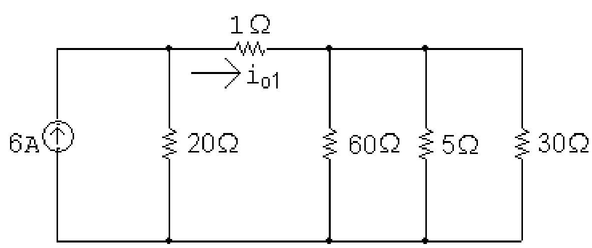
$$\frac{v_3 - v_2}{20} + \frac{v_3}{25} + \frac{v_3}{40} = 0$$

$$\therefore \quad v_2 = 441.6\text{ V}; \quad v_3 = 192\text{ V}; \quad i_{\ 2} = 192/40 = 4.8\text{ A}$$

$$\therefore \quad v \ = v_{\ 1} + v_{\ 2} = -90 + 360 = 270\text{ V}$$

$$i \ = i_{\ 1} + i_{\ 2} = -2.25 + 4.8 = 2.55\text{ A}$$

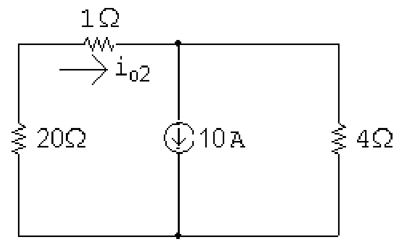
P 4.92 6 A source:



$$30\,\Omega \| 5\,\Omega \| 60\,\Omega = 4\,\Omega$$

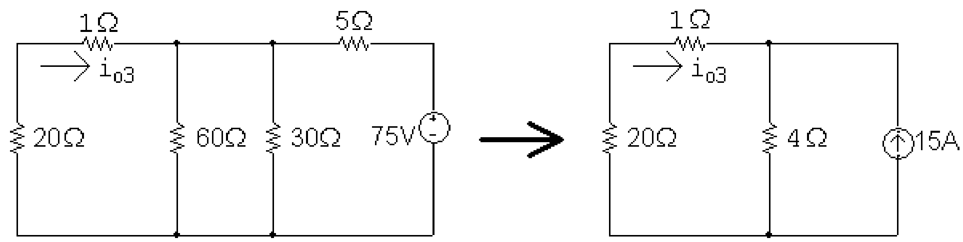
$$\therefore \quad i_{\ 1} = \frac{20}{20+5}(6) = 4.8\text{ A}$$

10 A source:



$$i_{2} = \frac{4}{25}(10) = 1.6 \text{ A}$$

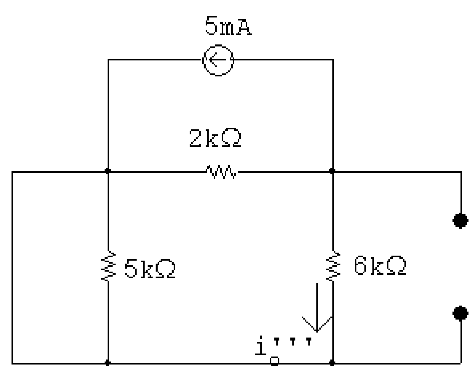
75 V source:



$$i_{3} = -\frac{4}{25}(15) = -2.4 \text{ A}$$

$$i = i_{1} + i_{2} + i_{3} = 4.8 + 1.6 - 2.4 = 4 \text{ A}$$

P 4.93 **[a]** By hypothesis $i' + i'' = 3.5$ mA.



$$i''' = \frac{2000}{8000}(-0.005) = -1.25 \text{ mA}; \qquad \therefore \quad i = 3.5 - 1.25 = 2.25 \text{ mA}$$

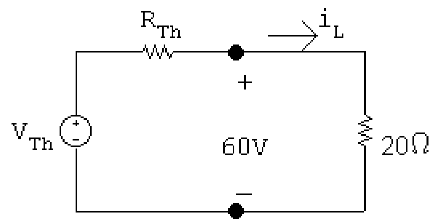
[b] With all three sources in the circuit write a single node voltage equation.

$$\frac{v - 8}{2000} + \frac{v}{6000} + 0.005 - 0.010 = 0$$

$$\therefore \quad v = 13.5 \text{ V}$$

$$i = \frac{v}{6000} = \frac{13.5}{6000} = 2.25 \text{ mA}$$

P 4.94 **[a]**



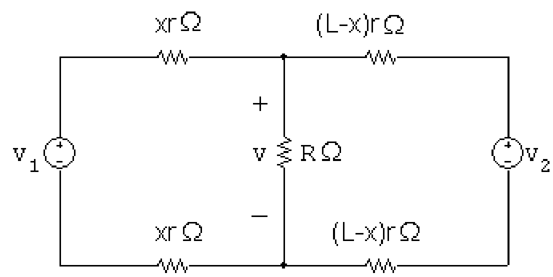
$$v_{\rm oc} = V_{\rm Th} = 75 \text{ V}; \qquad i = \frac{60}{20} = 3 \text{ A}; \qquad i = \frac{75-60}{R_{\rm Th}} = \frac{15}{R_{\rm Th}}$$

Therefore $R_{\rm Th} = \frac{15}{3} = 5\,\Omega$

[b] $i = \frac{v}{R} = \frac{V_{\rm Th} - v}{R_{\rm Th}}$

Therefore $R_{\rm Th} = \frac{V_{\rm Th} - v}{v\ /R} = \left(\frac{V_{\rm Th}}{v} - 1\right) R$

P 4.95 **[a]**



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L-x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L-x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L-x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$\frac{dv}{dx} = 0$ when numerator is zero.

The numerator simplifies to

$$x^2 + \frac{2L - v_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right\}$$

$v_2 = 1200$ V, $\quad v_1 = 1000$ V, $\quad L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1 v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$v_{\min} = \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}$$

$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$

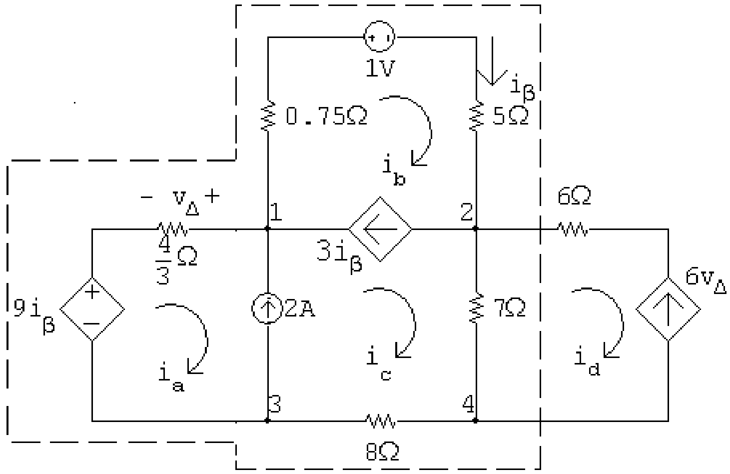
$$= 975 \text{ V}$$

P 4.96 **[a]** In studying the circuit in Fig. P4.96 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving v_Δ and i .

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c And i_d.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-9i\quad + \frac{4}{3}i_a + 0.75i_b + 1 + 5i_b + 7(i_c - i_d) + 8i_c = 0$$

Note that $i\quad = i_b$ And multiply the equation by 12:

$$-108i_b + 16i_a + 9i_b + 12 + 60i_b + 84(i_c - i_d) + 96i_c = 0$$

or

$$16i_a - 39i_b + 180i_c - 84i_d = -12$$

Now note:

$$i_b - i_c = 3i\quad = 3i_b; \qquad \therefore \quad i_c = -2i_b$$

whence

$$16i_a - 39i_b - 360i_b - 84i_d = -12$$

Now use the constraint that

$$i_a - i_c = -2$$

$$i_a = -2 + i_c = -2 - 2i_b$$

Therefore

$$-32 - 32i_b - 399i_b - 84i_d = -12$$

$$-431i_b - 84i_d = 20$$

Now use the constraint

$$i_d = -6v_\Delta = -6\left(\frac{-4}{3}i_a\right) = 8i_a = -16 - 16i_b$$

Therefore

$$-431i_\text{b} - 84(-16 - 16i_\text{b}) = 20$$

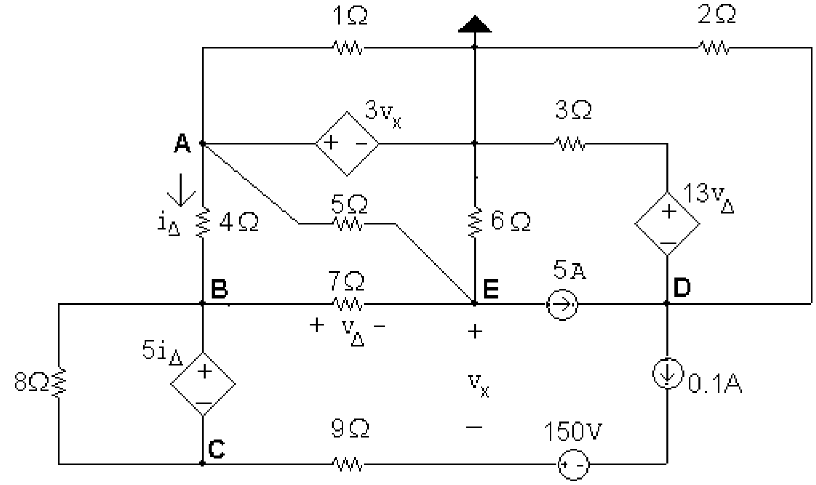
or

$$913i_\text{b} = -1324$$

$$\therefore \quad i_\text{b} \approx -1.45 \text{ A}$$

$$p_{1\text{V}} = 1i_\text{b} \cong -1.45 \text{ W}; \qquad \therefore \quad p_{1\text{V}} \text{ (developed) } \cong 1.45 \text{ W}$$

P 4.97



B–C supernode: $$\frac{v_\text{B} - 3v}{4} + \frac{v_\text{B} - v_\text{E}}{7} - 0.1 = 0$$

At node E: $$\frac{v_\text{E}}{6} + \frac{v_\text{E} - 3v}{5} + \frac{v_\text{E} - v_\text{B}}{7} + 5 = 0$$

At node D: $$\frac{v_\text{D} + 13v_\Delta}{3} - 5 + 0.1 + \frac{v_\text{D}}{2} = 0$$

Constraint: $v_\Delta = v_\text{B} - v_\text{E}$

Constraint: $v \ = -v_\Delta + 5i_\Delta - 0.9$

Constraint: $i_\Delta = (3v \ - v_\text{B})/4$

In standard form:

$$v_\text{B}\left(\frac{1}{4} + \frac{1}{7}\right) + v_\text{D}(0) + v_\text{E}\left(-\frac{1}{7}\right) + v_\Delta(0) + v \ \left(-\frac{3}{4}\right) + i_\Delta(0) = 0.1$$

$$v_\text{B}(0) + v_\text{D}\left(\frac{1}{2} + \frac{1}{3}\right) + v_\text{E}(0) + v_\Delta\left(\frac{13}{3}\right) + v \ (0) + i_\Delta(0) = 4.9$$

$$v_\text{B}\left(-\frac{1}{7}\right) + v_\text{D}(0) + v_\text{E}\left(\frac{1}{6} + \frac{1}{5} + \frac{1}{7}\right) + v_\Delta(0) + v \ \left(-\frac{3}{5}\right) + i_\Delta(0) = -5$$

$$v_\text{B}(-1) + v_\text{D}(0) + v_\text{E}(1) + v_\Delta(1) + v \ (0) + i_\Delta(0) = 0$$

$$v_\text{B}(0) + v_\text{D}(0) + v_\text{E}(0) + v_\Delta(1) + v \ (1) + i_\Delta(-5) = -0.9$$

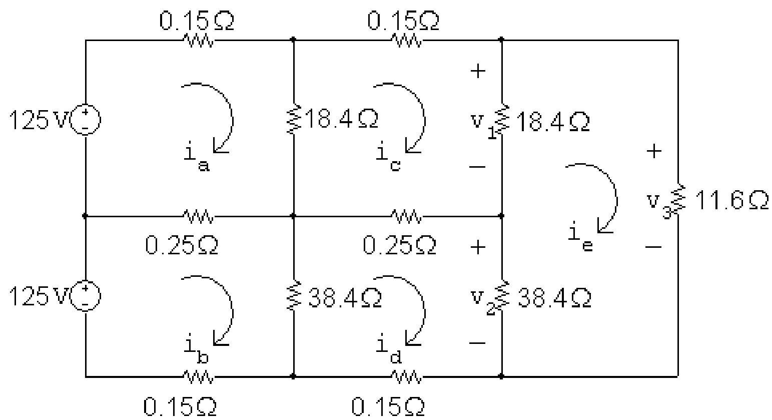
$$v_\text{B}(1) + v_\text{D}(0) + v_\text{E}(0) + v_\Delta(0) + v \ (-3) + i_\Delta(4) = 0$$

Solving, $v_B = -11.17$ V; $v_D = -20.95$ V; $v_E = -16.33$ V;
$v_\Delta = 5.16$ V; $v = -2.87$ V; $i_\Delta = 0.64$ A

$p_{5A} = (v_E - v_D)(5) = 23.1$ W

The 5 A source absorbs 23.1 W

P 4.98



The mesh equations are:

$$-125 + 0.15i_a + 18.4(i_a - i_c) + 0.25(i_a - i_b) = 0$$
$$-125 + 0.25(i_b - i_a) + 38.4(i_b - i_d) + 0.15i_b = 0$$
$$0.15i_c + 18.4(i_c - i_e) + 0.25(i_c - i_d) + 18.4(i_c - i_a) = 0$$
$$0.15i_d + 38.4(i_d - i_b) + 0.25(i_d - i_c) + 38.4(i_d - i_e) = 0$$
$$11.6i_e + 38.4(i_e - i_d) + 18.4(i_e - i_c) = 0$$

Place these equations in standard form:

$$i_a(18.8) + i_b(-0.25) + i_c(-18.4) + i_d(0) + i_e(0) = 125$$
$$i_a(-0.25) + i_b(38.8) + i_c(0) + i_d(-38.4) + i_e(0) = 125$$
$$i_a(-18.4) + i_b(0) + i_c(37.2) + i_d(-0.25) + i_e(-18.4) = 0$$
$$i_a(0) + i_b(-38.4) + i_c(-0.25) + i_d(77.2) + i_e(-38.4) = 0$$
$$i_a(0) + i_b(0) + i_c(-18.4) + i_d(-38.4) + i_e(68.4) = 0$$

Solving,

$i_a = 32.77$ A; $i_b = 26.46$ A; $i_c = 26.33$ A; $i_d = 23.27$ A; $i_e = 20.14$ A

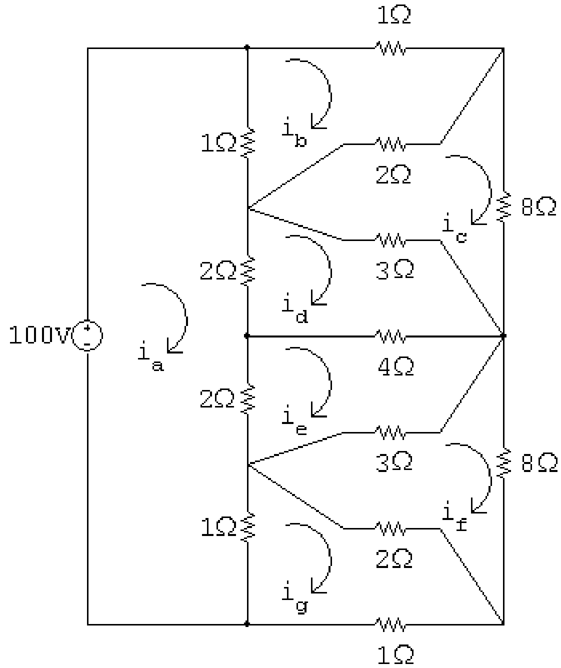
Find the requested voltages:

$v_1 = 18.4(i_c - i_e) = 113.90$ V

$v_2 = 38.4(i_d - i_e) = 120.19$ V

$v_3 = 11.6i_e = 233.62$ V

P 4.99



$$100 = 6i \;\; - 1i \;\; + 0i \;\; - 2i \;\; - 2i \;\; + 0i \;\; - 1i$$
$$0 = -1i \;\; + 4i \;\; - 2i \;\; + 0i \;\; + 0i \;\; + 0i \;\; + 0i$$
$$0 = 0i \;\; - 2i \;\; + 13i \;\; - 3i \;\; + 0i \;\; + 0i \;\; + 0i$$
$$0 = -2i \;\; + 0i \;\; - 3i \;\; + 9i \;\; - 4i \;\; + 0i \;\; + 0i$$
$$0 = -2i \;\; + 0i \;\; + 0i \;\; - 4i \;\; + 9i \;\; - 3i \;\; + 0i$$
$$0 = 0i \;\; + 0i \;\; + 0i \;\; + 0i \;\; - 3i \;\; + 13i \;\; - 2i$$
$$0 = -1i \;\; + 0i \;\; + 0i \;\; + 0i \;\; + 0i \;\; - 2i \;\; + 4i$$

A computer solution yields

$i \;\; = 30$ A; $\qquad i \;\; = 15$ A;

$i \;\; = 10$ A; $\qquad i \;\; = 5$ A;

$i \;\; = 5$ A; $\qquad i \;\; = 10$ A;

$i \;\; = 15$ A

$\therefore \quad i = i \;\; - i \;\; = 0$ A

CHECK: $\qquad p_{1T} = p_{1B} = (i\;\;)^2 = (i\;\;)^2 = 100$ W

$$p_{1L} = (i \;\; - i\;\;)^2 = (i \;\; - i\;\;)^2 = 400 \text{ W}$$
$$p_{2C} = 2(i \;\; - i\;\;)^2 = (i \;\; - i\;\;)^2 = 50 \text{ W}$$
$$p_3 = 3(i \;\; - i\;\;)^2 = 3(i \;\; - i\;\;)^2 = 300 \text{ W}$$
$$p_4 = 4(i \;\; - i\;\;)^2 = 0 \text{ W}$$
$$p_8 = 8(i\;\;)^2 = 8(i\;\;)^2 = 200 \text{ W}$$
$$p_{2L} = 2(i \;\; - i\;\;)^2 = 2(i \;\; - i\;\;)^2 = 450 \text{ W}$$

$$\sum p_{\text{abs}} = 100 + 400 + 50 + 200 + 300 + 450 + 0 + 450 + 300 + 200 + 50 + 400 + 100 = 3000 \text{ W}$$

$$\sum p_{\text{gen}} = 100i = 100(30) = 3000 \text{ W (CHECKS)}$$

P 4.100 $$\frac{dv_1}{dI_1} = \frac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dI_2} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_1} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_2} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 4.101 From the solution to Problem 4.100 we have

$$\frac{dv_1}{dI_1} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A}$$

and

$$\frac{dv_2}{dI_1} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_1 = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.5833 \text{ V}$$

Thus, $v_1 = 25 + 14.5833 = 39.5833$ V
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$$v_1 = 39.5830 \text{ V}$$

and

$$v_2 = 102.5000 \text{ V}$$

These values are in agreement with our predicted values.

P 4.102 From the solution to Problem 4.100 we have

$$\frac{dv_1}{dI_2} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_2} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_2 = 17 - 16 = 1$ A

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5$ V
Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 4.103 From the solutions to Problems 4.100 — 4.102 we have

$$\frac{dv_1}{dI_1} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_2} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_1} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_2} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_1 = 11 - 12 = -1 \text{ A}$$

$$\Delta I_2 = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$\Delta v_2 = 12.5 + 15 = 27.5$ V

Hence

$v_1 = 25 + 27.0833 = 52.0833$ V

$v_2 = 90 + 27.5 = 117.5$ V

The PSpice solution is

$v_1 = 52.0830$ V

and

$v_2 = 117.5$ V

These values are in agreement with our predicted values.

P 4.104 By hypothesis,

$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$

$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$

$\Delta R_3 = 55 - 50 = 5\,\Omega$

$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168$ V

$\therefore\quad v_1 = 25 + 4.9168 = 29.9168$ V

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1$ V

$\therefore\quad v_2 = 90 - 1.1 = 88.9$ V

The PSpice solution is

$v_1 = 29.6710$ V

and

$v_2 = 88.5260$ V

Note our predicted values are within a fraction of a volt of the actual values.

5

The Operational Amplifier

Assessment Problems

AP 5.1 **[a]** This is an inverting amplifier, so

$$v\ = (-R\ /R\ )v\ = (-80/16)v\ , \qquad \text{so} \qquad v\ = -5v$$

v (V)	0.4	2.0	3.5	-0.6	-1.6	-2.4
v (V)	-2.0	-10.0	-15.0	3.0	8.0	10.0

Two of the v values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v\ , \quad v\ = 3\ \text{V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v\ , \qquad v\ = -2\ \text{V}$$

Therefore $\quad -2 \leq v\ \leq 3\ \text{V}$

AP 5.2 From Assessment Problem 5.1

$$v\ = (-R\ /R\ )v\ = (-R\ /16{,}000)v$$
$$= (-R\ /16{,}000)(-0.640) = 0.64R\ /16{,}000 = 4\times10^{-5}R$$

Use the negative power supply value to determine one limit on the value of R :

$$4\times10^{-5}R\ = -15 \qquad \text{so} \qquad R\ = -15/4\times10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R is 0. Now use the positive power supply value to determine the upper limit on the value of R :

$$4\times10^{-5}R\ = 10 \qquad \text{so} \qquad R\ = 10/4\times10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \leq R\ \leq 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v \;\; = (-R \;\; /R_{\rm a})v_{\rm a} + (-R \;\; /R_{\rm b})v_{\rm b} = -(250/5)v_{\rm a} - (250/25)v_{\rm b} = -50v_{\rm a} - 10v_{\rm b}$$

Substituting the values for $v_{\rm a}$ and $v_{\rm b}$:

$$v \;\; = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for $v_{\rm b}$ into the equation for v from part (a) and use the negative power supply value:

$$v \;\; = -50v_{\rm a} - 10(0.25) = -50v_{\rm a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_{\rm a} = 7.5, \quad \text{so} \quad v_{\rm a} = 0.15 \text{ V}$$

[c] Substitute the value for $v_{\rm a}$ into the equation for v from part (a) and use the negative power supply value:

$$v \;\; = -50(0.10) - 10v_{\rm b} = -5 - 10v_{\rm b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_{\rm b} = 5, \quad \text{so} \quad v_{\rm b} = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the $v_{\rm b}$ term in each equation from negative to positive.

Repeat part (a):

$$v \;\; = -50v_{\rm a} + 10v_{\rm b} = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v \;\; = -50v_{\rm a} + 2.5 = -10 \text{ V}; \qquad 50v_{\rm a} = 12.5, \quad v_{\rm a} = 0.25 \text{ V}$$

Repeat part (c):

$$v \;\; = -5 + 10v_{\rm b} = 15 \text{ V}; \qquad 10v_{\rm b} = 20; \qquad v_{\rm b} = 2.0 \text{ V}$$

AP 5.4 **[a]** Write a node voltage equation at v ; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v}{4500} + \frac{v \;\; - v}{63{,}000} = 0$$

Solve for v in terms of v by multiplying both sides by 63,000 and collecting terms:

$$14v \;\; + v \;\; - v \;\; = 0 \qquad \text{so} \qquad v \;\; = 15v$$

Now use voltage division to calculate v . We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R resistor:

$$v \;\; = \frac{R}{15{,}000 + R}(0.400)$$

Now substitute the value $R \; = 60$ kΩ:

$$v \; = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v \; = v$, so substitute the value of v into the equation for v_0

$$v \; = 15v \; = 15v \; = 15(0.32) = 4.8 \text{ V}$$

[b] Substitute the expression for v into the equation for v and set the resulting equation equal to the positive power supply value:

$$v \; = 15\left(\frac{0.4R}{15{,}000 + R}\right) = 5$$

$$15(0.4R \;) = 5(15{,}000 + R \;) \quad \text{so} \quad R \; = 75\,\text{k}\Omega$$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v \; = \frac{20(60)}{10(24)}v_{\text{b}} - \frac{50}{10}v_{\text{a}}$$

Simplify this expression and substitute in the value for v_{b}:

$$v \; = 5(v_{\text{b}} - v_{\text{a}}) = 20 - 5v_{\text{a}}$$

Set this expression for v to the positive power supply value:

$$20 - 5v_{\text{a}} = 10 \text{ V} \quad \text{so} \quad v_{\text{a}} = 2 \text{ V}$$

Now set the expression for v to the negative power supply value:

$$20 - 5v_{\text{a}} = -10 \text{ V} \quad \text{so} \quad v_{\text{a}} = 6 \text{ V}$$

Therefore $\quad 2 \leq v_{\text{a}} \leq 6 \text{ V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v \; = \frac{8(60)}{10(12)}v_{\text{b}} - 5v_{\text{a}} = 4v_{\text{b}} - 5v_{\text{a}}$$

Now substitute the value for v_{b}:

$$v \; = 4(4) - 5v_{\text{a}} = 16 - 5v_{\text{a}}$$

Set this expression for v to the positive power supply value:

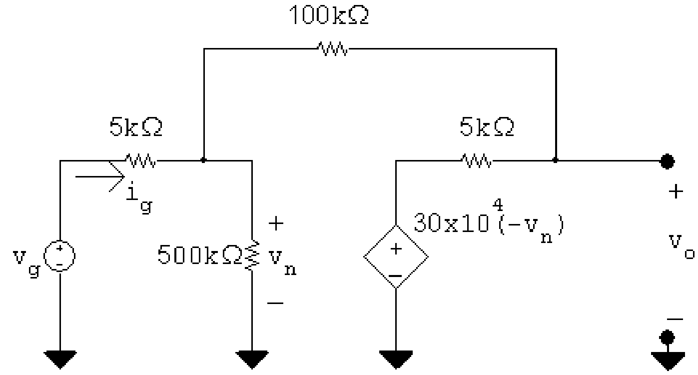
$$16 - 5v_{\text{a}} = 10 \text{ V} \quad \text{so} \quad v_{\text{a}} = 1.2 \text{ V}$$

Now set the expression for v to the negative power supply value:

$$16 - 5v_{\text{a}} = -10 \text{ V} \quad \text{so} \quad v_{\text{a}} = 5.2 \text{ V}$$

Therefore $\quad 1.2 \leq v_{\text{a}} \leq 5.2 \text{ V}$

AP 5.6 **[a]** Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v}{500{,}000} + \frac{v \;\; - v}{5000} + \frac{v \;\; - v}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v \;\; + 100v \;\; - 100v \;\; + 5v \;\; - 5v_0 = 0 \quad \text{so} \quad 21.2v \;\; - v \;\; = 20v$$

Write the node voltage equation at the right hand node:

$$\frac{v \;\; - 300{,}000(-v \;\;)}{5000} + \frac{v \;\; - v}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v \;\; + 6 \times 10^6 v \;\; + v \;\; - v \;\; = 0 \quad \text{so} \quad 6 \times 10^6 v \;\; + 21v \;\; = 0$$

Use Cramer's method to solve for v :

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N \;\; = \begin{vmatrix} 21.2 & 20v \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v$$

$$v \;\; = \frac{N}{\Delta} = -19.9985v \;\; ; \qquad \text{so} \; \frac{v}{v} = -19.9985$$

[b] Use Cramer's method again to solve for v :

$$N_1 = \begin{vmatrix} 20v & -1 \\ 0 & 21 \end{vmatrix} = 420v$$

$$v \;\; = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v$$

$$v \;\; = 1 \text{ V}, \qquad v \;\; = 69.995\,\mu\text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i \ = \frac{v \ - v}{5000} = \frac{v \ - 6.9995 \times 10^{-5} v}{5000}$$

Solve for the ratio of v to i to get the input resistance:

$$R \ = \frac{v}{i} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

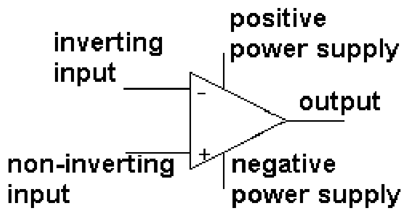
$$\frac{v}{v} = -\frac{100{,}000}{5000} = -20$$

Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v \ = 0$, $v \ = 0$
Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R \ = 5000\,\Omega$$

Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i \ = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v \ - v \ ) = 0$.

[d] Write a node voltage equation at v :

$$\frac{v \ - 2.5}{10{,}000} + \frac{v \ - v}{40{,}000} = 0$$

But $v \ = 0$ and $v \ = v \ = 0$. Thus,

$$\frac{-2.5}{10{,}000} - \frac{v}{40{,}000} = 0 \quad \text{so} \quad v \ = -10 \text{ V}$$

P 5.2 $$\frac{v_b - v_a}{20} + \frac{v_b - v}{100} = 0, \qquad \text{therefore} \quad v \ = 6v_b - 5v_a$$

[a] $v_a = 4$ V, $v_b = 0$ V, $v \ = -15$ V (sat)

[b] $v_a = 2$ V, $v_b = 0$ V, $v \ = -10$ V

[c] $v_a = 2$ V, $v_b = 1$ V, $v \ = -4$ V

[d] $v_a = 1$ V, $v_b = 2$ V, $v \ = 7$ V

[e] If $v_b = 1.6$ V, $v \ = 9.6 - 5v_a = \pm 15$

$$\therefore \quad -1.08 \le v_a \le 4.92 \text{ V}$$

P 5.3 $$v \ = -(0.5 \times 10^{-3})(10 \times 10^3) = -5 \text{ V}$$

$$\therefore \quad i \ = \frac{-5}{5000} = -1 \text{ mA}$$

P 5.4 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\,\text{M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.5 **[a]** $i_a = \dfrac{25 \times 10^{-3}}{5000} = 5\,\mu\text{A}$

$v_a = -50 \times 10^3 i_a = -250\,\text{mV}$

[b] $\dfrac{v_a}{50{,}000} + \dfrac{v_a}{10{,}000} + \dfrac{v_a - v}{40{,}000} = 0$

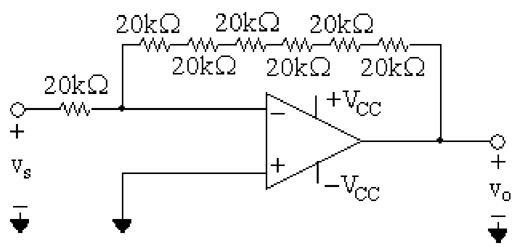
$\therefore\ \ 4v_a + 20v_a + 5v_a - 5v\ = 0$

$\therefore\ \ v\ = 29v_a/5 = -1.45\ \text{V}$

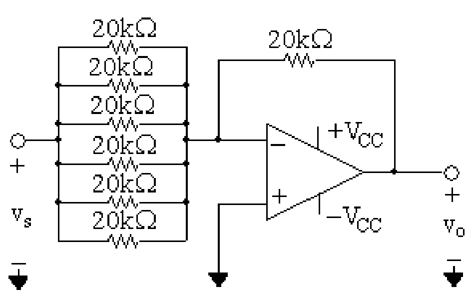
[c] $i_a = 5\,\mu\text{A}$

[d] $i\ = \dfrac{-v}{30{,}000} + \dfrac{v\ -v}{40{,}000} = 78.33\,\mu\ \text{A}$

P 5.6 **[a]** The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 6, the feedback resistor must be 6 times as large as the input resistor. There are many possible designs that use only 20 kΩ resistors. We present two here. Use a single 20 kΩ resistor as the input resistor, and use six 20 kΩ resistors in series as the feedback resistor to give a total of 120 kΩ.



Alternately, Use a single 20 kΩ resistor as the feedback resistor and use six 20 kΩ resistors in parallel as the input resistor to give a total of 3.33 kΩ.



[b] To amplify a 3 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(3)(6) = 18$ V.

P 5.7 **[a]** The circuit shown is a non-inverting amplifier.

[b] We assume the op amp to be ideal, so $v \;\; = v \;\; = 3$V. Write a KCL equation at v :

$$\frac{3}{40{,}000} + \frac{3 - v}{80{,}000} = 0$$

Solving,

$v \;\; = 9$ V.

P 5.8 $v \;\; = \frac{18}{24}(12) = 9 \;\; \text{V} = v$

$$\frac{v \;\; - 24}{30} + \frac{v \;\; - v}{20} = 0$$

$$v \;\; = (45 - 48)/3 = -1.0 \;\; \text{V}$$

$$i_{\text{L}} = \frac{v}{5} \times 10^{-3} = -\frac{1}{5} \times 10^{-3} = -200 \times 10^{-6}$$

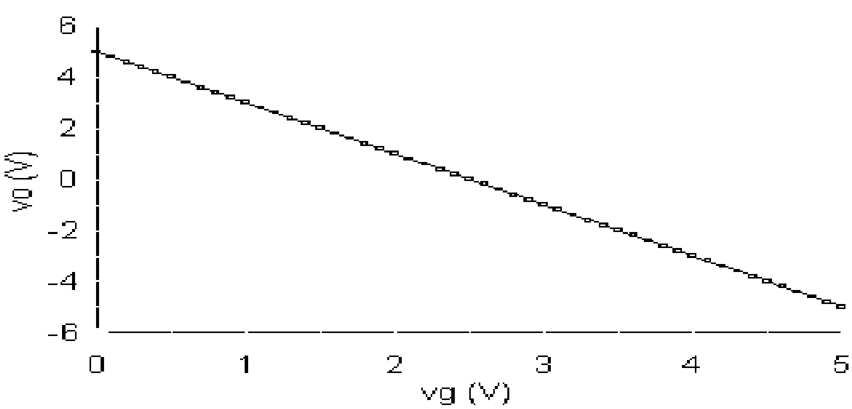
$$i_{\text{L}} = -200\,\mu\text{A}$$

P 5.9 **[a]** First, note that $v \;\; = v \;\; = 2.5$ V

Let $v_{\;1}$ equal the voltage output of the op-amp. Then

$$\frac{2.5 - v}{5000} + \frac{2.5 - v_{\;1}}{10{,}000} = 0, \qquad \therefore \;\; v_{\;1} = 7.5 - 2v$$

Also note that $v_{\;1} - 2.5 = v$, $\qquad \therefore \;\; v \;\; = 5 - 2v$



[b] Yes, the circuit designer is correct!

P 5.10 **[a]** Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0-v}{2000}+\frac{0-v_\Delta}{50{,}000}=0$$

$$-25v - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -25(40\times 10^{-3}) = -1 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta}+\frac{v_\Delta - 0}{50{,}000}+\frac{v_\Delta - v}{(1-\alpha)R_\Delta}=0$$

$$\frac{v_\Delta}{\alpha}+2v_\Delta+\frac{v_\Delta - v}{1-\alpha}=0$$

$$v_\Delta\left(\frac{1}{\alpha}+2+\frac{1}{1-\alpha}\right)=\frac{v}{1-\alpha}$$

$$\therefore \quad v = -1\left[1+2(1-\alpha)+\frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v = -1(1+1.6+4) = -6.6$ V

When $\alpha = 1, \qquad v = -1(1+0+0) = -1$ V

$\therefore \quad -6.6 \text{ V} \le v \le -1 \text{ V}$

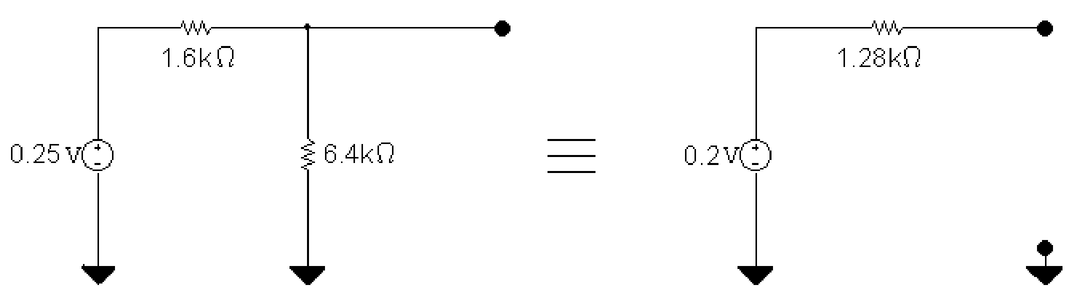
[b] $$-1\left[1+2(1-\alpha)+\frac{(1-\alpha)}{\alpha}\right]=-7$$

$$\alpha+2\alpha(1-\alpha)+(1-\alpha)=7\alpha$$

$$\alpha+2\alpha-2\alpha^2+1-\alpha=7\alpha$$

$\therefore \quad 2\alpha^2+5\alpha-1=0 \quad$ so $\quad \alpha \cong 0.186$

P 5.11 **[a]** Replace the combination of v, $1.6\,\text{k}\Omega$, and the $6.4\,\text{k}\Omega$ resistors with its Thévenin equivalent.



Then $$v = \frac{-[12+\sigma 50]}{1.28}(0.2)$$

At saturation $v = -5$ V; therefore

$$-\left(\frac{12+\sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

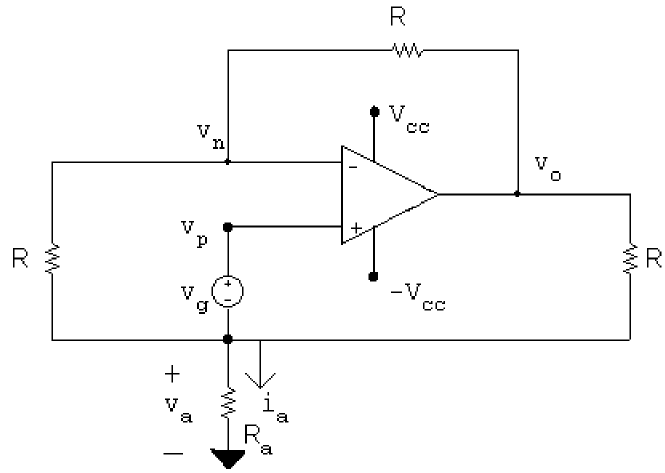
Thus for $0 \le \sigma < 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272, \qquad v \ = \frac{-(12+13.6)}{1.28}(0.2) = -4\ \text{V}$

Also $\frac{v}{10} + \frac{v}{25.6} + i \ = 0$

$\therefore \ i \ = -\frac{v}{10} - \frac{v}{25.6} = \frac{4}{10} + \frac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$

P 5.12 **[a]**



$$\frac{v \ - v_a}{R} + \frac{v \ - v}{R} = 0$$

$$2v \ - v_a = v$$

$$\frac{v_a}{R_a} + \frac{v_a - v}{R} + \frac{v_a - v}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v}{R} = \frac{v}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v \ = v$$

$$v \ = v \ = v_a + v$$

$$\therefore \ 2v \ - v_a = 2v_a + 2v \ - v_a = v_a + 2v$$

$$\therefore \ v_a - v \ = -2v \qquad (1)$$

$$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v \ = v$$

$$\therefore \ v_a\left(1 + \frac{R}{R_a}\right) - v \ = v \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_a\frac{R}{R_a} = -3v$$

or $v_a = 3v\,\dfrac{R_a}{R}$

Hence $i_a = \dfrac{v_a}{R_a} = \dfrac{3v}{R}$ Q.E.D.

[b] At saturation $V\ \ = \pm\,V_{cc}$

$\therefore\ \ v_a = \pm\,V_{cc} - 2v \qquad (3)$

and

$\therefore\ \ v_a\left(1 + \dfrac{R}{R_a}\right) = \pm\,V_{cc} + v \qquad (4)$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm\,V_{cc} + v}{\pm\,V_{cc} - 2v}$$

$$\therefore\ \ \frac{R}{R_a} = \frac{\pm\,V_{cc} + v}{\pm\,V_{cc} - 2v} - 1 = \frac{3v}{\pm\,V_{cc} - 2v}$$

or $R_a = \dfrac{(\pm\,V_{cc} - 2v\ )}{3v}R$ Q.E.D.

P 5.13 **[a]** Assume the op-amp is operating within its linear range, then

$i\ \ = \dfrac{8}{4000} = 2\,\text{mA}$

For $R\ \ = 4\,\text{k}\Omega \qquad v\ \ = (4+4)(2) = 16\ \ \text{V}$

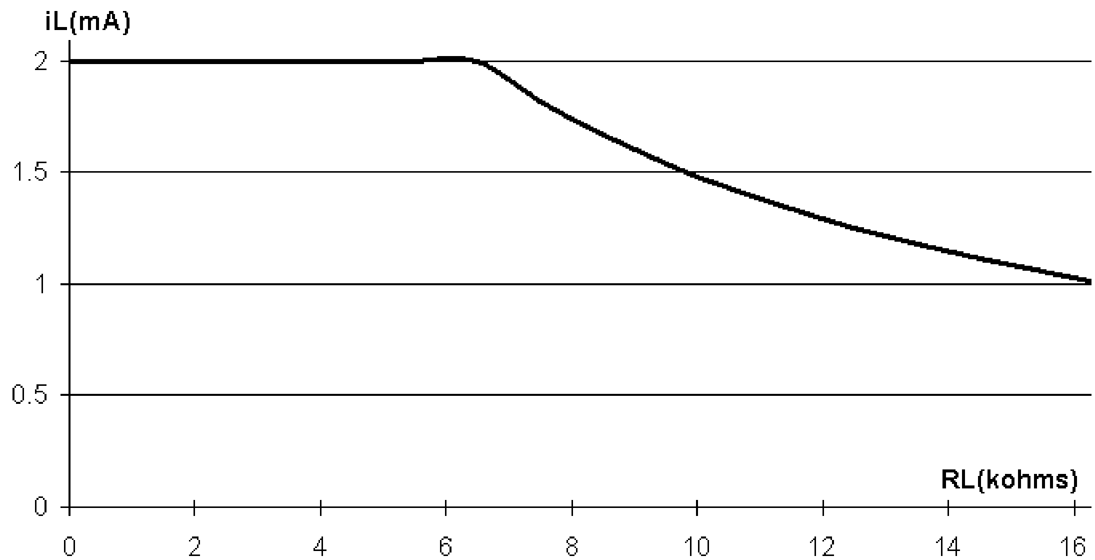
Now since $v\ \ < 20$ V our assumption of linear operation is correct, therefore

$i\ \ = 2\,\text{mA}$

[b] $20 = 2(4 + R\ \ ); \qquad R\ \ = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i is independent of R . From (b) we found the op-amp is operating in its linear region as long as $R\ \ \leq 6\,\text{k}\Omega$. Therefore when $R\ \ = 16\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i by assuming $i\ \ = i\ \ \ll i$. Then $i\ \ = 20/(4{,}0000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, and the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i\ \ = i\ \ = (8-4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.14 **[a]** Let v_1 = output voltage of the amplifier on the left. Let v_2 = output voltage of the amplifier on the right. Then

$$v_1 = \frac{-47}{10}(1) = -4.7\ \text{V}; \qquad v_2 = \frac{-220}{33}(-0.15) = 1.0\ \text{V}$$

$$i_a = \frac{v_2 - v_1}{1000} = 5.7\,\text{mA}$$

[b] $i_a = 0$ when $v_1 = v_{02}$ so from (a) $v_2 = 1\ \text{V}$

Thus

$$\frac{-47}{10}(v_L) = 1$$

$$v_L = -\frac{10}{47} = -212.77\ \text{mV}$$

P 5.15 **[a]** $p_{600\Omega} = \dfrac{(60 \times 10^{-3})^2}{(600)} = 6\,\mu\text{W}$

[b] $v_{600\Omega} = \dfrac{600}{30{,}000}(60 \times 10^{-3}) = 1.2\,\text{mV}$

$$p_{600\Omega} = \frac{(1.2 \times 10^{-3})^2}{(600)} = 2.4\ \text{nW}$$

[c] $\dfrac{p_a}{p_b} = \dfrac{6 \times 10^{-6}}{2.4 \times 10^{-9}} = 2500$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control $2,500$ times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.16 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v = -\frac{220}{33}v_a - \frac{220}{22}v_b - \frac{220}{80}v_c = -8 + 15 - 11 = -4\ \text{V}$

[c] $v = -19 - 10v_b = \pm 6$

$$\therefore \quad v_b = -1.3\ \text{V} \quad \text{when} \quad v = -6\ \text{V};$$

$$v_b = -2.5\ \text{V} \quad \text{when} \quad v = 6\ \text{V}$$

$$\therefore \quad -2.5\ \text{V} \leq v_b \leq -1.3\ \text{V}$$

P 5.17 **[a]** Write a KCL equation at the inverting input to the op amp:

$$\frac{v_d - v_a}{40{,}000} + \frac{v_d - v_b}{22{,}000} + \frac{v_d - v_c}{100{,}000} + \frac{v_d}{352{,}000} + \frac{v_d - v}{220{,}000} = 0$$

Multiply through by 220,000, plug in the values of input voltages, and rearrange to solve for v :

$$v = 220{,}000\left(\frac{4}{40{,}000} + \frac{-1}{22{,}000} + \frac{-5}{100{,}000} + \frac{8}{352{,}000} + \frac{8}{220{,}000}\right) = 14\ \text{V}$$

[b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{8 - v_a}{40{,}000} + \frac{8 - 9}{22{,}000} + \frac{8 - 13}{100{,}000} + \frac{8}{352{,}000} + \frac{8 - v}{220{,}000} = 0$$

Simplify and solve for v :

$$44 - 5.5v_a - 10 - 11 + 5 + 8 - v = 0 \quad \text{so} \quad v = 36 - 5.5v_a$$

Set v to the positive power supply voltage and solve for v_a:

$$36 - 5.5v_a = 15 \quad \therefore \quad v_a = 3.818\ \text{V}$$

Set v to the negative power supply voltage and solve for v_a:

$$36 - 5.5v_a = -15 \quad \therefore \quad v_a = 9.273\ \text{V}$$

Therefore,

$3.818\ \text{V} \le v_\text{a} \le 9.273\ \text{V}$

P 5.18 **[a]** $\dfrac{8-4}{40{,}000} + \dfrac{8-9}{22{,}000} + \dfrac{8-13}{100{,}000} + \dfrac{8}{352{,}000} + \dfrac{8-v_0}{R} = 0$

$$\frac{8-v}{R} = -2.7272 \times 10^{-5} \quad \text{so} \quad R = \frac{8-v}{-2.727 \times 10^{-5}}$$

For $v = 15\,\text{V}$, $R = 256.7\,\text{k}\Omega$

For $v = -15\,\text{V}$, $R < 0$ so this solution is not possible.

[b] $i = -(i + i_{10\text{k}}) = -\left[\dfrac{15-8}{256.7 \times 10^3} + \dfrac{15}{10{,}000}\right] = -1.527\,\text{mA}$

P 5.19 We want the following expression for the output voltage:

$$v = -(2v + 4v + 6v + 8v)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

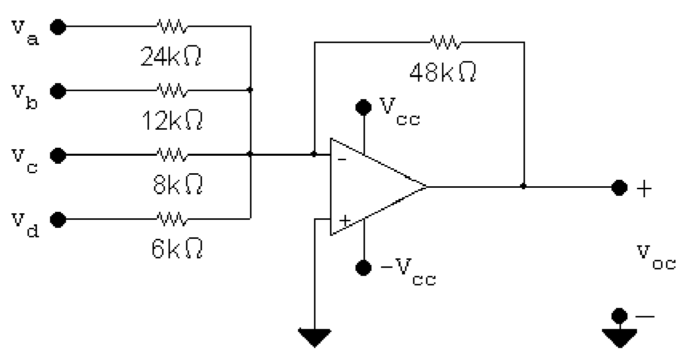
$$v = -\left[\frac{48}{R_\text{a}}v_\text{a} + \frac{48}{R_\text{b}}v_\text{b} + \frac{48}{R_\text{c}}v_\text{c} + \frac{48}{R_\text{d}}v_\text{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$\therefore \quad R_\text{a} = 48{,}000/2 = 24\,\text{k}\Omega \qquad R_\text{c} = 48{,}000/6 = 8\,\text{k}\Omega$

$R_\text{b} = 48{,}000/4 = 12\,\text{k}\Omega \qquad R_\text{d} = 48{,}000/8 = 6\,\text{k}\Omega$

The final circuit is shown here:



P 5.20 **[a]** $v = v, \qquad v = \dfrac{R_1 v}{R_1 + R_2}, \qquad v = v$

Therefore $\quad v = \left(\dfrac{R_1 + R_2}{R_1}\right) v = \left(1 + \dfrac{R_2}{R_1}\right) v$

[b] $v \ = v$

[c] Because $v \ = v$, thus the output voltage follows the signal voltage.

P 5.21 $$v \ = -\left[\frac{R_\text{f}}{3000}(0.15) + \frac{R_\text{f}}{5000}(0.1) + \frac{R_\text{f}}{25{,}000}(0.25)\right]$$

$$-6 = -8 \times 10^{-5} R_\text{f}; \qquad R_\text{f} = 75\,\text{k}\Omega; \qquad \therefore \quad 0 \le R_\text{f} \le 75\,\text{k}\Omega$$

P 5.22 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v :

$$v \ = \frac{10{,}000}{10{,}000 + 30{,}000} v \ = \frac{v}{4}$$

Write a KCL equation at $v \ = v \ = v \ /4$:

$$\frac{v \ /4}{4000} + \frac{v \ /4 - v}{28{,}000} = 0$$

Solving,

$$v \ = 7v \ /4 + v \ /4 = 2v$$

[c] $2v \ = 8$ so $v \ = 4$ V

$2v \ = -12$ so $v \ = -6$ V

Thus, -6 V $\le v \ \le 4$ V.

P 5.23 **[a]** $$v \ = v \ = \frac{68{,}000}{80{,}000} v \ = 0.85v$$

$$\therefore \quad \frac{0.85v}{30{,}000} + \frac{0.85v \ - v}{63{,}000} = 0;$$

$$\therefore \quad v \ = 2.635v \ = 2.635(4), \quad v \ = 10.54 \text{ V}$$

[b] $v \ = 2.635v \ = \pm 12$

$v \ = \pm 4.554$ V, $\qquad -4.554 \le v \ \le 4.554$ V

[c] $$\frac{0.85v}{30{,}000} + \frac{0.85v \ - v}{R_\text{f}} = 0$$

$$\left(\frac{0.85R_\text{f}}{30{,}000} + 0.85\right) v \ = v \ = \pm 12$$

$$\therefore \quad 1.7{\times}10^{-3} R_\text{f} + 51 = \pm 360; \quad 1.7{\times}10^{-3} R_\text{f} = 360 - 51; \quad R_\text{f} = 181.76\,\text{k}\Omega$$

P 5.24 **[a]** This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v and solve for v in terms of v:

$$\frac{v - v}{15{,}000} + \frac{v - 6}{30{,}000} = 0$$

$$2v - 2v + v - 6 = 0 \qquad \text{so} \qquad v = 2v/3 + 2$$

Now write a KCL equation at v and solve for v:

$$\frac{v}{20{,}000} + \frac{v - v}{60{,}000} = 0 \qquad \text{so} \qquad v = 4v$$

Since we assume the op amp is ideal, $v = v$. Thus,

$$v = 4(2v/3 + 2) = 8v/3 + 8$$

[c] $8v/3 + 8 = 16 \qquad \text{so} \qquad v = 3\ \text{V}$

$$8v/3 + 8 = -12 \qquad \text{so} \qquad v = -7.5\ \text{V}$$

Thus, $-7.5\ \text{V} \leq v \leq 3\ \text{V}$.

P 5.25 **[a]** The circuit is a non-inverting summing amplifier.

[b] $\dfrac{v - v_a}{3.3 \times 10^3} + \dfrac{v - v_b}{4.7 \times 10^3} = 0$

$$\therefore \quad v = 0.5875v_a + 0.4125v_b$$

$$\frac{v}{10{,}000} + \frac{v - v}{100{,}000} = 0$$

$$\therefore \quad v = 11v = 11v = 6.4625v_a + 4.5375v_b = 8.03\ \text{V}$$

[c] $v = v = \dfrac{v}{11} = 730\ \text{mV}$

$$i_a = \frac{v_a - v}{3.3 \times 10^3} = -100\,\mu\text{A}$$

$$i_b = \frac{v_b - v}{4.7 \times 10^3} = 100\,\mu\text{A}$$

[d] 6.4625 for v_a

4.5375 for v_b

P 5.26 **[a]** $\dfrac{v - v_a}{R_a} + \dfrac{v - v_b}{R_b} + \dfrac{v - v_c}{R_c} + \dfrac{v}{R} = 0$

$$\therefore \quad v = \frac{R_b R_c R}{D} v_a + \frac{R_a R_c R}{D} v_b + \frac{R_a R_b R}{D} v_c$$

where $D = R_b R_c R + R_a R_c R + R_a R_b R + R_a R_b R_c$

$$\frac{v}{R} + \frac{v \quad - v}{R_{\mathrm{f}}} = 0$$

$$v \quad \left(\frac{1}{R} + \frac{1}{R_{\mathrm{f}}}\right) = \frac{v}{R_{\mathrm{f}}}$$

$$\therefore \quad v \quad = \left(1 + \frac{R_{\mathrm{f}}}{R}\right) v \quad = kv$$

where $k = \left(1 + \frac{R_{\mathrm{f}}}{R}\right)$

$$v \quad = v$$

$$\therefore \quad v \quad = kv$$

or

$$v \quad = \frac{kR \;\, R_{\mathrm{b}}R_{\mathrm{c}}}{D}v_{\mathrm{a}} + \frac{kR \;\, R_{\mathrm{a}}R_{\mathrm{c}}}{D}v_{\mathrm{b}} + \frac{kR \;\, R_{\mathrm{a}}R_{\mathrm{b}}}{D}v_{\mathrm{c}}$$

$$\frac{kR \;\, R_{\mathrm{b}}R_{\mathrm{c}}}{D} = 3 \qquad \therefore \quad \frac{R_{\mathrm{b}}}{R_{\mathrm{a}}} = 1.5$$

$$\frac{kR \;\, R_{\mathrm{a}}R_{\mathrm{c}}}{D} = 2 \qquad \therefore \quad \frac{R_{\mathrm{c}}}{R_{\mathrm{b}}} = 2$$

$$\frac{kR \;\, R_{\mathrm{a}}R_{\mathrm{b}}}{D} = 1 \qquad \therefore \quad \frac{R_{\mathrm{c}}}{R_{\mathrm{a}}} = 3$$

Since $R_{\mathrm{a}} = 2\,\mathrm{k\Omega}$ $\quad R_{\mathrm{b}} = 3\,\mathrm{k\Omega}$ $\quad R_{\mathrm{c}} = 6\,\mathrm{k\Omega}$

$$\therefore \quad D = [(3)(6)(4) + (2)(6)(4) + (2)(3)(4) + (2)(3)(6)] \times 10^9 = 180 \times 10^9$$

$$\frac{k(4)(3)(6) \times 10^9}{180 \times 10^9} = 3$$

$$k = \frac{540 \times 10^9}{72 \times 10^9} = 7.5$$

$$\therefore \quad 7.5 = 1 + \frac{R_{\mathrm{f}}}{R}$$

$$\frac{R_{\mathrm{f}}}{R} = 6.5$$

$$R_{\mathrm{f}} = (6.5)(12{,}000) = 78\,\mathrm{k\Omega}$$

[b] $v \quad = 3(0.8) + 2(1.5) + 2.10 = 7.5 \;\; \mathrm{V}$

$$v \quad = v \quad = \frac{7.5}{7.5} = 1.0 \;\; \mathrm{V}$$

$$i_{\mathrm{a}} = \frac{0.8 - 1}{2000} = \frac{-0.2}{2000} = -0.1\,\mathrm{mA} = -100\,\mu\mathrm{A}$$

$$i_b = \frac{1.5 - 1.0}{3000} = \frac{0.5}{3000} = 166.67\,\mu\text{A}$$

$$i_c = \frac{2.10 - 1.0}{6000} = \frac{1.1}{6000} = 183.33\,\mu\text{A}$$

$$i \;=\; \frac{1}{4000} = 250\,\mu\text{A}$$

$$i \;=\; \frac{v}{12{,}000} = \frac{1}{12{,}000} = 83.33\,\mu\text{A}$$

P 5.27 **[a]** $$\frac{v \;\; - v_a}{R_a} + \frac{v \;\; - v_b}{R_b} + \frac{v \;\; - v_c}{R_c} = 0$$

$$\therefore \quad v \;\; = \frac{R_bR_c}{D}v_a + \frac{R_aR_c}{D}v_b + \frac{R_aR_b}{D}v_c$$

where $D = R_bR_c + R_aR_c + R_aR_b$

$$\frac{v}{10{,}000} + \frac{v \;\; - v}{R_f} = 0$$

$$\left(\frac{R_f}{10{,}000} + 1\right)v \;\; = v$$

Let $\dfrac{R_f}{10{,}000} + 1 = k$

$$v \;\; = kv \;\; = kv$$

$$\therefore \quad v \;\; = \frac{kR_bR_c}{D}v_a + \frac{kR_aR_c}{D}v_b + \frac{kR_aR_b}{D}v_c$$

$$\therefore \quad \frac{kR_bR_c}{D} = 5 \qquad\qquad \therefore \quad \frac{R_c}{R_a} = 5$$

$$\frac{kR_aR_c}{D} = 4$$

$$\frac{kR_aR_b}{D} = 1 \qquad\qquad \therefore \quad \frac{R_c}{R_b} = 4$$

$$\therefore \quad R_c = 5R_a = 5\,\text{k}\Omega$$

$$R_b = R_c/4 = 1.25\,\text{k}\Omega$$

$$\therefore \quad D = (1.25)(5) + (1)(5) + (1.25)(1) = 12.5 \times 10^6$$

$$\therefore \quad k = \frac{5D}{R_bR_c} = \frac{(5)(12.5) \times 10^6}{(1.25)(5) \times 10^6} = 10$$

$$\therefore \quad \frac{R_f}{10{,}000} + 1 = 10, \qquad R_f = 90\,\text{k}\Omega$$

[b] $v \;\; = 5(0.5) + 4(1) + 1.5 = 8\ \text{V}$

$$v \;\; = v \;\; /10 = 0.8\ \text{V} = v$$

$$i_\text{a} = \frac{v_\text{a} - v}{1000} = \frac{0.5 - 0.8}{1000} = -300\,\mu\text{A}$$

$$i_\text{b} = \frac{v_\text{b} - v}{1250} = \frac{1 - 0.8}{1250} = 160\,\mu\text{A}$$

$$i_\text{c} = \frac{v_\text{c} - v}{5000} = \frac{1.5 - 0.8}{5000} = 140\,\mu\text{A}$$

P 5.28 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v \;\; = 0$, therefore $v \;\; = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v'}{R_\text{b}} + i \;\; = 0, \qquad i \;\; = 0$$

Therefore

$$\frac{v'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v' = -\frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v \;\; = v \;\; = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v}{R_\text{a}} + \frac{v \;\; - v''}{R_\text{b}} + i \;\; = 0, \qquad i \;\; = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v''}{R_\text{b}} = 0$$

$$v'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v \;\; = v' + v'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$, therefore $R_\text{d}(R_\text{a} + R_\text{b}) = R_\text{b}(R_\text{c} + R_\text{d})$

$R_\text{d} R_\text{a} = R_\text{b} R_\text{c}$, therefore $\dfrac{R_\text{a}}{R_\text{b}} = \dfrac{R_\text{c}}{R_\text{d}}$

When $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$

Eq. (5.22) reduces to $v \;\; = \dfrac{R_\text{b}}{R_\text{a}} v_\text{b} - \dfrac{R_\text{b}}{R_\text{a}} v_\text{a} = \dfrac{R_\text{b}}{R_\text{a}}(v_\text{b} - v_\text{a})$.

P 5.29 Use voltage division to find v :

$$v \; = \frac{2000}{2000+8000}(5) = 1 \;\text{ V}$$

Write a KCL equation at v and solve it for v :

$$\frac{v \; - v}{5000} + \frac{v \; - v}{R} = 0 \qquad \text{so} \qquad \left(\frac{R}{5000}+1\right)v \; - \frac{R}{5000}v \; = v$$

Since the op amp is ideal, $v \; = v \; = 1$V, so

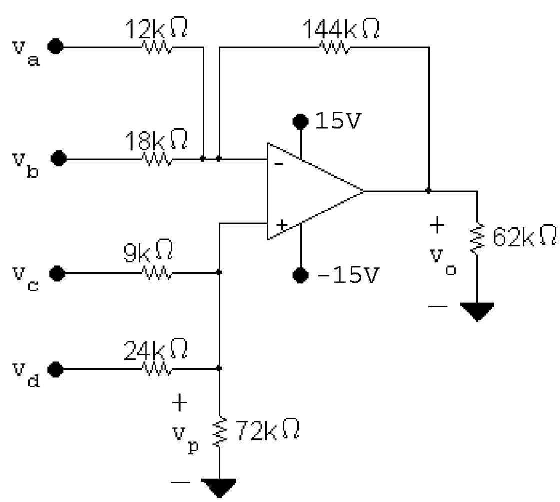
$$v \; = \left(\frac{R}{5000}+1\right) - \frac{R}{5000}v$$

To satisfy the equation,

$$\left(\frac{R}{5000}+1\right) = 5 \qquad \text{and} \qquad \frac{R}{5000} = 4$$

Thus, $R \; = 20$ kΩ.

P 5.30 **[a]**



$$\frac{v}{72{,}000} + \frac{v \; - v_\text{c}}{9{,}000} + \frac{v \; - v_\text{d}}{24{,}000} = 0$$

$$\therefore \; v \; = (2/3)v_\text{c} + 0.25v_\text{d} = v$$

$$\frac{v \; - v_\text{a}}{12{,}000} + \frac{v \; - v_\text{b}}{18{,}000} + \frac{v \; - v}{144{,}000} = 0$$

$$\begin{aligned} \therefore \; v \; &= 21v \; - 12v_\text{a} - 8v_\text{b} \\ &= 21[(2/3)v_\text{c} + 0.25v_\text{d}] - 12v_\text{a} - 8v_\text{b} \\ &= 21(0.4+0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

[b] $v \ = 14v_{\rm c} + 4.2 - 6 - 2.4$

$\pm 15 = 14v_{\rm c} - 4.2$

$\therefore \quad 14v_{\rm c} = \pm 15 + 4.2$

$\therefore \quad v_{\rm c} = 1.371 \;\text{ V} \qquad \text{and} \qquad v_{\rm c} = -0.771 \;\text{ V}$

$\therefore \quad -771 \leq v_{\rm c} \leq 1371\,\text{mV}$

P 5.31 **[a]** $$v \ = \frac{R_{\rm d}(R_{\rm a} + R_{\rm b})}{R_{\rm a}(R_{\rm c} + R_{\rm d})}v_{\rm b} - \frac{R_{\rm b}}{R_{\rm a}}v_{\rm a} = \frac{47(110)}{10(80)}(0.80) - 10(0.67)$$

$v \ = 5.17 - 6.70 = -1.53$ V

[b] $$v \ = v \ = \frac{(800)(47)}{80} = 470\,\text{mV}$$

$$i_{\rm a} = \frac{(670 - 470)10^{-3}}{10 \times 10^{3}} = 20\,\mu\text{A}$$

$$R_{\rm a} = \frac{v_{\rm a}}{i_{\rm a}} = \frac{670 \times 10^{-3}}{20 \times 10^{-6}} = 33.5\,\text{k}\Omega$$

[c] $R_{\rm in\,b} = R_{\rm c} + R_{\rm d} = 80\,\text{k}\Omega$

P 5.32 $$v \ = \frac{v_{\rm b}R_{\rm b}}{R_{\rm a} + R_{\rm b}} = v$$

$$\frac{v \ - v_{\rm a}}{4700} + \frac{v \ - v}{R_{\rm f}} = 0$$

$$v \ \left(\frac{R_{\rm f}}{4700} + 1\right) - \frac{v_{\rm a}R_{\rm f}}{4700} = v$$

$$\therefore \quad \left(\frac{R_{\rm f}}{4700} + 1\right)\frac{R_{\rm b}}{R_{\rm a} + R_{\rm b}}v_{\rm b} - \frac{R_{\rm f}}{4700}v_{\rm a} = v$$

$$\therefore \quad \frac{R_{\rm f}}{4700} = 10; \qquad R_{\rm f} = 47\,\text{k}\Omega$$

$$\therefore \quad \frac{R_{\rm f}}{4700} + 1 = 11$$

$$\therefore \quad 11\left(\frac{R_{\rm b}}{R_{\rm a} + R_{\rm b}}\right) = 10$$

$11R_{\rm b} = 10R_{\rm b} + 10R_{\rm a} \qquad R_{\rm b} = 10R_{\rm a}$

$R_a + R_b = 220\,\text{k}\Omega$

$11R_a = 220\,\text{k}\Omega$

$R_a = 20\,\text{k}\Omega$

$R_b = 220 - 20 = 200\,\text{k}\Omega$

P 5.33 $v \ \ = v \ \ = R_b i_b$

$$\frac{R_b i_b - 3000 i_a}{3000} + \frac{R_b i_b - v \ }{R_f} = 0$$

$$\left(\frac{R_b}{3000} + \frac{R_b}{R_f}\right) i_b - i_a = \frac{v \ }{R_f}$$

$$v \ \ = \left[\frac{R_b R_f}{3000} + R_b\right] i_b - R_f i_a$$

$\therefore \quad R_f = 2000\,\Omega$

$(2/3)R_b + R_b = 2000$

$\therefore \quad R_b = 1200\,\Omega$

P 5.34 $v \ \ = \dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \dfrac{R_b}{R_a} v_a$

By hypothesis: $R_b/R_a = 4$; $R_c + R_d = 470\,\text{k}\Omega$; $\dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 3$

$\therefore \quad \dfrac{R_d}{R_a}\dfrac{(R_a + 4R_a)}{470{,}000} = 3$ so $R_d = 282\,\text{k}\Omega$; $R_c = 188\,\text{k}\Omega$

Also, when $v \ \ = 0$ we have

$$\frac{v \ \ - v_a}{R_a} + \frac{v \ }{R_b} = 0$$

$$\therefore \quad v \ \left(1 + \frac{R_a}{R_b}\right) = v_a; \qquad v \ \ = 0.8 v_a$$

$$i_a = \frac{v_a - 0.8v_a}{R_a} = 0.2\frac{v_a}{R_a}; \qquad R_{in} = \frac{v_a}{i_a} = 5R_a = 22\,\text{k}\Omega$$

$\therefore \quad R_a = 4.4\,\text{k}\Omega; \qquad R_b = 17.6\,\text{k}\Omega$

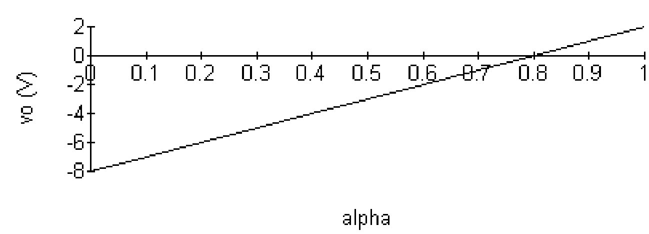
P 5.35 **[a]** $v\ = v\ = \alpha v$ $\qquad\qquad v = (\alpha v\ - v\ )4 + \alpha v$

$$\frac{v\ - v}{R_1} + \frac{v\ - v}{R_\mathrm{f}} = 0 \qquad = [(\alpha - 1)4 + \alpha]v$$

$$(v\ - v\ )\frac{R_\mathrm{f}}{R_1} + v\ - v\ = 0 \qquad = (5\alpha - 4)v$$

$$= (5\alpha - 4)(2) = 10\alpha - 8$$

α	v	α	v	α	v
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v from (a) gives

$$v\ = \left(\frac{R}{R_1} + 1\right) v\ \alpha + -\left(\frac{R}{R_1}\right) v$$

Therefore,

$$\text{slope } = \left(\frac{R}{R_1} + 1\right) v\ ; \qquad \text{intercept } = -\left(\frac{R}{R_1}\right) v$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R}{R_1} + 1\right) v\ ; \qquad 4 = -\left(\frac{R}{R_1}\right) v$$

Solving,

$$v\ = -2\text{ V}; \qquad \frac{R}{R_1} = 2$$

P 5.36 $v\ = \dfrac{1500}{9000}(-18) = -3\ \text{V} = v$

$$\frac{18 - 3}{1600} + \frac{-3 - v}{R_\mathrm{f}} = 0$$

$$\therefore \quad v = \frac{15}{1600}R_f - 3$$

$$v = 9 \text{ V}; \qquad R_f = 1280\,\Omega$$

$$v = -9 \text{ V}; \qquad R_f = -640\,\Omega$$

But $\quad R_f \geq 0, \qquad \therefore \quad R_f = 1280\,\Omega$

P 5.37 **[a]** $A_{dm} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{cm} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] CMRR $= \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.38 $A_{cm} = \dfrac{(20)(50) - (50)R}{20(50 + R\ )}$

$$A_{dm} = \frac{50(20 + 50) + 50(50 + R\ )}{2(20)(50 + R\ )}$$

$$\frac{A_{dm}}{A_{cm}} = \frac{R\ + 120}{2(20 - R\ )}$$

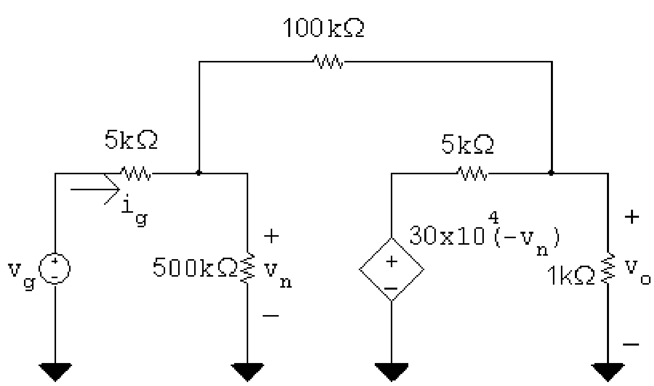
$$\therefore \quad \frac{R\ + 120}{2(20 - R\ )} = \pm 1000 \text{ for the limits on the value of } R$$

If we use $+1000 \quad R\ = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R\ = 20.07\,\text{k}\Omega$

$19.93\,\text{k}\Omega \leq R\ \leq 20.07\,\text{k}\Omega$

P 5.39 **[a]** Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v \;\; - v}{5000} + \frac{v \;\; - v}{100{,}000} + \frac{v}{500{,}000} = 0$$

$$\frac{v \;\; + 3 \times 10^5 v}{5000} + \frac{v \;\; - v}{100{,}000} + \frac{v}{1000} = 0$$

Simplify and place in standard form:

$$106v \;\; - 5v \;\; = 100v$$

$$(6{\times}10^6 - 1)v \;\; + 121v \;\; = 0$$

Let $v \;\; = 1$ V and solve the two simultaneous equations:

$v \;\; = -19.9915$ V; $\qquad v \;\; = 403.2\,\mu$V

[b] From the solution in part (a), $v \;\; = 403.2\,\mu$V.

[c] $i \;\; = \dfrac{v \;\; - v}{5000} = \dfrac{v \;\; - 403.2 \times 10^{-6} v}{5000}$

$$R \;\; = \frac{v}{i} = \frac{5000}{1 - 403.2 \times 10^{-6}} = 5002.02\,\Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v}{v} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$v \;\; = v \;\; = 0$ V; $\qquad R \;\; = 5000\,\Omega$

P 5.40 Note – the load resistor should have the value $4\,\text{k}\Omega$.

[a] Replace the op amp with the model shown in Fig. 5.15. The node voltage equation at the inverting input:

$$\frac{v}{40{,}000} + \frac{v \;\; - v}{500{,}000} + \frac{v \;\; - v}{80{,}000} = 0$$

Simplify:

$$12.5v \;\; + v \;\; - v \;\; + 6.25v \;\; - 6.25v \;\; = 0$$

The node voltage equation at the op amp output:

$$\frac{v}{4000} + \frac{v \;\; - 20{,}000(v \;\; - v \;\;)}{5000} + \frac{v \;\; - v}{80{,}000} = 0$$

Simplify:

$$20v \; + 16v \; - 320{,}000(v \; - v \;) + v \; - v \; = 0$$

From the input,

$$v \; - v \; = 0.8(v \; - v \;)$$

Substituting into the equation written at the output,

$$20v \; + 16v \; - 256{,}000(v \; - v \;) + v \; - v \; = 0$$

Now let $v \; = 1$ V; plug this value into both the input and output equations and simplify into two simultaneous equations:

$$19.75v \; - 6.25v \; = 1$$

$$255{,}999v \; + 37v \; = 256{,}000$$

These equations are in standard form, so solve them to yield
$v \; = 2.9986$ V; $v \; = 999.571$ mV
Thus,

$$\frac{v}{v} = \frac{2.9986}{1} = 2.9986$$

[b] From part (a), $v \; = 999.571$ mV. Use this value to solve for v :

$$v \; = 0.8(1 - v \;) + v \; = 999.914 \text{ mV}$$

[c] $v \; - v \; = 343.6\,\mu$ V

[d] $i \; = \dfrac{v \; - v}{100{,}000} = \dfrac{1 - 999.914 \times 10^{-3}}{100{,}000} = 859$ pA

[e] For an ideal op amp, $v \; = v \; = v$, so the KVL equation at the inverting node is

$$\frac{v}{40{,}000} + \frac{v \; - v}{80{,}000} = 0$$

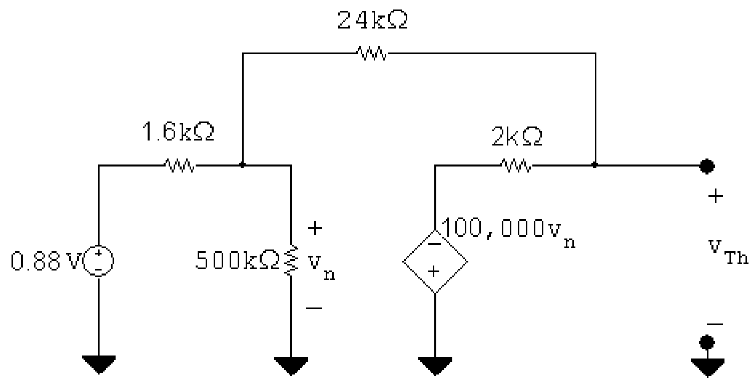
Then,

$$v \; = 3v \quad \text{so} \quad \frac{v}{v} = 3$$

Also,

$$v \; = v \; = 1 \text{ V}; \quad v \; - v \; = 0 \text{ V}; \quad i \; = 0 \text{ A}$$

P 5.41 **[a]**

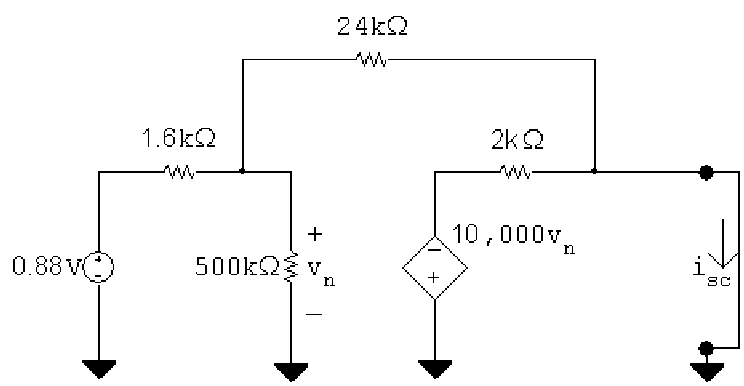


$$\frac{v \quad - 0.88}{1600} + \frac{v}{500{,}000} + \frac{v \quad - v_{\text{Th}}}{24{,}000} = 0$$

$$\frac{v_{\text{Th}} + 10^5 v}{2000} + \frac{v_{\text{Th}} - v}{24{,}000} = 0$$

Solving, $v_{\text{Th}} = -13.198\ \text{V}$

Short-circuit current calculation:

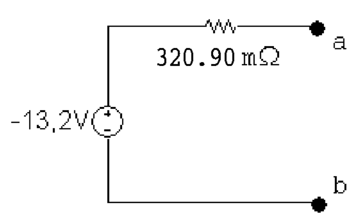


$$\frac{v}{500{,}000} + \frac{v \quad - 0.88}{1600} + \frac{v \quad - 0}{24{,}000} = 0$$

$$\therefore \quad v \quad = 0.823\ \text{V}$$

$$i_{\text{sc}} = \frac{v}{24{,}000} - \frac{10^5}{2000} v \quad = -41.13\ \text{A}$$

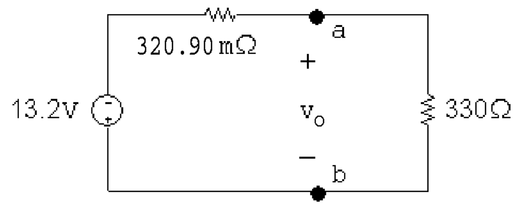
$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 320.90\,\text{m}\Omega$$



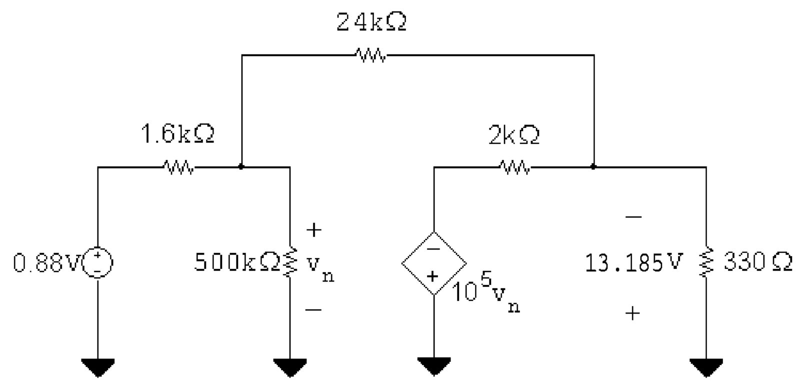
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R \;= R_{\text{Th}} = 320.90\,\text{m}\Omega$$

[c]



$$v \;= \left(\frac{330}{330.32}\right)(-13.198) = -13.185\ \text{V}$$



$$\frac{v \;- 0.88}{1600} + \frac{v}{500{,}000} + \frac{v \;+ 13.185}{24{,}000} = 0$$

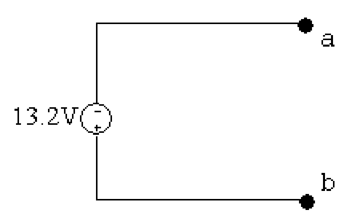
$$\therefore \;\; v \;= 941.92\,\mu\text{V}$$

$$i \;= \frac{0.88 - 941.92 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R \;= \frac{0.88}{0.88 - 941.92 \times 10^{-6}}(1600) = 1601.7\,\Omega$$

P 5.42 **[a]** $v_{\text{Th}} = \frac{-24}{1.6}(0.88) = -13.2\ \text{V}$

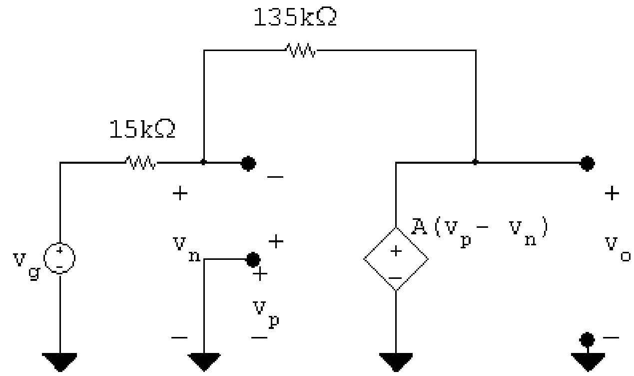
$R_{\text{Th}} = 0,$ since op-amp is ideal



[b] $R \;\; = R_{\mathrm{Th}} = 0\,\Omega$

[c] $R \;\; = 1.6\,\mathrm{k}\Omega$ since $v \;\; = 0$

P 5.43 **[a]**



$$\frac{v \;\; - v}{15{,}000} + \frac{v \;\; - v}{135{,}000} = 0$$

$$\therefore \quad v \;\; = 10v \;\; - 9v$$

Also $\quad v \;\; = A(v \;\; - v \;\;) = -Av$

$$\therefore \quad v \;\; = \frac{-v}{A}$$

$$\therefore \quad v \;\; \left(1 + \frac{10}{A}\right) = -9v$$

$$v \;\; = \frac{-9A}{(10 + A)} v$$

[b] $v \;\; = \dfrac{-9(90)(0.4)}{(10 + 90)} = -3.24\;\;\mathrm{V}$

[c] $v \;\; = -9(0.4) = -3.60\;\;\mathrm{V}$

[d] $-3.42 = \dfrac{-9(0.4)A}{10 + A}$

$$\therefore \quad A = 190$$

P 5.44 From Eq. 5.57,

$$\frac{v_{\mathrm{ref}}}{R + \Delta R} = v \;\; \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R}\right) - \frac{v}{R}$$

Substituting Eq. 5.59 for $v \;\; = v \;\;$:

$$\frac{v_{\mathrm{ref}}}{R + \Delta R} = \frac{v_{\mathrm{ref}}\left(\frac{1}{+\Delta} + \frac{1}{-\Delta} + \frac{1}{f}\right)}{(R - \Delta R)\left(\frac{1}{+\Delta} + \frac{1}{-\Delta} + \frac{1}{f}\right)} - \frac{v}{R}$$

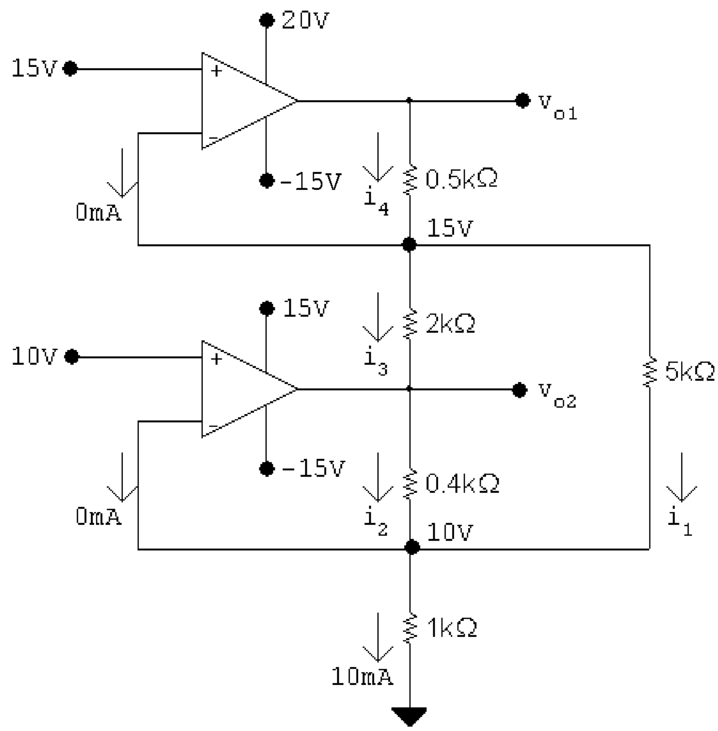
Rearranging,

$$\frac{v}{R} = v_{\text{ref}}\left(\frac{1}{R-\Delta R} - \frac{1}{R+\Delta R}\right)$$

Thus,

$$v = v_{\text{ref}}\left(\frac{2\Delta R}{R^2 - \Delta R^2}\right)R$$

P 5.45



$$i_1 = \frac{15-10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{\ 2} = 10 + (400)(9)\times 10^{-3} = 13.6\ \text{V}$$

$$i_3 = \frac{15-13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{\ 1} = 15 + 1.7(0.5) = 15.85\ \text{V}$$

P 5.46 $v \;\; = \dfrac{5.6}{8.0}v \;\; = 0.7v \;\; = 7\sin(\pi/3)t \;$ V

$$\frac{v}{15{,}000} + \frac{v \;\; - v}{75{,}000} = 0$$

$$6v \;\; = v \;\; ; \qquad v \;\; = v$$

$$\therefore \quad v \;\; = 42\sin(\pi/3)t \;\text{ V} \qquad 0 \le t \le \infty$$

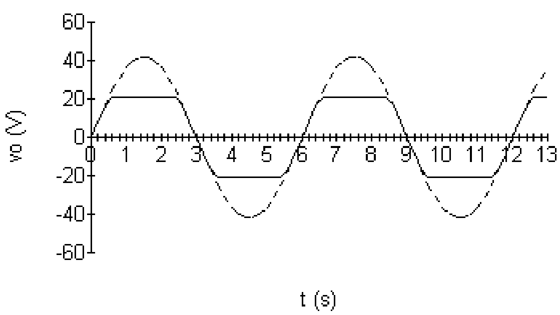
$$v \;\; = 0 \qquad t \le 0$$

At saturation

$$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$$

$$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad \text{etc.}$$



P 5.47 It follows directly from the circuit that $v \;\; = -16v$
From the plot of v we have $\quad v \;\; = 0, \quad t < 0$

$$v = t \qquad 0 \le t \le 0.5$$

$$v = -t + 1 \qquad 0.5 \le t \le 1.5$$

$$v = t - 2 \qquad 1.5 \le t \le 2.5$$

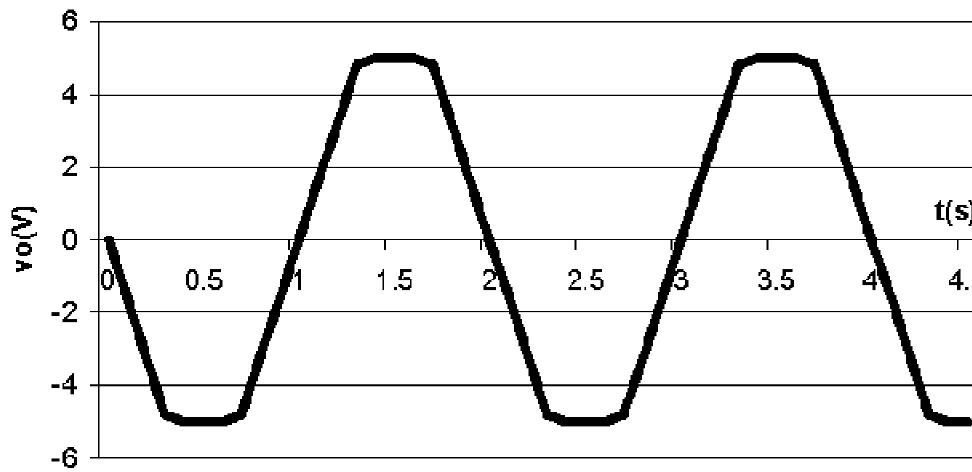
$$v = -t + 3 \qquad 2.5 \le t \le 3.5$$

$$v = t - 4 \qquad 3.5 \le t \le 4.5, \quad \text{etc.}$$

Therefore

$$v = -16t \qquad 0 \le t \le 0.5$$
$$v = 16t - 16 \qquad 0.5 \le t \le 1.5$$
$$v = -16t + 32 \qquad 1.5 \le t \le 2.5$$
$$v = 16t - 48 \qquad 2.5 \le t \le 3.5$$
$$v = -16t + 64 \qquad 3.5 \le t \le 4.5, \quad \text{etc.}$$

These expressions for v are valid as long as the op amp is not saturated. Since the peak values of v are ± 5, the output is clipped at ± 5. The plot is shown below.



P 5.48 **[a]** Use Eq. 5.61 to solve for R ; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R = \frac{v\ R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

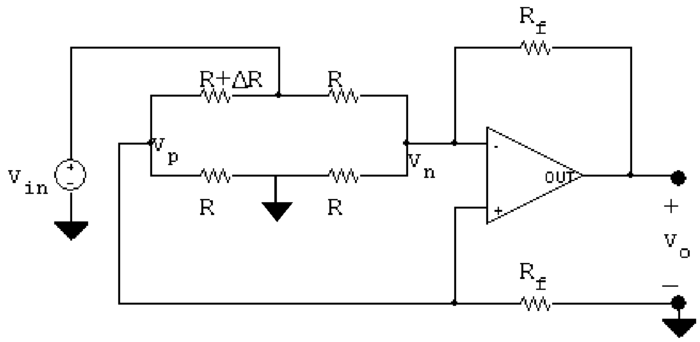
[b] Now solve for Δ given $v = 50$ mV:

$$\Delta = \frac{v\ R}{2R\ v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.49 **[a]**



Let $R_1 = R + \Delta R$

$$\frac{v}{R} + \frac{v}{R} + \frac{v\ \ - v_{in}}{R_1} = 0$$

$$\therefore \quad v \quad \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{in}}{R_1}$$

$$\therefore \quad v \quad = \frac{RR\ v_{in}}{RR_1 + R\ \ R_1 + R\ \ R} = v$$

$$\frac{v}{R} + \frac{v\ \ - v_{in}}{R} + \frac{v\ \ - v}{R} = 0$$

$$v \quad \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right] - \frac{v}{R} = \frac{v_{in}}{R}$$

$$\therefore \quad v \quad \left[\frac{R + 2R}{RR}\right] - \frac{v_{in}}{R} = \frac{v}{R}$$

$$\therefore \quad \frac{v}{R} = \left[\frac{R + 2R}{RR}\right] \frac{RR\ v_{in}}{[RR_1 + R\ \ R_1 + R\ \ R]} - \frac{v_{in}}{R}$$

$$\therefore \quad \frac{v}{R} = \left[\frac{R + 2R}{RR_1 + R\ \ R_1 + R\ \ R} - \frac{1}{R}\right] v_{in}$$

$$\therefore \quad v \quad = \frac{[R^2 + 2RR\ \ - R_1(R + R\ \ ) - RR\ \ ]R}{R[R_1(R + R\ \ ) + RR\ \ ]} v_{in}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v \quad = \frac{-\Delta R(R + R\ \ )R\ \ v_{in}}{R[(R + \Delta R)(R + R\ \ ) + RR\ \ ]}$$

If $\Delta R \ll R$

$$v \quad \approx \frac{(R + R\ \ )R\ \ (-\Delta R)v_{in}}{R^2(R + 2R\ \ )}$$

[b] $v \quad \approx \dfrac{47 \times 10^4(48 \times 10^4)(-95)15}{10^8(95 \times 10^4)} \approx -3.384\ \text{V}$

[c] $v \;=\; \dfrac{-95(48\times10^4)(47\times10^4)15}{10^4[(1.0095)10^4(48\times10^4)+47\times10^8]} = -3.368 \;\text{V}$

P 5.50 **[a]** $v \;\approx\; \dfrac{(R+R\;\;)R\;\;(-\Delta R)v_{\text{in}}}{R^2(R+2R\;\;)}$

$$v \;=\; \frac{(R+R\;\;)(-\Delta R)R\;\;v_{\text{in}}}{R[(R+\Delta R)(R+R\;\;)+RR\;\;]}$$

$$\therefore\;\; \frac{\text{approx value}}{\text{true value}} = \frac{R[(R+\Delta R)(R+R\;\;)+RR\;\;]}{R^2(R+2R\;\;)}$$

$$\therefore\;\; \text{Error} \;=\; \frac{R[(R+\Delta R)(R+R\;\;)+RR\;\;]-R^2(R+2R\;\;)}{R^2(R+2R\;\;)}$$

$$= \frac{\Delta R}{R}\frac{(R+R\;\;)}{(R+2R\;\;)}$$

$$\therefore\;\; \%\ \text{error} \;=\; \frac{\Delta R(R+R\;\;)}{R(R+2R\;\;)}\times 100$$

[b] $\%\ \text{error} = \dfrac{95(48\times10^4)\times100}{10^4(95\times10^4)} = 0.48\%$

P 5.51 $1 = \dfrac{\Delta R(48\times10^4)}{10^4(95\times10^4)}\times100$

$$\therefore\;\; \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore\;\; \%\ \text{change in } R = \frac{197.19667}{10^4}\times100 \approx 1.98\%$$

P 5.52 **[a]** It follows directly from the solution to Problem 5.49 that

$$v \;=\; \frac{[R^2+2RR\;\;-R_1(R+R\;\;)-RR\;\;]R\;\;v_{\text{in}}}{R[R_1(R+R\;\;)+RR\;\;]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v \;=\; \frac{(R+R\;\;)R\;\;(\Delta R)v_{\text{in}}}{R[(R-\Delta R)(R+R\;\;)+RR\;\;]}$$

Now let $\Delta R \ll R$ and get

$$v \;\approx\; \frac{(R+R\;\;)R\;\;\Delta R v_{\text{in}}}{R^2(R+2R\;\;)}$$

[b] It follows directly from the solution to Problem 5.49 that

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R\ \ ) + RR\ \ ]}{R^2(R + 2R\ \ )}$$

$$\therefore \quad \text{Error} = \frac{(R - \Delta R)(R + R\ \ ) + RR\ \ - R(R + 2R\ \ )}{R(R + 2R\ \ )}$$

$$= \frac{-\Delta R(R + R\ \ )}{R(R + 2R\ \ )}$$

$$\therefore \quad \%\ \text{error} = \frac{-\Delta R(R + R\ \ )}{R(R + 2R\ \ )} \times 100$$

[c] $R - \Delta R = 9810\,\Omega \qquad \therefore \quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore \quad v \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768\ \text{V}$$

[d] $\%\ \text{error} = \dfrac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$

6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i \;= 8e^{-300} \;- 8e^{-1200}$ A

$$v = L\frac{di}{dt} = -9.6e^{-300} \;+ 38.4e^{-1200}\text{ V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\text{ V}$$

[b] $v = 0$ when $38.4e^{-1200} \;= 9.6e^{-300}$ or $t = (\ln 4)/900 = 1.54$ ms

[c] $p = vi = 384e^{-1500} \;- 76.8e^{-600} \;- 307.2e^{-2400}$ W

[d] $\frac{dp}{dt} = 0$ when $e^{1800} \;- 12.5e^{900} \;+ 16 = 0$

Let $x = e^{900}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.45, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.05, \qquad t = \frac{\ln 11.05}{900} = 2.67\text{ ms}$$

p is maximum at $t = 411.05\,\mu$s

[e] $p_{\text{max}} = 384e^{-1\;5(0\;41105)} - 76.8e^{-0\;6(0\;41105)} - 307.2e^{-2\;4(0\;41105)} = 32.72$ W

[f] $i_{\text{max}} = 8[e^{-0\;3(1\;54)} - e^{-1\;2(1\;54)}] = 3.78$ A

$$w_{\text{max}} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\text{ mJ}$$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54$ ms.

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15\,000} \ \sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15\,000} \ \text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$$p = vi = -649.23\,\text{mW}$$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}(-0.03e^{-5} \ )\,dx - 2 = 0.125e^{-5} \ - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}(-0.03e^{-5} \ )\,dx + 3 = 0.1e^{-5} \ + 2.9\,\text{A}$

$$i_2 = \frac{25}{6}\int_{0^+}(-0.03e^{-5} \ )\,dx - 5 = 0.025e^{-5} \ - 5.025\,\text{A}$$

$$i_1 + i_2 = i$$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+} 240 \times 10^{-6}e^{-10} \ \ dx - 10 = -12e^{-10} \ + 2\,\text{V}$

$$v_2 = 0.125 \times 10^6\int_{0^+} 240 \times 10^{-6}e^{-10} \ \ dx - 5 = -3e^{-10} \ - 2\,\text{V}$$

$$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$$

$$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$$

AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i\ \ )}{dt} + 20(i_1 - i_2) + 5(i_1 + i\ \ ) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i\ \ + 8\frac{di}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i\ \ )}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di}{dt}$$

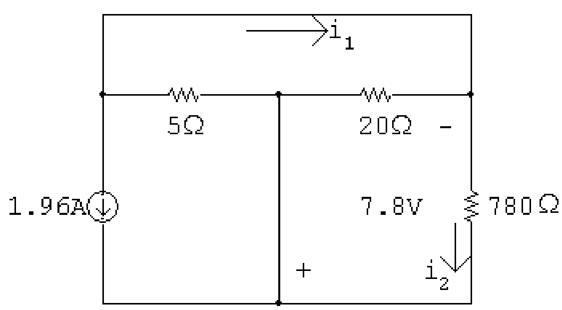
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4}\ \ - 60e^{-5}$$

$$\frac{di_2}{dt} = 3.96e^{-4}\ \ - 5e^{-5}$$

Also, $$\frac{di}{dt} = 7.84e^{-4}$$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4}\ \ - 240e^{-5}$$

$$25i_1 = -10 - 290e^{-4} + 300e^{-5}$$

$$8\frac{di_2}{dt} = 31.68e^{-4} - 40e^{-5}$$

$$20i_2 = -0.20 - 19.80e^{-4} + 20e^{-5}$$

$$5i = 9.8 - 9.8e^{-4}$$

$$8\frac{di}{dt} = 62.72e^{-4}$$

Test:

$$185.60e^{-4} - 240e^{-5} - 10 - 290e^{-4} + 300e^{-5} + 31.68e^{-4} - 40e^{-5}$$
$$+0.20 + 19.80e^{-4} - 20e^{-5} \stackrel{?}{=} -[9.8 - 9.8e^{-4} + 62.72e^{-4}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5}$$
$$+(185.60 - 290 + 31.68 + 19.80)e^{-4} \stackrel{?}{=} -(9.8 + 52.92e^{-4})$$

$$-9.8 + 0e^{-5} + (237.08 - 290)e^{-4} \stackrel{?}{=} -9.8 - 52.92e^{-4}$$

$$-9.8 - 52.92e^{-4} = -9.8 - 52.92e^{-4} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4} - 480e^{-5}$$

$$20i_1 = -8 - 232e^{-4} + 240e^{-5}$$

$$16\frac{di_2}{dt} = 63.36e^{-4} - 80e^{-5}$$

$$800i_2 = -8 - 792e^{-4} + 800e^{-5}$$

$$16\frac{di}{dt} = 125.44e^{-4}$$

Test:

$$371.20e^{-4} - 480e^{-5} + 8 + 232e^{-4} - 240e^{-5} + 63.36e^{-4} - 80e^{-5}$$
$$-8 - 792e^{-4} + 800e^{-5} \stackrel{?}{=} -125.44e^{-4}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5}$$
$$+(371.20 + 232 + 63.36 - 792)e^{-4} \stackrel{?}{=} -125.44e^{-4}$$

$$(800 - 800)e^{-5} + (666.56 - 792)e^{-4} \stackrel{?}{=} -125.44e^{-4}$$

$$-125.44e^{-4} = -125.44e^{-4} \quad \text{(OK)}$$

Problems

P 6.1 **[a]**

$$\begin{aligned} i &= 0 & t &< 0 \\ i &= 50t\,\text{A} & 0 \le t &\le 5\,\text{ms} \\ i &= 0.5 - 50t\,\text{A} & 5 \le t &\le 10\,\text{ms} \\ i &= 0 & 10\,\text{ms} &< t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 20 \times 10^{-3}(50) = 1\,\text{V} \qquad 0 \le t \le 5\,\text{ms}$

$v = 20 \times 10^{-3}(-50) = -1\,\text{V} \qquad 5 \le t \le 10\,\text{ms}$

$$\begin{aligned} v &= 0 & t &< 0 \\ v &= 1\,\text{V} & 0 < t &< 5\,\text{ms} \\ v &= -1\,\text{V} & 5 < t &< 10\,\text{ms} \\ v &= 0 & 10\,\text{ms} &< t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 & t &< 0 \\ p &= (50t)(1) = 50t\,\text{W} & 0 < t &< 5\,\text{ms} \\ p &= (0.5 - 50t)(-1) = 50t - 0.5\,\text{W} & 5 < t &< 10\,\text{ms} \\ p &= 0 & 10\,\text{ms} &< t \end{aligned}$$

$$\begin{aligned} w &= 0 & & t < 0 \\ w &= \int_0 (50x)\,dx = 50\frac{x^2}{2}\Big|_0 = 25t^2\,\text{J} & & 0 < t < 5\,\text{ms} \\ w &= \int_{0\,005} (50x - 0.5)\,dx + 0.625 \times 10^{-3} & & \\ &= 25x^2 - 0.5x\Big|_{0\,005} + 0.625 \times 10^{-3} & & \\ &= 25t^2 - 0.5t + 2.5 \times 10^{-3}\,\text{J} & & 5 < t < 10\,\text{ms} \\ w &= 0 & & 10\,\text{ms} < t \end{aligned}$$

P 6.2 **[a]** $0 \le t \le 2\,\text{ms}$:

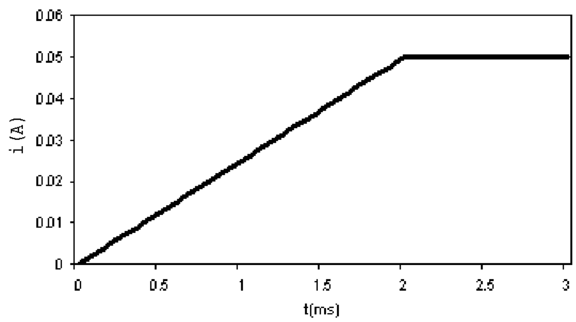
$$i = \frac{1}{L}\int_0 v\ \ dx + i(0) = \frac{1}{200 \times 10^{-6}}\int_0 5 \times 10^{-3}\,dx + 0$$

$$= 25x \Big|_0 = 25t\,\text{A}$$

$2\,\text{ms} \le t < \infty$:

$$i = \frac{1}{200 \times 10^{-6}} \int_{2\times 10^{-3}} (0)\,dx + 25(2 \times 10^{-3}) = 50\,\text{mA}$$

[b]



P 6.3 Note – the initial current should be 1 A.

$0 \le t \le 2$ s

$$i \;\; = \frac{1}{2.5 \times 10^{-4}} \int_0 3 \times 10^{-3} e^{-4} \;\; dx + 0 = 1.2 \frac{e^{-4}}{-4} \Big|_0 \;\; + 0$$

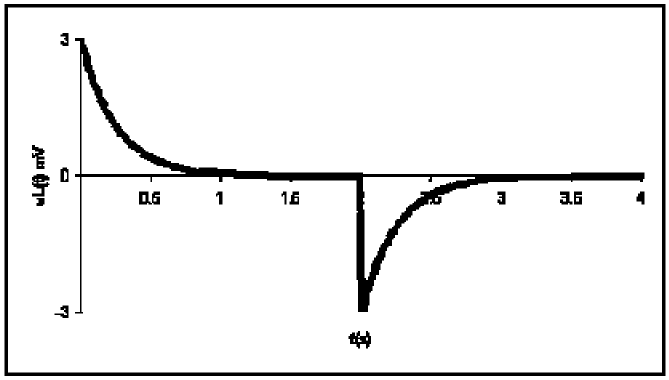
$$= 0.3 - 0.3e^{-4} \;\; \text{A}, \qquad 0 \le t \le 2 \text{ s}$$

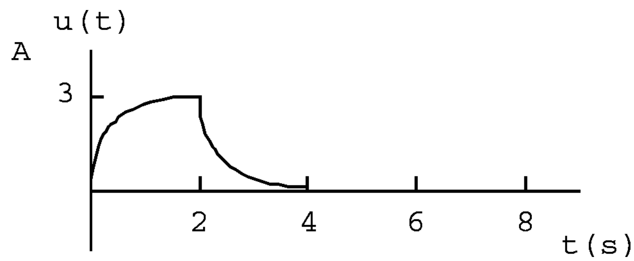
$i \;\; (2) = 0.3\text{A}$

$2 \text{ s} < t < \infty$

$$i \;\; = -1.2 \left(\frac{e^{-4(\;\; -2)}}{-4} \Big|_2 \;\; + 0.3 \right)$$

$$= 0.3e^{-4(\;\; -2)}\,\text{A}, \qquad 2 \text{ s} \le t < \infty$$





P 6.4 **[a]** $v = L\dfrac{di}{dt}$

$$\frac{di}{dt} = 18[t(-10e^{-10}\ ) + e^{-10}\ ] = 18e^{-10}\ (1 - 10t)$$

$$v = (50 \times 10^{-6})(18)e^{-10}\ (1 - 10t)$$

$$= 0.9e^{-10}\ (1 - 10t)\,\text{mV}, \quad t > 0$$

[b] $p = vi$

$$v(200\,\text{ms}) = 0.9e^{-2}(1 - 2) = -121.8\,\mu\text{V}$$

$$i(200\,\text{ms}) = 18(0.2)e^{-2} = 487.2\,\text{mA}$$

$$p(200\,\text{ms}) = (-121.8 \times 10^{-6})(487.2 \times 10^{-3}) = -59.34\,\mu\text{W}$$

[c] delivering

[d] $w = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(50 \times 10^{-6})(487.2 \times 10^{-3})^2 = 5.93\,\mu\text{J}$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di}{dt} = 18[t(-10)e^{-10}\ + e^{-10}\ ) = 18e^{-10}\ (1 - 10t)$$

$$\frac{di}{dt} = 0 \quad \text{when} \quad t = 0.1\,\text{s}$$

$$i_{\max} = 18(0.1)e^{-1} = 662.2\,\mathrm{mA}$$

$$w_{\max} = \frac{1}{2}(50 \times 10^{-6})(662.2 \times 10^{-3})^2 = 10.96\,\mu\mathrm{J}$$

P 6.5 **[a]** $0 \le t \le 1\,\mathrm{s}:$

$$v = -100t$$

$$i = \frac{1}{5}\int_0 -100x\,dx + 0 = -20\frac{x^2}{2}\Big|_0$$

$$i = -10t^2\,\mathrm{A}$$

$$1\,\mathrm{s} \le t \le 3\,\mathrm{s}:$$

$$v = -200 + 100t$$

$$i(1) = -10\,\mathrm{A}$$

$$\begin{aligned}
\therefore\ i &= \frac{1}{5}\int_1 (100x - 200)\,dx - 10\\
&= 20\int_1 x\,dx - 40\int_1 dx - 10\\
&= 10(t^2 - 1) - 40(t - 1) - 10\\
&= 10t^2 - 40t + 20\,\mathrm{A}
\end{aligned}$$

$$3\,\mathrm{s} \le t \le 5\,\mathrm{s}:$$

$$v = 100$$

$$i(3) = 10(9) - 120 + 20 = -10\,\mathrm{A}$$

$$\begin{aligned}
i &= \frac{1}{5}\int_3 100\,dx - 10\\
&= 20t - 60 - 10 = 20t - 70\,\mathrm{A}
\end{aligned}$$

$$5\,\mathrm{s} \le t \le 6\,\mathrm{s}:$$

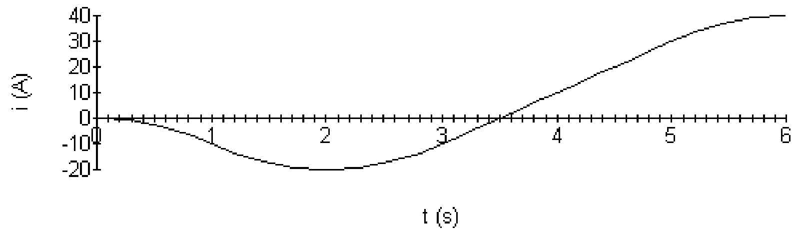
$$v = -100t + 600$$

$$i(5) = 100 - 70 = 30$$

$$\begin{aligned}
i &= \frac{1}{5}\int_5 (-100x + 600)\,dx + 30\\
&= -20\int_5 x\,dx + 120\int_5 dx + 30\\
&= -10(t^2 - 25) + 120(t - 5) + 30\\
&= -10t^2 + 120t - 320\,\mathrm{A}
\end{aligned}$$

[b] $i(6) = -10(36) + 120(6) - 320 = 720 - 680 = 40\,\text{A}, \qquad 6 \le t < \infty$

[c]



P 6.6 **[a]** $v = L\dfrac{di}{dt} = [125\sin 400t]e^{-200}\ \text{V}$

$$\therefore \quad \frac{dv}{dt} = 25{,}000(2\cos 400t - \sin 400t)e^{-200}\ \text{V/s}$$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad \tan 400t = 2$$

$$\therefore \quad t = 2.77\,\text{ms}$$

Also $\quad 400t = 1.107 + \pi \qquad$ etc.

Because of the decaying exponential v will be maximum the first time the derivative is zero.

[b] $v\ (\max) = [125\sin 1.107]e^{-0\ 554} = 64.27\,\text{V}$

v max $= 64.27\,\text{V}$

Note: When $\quad t = (1.107 + \pi)/400; \qquad v = -13.36\,\text{V}$

P 6.7 **[a]**

$$i = \frac{1}{15 \times 10^{-3}}\int_0 30\sin 500x\,dx - 4$$

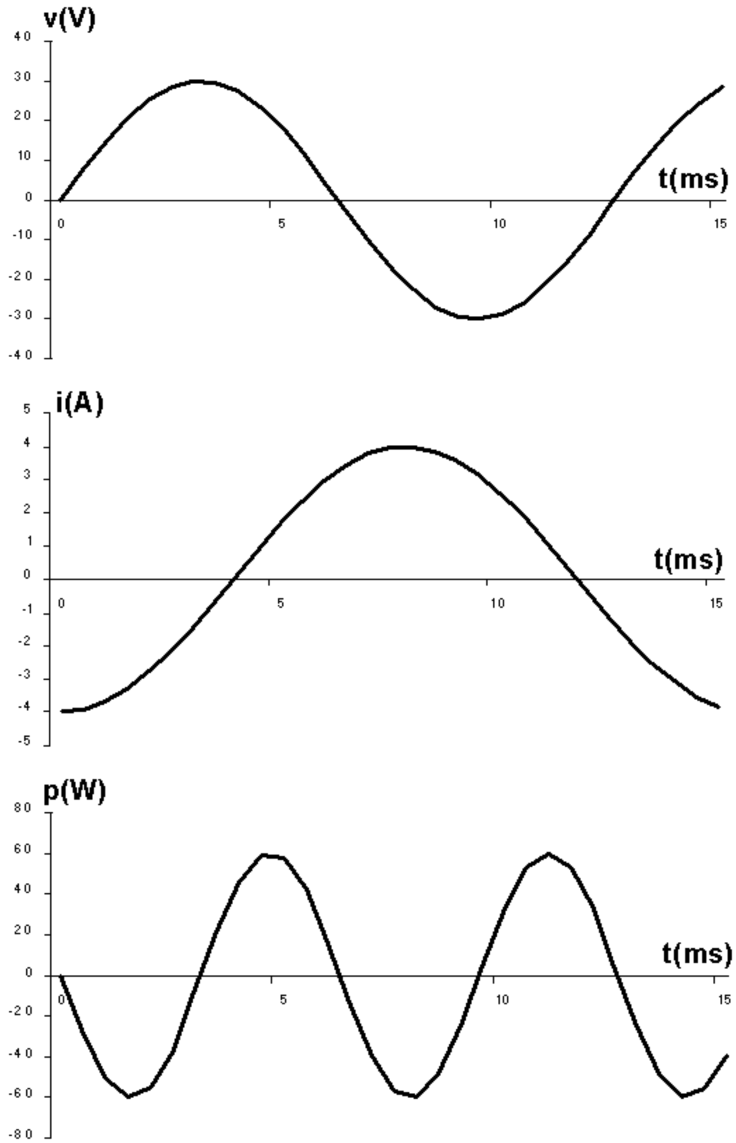
$$= 2000\int_0 \sin 500x\,dx - 4$$

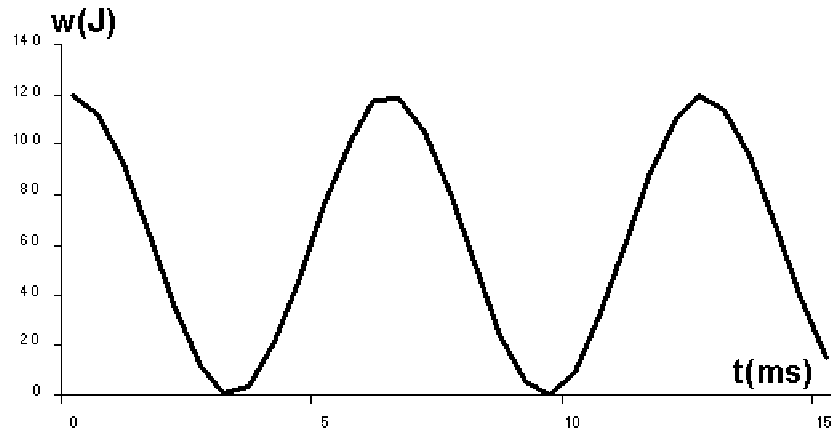
$$= 2000\left[\frac{-\cos 500x}{500}\right|_0 - 4$$

$$= 4(1 - \cos 500t) - 4$$

$$i = -4\cos 500t\,\text{A}$$

[b]
$$p = vi = (30\sin 500t)(-4\cos 500t)$$
$$= -120\sin 500t\cos 500t$$
$$p = -60\sin 1000t\,\mathrm{W}$$
$$w = \frac{1}{2}Li^2$$
$$= \frac{1}{2}(15\times 10^{-3})16\cos^2 500t$$
$$= 120\cos^2 500t\,\mathrm{mJ}$$
$$w = [60 + 60\cos 1000t]\,\mathrm{mJ}.$$





[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$\pi \leq t \leq 2\pi$ ms	$0 \leq t \leq \pi$ ms
$3\pi \leq t \leq 4\pi$ ms	$2\pi \leq t \leq 3\pi$ ms

P 6.8 **[a]** $i(0) = A_1 + A_2 = 0.04$

$$\frac{di}{dt} = -10{,}000A_1e^{-10\,000} - 40{,}000A_2e^{-40\,000}$$

$$v = -200A_1e^{-10\,000} - 800A_2e^{-40\,000} \text{ V}$$

$$v(0) = -200A_1 - 800A_2 = 28$$

Solving, $A_1 = 0.1$ and $A_2 = -0.06$

Thus,

$$i = 0.1e^{-10\,000} - 0.06e^{-40\,000} \text{ A}, \qquad t \geq 0$$

$$v = -20e^{-10\,000} + 48e^{-40\,000} \text{ V}, \qquad t \geq 0$$

[b] If $p = 0$ then either $i = 0$ or $v = o$. Suppose $i = 0$:

$$i = 0.1e^{-10\,000} - 0.06e^{-40\,000} = 0$$

$$\therefore \quad 0.1e^{30\,000} = 0.06 \quad \text{so} \quad t = -17.03\,\mu\text{s}$$

This answer is impossible! So assume that $v = 0$:

$$v = -20e^{-10\,000} + 48e^{-40\,000} = 0$$

Then, $\quad -20e^{30\,000} = -48 \quad \therefore \quad t = 29.18\,\mu\text{s}$

This answer makes sense; therefore, the power is 0 at $t = 29.18\,\mu$s.

P 6.9 **[a]** From Problem 6.8 we have

$A_1 + A_2 = 0.04$

Now, we add the second equation for the coefficients:

$-200A_1 - 800A_2 = -68$

Solving, $A_1 = -0.06;\qquad A_2 = 0.1$
Thus,

$i = -0.06e^{-10\,000} + 0.1e^{-40\,000}\ \text{A} \quad t \geq 0$

$v = 12e^{-10\,000} - 80e^{-40\,000}\ \text{A} \quad t \geq 0$

[b] $i = 0 \quad$ when $\quad 0.06e^{-10\,000} = 0.1e^{-40\,000}$

$\therefore \quad e^{30\,0000} = 5/3 \quad$ so $\quad t = 17.03\,\mu\text{s}$

Thus,

$i > 0 \quad$ for $\quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad$ and $\qquad i < 0 \quad$ for $\quad 17.03\,\mu\text{s} \leq t < \infty$

$v = 0 \quad$ when $\quad 12e^{-10\,000} = 80e^{-40\,000}$

$\therefore \quad e^{30\,0000} = 20/3 \quad$ so $\quad t = 63.24\,\mu\text{s}$

Thus,

$v < 0 \quad$ for $\quad 0 \leq t \leq 63.24\,\mu\text{s} \qquad$ and $\qquad v > 0 \quad$ for $\quad 63.24\,\mu\text{s} \leq t < \infty$

Therefore,

$p < 0 \quad$ for $\quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad$ and $\qquad 63.24\,\mu\text{s} \leq t < \infty$

(inductor delivers energy)

$p > 0 \quad$ for $\quad 17.03\,\mu\text{s} \leq t \leq 63.24\,\mu\text{s}$ (inductor stores energy)

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(20 \times 10^{-3})(40 \times 10^{-3})^2 = 16\,\mu\text{J}$$

The power for $t > 0$ is

$p = vi = 6e^{-50\,000} - 8e^{-80\,000} - 0.72e^{-20\,000}$

The energy for $t > 0$ is

$$w = \int_0^\infty p\,dt = \int_0^\infty 6e^{-50\,000}\ dx - \int_0^\infty 8e^{-80\,000}\ dx - \int_0^\infty 0.72e^{-20\,000}\ dx$$

$$= \frac{6}{50{,}000} - \frac{8}{80{,}000} - \frac{0.72}{20{,}000} = -16\,\mu\text{J}$$

Thus, the energy stored at $t = 0$ equals the energy extracted for $t > 0$.

P 6.10 $i = (B_1 \cos 1.6t + B_2 \sin 1.6t)e^{-0\ 4}$

$i(0) = B_1 = 5\,\text{A}$

$$\frac{di}{dt} = (B_1 \cos 1.6t + B_2 \sin 1.6t)(-0.4e^{-0\ 4}\ ) + e^{-0\ 4}\ (-1.6B_1 \sin 1.6t + 1.6B_2 \cos 1.6t)$$

$$= [(1.6B_2 - 0.4B_1)\cos 1.6t - (1.6B_1 + 0.4B_2)\sin 1.6t]e^{-0\ 4}$$

$$v = 2\frac{di}{dt} = [(3.2B_2 - 0.8B_1)\cos 1.6t - (3.2B_1 + 0.8B_2)\sin 1.6t]e^{-0\ 4}$$

$$v(0) = 28 = 3.2B_2 - 0.8B_1 = 3.2B_2 - 4 \quad \therefore \quad B_2 = 32/3.2 = 10\,\text{A}$$

Thus,

$$i = (5\cos 1.6t + 10\sin 1.6t)e^{-0\ 4}\ \ \text{A}, \qquad t \geq 0$$

$$v = (28\cos 1.6t - 24\sin 1.6t)e^{-0\ 4}\ \ \text{V}, \qquad t \geq 0$$

$i(5) = 1.24\,\text{A}; \qquad v(5) = -3.76\,\text{V}$

$p(5) = (1.24)(-3.76) = -4.67\,\text{W}$

The power delivered is 4.67 W.

P 6.11 For $0 \leq t \leq 1.6\,\text{s}$:

$$i \ \ = \frac{1}{5}\int_0 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i\ \ (1.6\,\text{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\text{mA}$$

$$R \ \ = (20)(1000) = 20\,\text{k}\Omega$$

$$v\ \ (1.6\,\text{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\text{V}$$

P 6.12 $p = vi = 40t[e^{-10}\ \ - 10te^{-20}\ \ - e^{-20}\ ]$

$$\text{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10}\ \ - 10xe^{-20}\ \ - e^{-20}\ ]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.13 **[a]**

$$v(20\,\mu\text{s}) = 12.5 \times 10^9 (20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)}$$

$$v(20\,\mu\text{s}) = 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10$$

$$= 5\,\text{V (start of second interval)}$$

$$v(40\,\mu\text{s}) = 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10$$

$$= 10\,\text{V (end of second interval)}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = 62.5\,\text{mW},$

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.14 $i = C(dv/dt)$

$0 < t < 0.5:$

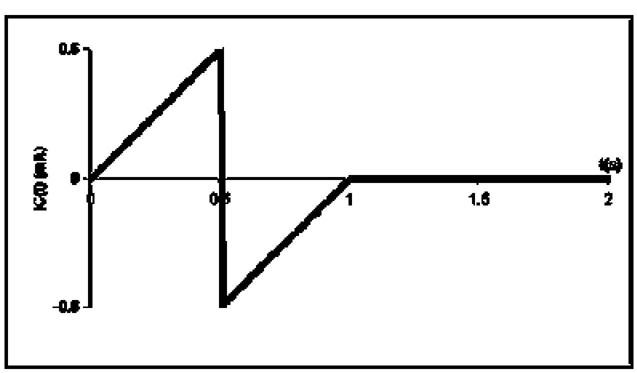
$$v = 30t^2\,\text{V}$$

$$i = 20 \times 10^{-6}(60)t = 1.2t\,\text{mA}$$

$0.5 < t < 1:$

$$v = 30(t-1)^2\,\text{V}$$

$$i = 20 \times 10^{-6}(60)(t-1) = 1.2(t-1)\,\text{mA}$$



P 6.15 **[a]** $0 \le t \le 5\,\mu\text{s}$

$$C = 5\,\mu\text{F} \qquad \frac{1}{C} = 2 \times 10^5$$

$$v = 2 \times 10^5 \int_0 4\,dx + 12$$

$$v = 8 \times 10^5 t + 12\,\text{V} \qquad 0 \le t \le 5\,\mu\text{s}$$

$$v(5\,\mu\text{s}) = 4 + 12 = 16\,\text{V}$$

[b] $5\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{5\times10^{-6}} -2\,dx + 16 = -4 \times 10^5 t + 2 + 16$$

$$v = -4 \times 10^5 t + 18\text{V} \qquad 5 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = -4 \times 10^5 (20 \times 10^{-6}) + 18 = 10\,\text{V}$$

[c] $20\,\mu\text{s} \le t \le 25\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{20\times10^{-6}} 6\,dx + 10 = 12 \times 10^5 t - 24 + 10$$

$$v = 12 \times 10^5 t - 14\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 25\,\mu\text{s}$$

$$v(25\,\mu\text{s}) = 12 \times 10^5 (25 \times 10^{-6}) - 14 = 16\,\text{V}$$

[d] $25\,\mu\text{s} \le t \le 35\,\mu\text{s}$

$$v = 2 \times 10^5 \int_{25\times10^{-6}} 4\,dx + 16 = 8 \times 10^5 t - 20 + 16$$

$$v = 8 \times 10^5 t - 4\,\text{V}, \qquad 25\,\mu\text{s} \le t \le 35\,\mu\text{s}$$

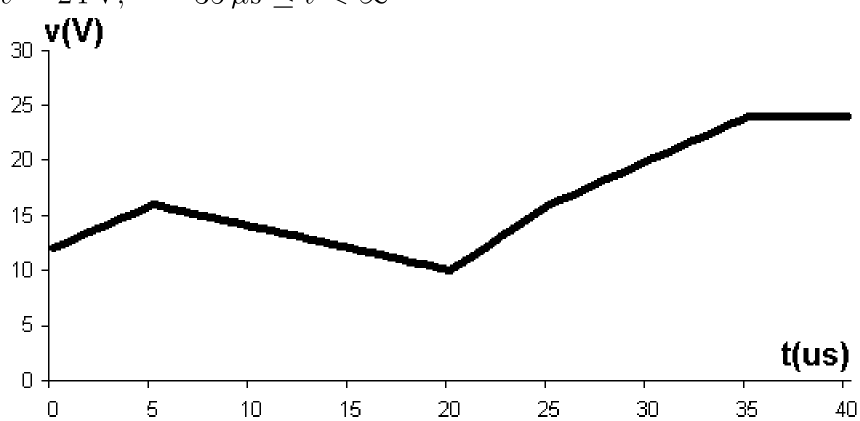
$$v(35\,\mu\text{s}) = 8 \times 10^5 (35 \times 10^{-6}) - 4 = 24\,\text{V}$$

[e] $35\,\mu\text{s} \le t < \infty$

$$v = 2 \times 10^5 \int_{35\times10^{-6}} 0\,dx + 24 = 24$$

$$v = 24\,\text{V}, \qquad 35\,\mu\text{s} \le t < \infty$$

[f]



P 6.16 $v = -10\,\text{V}, \quad t \leq 0; \qquad C = 0.8\,\mu\text{F}$

$$v = 40 - e^{-1000}\,(50\cos 500t + 20\sin 500t)\text{V}, \quad t \geq 0$$

[a] $i = 0, \quad t < 0$

[b]

$$\begin{aligned} \frac{dv}{dt} &= 1000e^{-1000}\,(50\cos 500t + 20\sin 500t) \\ &\quad -e^{-1000}\,(-25{,}000\sin 500t + 10{,}000\cos 500t) \\ &= e^{-1000}\,(50{,}000\cos 500t + 20{,}000\sin 500t \\ &\quad +25{,}000\sin 500t - 10{,}000\cos 500t) \\ &= (40{,}000\cos 500t + 45{,}000\sin 500t)e^{-1000} \\ i &= C\frac{dv}{dt} = (32\cos 500t + 36\sin 500t)e^{-1000}\ \text{mA} \end{aligned}$$

[c] no

[d] yes, from 0 to 32 mA

[e] $v(\infty) = 40\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}$$

P 6.17 **[a]** $i = \dfrac{400 \times 10^{-3}}{5 \times 10^{-6}}t = 8 \times 10^4 t \qquad 0 \leq t \leq 5\,\mu\text{s}$

$$i = 400 \times 10^{-3} \qquad 5 \leq t \leq 20\,\mu\text{s}$$

$$\begin{aligned} q &= \int_0^{5\times10^{-6}} 8 \times 10^4 t\,dt + \int_{5\times10^{-6}}^{15\times10^{-6}} 400 \times 10^{-3}\,dt \\ &= 8 \times 10^4 \frac{t^2}{2}\Big|_0^{5\times10^{-6}} + 400 \times 10^{-3}(10 \times 10^{-6}) \\ &= 8 \times 10^4(\tfrac{1}{2})(25 \times 10^{-12}) + 4 \times 10^{-6} \\ &= 5\,\mu\text{C} \end{aligned}$$

[b] $v = \dfrac{1}{0.25 \times 10^{-6}}\left[\int_0^{5\ \text{s}} 8 \times 10^4 x\,dx + \int_{5\ \text{s}}^{20\ \text{s}} 0.4x\,dx + \int_{20\ \text{s}}^{30\ \text{s}} (10^4 x - 0.5)\,dx\right]$

$$= \frac{1}{0.25 \times 10^{-6}}\left[4 \times 10^4 t^2\Big|_0^{5\ \text{s}} + 0.4t\Big|_{5\ \text{s}}^{20\ \text{s}} + (5000t^2 - 0.5t)\Big|_{20\ \text{s}}^{30\ \text{s}}\right]$$

$$= \frac{1}{0.25 \times 10^{-6}}[1 \times 10^{-6} + 6 \times 10^{-6} - 10.5 \times 10^{-6} + 8 \times 10^{-6}] = 18\text{V}$$

[c] $v(50\,\mu\text{s}) = 18 + \frac{1}{0.25 \times 10^{-6}}(5000t^2 - 0.5t)\Big|_{30\ \ \text{s}}^{50\ \ \text{s}}$

$$= 18 + \frac{1}{0.25 \times 10^{-6}}(-12.5 \times 10^{-6} + 10.5 \times 10^{-6}) = 10\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25 \times 10^{-6})(10)^2 = 12.5\,\mu\text{J}$$

P 6.18 **[a]**

$$v = \frac{1}{0.5 \times 10^{-6}}\int_0^{500\times10^{-6}} 50 \times 10^{-3}e^{-2000}\ dt - 20$$

$$= 100 \times 10^3 \frac{e^{-2000}}{-2000}\Big|_0^{500\times10^{-6}} -20$$

$$= 50(1 - e^{-1}) - 20 = 11.61\,\text{V}$$

$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5 \times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.19 **[a]** $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.25) \times 10^{-6}(50)^2 = 312.5\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-4000}$

$$v(0) = A_2 = 50\,\text{V}$$

$$\frac{dv}{dt} = -4000e^{-4000}\ (A_1t + A_2) + e^{-4000}\ (A_1)$$

$$= (-4000A_1t - 4000A_2 + A_1)e^{-4000}$$

$$\frac{dv}{dt}(0) = A_1 - 4000A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{400 \times 10^{-3}}{0.25 \times 10^{-6}} = 16 \times 10^5$$

$$\therefore\quad 16 \times 10^5 = A_1 - 4000(50)$$

Thus, $A_1 = 16 \times 10^5 + 2 \times 10^5 = 18 \times 10^5\,\frac{\text{V}}{\text{s}}$

[c] $v = (18 \times 10^5 t + 50)e^{-4000}$

$$i = C\frac{dv}{dt} = 0.25 \times 10^{-6}\frac{d}{dt}(18 \times 10^5 t + 50)e^{-4000}$$

$$\begin{aligned} i &= \frac{d}{dt}[(0.45t + 12.5 \times 10^{-6})e^{-4000}] \\ &= (0.45t + 12.5 \times 10^{-6})(-4000)e^{-4000} + e^{-4000}(0.45) \\ &= (-1800t - 0.05 + 0.45)e^{-4000} \\ &= (0.40 - 1800t)e^{-4000}\ \text{A}, \qquad t \geq 0 \end{aligned}$$

P 6.20 $5\|(12 + 8) = 4\,\text{H}$

$4\|4 = 2\,\text{H}$

$15\|(8 + 2) = 6\,\text{H}$

$3\|6 = 2\,\text{H}$

$6 + 2 = 8\,\text{H}$

P 6.21 $30\|20 = 12\,\text{H}$

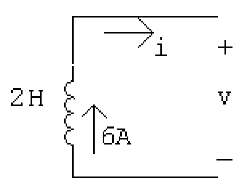
$80\|(8 + 12) = 16\,\text{H}$

$60\|(14 + 16) = 20\,\text{H}$

$15\|(20 + 10) = 20\,\text{H}$

$L_{\text{ab}} = 5 + 10 = 15\,\text{H}$

P 6.22 **[a]**



$$\begin{aligned} i(t) &= -\frac{1}{2}\int_0 12e^{-}\ dx + 6 \\ &= 6e^{-}\Big|_0 + 6 \\ &= 6e^{-} - 6 + 6 \\ i(t) &= 6e^{-}\ \text{A}, \qquad t \geq 0 \end{aligned}$$

[b]
$$\begin{aligned} i_1(t) &= -\frac{1}{3}\int_0 12e^{-} \quad dx + 2 \\ &= 4e^{-} \Big|_0 +2 \\ &= 4(e^{-} \; - 1) + 2 \\ i_1(t) &= 4e^{-} \; - 2\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[c]
$$\begin{aligned} i_2(t) &= -\frac{1}{6}\int_0 12e^{-} \quad dx + 4 \\ &= 2e^{-} \Big|_0 +4 \\ &= 2(e^{-} \; - 1) + 4 \\ i_2(t) &= 2e^{-} \; + 2\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[d] $p = vi = (12e^{-}\;)(6e^{-}\;) = 72e^{-2} \;\; \text{W}$
$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 72e^{-2} \;\; dt \\ &= 72\frac{e^{-2}}{-2} \Big|_0^\infty \\ &= 36\,\text{J} \end{aligned}$$

[e] $w = \frac{1}{2}(3)(2)^2 + \frac{1}{2}(6)(4)^2 = 54\,\text{J}$

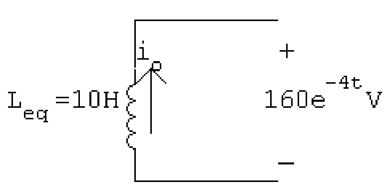
[f] $w_{\text{trapped}} = \frac{1}{2}(3)(-2)^2 + \frac{1}{2}(6)(2)^2 = 18\,\text{J}$

$w_{\text{trapped}} = 54 - 36 = 18\,\text{J}$ checks

[g] Yes, they agree.

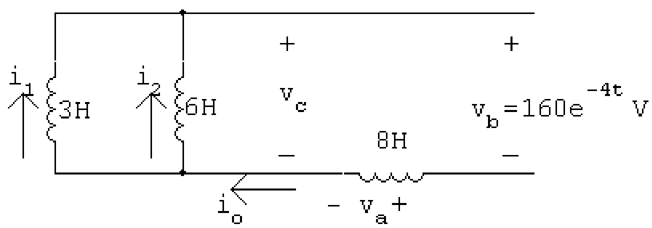
P 6.23 **[a]** $i\;(0) = i_1(0) + i_2(0) = 4\,\text{A}$

[b]



$$\begin{aligned} i &= -\frac{1}{10}\int_0 160e^{-4} \;\; dx + 4 = -16\left[\frac{e^{-4}}{-4}\Big|_0\right] + 4 \\ &= 4(e^{-4} \;\; - 1) + 4 = 4e^{-4} \;\; \text{A}, \qquad t \geq 0 \end{aligned}$$

[c]



$$v = 8\frac{d}{dt}(4e^{-4}) = -128e^{-4}\ \text{V}$$

$$v = v + v = -128e^{-4} + 160e^{-4}$$

$$= 32e^{-4}\ \text{V}$$

$$i_1 = -\frac{1}{3}\int_0 32e^{-4}\ dx + 1$$

$$= 2.67e^{-4} - 2.67 + 1$$

$$i_1 = 2.67e^{-4} - 1.67\,\text{A}, \qquad t \geq 0$$

[d] $$i_2 = -\frac{1}{6}\int_0 32e^{-4}\ dx + 3$$

$$= 1.33e^{-4} - 1.33 + 3$$

$$i_2 = 1.33e^{-4} + 1.67\,\text{A}, \qquad t \geq 0$$

[e] $w(0) = \frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(3)^2 + \frac{1}{2}(8)(4)^2 = 92.5\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(4)^2 = 80\,\text{J}$

[g] $w_{\text{trapped}} = 92.5 - 80 = 12.5\,\text{J}$

P 6.24 $v_{\text{b}} = 160e^{-4}\ \text{V}$

$i = 4e^{-4}\ \text{A}$

$p = 640e^{-8}\ \text{W}$

$$w = \int_0 640e^{-8}\ dx = 640\frac{e^{-8}}{-8}\Big|_0 = 80(1 - e^{-8})\,\text{W}$$

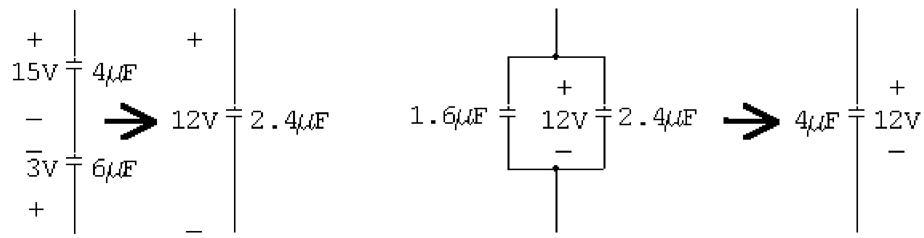
$w_{\text{total}} = 80\,\text{J}$

$w(0.2) = 80(1 - e^{-1\ 6}) = 63.85\,\text{J}$

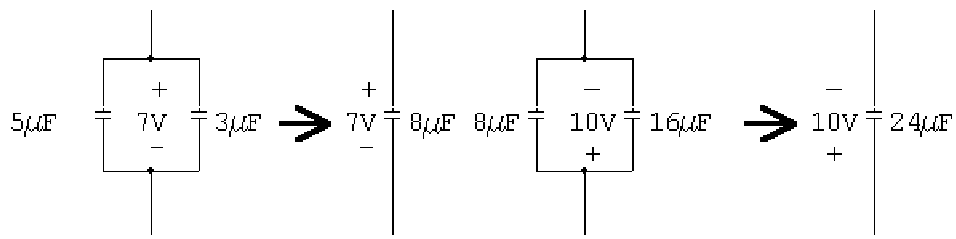
Thus,

$$\%\ \text{delivered} = \frac{63.85}{80}(100) = 79.8\%$$

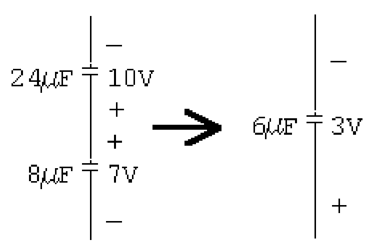
P 6.25 $\frac{1}{4}+\frac{1}{6}=\frac{5}{12} \quad \therefore \quad C_{\rm eq}=2.4\,\mu\text{F}$



$$\frac{1}{4}+\frac{1}{12}=\frac{4}{12} \quad \therefore \quad C_{\rm eq}=3\,\mu\text{F}$$



$$\frac{1}{24}+\frac{1}{8}=\frac{4}{24} \quad \therefore \quad C_{\rm eq}=6\,\mu\text{F}$$



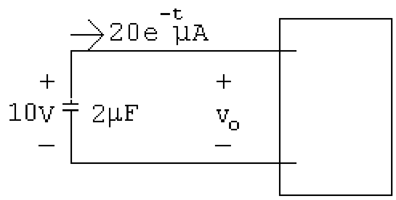
P 6.26 Work from the right hand side of the circuit, simplifying step by step:

1. $48\,\mu$F in series with $16\,\mu$F : $\quad 1/C = 1/16\,\mu + 1/48\,\mu \quad \therefore \quad C = 12\,\mu$F
   The voltages add in series, so the $12\,\mu$F capacitor has a voltage of 20 V, negative at the top.
2. Previous $12\,\mu$F in parallel with $3\,\mu$F : $\quad C = 12\,\mu + 3\,\mu = 15\,\mu$F
   The voltage is 20 V, negative at the top.
3. Previous $15\,\mu$F in series with $30\,\mu$F :
   $1/C = 1/15\,\mu + 1/30\,\mu \quad \therefore \quad C = 10\,\mu$F
   The voltages add in series, so the $10\,\mu$F capacitor has a voltage of 10 V, positive at the right.

4. Previous $10\,\mu$F in parallel with $10\,\mu$F : $C = 10\,\mu + 10\,\mu = 20\,\mu$F
   The voltage is 10 V, negative at the top.
5. Previous $20\,\mu$F in series with $5\,\mu$F and $4\,\mu$F :
   $1/C = 1/20\,\mu + 1/5\,\mu + 1/4\,\mu \quad \therefore \quad C = 2\,\mu$F
   The voltages in series add: $5\text{V} - 10V + 30\text{V} = 25\text{V}$ positive at the top.

The equivalent capacitance is $2\,\mu$F with a voltage of 25 V, positive at the top.

P 6.27 **[a]**



$$v = -\frac{1}{2\times 10^{-6}}\int_0 20\times 10^{-6}e^{-}\ dx + 10$$

$$= 10e^{-}\Big|_0 +10$$

$$= 10e^{-}\ \text{V}, \qquad t \geq 0$$

[b]
$$v_1 = -\frac{1}{3\times 10^{-6}}(20\times 10^{-6})e^{-}\Big|_0 +4$$

$$= 6.67e^{-} - 2.67\,\text{V}, \qquad t \geq 0$$

[c]
$$v_2 = -\frac{1}{6\times 10^{-6}}(20\times 10^{-6})e^{-}\Big|_0 +6$$

$$= 3.33e^{-} + 2.67\,\text{V}, \qquad t \geq 0$$

[d]
$$p = vi = (10e^{-})(20\times 10^{-6})e^{-}$$

$$= 200\times 10^{-6}e^{-2}$$

$$w = \int_0^{\infty} 200\times 10^{-6}e^{-2}\ dt$$

$$= 200\times 10^{-6}\frac{e^{-2}}{-2}\Big|_0^{\infty}$$

$$= -100\times 10^{-6}(0-1) = 100\,\mu\text{J}$$

[e]
$$w = \tfrac{1}{2}(3\times 10^{-6})(4)^2 + \tfrac{1}{2}(6\times 10^{-9})(6)^2$$

$$= 132\,\mu\text{J}$$

[f]
$$w_{\text{trapped}} = \tfrac{1}{2}(3\times 10^{-6})(8/3)^2 + \tfrac{1}{2}(6\times 10^{-6})(8/3)^2$$

$$= 32\,\mu\text{J}$$

CHECK: $100 + 32 = 132\,\mu\text{J}$

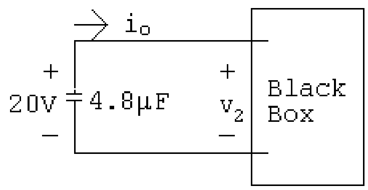
[g] Yes, they agree.

P 6.28 $C_1 = 10 + 2 = 12\,\mu\text{F}$

$$\frac{1}{C_2} = \frac{1}{12\,\mu} + \frac{1}{8\,\mu} \qquad \therefore \qquad C_2 = 4.8\,\mu\text{F}$$

$$v\ (0) + v_1(0) = -5 + 25 = 20\,\text{V}$$

[a]



$$\begin{aligned} v_2 &= -\frac{1}{4.8\times10^{-6}}\int_0 1.92\times10^{-3}e^{-20}\ dx + 20 \\ &= -400\frac{e^{-20}}{-20}\Big|_0 +20 \\ &= 20(e^{-20}\ -1)+20 \\ &= 20e^{-20}\ \text{V}, \qquad t\geq 0 \end{aligned}$$

[b]

$$\begin{aligned} v &= -\frac{1}{8\times10^{-6}}\int_0 1.92\times10^{-3}e^{-20}\ dx - 5 \\ &= -240\frac{e^{-20}}{-20}\Big|_0 -5 \\ &= 12(e^{-20}\ -1)-5 \\ &= 12e^{-20}\ -17\,\text{V}, \qquad t\geq 0 \end{aligned}$$

[c]

$$\begin{aligned} v_1 &= -\frac{1}{12\times10^{-6}}\int_0 1.92\times10^{-3}e^{-20}\ dx + 25 \\ &= -160\frac{e^{-20}}{-20}\Big|_0 +25 \\ &= 8(e^{-20}\ -1)+25 \\ &= 8e^{-20}\ +17\,\text{V}, \qquad t\geq 0 \end{aligned}$$

[d]
$$i_1 = -10 \times 10^{-6} \frac{d}{dt}[8e^{-20} + 17]$$
$$= -10 \times 10^{-6}(-20)8e^{-20}$$
$$= 1.6e^{-20} \text{ mA}, \qquad t > 0$$

[e]
$$i_2 = -2 \times 10^{-6} \frac{d}{dt}[8e^{-20} + 17]$$
$$= -2 \times 10^{-6}(-20)8e^{-20}$$
$$= 0.32e^{-20} \text{ mA}, \qquad t > 0$$

CHECK: $i_1 + i_2 = 1.92e^{-20} \text{ mA} = i$

P 6.29 **[a]**
$$w(0) = [\tfrac{1}{2}(8 \times 10^{-6})(-5)^2 + \tfrac{1}{2}(10 \times 10^{-6})(25)^2 + \tfrac{1}{2}(2 \times 10^{-6})(25)^2]$$
$$= 3850\,\mu\text{J}$$

[b]
$$v\,(\infty) = -17\,\text{V}$$
$$v_1(\infty) = 17\,\text{V}$$
$$w(\infty) = [\tfrac{1}{2}(8 \times 10^{-6})(-17)^2 + \tfrac{1}{2}(12 \times 10^{-6})(17)^2]$$
$$= 2890\,\mu\text{J}$$

[c] $w = \int_0^{\infty} (20e^{-20})(1.92 \times 10^{-3}e^{-20})\,dt = 960\,\mu\text{J}$

CHECK: $3850 - 2890 = 960\,\mu\text{J}$

[d] $\%$ delivered $= \dfrac{960}{3850} \times 100 = 24.9\%$

[e]
$$w(40\,\text{ms}) = \int_0^{0\ 04} (20e^{-20})(1.92 \times 10^{-3}e^{-20})\,dt$$
$$= 0.0384\frac{e^{-40}}{-40}\Big|_0^{0\ 04}$$
$$= 960 \times 10^{-6}(1 - e^{-1\ 6}) = 766.2\,\mu\text{J}$$

% delivered $= \dfrac{766.2}{960}(100) = 79.8\%$

P 6.30 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0 i + v_1(0) + \frac{1}{C_2}\int_0 i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0 i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\frac{1}{C_{eq}} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right], \qquad v_{eq}(0) = v_1(0) + v_2(0) + \cdots$

P 6.31 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{eq} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.32

$$\begin{aligned}\frac{di}{dt} &= 5\{e^{-2000}\,[-8000\sin 4000t + 4000\cos 4000t] \\ &\quad -2000e^{-2000}\,[2\cos 4000t + \sin 4000t]\} \\ \frac{di}{dt}(0^+) &= 5[1(4000) + (-2000)(2)] = 0 \\ v_2(0^+) &= 10\times 10^{-3}\frac{di}{dt}(0^+) = 0 \\ v_1(0^+) &= 40i\ (0^+) + v_2(0^+) = 40(10) + 0 = 400\text{V}\end{aligned}$$

P 6.33

$$\begin{aligned}v_c &= -\frac{1}{0.625\times 10^{-6}}\left(\int_0 1.5e^{-16\,000}\ dx - \int_0 0.5e^{-4000}\ dx\right) - 50 \\ &= 150(e^{-16\,000} - 1) - 200(e^{-4000} - 1) - 50 \\ &= 150e^{-16\,000} - 200e^{-4000}\ \text{V} \\ v &= 25\times 10^{-3}\frac{di}{dt} \\ &= 25\times 10^{-3}(-24{,}000e^{-16\,000} + 2000e^{-4000}) \\ &= -600e^{-16\,000} + 50e^{-4000}\ \text{V} \\ v &= v_c - v \\ &= (150e^{-16\,000} - 200e^{-4000}) - (-600e^{-16\,000} + 50e^{-4000}) \\ &= 750e^{-16\,000} - 250e^{-4000}\ \text{V}, \qquad t > 0\end{aligned}$$

P 6.34 **[a]** $-2\frac{di}{dt} + 16\frac{di_2}{dt} + 32i_2 = 0$

$$16\frac{di_2}{dt} + 32i_2 = 2\frac{di}{dt}$$

[b] $i_2 = e^{-} - e^{-2}\ \text{A}$

$$\frac{di_2}{dt} = -e^{-} + 2e^{-2}\ \text{A/s}$$

$$i = 8 - 8e^{-} \text{ A}$$

$$\frac{di}{dt} = 8e^{-} \text{ A/s}$$

$$\therefore \quad -16e^{-} + 32e^{-2} + 32e^{-} - 32e^{-2} = 16e^{-}$$

[c]
$$\begin{aligned} v_1 &= 4\frac{di}{dt} - 2\frac{di_2}{dt} \\ &= 4(8e^{-}) - 2(-e^{-} + 2e^{-2}) \\ &= 34e^{-} - 4e^{-2} \text{ V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = 34 - 4 = 30\,\text{V}$; Also

$$\begin{aligned} v_1(0) &= 4\frac{di}{dt}(0) - 2\frac{di_2}{dt}(0) \\ &= 4(8) - 2(-1+2) = 32 - 2 = 30\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.35 **[a]** Yes, $v = 20(i_2 - i_1) + 60i_2$

[b]
$$\begin{aligned} v &= 20(1 - 52e^{-5} + 51e^{-4} - 4 - 64e^{-5} + 68e^{-4})+ \\ &\quad 60(1 - 52e^{-5} + 51e^{-4}) \\ &= 20(-3 - 116e^{-5} + 119e^{-4}) + 60 - 3120e^{-5} + 3060e^{-4} \\ v &= -5440e^{-5} + 5440e^{-4} \text{ V} \end{aligned}$$

[c]
$$\begin{aligned} v &= L_2\frac{d}{dt}(i - i_2) + M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15 + 36e^{-5} - 51e^{-4}) + 8\frac{d}{dt}(4 + 64e^{-5} - 68e^{-4}) \\ &= -2880e^{-5} + 3264e^{-4} - 2560e^{-5} + 2176e^{-4} \\ v &= -5440e^{-5} + 5440e^{-4} \text{ V} \end{aligned}$$

P 6.36 **[a]**
$$\begin{aligned} v &= 5(i - i_1) + 20(i_2 - i_1) + 60i_2 \\ &= 5(16 - 16e^{-5} - 4 - 64e^{-5} + 68e^{-4})+ \\ &\quad 20(1 - 52e^{-5} + 51e^{-4} - 4 - 64e^{-5} + 68e^{-4})+ \\ &\quad 60(1 - 52e^{-5} + 51e^{-4}) \\ &= 60 + 5780e^{-4} - 5840e^{-5} \text{ V} \end{aligned}$$

[b] $v\ (0) = 60 + 5780 - 5840 = 0\,\text{V}$

[c] $p_{\text{dev}} = v\ i$

$= 960 + 92{,}480e^{-4} - 94{,}400e^{-5} - 92{,}480e^{-9} + 93{,}440e^{-10}$ W

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i\ (\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.37 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i\ - 8\frac{di}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5} + 272e^{-4}$$

$$\frac{di_2}{dt} = 260e^{-5} - 204e^{-4}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5} + 1088e^{-4}$$

$$25i_1 = 100 + 1600e^{-5} - 1700e^{-4}$$

$$8\frac{di_2}{dt} = 2080e^{-5} - 1632e^{-4}$$

$$20i_2 = 20 - 1040e^{-5} + 1020e^{-4}$$

$$5i\ = 80 - 80e^{-5}$$

$$8\frac{di}{dt} = 640e^{-5}$$

Thus,

$$-1280e^{-5} + 1088e^{-4} + 100 + 1600e^{-5} - 1700e^{-4} - 2080e^{-5}$$
$$+1632e^{-4} - 20 + 1040e^{-5} - 1020e^{-4} \stackrel{?}{=} 80 - 80e^{-5} - 640e^{-5}$$
$$80 + (1088 - 1700 + 1632 - 1020)e^{-4}$$
$$+(1600 - 1280 - 2080 + 1040)e^{-5} \stackrel{?}{=} 80 - 720e^{-5}$$
$$80 + (2720 - 2720)e^{-4} + (2640 - 3360)e^{-5} = 80 - 720e^{-5} \quad \text{(OK)}$$
$$8\frac{di_1}{dt} = -2560e^{-5} + 2176e^{-4}$$
$$20i_1 = 80 + 1280e^{-5} - 1360e^{-4}$$
$$16\frac{di_2}{dt} = 4160e^{-5} - 3264e^{-4}$$
$$80i_2 = 80 - 4160e^{-5} + 4080e^{-4}$$
$$16\frac{di}{dt} = 1280e^{-5}$$
$$2560e^{-5} - 2176e^{-4} - 80 - 1280e^{-5} + 1360e^{-4} + 4160e^{-5} - 3264e^{-4}$$
$$+80 - 4160e^{-5} + 4080e^{-4} \stackrel{?}{=} 1280e^{-5}$$
$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5}$$
$$+(1360 - 2176 - 3264 + 4080)e^{-4} \stackrel{?}{=} 1280e^{-5}$$
$$0 + 1280e^{-5} + 0e^{-4} = 1280e^{-5} \quad \text{(OK)}$$

P 6.38 **[a]** $L_2 = \left(\frac{M^2}{k^2 L_1}\right) = \frac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{0.288}{(1200)^2} = 0.2 \times 10^{-6}$ Wb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6} \text{ Wb/A}$$

P 6.39 $\mathcal{P}_1 = \frac{L_1}{N_1^2} = 2$ nWb/A; $\mathcal{P}_2 = \frac{L_2}{N_2^2} = 2$ nWb/A; $M = k\sqrt{L_1 L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.40 **[a]** $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{7.2}{\sqrt{81}} = 0.8$

[b] $M = \sqrt{81} = 9\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{27}{3} = 9$$

$$\frac{N_1}{N_2} = 3$$

P 6.41 **[a]** $M = k\sqrt{L_1L_2} = 0.8\sqrt{324} = 14.4\,\text{mH}$

$$\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{36\times10^{-3}}{(200)^2} = 900 \text{ nWb/A}$$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.1; \qquad \mathcal{P}_{21} = 10\mathcal{P}_{11}$$

$$\mathcal{P}_1 = \mathcal{P}_{11} + \mathcal{P}_{21} = 11\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = \frac{1}{11}\mathcal{P}_1 = 81.82 \text{ nWb/A}$$

$$\mathcal{P}_{21} = 10\mathcal{P}_{11} = 818.18 \text{ nWb/A}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{14.4\times10^{-3}}{(200)(818.18\times10^{-9})} = 88 \text{ turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{9\times10^{-3}}{(88)^2} = 1162.19 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 81.82$ nWb/A [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{P_{22}}{P_{12}}$

$$P_{12} = P_{21} = 818.18 \text{ nWb/A}$$

$$P_{22} = P_2 - P_{12} = 1162.19\times10^{-9} - 818.18\times10^{-9} = 344.01 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{344.01}{818.18} = 0.4205$$

P 6.42 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.43 **[a]** $\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.44 **[a]** $v_{\text{ab}} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{\text{ab}} = (L_1 + L_2 + 2M)$

[b] $v_{\text{ab}} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{\text{ab}} = (L_1 + L_2 - 2M)$

P 6.45 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks upscale, the induced voltage across the voltmeter must be positive at its positive terminal. Therefore, the voltage is negative at the negative terminal of the voltmeter.

Thus, the lower terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the lower terminal of the unmarked coil.

P 6.46 **[a]** $v_{\rm ab} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\rm ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\rm ab}$$

from which we have

$$v_{\rm ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\rm ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.47 **[a]** $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.48 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

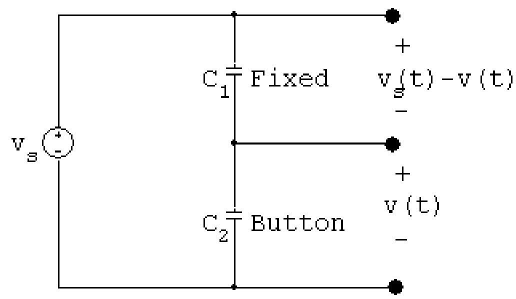
Therefore $\quad 16i_2^2 + 144i_2 + 324 = 0, \qquad i_2^2 + 9i_2 + 20.25 = 0$

Therefore $\quad i_2 = -\left(\frac{9}{2}\right) \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$

Therefore $\quad i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 When the button is not pressed we have



$$C_2\frac{dv}{dt} = C_1\frac{d}{dt}(v\ \ - v)$$

or

$$(C_1 + C_2)\frac{dv}{dt} = C_1\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{C_1}{(C_1 + C_2)}\frac{dv}{dt}$$

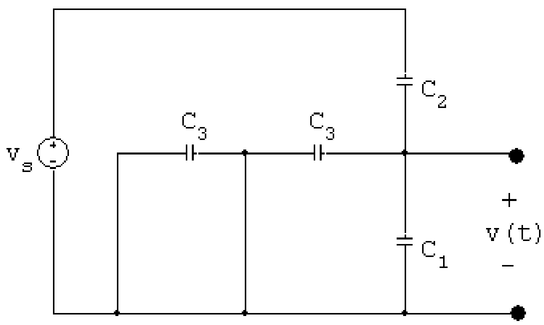
Assuming $\quad C_1 = C_2 = C$

$$\frac{dv}{dt} = 0.5\frac{dv}{dt}$$

or

$$v = 0.5v\ \ (t) + v(0)$$

When the button is pressed we have



$$C_1\frac{dv}{dt} + C_3\frac{dv}{dt} + C_2\frac{d(v - v\ )}{dt} = 0$$

$$\therefore \quad \frac{dv}{dt} = \frac{C_2}{C_1 + C_2 + C_3}\,\frac{dv}{dt}$$

Assuming $\quad C_1 = C_2 = C_3 = C$

$$\frac{dv}{dt} = \frac{1}{3}\,\frac{dv}{dt}$$

$$v = \frac{1}{3}v\ (t) + v(0)$$

Therefore interchanging the fixed capacitor and the button has no effect on the change in $v(t)$.

P 6.50 With no finger touching and equal 10 pF capacitors

$$v(t) = \frac{10}{20}(v\ (t)) + 0 = 0.5v\ (t)$$

With a finger touching

Let $\quad C\ $ = equivalent capacitance of person touching lamp

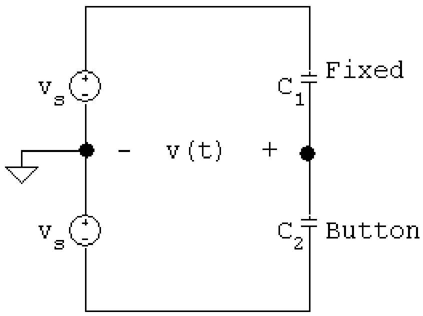
$$C\ = \frac{(10)(100)}{110} = 9.091 \text{ pF}$$

Then $\qquad C + C\ = 10 + 9.091 = 19.091$ pF

$$\therefore \quad v(t) = \frac{10}{29.091}v\ = 0.344v$$

$$\therefore \quad \Delta v(t) = (0.5 - 0.344)v\ = 0.156v$$

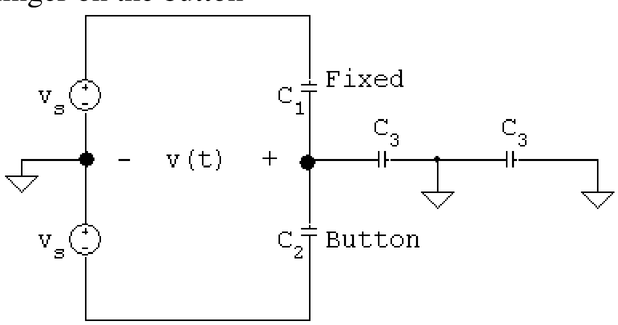
P 6.51 With no finger on the button the circuit is



$$C_1 \frac{dv}{dt}(v - v\ ) + C_2 \frac{d}{dt}(v + v\ ) = 0$$

when $\quad C_1 = C_2 = C \qquad (2C)\frac{dv}{dt} = 0$

With a finger on the button



$$C_1 \frac{d(v - v\ )}{dt} + C_2 \frac{d(v + v\ )}{dt} + C_3 \frac{dv}{dt} = 0$$

$$(C_1 + C_2 + C_3)\frac{dv}{dt} + C_2 \frac{dv}{dt} - C_1 \frac{dv}{dt} = 0$$

when $\quad C_1 = C_2 = C_3 = C \qquad (3C)\frac{dv}{dt} = 0$

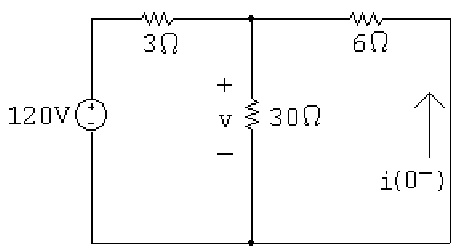
$\therefore$ there is no change in the output voltage of this circuit.

7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$
Now find the current using Ohm's law:
$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

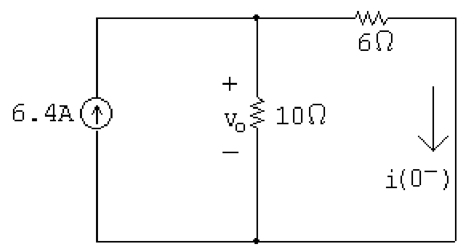
[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

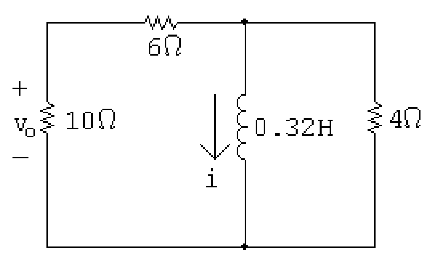
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$
$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$
$\%\text{ dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,
$i(0^-) = \dfrac{10}{10+6}(6.4) = 4\,\text{A}$
Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \dfrac{L}{R_{\text{eq}}} = \dfrac{0.32}{3.2} = 0.1\,\text{s}$
Use the initial inductor current and the time constant to find the current in the inductor:
$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$
Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:
$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

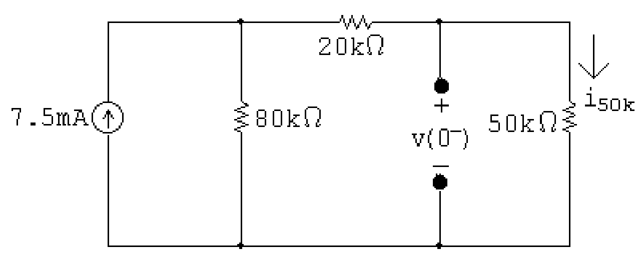
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^{\infty} 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

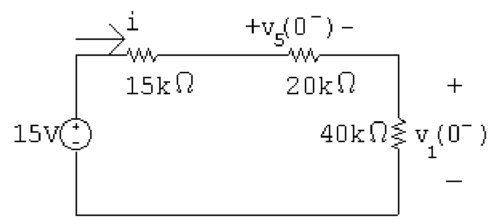
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 **[a]** This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

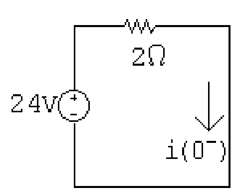
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

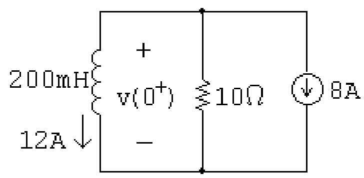
$\%$ dissipated $= (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$
Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

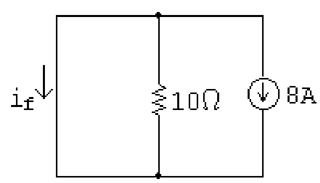
[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.



$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,
$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



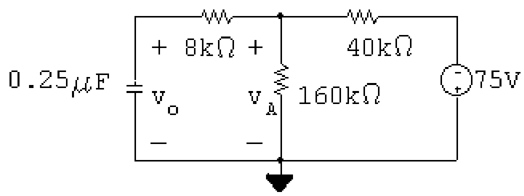
Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,
$i_f = -8\,\text{A}$
Now,

$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$$
$$= -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}$ V

Write a KVL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

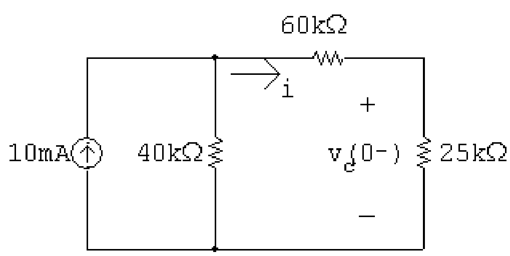
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

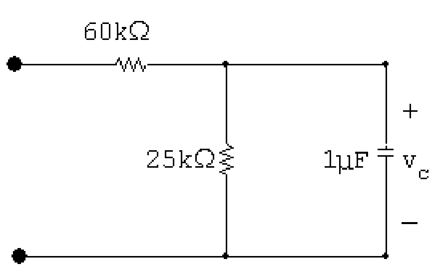
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10$ ms, to calculate $v_c(t)$ for that interval:

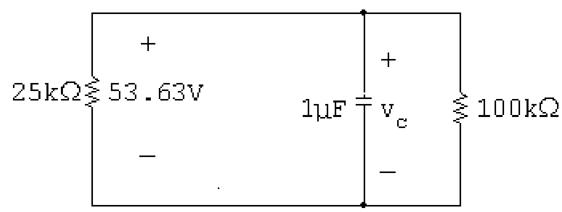


For $0 \le t \le 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \ge 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \ge 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $t \ge 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \ge 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000} dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000} dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000} dt = 0.29\,\text{mJ}$$

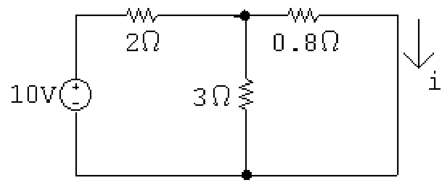
We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the $25\,\text{k}\Omega$ resistor and the $100\,\text{k}\Omega$ resistor.

Check: $w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$$

AP 7.8 **[a]** Note – the $30\,\Omega$ resistor should be a $3\,\Omega$ resistor; the resistor in parallel with the 8 A current source should be $9\,\Omega$.
Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.

For $0 \leq t \leq 1$ s,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the $0.8\,\Omega$ resistor is

$$i(\infty) = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$0.8 + (2\|3) = 2\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2} = 1\,\text{s}$$

Therefore,

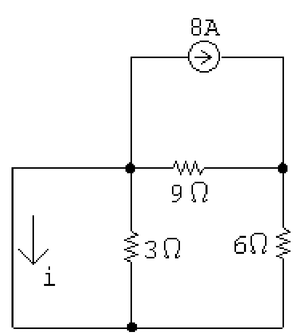
$$i = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = 3 - 3e^{-t}\,\text{A}, \quad 0 \leq t \leq 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-1} = 1.896\,\text{A}$$

.

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9 + 6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

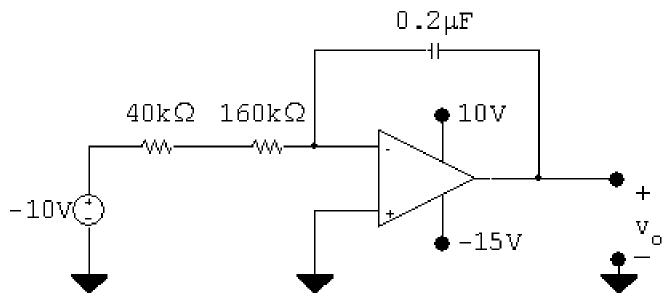
$$3\|(9 + 6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i(\infty) + [i(1^+) - i(\infty)]e^{-(t-1)/\tau}$$

$$= -4.8 + 6.696e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

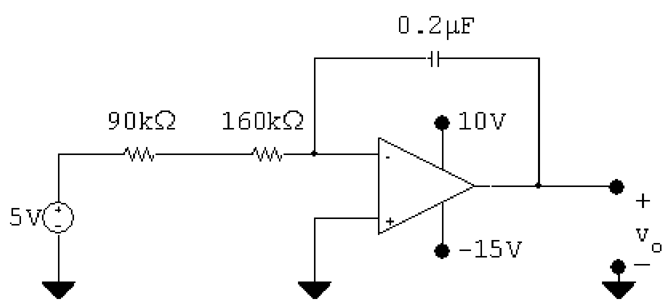


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32$ ms:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32\times10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 **[a]** Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$$

$$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t}\ \text{V}$$

The analysis for v_o is the same as in part (a):

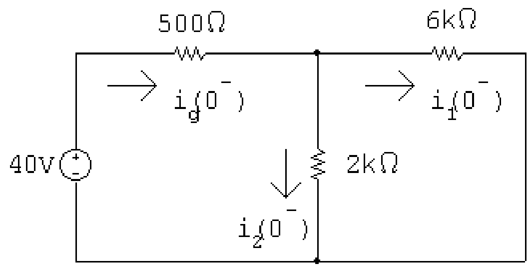
$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 **[a]** $t < 0$



$2\,\mathrm{k}\Omega \| 6\,\mathrm{k}\Omega = 1.5\mathrm{k}\Omega$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\mathrm{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\mathrm{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\mathrm{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\mathrm{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\mathrm{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.4 \times 10^{-3}}{8 \times 10^3} = 5 \times 10^{-5}\,\mathrm{s}; \qquad \dfrac{1}{\tau} = 20{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\mathrm{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\mathrm{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal $15\,\mathrm{mA}$ and $i_2(0^+) = -5\,\mathrm{mA}$.

P 7.2 **[a]** $i(0) = 60\,\mathrm{V}/(10\,\Omega + 5\,\Omega) = 4\,\mathrm{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{45 + 5} = 80\,\mathrm{ms}$

[c] $i = 4e^{-t/0.08} = 4e^{-12.5t}\,\text{A}, \qquad t \geq 0$

$$v_1 = -45i = -180e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = L\frac{di}{dt} = (4)(-12.5)(4e^{-12.5t}) = -200e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

[d] $p_{\text{diss}} = i^2(45) = 720e^{-25t}\,\text{W}$

$$w_{\text{diss}} = \int_0^t 720e^{-25x}\,dx = 720\frac{e^{-25x}}{-25}\Big|_0^t = 28.8 - 28.8e^{-25t}\,\text{J}$$

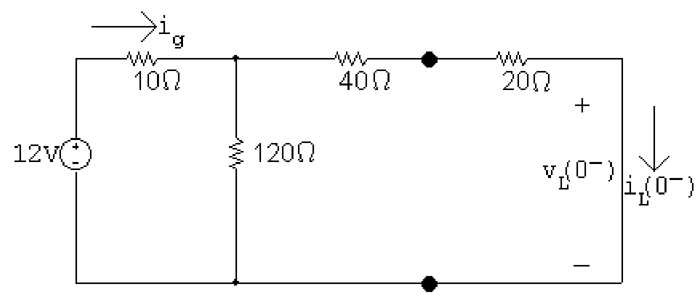
$$w_{\text{diss}}(40\,\text{ms}) = 28.8 - 28.8e^{-1} = 18.205\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(4)^2 = 32\,\text{J}$$

$$\%\text{ dissipated } = \frac{18.205}{32}(100) = 56.89\%$$

P 7.3 **[a]** $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

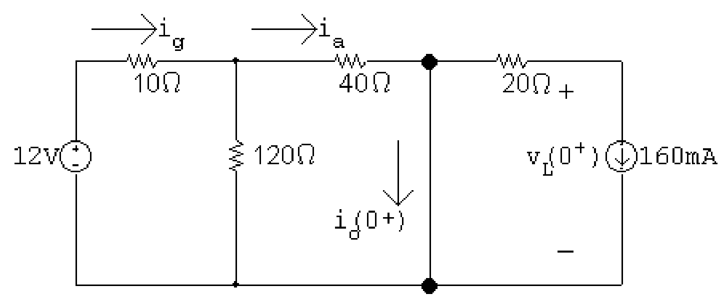


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_{\text{a}} = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_{\text{a}} = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_{\text{a}} - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

P 7.4 **[a]** $\dfrac{v}{i} = R = \dfrac{400e^{-5t}}{10e^{-5t}} = 40\,\Omega$

[b] $\tau = \dfrac{1}{5} = 200\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 200 \times 10^{-3}$

$$L = (200 \times 10^{-3})(40) = 8\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(8)(10)^2 = 400\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 4000e^{-10x}\,dx = 400 - 400e^{-10t}$

$$0.8w(0) = (0.8)(400) = 320\,\text{J}$$

$$400 - 400e^{-10t} = 320 \qquad \therefore \quad e^{10t} = 5$$

Solving, $t = 160.9$ ms.

P 7.5 **[a]** $i_L(0) = \frac{12}{6} = 2\,\text{A}$

$$i_o(0^+) = \frac{12}{2} - 2 = 6 - 2 = 4\,\text{A}$$

$$i_o(\infty) = \frac{12}{2} = 6\,\text{A}$$

[b] $i_L = 2e^{-t/\tau}; \qquad \tau = \frac{L}{R} = \frac{1}{4}\,\text{s}$

$$i_L = 2e^{-4t}\,\text{A}$$

$$i_o = 6 - i_L = 6 - 2e^{-4t}\,\text{A}, \qquad t \geq 0^+$$

[c] $6 - 2e^{-4t} = 5$

$$1 = 2e^{-4t}$$

$$e^{6t} = 2 \qquad \therefore \quad t = 173.3\,\text{ms}$$

P 7.6 $w(0) = \frac{1}{2}(30 \times 10^{-3})(3^2) = 135\,\text{mJ}$

$$\frac{1}{5}w(0) = 27\,\text{mJ}$$

$$i_R = 3e^{-t/\tau}$$

$$p_{\text{diss}} = i_R^2 R = 9Re^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^t R(9)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 9R\frac{e^{-2x/\tau}}{-2/\tau}\bigg|_0^{t_o} = -4.5\tau R(e^{-2t_o/\tau} - 1) = 4.5L(1 - e^{-2t_o/\tau})$$

$$4.5L = (4.5)(30) \times 10^{-3} = 0.135; \qquad t_o = 15\,\mu\text{s}$$

$$1 - e^{-2t_o/\tau} = \frac{1}{5}$$

$$e^{2t_o/\tau} = 1.25; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 1.25$$

$$R = \frac{L\ln 1.25}{2t_o} = \frac{30 \times 10^{-3}\ln 1.25}{30 \times 10^{-6}} = 223.14\,\Omega$$

P 7.7 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\rm diss} = \int_0^{t_o} I_g^2 Re^{-2t/\tau}\,dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\rm diss} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \tau\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

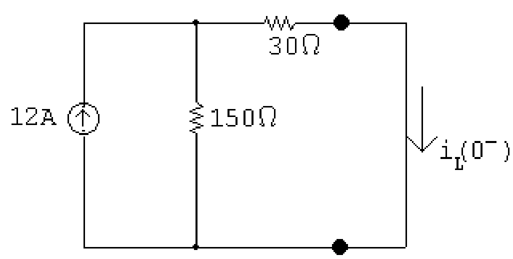
$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(30\times10^{-3})\ln[1/0.8]}{30\times10^{-6}}$

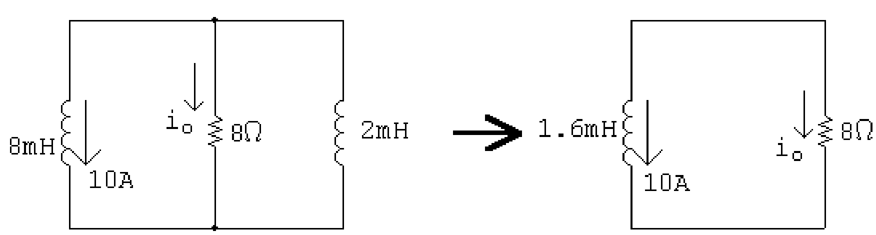
$$R = 223.14\,\Omega$$

P 7.8 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$$t \geq 0$$



$$\tau = \frac{1.6\times10^{-3}}{8} = 200\times10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \frac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95w_{\text{del}} = 76\,\text{mJ}$

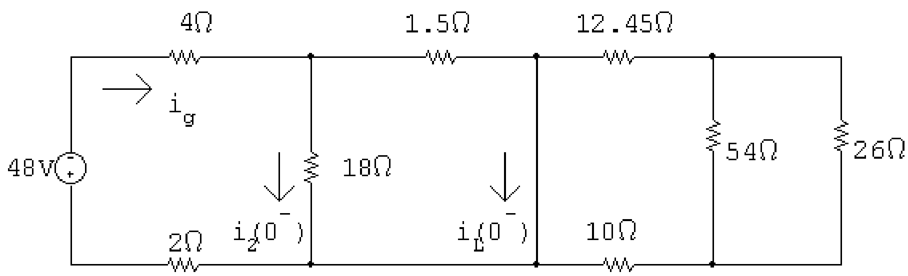
$$\therefore \quad 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76 \times 10^{-3} = -80 \times 10^{-3}e^{-10{,}000t}\Big|_0^{t_o} = 80 \times 10^{-3}(1 - e^{-10{,}000t_o})$$

$$\therefore \quad e^{-10{,}000t_o} = 4 \times 10^{-3} \quad \text{so} \quad t_o = 552.1\,\mu\text{s}$$

$$\therefore \quad \frac{t_o}{\tau} = \frac{552.1 \times 10^{-6}}{200 \times 10^{-6}} = 2.76 \quad \text{so} \quad t_o \approx 2.76\tau$$

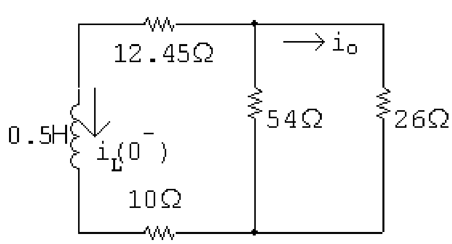
P 7.9 For $t < 0^+$



$$i_g = \frac{-48}{6 + (18\|1.5)} = -6.5\,\text{A}$$

$$i_L(0^-) = \frac{18}{18 + 1.5}(-6.5) = -6\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10 + 12.45 + (54\|26)} = 0.0125\,\text{s}; \qquad \frac{1}{\tau} = 80$$

$$i_L(t) = -6e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

P 7.10 From the solution to Problem 7.9,

$$i_{54\Omega} = \frac{26}{80}(-i_L) = -1.95e^{-80t}\,\text{A}$$

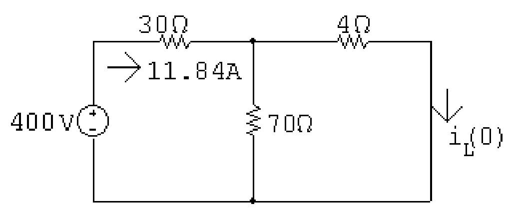
$$P_{54\Omega} = 54(i_{54\Omega})^2 = 205.335e^{-160t}\text{W}$$

$$w_{\text{diss}} = \int_0^{0.0125} 205.335e^{-160t}\,dt$$
$$= \frac{205.335}{-160}e^{-160t}\Big|_0^{0.0125}$$
$$= 1.28(1 - e^{-2}) = 1.11\,\text{J}$$

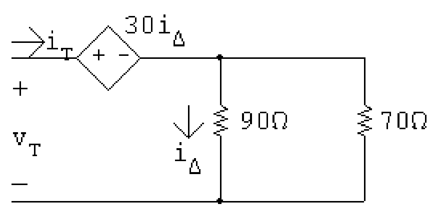
$$w_{\text{stored}} = \frac{1}{2}(0.5)(-6)^2 = 9\,\text{mJ}.$$

$$\%\ \text{diss} = \frac{1.11}{9} \times 100 = 12.3\%$$

P 7.11 **[a]** $t < 0$:



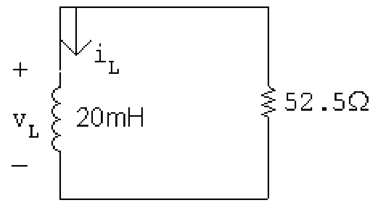
$$i_L(0^-) = i_L(0^+) = \frac{70}{70+4}(11.84) = 11.2\,\text{A}$$



$$i_\Delta = \frac{70}{160}i_T = 0.4375i_T$$

$$v_T = 30i_\Delta + i_T\frac{(90)(70)}{160} = 30(0.4375)i_T + \frac{(90)(70)}{160}i_T = 52.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 52.5\,\Omega$$

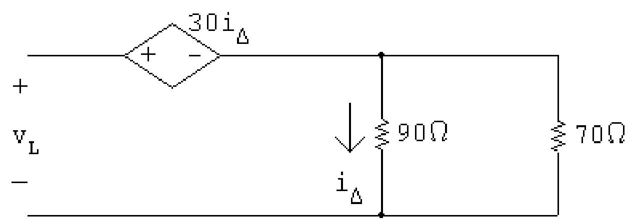


$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{52.5} = \quad \therefore \qquad \frac{1}{\tau} = 2625$$

$$i_L = 11.2e^{-2625t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 20 \times 10^{-3}(-2625)(11.2e^{-2625t}) = -588e^{-2625t}\,\text{V}, \quad t \geq 0^+$

[c]



$$v_L = 30i_\Delta + 90i_\Delta = 120i_\Delta$$

$$i_\Delta = \frac{v_L}{120} = -4.9e^{-2625t}\,\text{A} \qquad t \geq 0^+$$

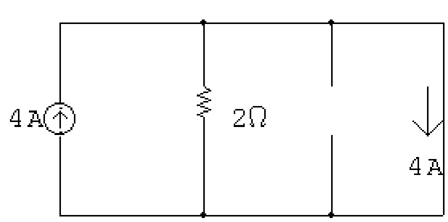
P 7.12 $w(0) = \dfrac{1}{2}(20 \times 10^{-3})(11.2)^2 = 1254.4\,\text{mJ}$

$$p_{30i_\Delta} = -30i_\Delta i_L = -30(-4.9e^{-2625t})(11.2e^{-2625t}) = 1646.4e^{-5250t}\,\text{W}$$

$$w_{30i_\Delta} = \int_0^\infty 1646.4e^{-5250t}\,dt = 1646.4\frac{e^{-5250t}}{-5250}\Big|_0^\infty = 313.6\text{mJ}$$

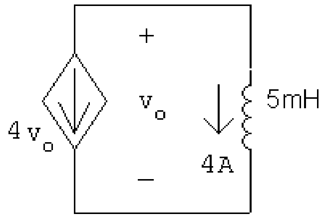
$$\%\ \text{dissipated} = \frac{313.6}{1254.4}(100) = 25\%$$

P 7.13 $t < 0$

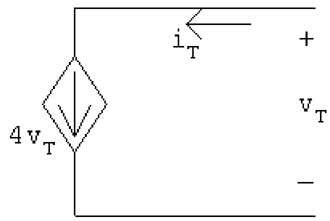


$$i_L(0^-) = i_L(0^+) = 4\,\text{A}$$

$t > 0$

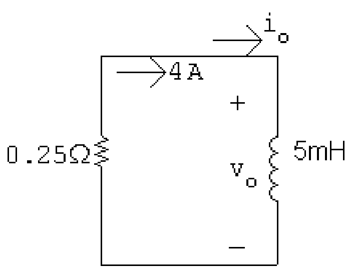


Find Thévenin resistance seen by inductor



$$i_T = 4v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{4} = 0.25\,\Omega$$

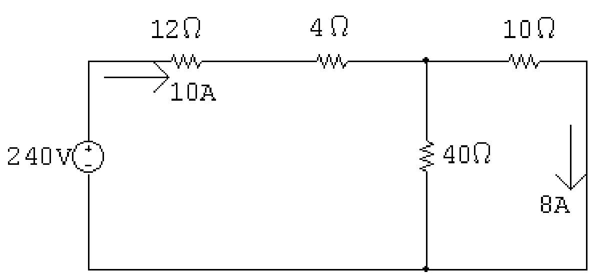
$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{0.25} = 20\,\text{ms}; \qquad 1/\tau = 50$$



$$i_o = 4e^{-50t}\,\text{A}, \qquad t \geq 0$$

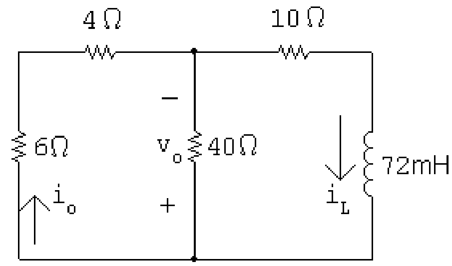
$$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-200e^{-50t}) = -e^{-50t}\,\text{V}, \quad t \geq 0^+$$

P 7.14 $t < 0$:



$$i_L(0^+) = 8\,\text{A}$$

$t > 0$:



$$R_e = \frac{(10)(40)}{50} + 10 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{0.072}{18} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 8e^{-250t}\,\text{A}$$

$$\therefore \quad v_o = -10i_L - 0.072\frac{di_L}{dt} = -80e^{-250t} + 144e^{-250t}$$

$$= 64e^{-250t}\,\text{A} \quad t \geq 0^+$$

P 7.15 $$w(0) = \frac{1}{2}(72 \times 10^{-3})(8)^2 = 2304\,\text{mJ}$$

$$p_{40\Omega} = \frac{v_o^2}{40} = \frac{64^2}{40}e^{-500t} = 102.4e^{-500t}\,\text{W}$$

$$w_{40\Omega} = \int_0^\infty 102.4e^{-500t}\,dt = 204.8\,\text{mJ}$$

$$\%\text{diss} = \frac{204.8}{2304}(100) = 8.89\%$$

P 7.16 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-1\times10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{1\times10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{1\times10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10\times10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)30 \times 10^{-3} = -30\,\text{mV}$

$$\therefore \qquad v_o = -0.03e^{-t/\tau}\,\text{V}, \quad t \geq 0^+$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

$$w_{10\Omega}(1\,\text{ms}) = \int_{0^+}^{10^{-3}} 9 \times 10^{-5}e^{-2t/\tau}\,dt$$

$$= 4.5\tau \times 10^{-5}(1 - e^{-2(0.001)/\tau})$$

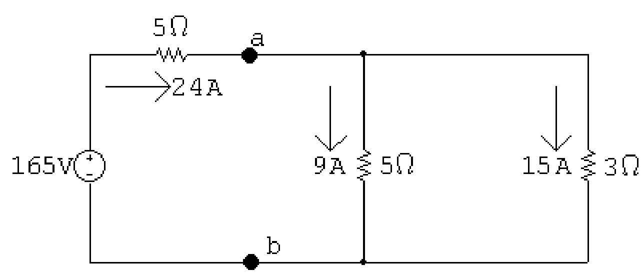
$$\tau = \frac{1}{1000\ln 2}$$

$$\therefore \qquad w_{10\Omega}(1\,\text{ms}) = 48.69\,\text{nJ}$$

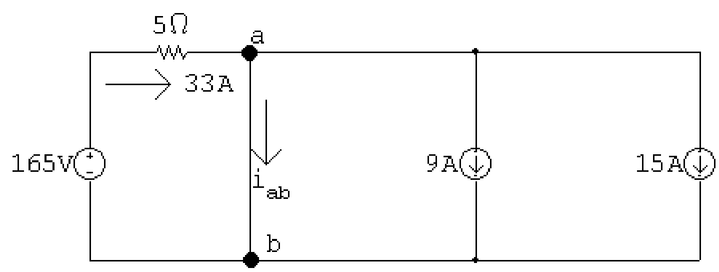
$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\text{dissipated in 1 ms} = \frac{48.69}{64.92}(100) = 75\%$$

P 7.17 **[a]** $t < 0$:

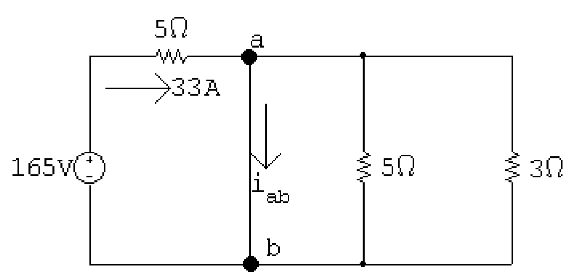


$t = 0^+$:

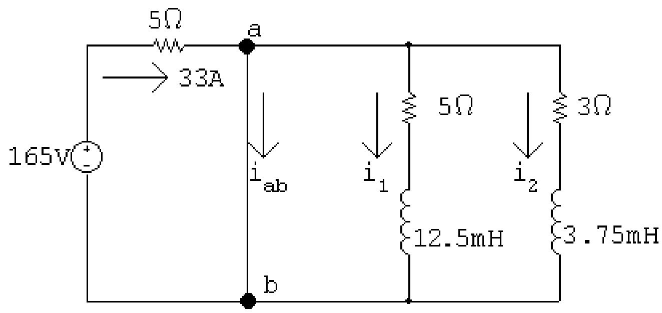


$$33 = i_{\text{ab}} + 9 + 15, \qquad i_{\text{ab}} = 9\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{\text{ab}} = 165/5 = 33\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 9, \qquad \tau_1 = \dfrac{12.5 \times 10^{-3}}{5} = 2.5\,\text{ms}$

$i_2(0) = 15, \qquad \tau_2 = \dfrac{3.75 \times 10^{-3}}{3} 1.25\,\text{ms}$

$i_1(t) = 9e^{-400t}\,\text{A}, \quad t \geq 0$

$i_2(t) = 15e^{-800t}\,\text{A}, \quad t \geq 0$

$i_{\text{ab}} = 33 - 9e^{-400t} - 15e^{-800t}\,\text{A}, \quad t \geq 0^+$

$33 - 9e^{-400t} - 15e^{-800t} = 19$

$14 = 9e^{-400t} + 15e^{-800t}$

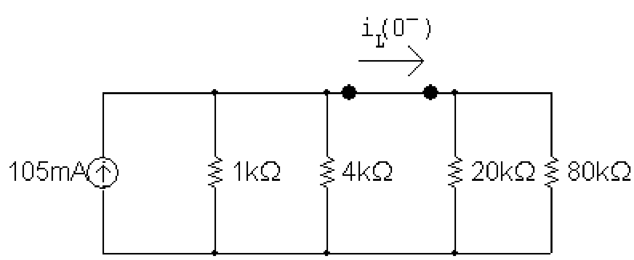
Let $\quad x = e^{-400t} \quad \therefore \quad x^2 = e^{-800t}$

Substituting,

$15x^2 + 9x - 14 = 0 \quad$ so $\quad x = 0.7116 = e^{-400t}$

$\therefore \qquad t = \dfrac{[\ln(1/0.7116)]}{400} = 850.6\,\mu\text{s}$

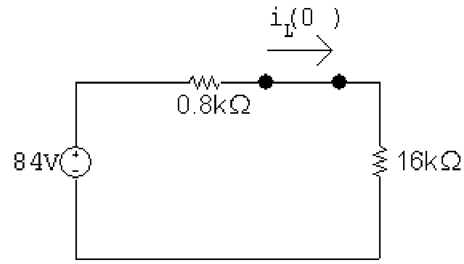
P 7.18 **[a]** $t < 0$



$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$

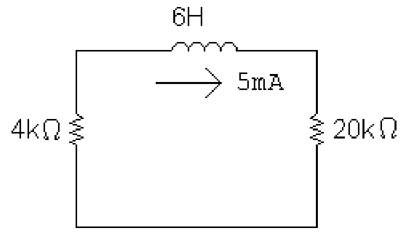
$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$

$(105 \times 10^{-3})(0.8 \times 10^3) = 84\,\text{V}$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

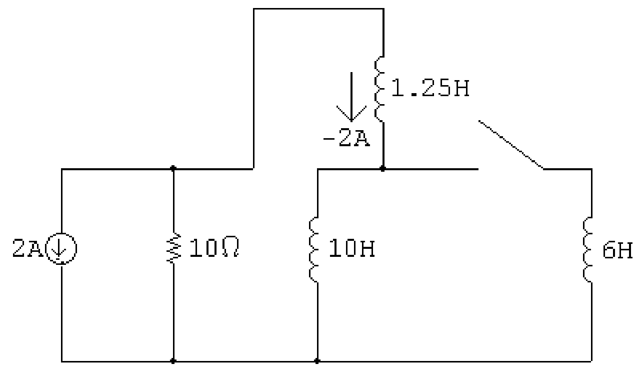
$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

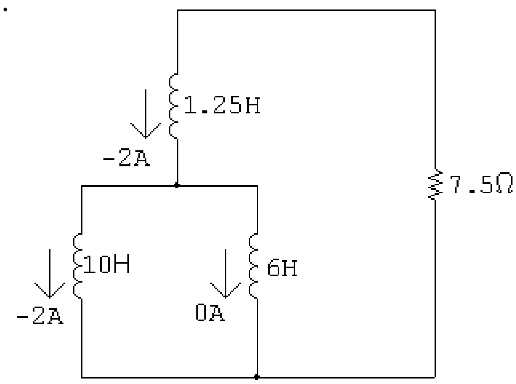
$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

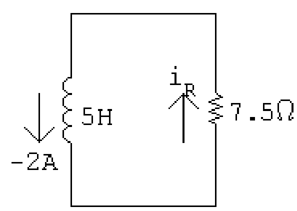
P 7.19 **[a]** $t < 0$:



$t = 0^+$:

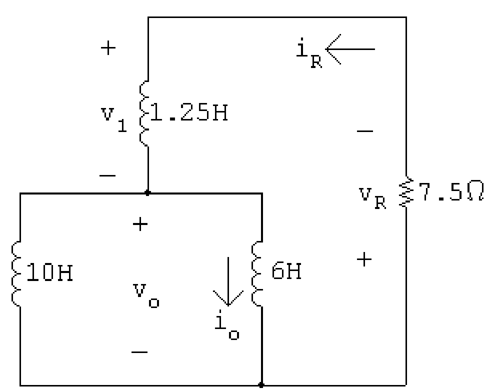


$t > 0$:



$$i_R = -2e^{-t/\tau}\,\text{A}; \qquad \tau = \frac{L}{R} = \frac{5}{7.5} = 666.67\,\text{ms} \quad \therefore \quad \frac{1}{\tau} = 1.5$$

$$i_R = -2e^{-1.5t}\,\text{A}$$



$$v_R = (7.5)(-2e^{-1.5t}) = -15e^{-1.5t}\,\text{V}$$

$$v_1 = 1.25[(-1.5)(-2e^{-1.5t})] = 3.75e^{-1.5t}\,\text{V},$$

$$v_o = -v_1 - v_R = 11.25e^{-1.5t}\,\text{V} \quad t \geq 0^+$$

[b] $i_o = \dfrac{1}{6}\displaystyle\int_0^t 11.25e^{-1.5x}\,dx + 0 = 1.25 - 1.25e^{-1.5t}\,\text{A} \quad t \geq 0$

P 7.20 **[a]** From the solution to Problem 7.19,

$$i_R = -2e^{-1.5t}\,\text{A}$$

$$p_R = (-2e^{-1.5t})^2(7.5) = 30e^{-3t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^\infty 30e^{-3t}\,dt$$

$$= 30\frac{e^{-3t}}{-3}\Big|_0^\infty = 10\,\text{J}$$

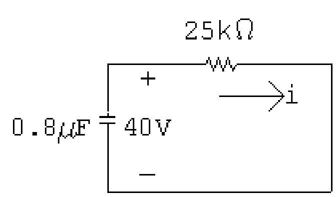
[b] $w_{\text{trapped}} = \frac{1}{2}(10)(-1.25)^2 + \frac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

CHECK: $w(0) = \frac{1}{2}(1.25)(2)^2 + \frac{1}{2}(10)(2)^2 = 22.5\,\text{J}$

$\therefore \quad w(0) = w_{\text{diss}} + w_{\text{trapped}}$

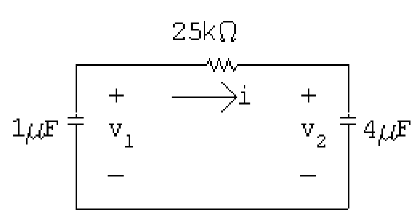
P 7.21 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V} \qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$$



$$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \frac{1}{\tau} = 50$$

$$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4 \times 10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4 \times 10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors (final voltage on the equivalent capacitor is zero):

$$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.22 **[a]** Calculate the initial voltage drop across the capacitor:

$$v(0) = (2.7\,\text{k}\|3.3\,\text{k})(40\,\text{mA}) = (1485)(40 \times 10^{-3}) = 59.4\,\text{V}$$

The equivalent resistance seen by the capacitor is

$$R_e = 3\,\text{k}\|(2.4\,\text{k} + 3.6\,\text{k}) = 3\,\text{k}\|6\,\text{k} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.5) \times 10^{-6} = 1000\,\mu\text{s}; \qquad \frac{1}{\tau} = 1000$$

$$v = v(0)e^{-t/\tau} = 59.4e^{-1000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{2.4\,\text{k} + 3.6\,\text{k}} = 9.9e^{-1000t}\,\text{mA}, \quad t \geq 0^+$$

[b] $w(0) = \frac{1}{2}(0.5 \times 10^{-6})(59.4)^2 = 882.09\,\mu\text{J}$

$$i_{3k} = \frac{59.4e^{-1000t}}{3000} = 19.8e^{-1000t}\,\text{mA}$$

$$p_{3k} = [(19.8 \times 10^{-3})e^{-1000t}]^2(3000) = 1.176e^{-2000t}$$

$$w_{3k}(500\,\mu\text{s}) = 1.176\frac{e^{-2000x}}{-2000}\Big|_0^{500\times10^{-6}} = \frac{1.176}{-2000}(e^{-1} - 1) = 371.72\,\mu\text{J}$$

$$\% = \frac{371.72}{882.09} \times 100 = 42.14\%$$

P 7.23 **[a]** $R = \frac{v}{i} = 4\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 25; \qquad C = \frac{1}{(25)(4 \times 10^3)} = 10\,\mu\text{F}$

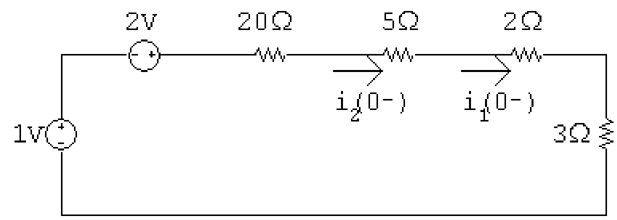
[c] $\tau = \frac{1}{25} = 40\,\text{ms}$

[d] $w(0) = \frac{1}{2}(10 \times 10^{-6})(48)^2 = 11.52\,\text{mJ}$

[e] $w_{\text{diss}}(60\,\text{ms}) = \int_0^{0.06} \frac{v^2}{R}dt = \int_0^{0.06} \frac{(48e^{-25t})^2}{(4\times 10^3)}dt$

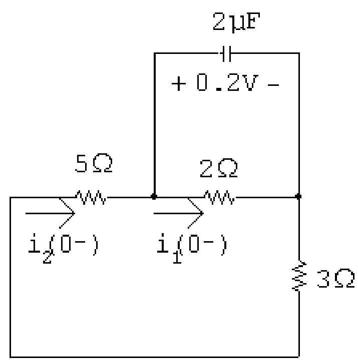
$$= 0.576\frac{e^{-50t}}{-50}\Big|_0^{0.06} = -5.74\times 10^{-4} + 0.01152 = 10.95\,\text{mJ}$$

P 7.24 **[a]** $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3\,\text{V}}{30\,\Omega} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = -25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \quad t \geq 0 \qquad R_e = 2||(5+3) = 1.6\,\Omega$

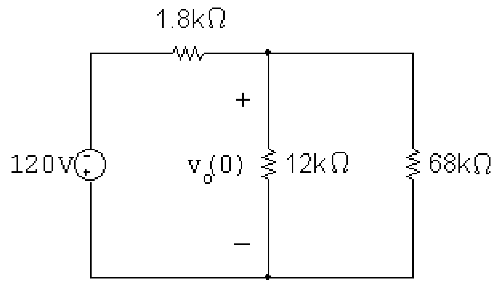
$$\tau = 1.6(2\times 10^{-6}) = 3.2\times 10^{-6}\,\text{s}$$

$$v_c = 0.2e^{-312{,}500t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}500t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \frac{-v_c}{8} = -25e^{-312{,}500t}\,\text{mA}, \qquad t \geq 0^+$

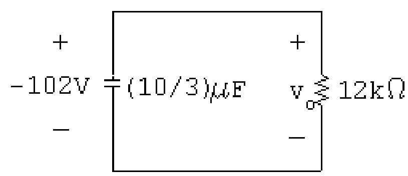
P 7.25 **[a]** $t < 0$:



$$R_e = 12\text{ k}||68\text{ k} = 10.2\text{ k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3}e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3}e^{-50t}\,dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7.82\,\text{mJ}$$

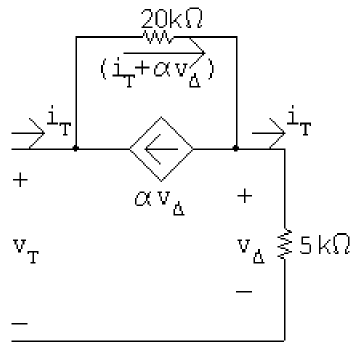
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$$0.75w(0) = 13\,\text{mJ}$$

$$\int_0^{t_o} 867 \times 10^{-3}e^{-50x}\,dx = 13 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,ms$$

P 7.26 **[a]**



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha(5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

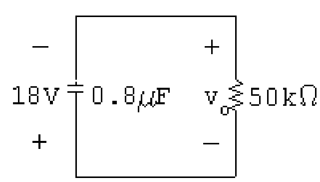
$$\tau = R_{\text{Th}}C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

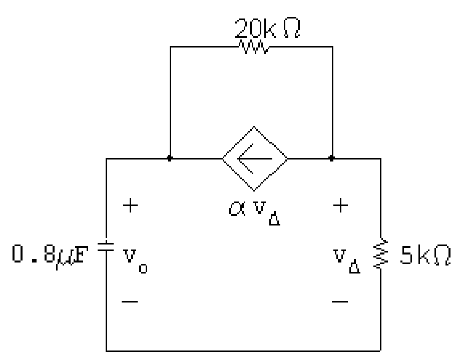
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

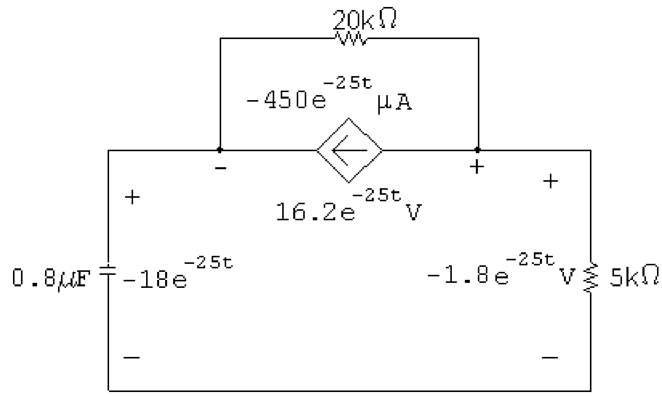


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.27 **[a]**



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J

[b] $w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$

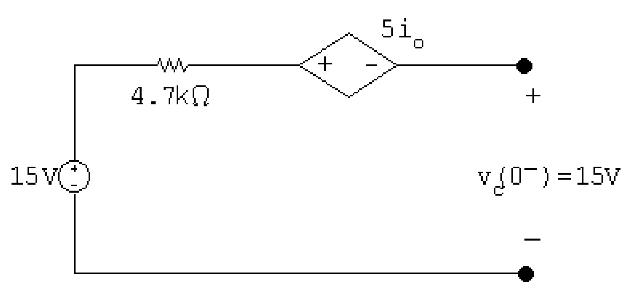
$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

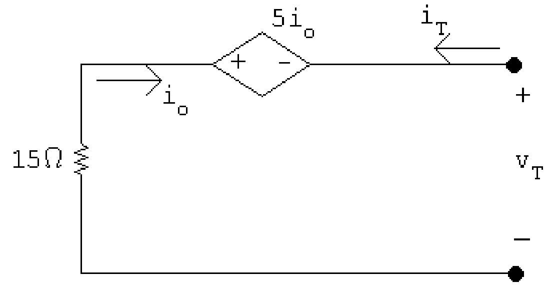
$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

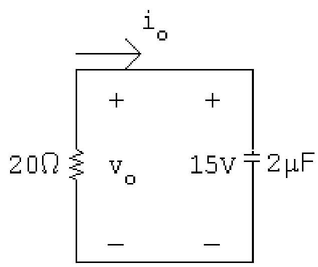
P 7.28 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$

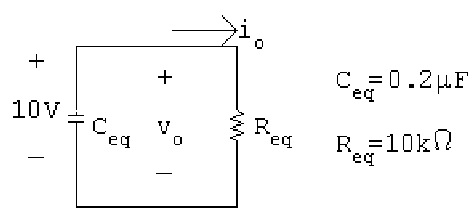


$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.29 **[a]** The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24k\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24k\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24k\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0 - 1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25\times10^{-6})(40)^2 + \frac{1}{2}(1\times10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84\times10^{-6}}{1.45\times10^{-3}}\times100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1\times10^{-3}e^{-500t})^2 = 0.4\times10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\Omega) = \frac{0.4\times10^{-6}}{1.45\times10^{-3}}\times100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t\ge0^+$$

$$p_{16k\Omega} = (0.6\times10^{-3}e^{-500t})^2(16{,}000) = 5.76\times10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76\times10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16\text{k}\Omega) = 0.4\%$$

[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45\times10^{-3} - 10\times10^{-6} = 1.44\,\text{mJ}$$

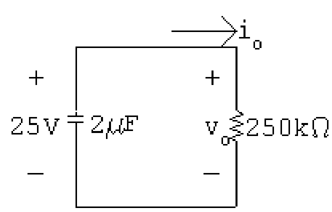
$$\%\ \text{trapped} = \frac{1.44}{1.45}\times100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.30 **[a]** $C_e = \dfrac{(2+1)6}{2+1+6} = 2\,\mu\text{F}$

$$v_o(0) = -5 + 30 = 25\,\text{V}$$

$$\tau = (2\times10^{-6})(250\times10^3) = 0.5\,\text{s}; \qquad \frac{1}{\tau} = 2$$



$$v_o = 25e^{-2t}\,\text{V}, \qquad t > 0^+$$

[b] $w_o = \frac{1}{2}(3 \times 10^{-6})(30)^2 + \frac{1}{2}(6 \times 10^{-6})(5)^2 = 1425\,\mu\text{J}$

$$w_{\text{diss}} = \frac{1}{2}(2 \times 10^{-6})(25)^2 = 625\,\mu\text{J}$$

$$\%\ \text{diss} = \frac{1425 - 625}{1425} \times 100 = 56.14\%$$

[c] $i_o = \frac{v_o}{250 \times 10^{-3}} = 100e^{-2t}\,\mu\text{A}$

$$v_1 = -\frac{1}{6 \times 10^{-6}}\int_0^t 100 \times 10^{-6} e^{-2x}\,dx - 5 = -16.67\int_0^t e^{-2x}\,dx - 5$$

$$= -16.67\frac{e^{-2x}}{-2}\Big|_0^t - 5 = 8.33e^{-2t} - 13.33\,\text{V} \qquad t \geq 0$$

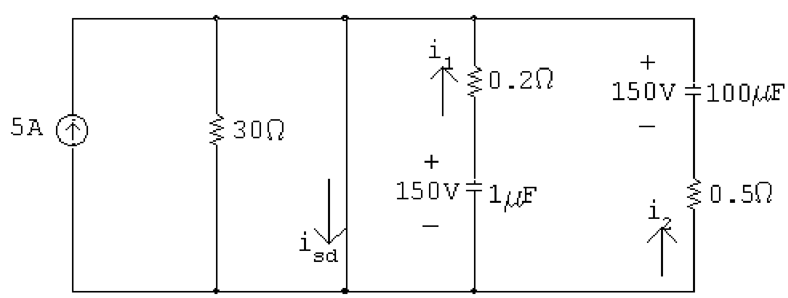
[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 25e^{-2t} - 8.33e^{-2t} + 13.33 = 16.67e^{-2t} + 13.33\,\text{V} \quad t \geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(6 \times 10^{-6})(13.33)^2 + \frac{1}{2}(3 \times 10^{-6})(13.33)^2 = 800\,\mu\text{J}$

$$w_{\text{diss}} + w_{\text{trapped}} = 625 + 800 = 1425\,\mu\text{J} \qquad \text{(check)}$$

P 7.31 **[a]** At $t = 0^-$ the voltage on each capacitor will be $150\,\text{V}(5 \times 30)$, positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5} = 1055\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 5\,\text{A}$$

[b] $i_{sd}(t) = 5 + i_1(t) + i_2(t)$

$$\tau_1 = 0.2(10^{-6}) = 0.2\,\mu\text{s}$$

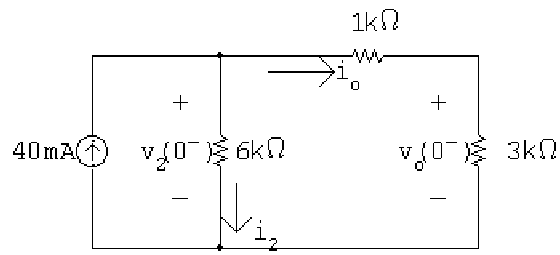
$$\tau_2 = 0.5(100 \times 10^{-6}) = 50\,\mu\text{s}$$

$$\therefore \quad i_1(t) = 750e^{-5\times10^6 t}\,\text{A}, \qquad t \geq 0^+$$

$$i_2(t) = 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 5 + 750e^{-5\times10^6 t} + 300e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 7.32 **[a]** $t < 0$:



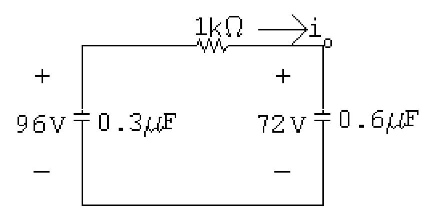
$$i_o(0^-) = \frac{6000}{6000+4000}(40\,\text{m}) = 24\,\text{mA}$$

$$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$$

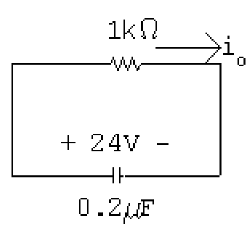
$$i_2(0^-) = 40 - 24 = 16\,\text{mA}$$

$$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$$

$t > 0$

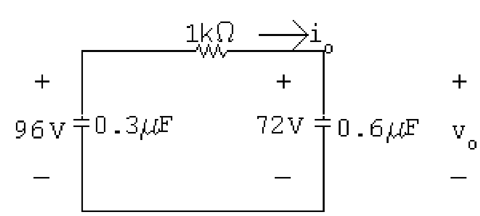


$$\tau = RC = (1000)(0.2\times10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{24}{1\times10^3}e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$v_o = \frac{1}{0.6 \times 10^{-6}}\int_0^t 24 \times 10^{-3} e^{-5000x}\,dx + 72$$
$$= (40{,}000)\frac{e^{-5000x}}{-5000}\Big|_0^t + 72$$
$$= -8e^{-5000t} + 8 + 72$$
$$v_o = [-8e^{-5000t} + 80]\,\mathrm{V}, \qquad t \geq 0$$

[c] $w_{\text{trapped}} = (1/2)(0.3 \times 10^{-6})(80)^2 + (1/2)(0.6 \times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\mathrm{J}.$

Check:

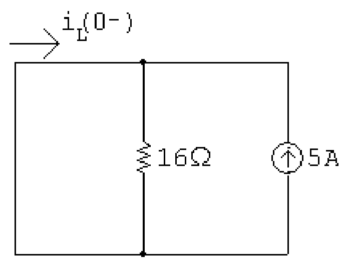
$$w_{\text{diss}} = \frac{1}{2}(0.2 \times 10^{-6})(24)^2 = 57.6\,\mu\mathrm{J}$$

$$w(0) = \frac{1}{2}(0.3 \times 10^{-6})(96)^2 + \frac{1}{2}(0.6 \times 10^{-6})(72)^2 = 2937.6\,\mu\mathrm{J}.$$

$w_{\text{trapped}} + w_{\text{diss}} = w(0)$

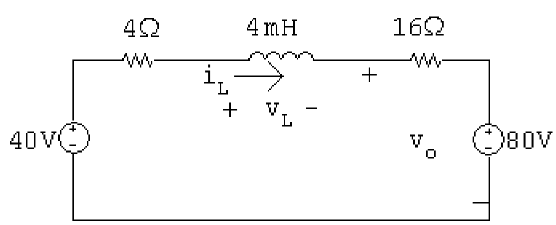
$2880 + 57.6 = 2937.6$ OK.

P 7.33 **[a]** $t < 0$



$i_L(0^-) = -5\,\mathrm{A}$

$t > 0$



$$i_L(\infty) = \frac{40 - 80}{4 + 16} = -2\,\mathrm{A}$$

$$\tau = \frac{L}{R} = \frac{4 \times 10^{-3}}{4 + 16} = 200\,\mu\mathrm{s}; \qquad \frac{1}{\tau} = 5000$$

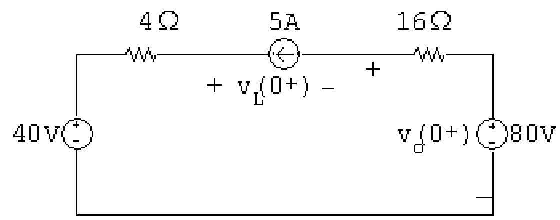
$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$
$$= -2 + (-5+2)e^{-5000t} = -2 - 3e^{-5000t}\,\text{A}, \qquad t \geq 0$$
$$v_o = 16i_L + 80 = 16(-2 - 3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_L = L\dfrac{di_L}{dt} = 4\times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = 60\,\text{V}$

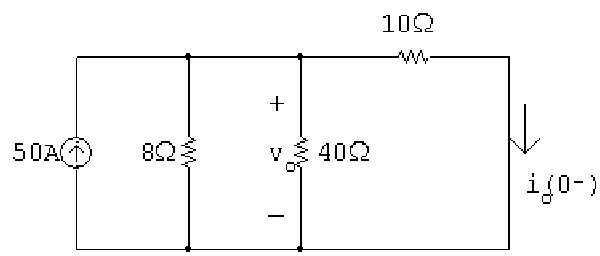
From part (a) $\quad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$

P 7.34 **[a]** $t < 0$



KVL equation at the top node:

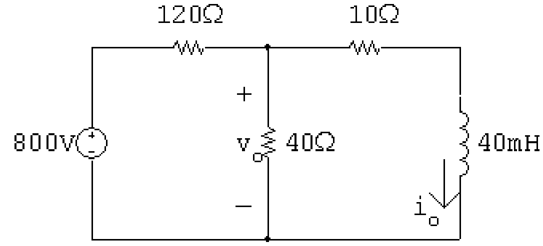
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$2000 = (5+1+4)v_o; \qquad v_o = 200\,\text{V}$

$\therefore \quad i_o(0^-) = \dfrac{v_o}{10} = 200/10 = 20\,\text{A}$

$t > 0$



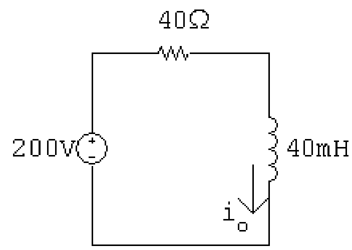
Use voltage division to find the Thévenin voltage:

$$V_{\rm Th} = v_o = \frac{40}{40+120}(800) = 200\,\rm V$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\rm Th} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{40\times10^{-3}}{40} = 1\,\rm ms; \qquad \frac{1}{\tau} = 1000$$

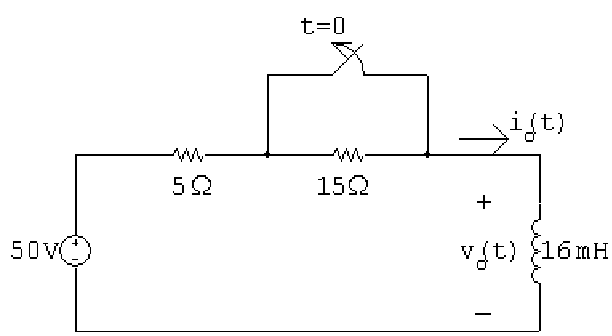
$$i_o(\infty) = \frac{200}{40} = 5\,\rm A$$

$$\therefore\quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 5 + (20-5)e^{-1000t} = 5 + 15e^{-1000t}\,\rm A, \qquad t \geq 0$$

[b]
$$v_o = 10i_o + L\frac{di_o}{dt}$$
$$= 10(5+15e^{-1000t}) + 0.04(-1000)(15e^{-1000t})$$
$$= 50 + 150e^{-1000t} - 600e^{-1000t}$$
$$v_o = 50 - 450e^{-1000t}\,\rm V, \qquad t \geq 0^+$$

P 7.35 After making a Thévenin equivalent we have



For $t < 0$, the $15\,\Omega$ resistor is bypassed:

$$i_o(0^-) = i_o(0^+) = 50/5 = 10\,\rm A$$

$$\tau = \frac{L}{R} = \frac{16\times 10^{-3}}{5+15} = 8\times 10^{-4}; \qquad \frac{1}{\tau} = 1250$$

$$i(\infty) = \frac{V}{R_{\text{eq}}} = \frac{50}{5+15} = 2.5\,\text{A}$$

$$i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau} = 2.5 + (10-2.5)e^{-1250t} = 2.5 + 7.5e^{-1250t}\,\text{A}, t \geq 0$$

$$v_o = L\frac{di_o}{dt} = 16\times 10^{-3}(-1250)(7.5e^{-1250t}) = -150e^{-1250t}\,\text{V}, \quad t \geq 0^+$$

P 7.36 **[a]** $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1+R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o = -(10)(15)e^{-\frac{(5+15)}{0.016}t} = -150e^{-1250t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o(0^+) \to \infty,$ and the duration of $v_o(t) \to$ zero

[d] $v_{sw} = R_2i_o; \qquad \tau = \dfrac{L}{R_1+R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1+R_2}$$

Therefore $\quad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[e] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.37 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [e] of Problem 7.36) causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.38 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

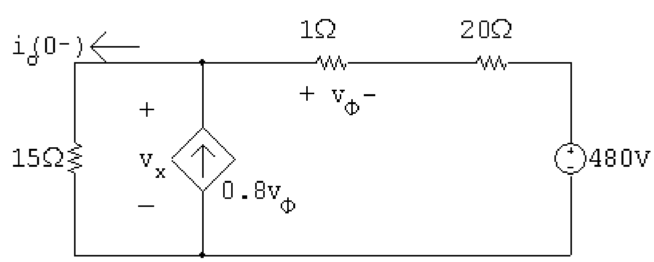
Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0$; Solving, $x = 0.5$; $x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$$

P 7.39 For $t < 0$



$$\frac{v_x}{15} - 0.8v_\phi + \frac{v_x - 480}{21} = 0$$

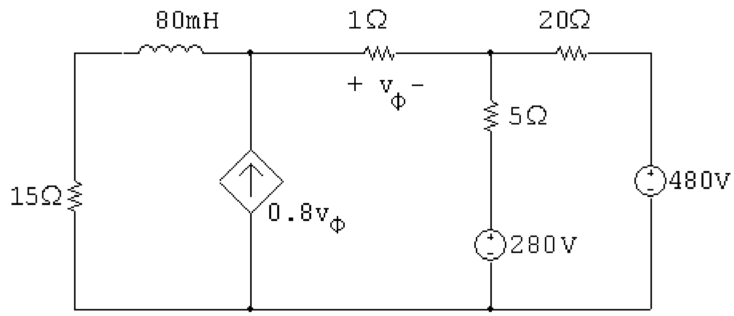
$$v_\phi = \frac{v_x - 480}{21}$$

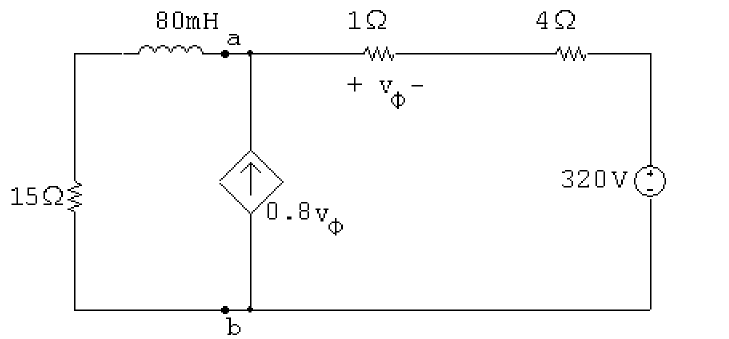
$$\frac{v_x}{15} - 0.8\left(\frac{v_x - 480}{21}\right) + \left(\frac{v_x - 480}{21}\right)$$

$$= \frac{v_x}{15} + 0.2\left(\frac{v_s - 480}{21}\right) = 21v_x + 3(v_x - 480) = 0$$

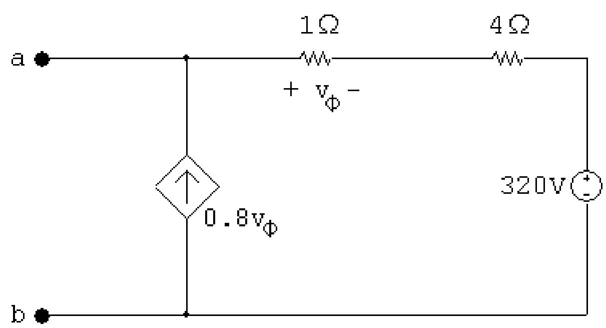
$$\therefore \qquad 24v_x = 1440 \quad \text{so} \quad v_x = 60\,\text{V} \quad i_o(0^-) = \frac{v_x}{15} = 4\text{ A}$$

$t > 0$

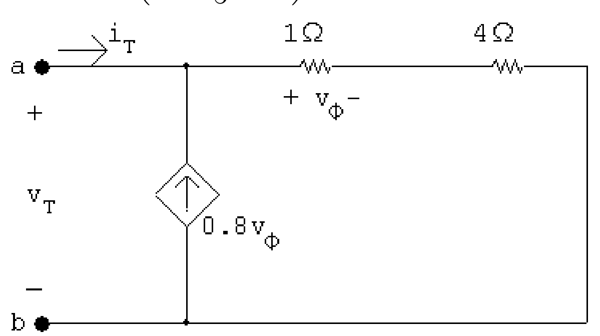




Find Thévenin equivalent with respect to a, b



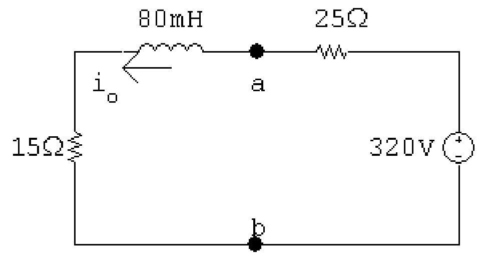
$$\frac{V_{\text{Th}} - 320}{5} - 0.8\left(\frac{V_{\text{Th}} - 320}{5}\right) = 0 \qquad V_{\text{Th}} = 320\,\text{V}$$



$$v_T = (i_T + 0.8v_\phi)(5) = \left(i_T + 0.8\frac{v_T}{5}\right)(5)$$

$v_T = 5i_T + 0.8v_T \qquad \therefore \quad 0.2v_T = 5i_T$

$$\frac{v_T}{i_T} = R_{\rm Th} = 25\,\Omega$$

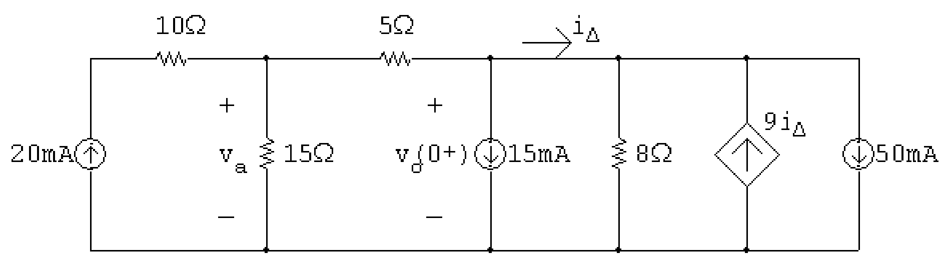


$i_o(\infty) = 320/40 = 8\,\text{A}$

$$\tau = \frac{80 \times 10^{-3}}{40} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$i_o = 8 + (4-8)e^{-500t} = 8 - 4e^{-500t}\,\text{A}, \qquad t \geq 0$

P 7.40 $t > 0$;



$$\frac{v_{\rm a}}{15} + \frac{v_{\rm a} - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$\therefore \quad v_{\rm a} = 0.75v_o(0^+) + 75 \times 10^{-3}$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_{\rm a}}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$13v_o(0^+) - 8v_{\rm a} - 360i_\Delta = -2600 \times 10^{-3}$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800 \times 10^{-3}$

$$8v_{\text{a}} = 6v_o(0^+) + 600 \times 10^{-3}$$

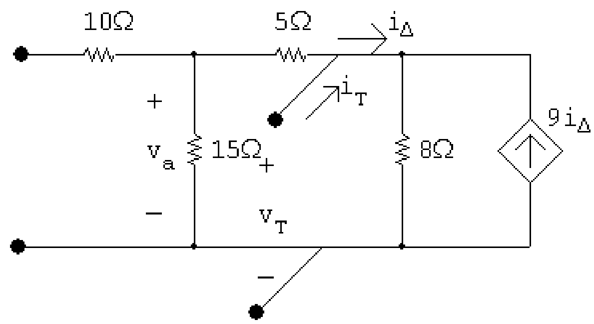
$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$

$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

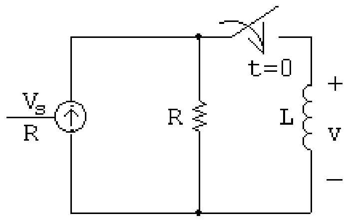
$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\text{S}$$

$$\therefore \quad R_{\text{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\text{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\text{mV}, \qquad t \geq 0^+$$

P 7.41 **[a]**



$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,dx = \frac{V_s}{R}$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $$\frac{dv}{dt} = -\frac{R}{L}v$$

$$\frac{dv}{dt}\,dt = -\frac{R}{L}v\,dt$$

$$\therefore \quad \frac{dv}{v} = -\frac{R}{L}\,dt$$

$$\int_{v(0^+)}^{v(t)} \frac{dy}{y} = -\frac{R}{L}\int_{0^+}^{t} dx$$

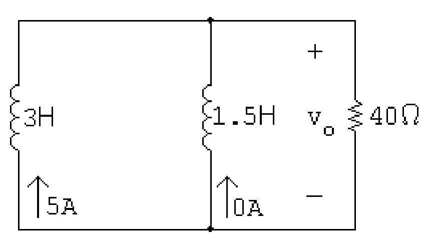
$$\ln y \Big|_{v(0^+)}^{v(t)} = -\left(\frac{R}{L}\right)t$$

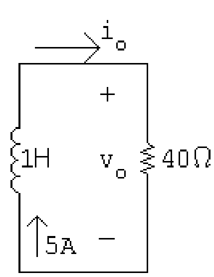
$$\ln\left[\frac{v(t)}{v(0^+)}\right] = -\left(\frac{R}{L}\right)t$$

$$v(t) = v(0^+)e^{-(R/L)t}; \qquad v(0^+) = \left(\frac{V_s}{R} - I_o\right)R = V_s - I_oR$$

$$\therefore \quad v(t) = (V_s - I_oR)e^{-(R/L)t}$$

P 7.42 $t > 0$





$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.43 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \frac{1}{3}\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

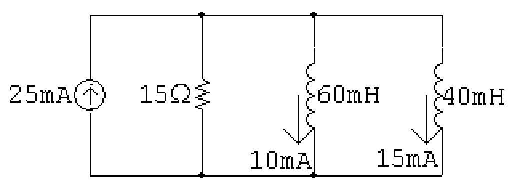
$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

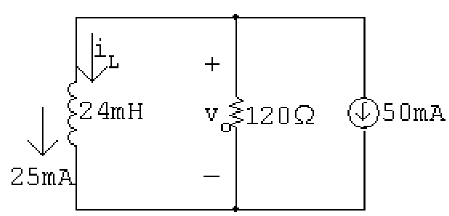
$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.44 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \frac{1}{60 \times 10^{-3}} \int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \frac{1}{40 \times 10^{-3}} \int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.45 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_g R_g e^{-t/\tau}; \qquad \tau = L_e/R_g$

Thus

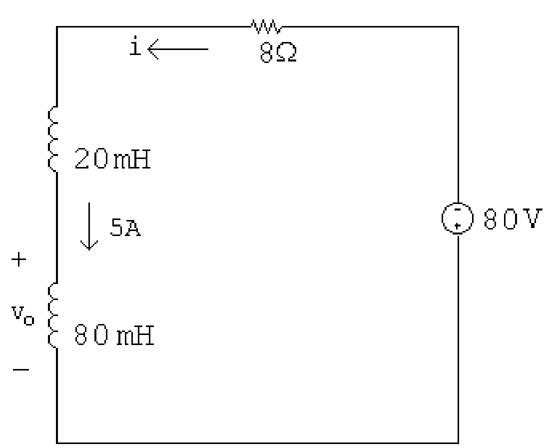
$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Bigg|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \frac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \frac{L_1}{L_1 + L_2}I_g$

P 7.46 For $t < 0$, $\quad i_{80\text{mH}}(0) = 50\,\text{V}/10\,\Omega = 5\,\text{A}$

For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

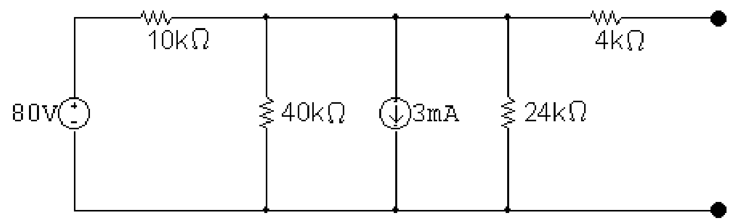
$$\frac{1}{\tau} = \frac{R}{L} = \frac{8}{100 \times 10^{-3}} = 80$$

$$I_o = 5\,\text{A}; \qquad I_f = \frac{V_s}{R} = \frac{-80}{8} = -10\,\text{A}$$

$$i = -10 + (5+10)e^{-80t} = -10 + 15e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.08\frac{di}{dt} = 0.08(-1200e^{-80t}) = -96e^{-80t}\,\text{V}, \qquad t > 0^+$$

P 7.47 For $t < 0$



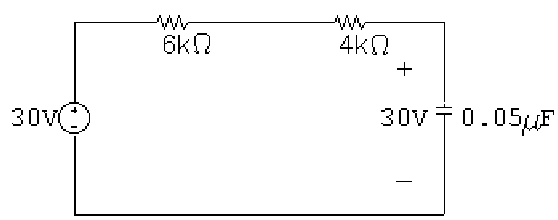
Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

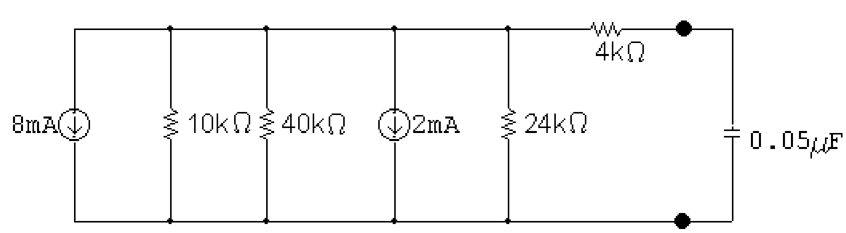
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



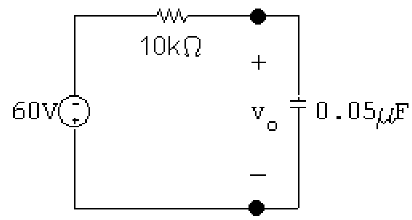
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



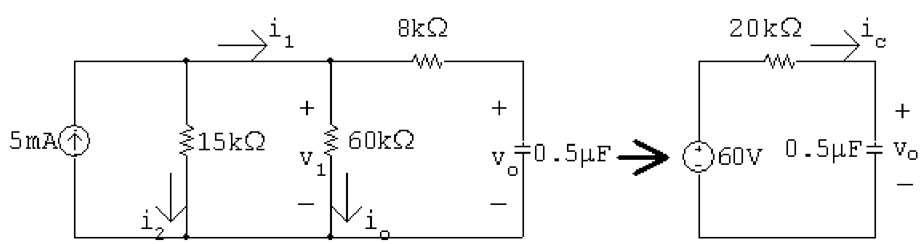
$v_o(\infty) = -60\,\text{V}$

$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$

$= -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$

P 7.48 **[a]** Simplify the circuit for $t > 0$ using source transformation:



Since there is no source connected to the capacitor for $t < 0$

$v_o(0^-) = v_o(0^+) = 0\,\text{V}$

From the simplified circuit,

$v_o(\infty) = 60\,\text{V}$

$\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms} \qquad 1/\tau = 100$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})\,\text{V}, \qquad t \geq 0$

[b] $i_c = C\dfrac{dv_o}{dt}$

$$i_c = 0.5 \times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t}\,\text{mA}$$

$$v_1 = 8000i_c + v_o = (8000)(3 \times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t}\,\text{V}$$

$$i_o = \frac{v_1}{60 \times 10^3} = 1 - 0.6e^{-100t}\,\text{mA}, \qquad t \geq 0^+$$

[c] $i_1(t) = i_o + i_c = 1 + 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_1}{15 \times 10^3} = 4 - 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[e] $i_1(0^+) = 1 + 2.4 = 3.4\,\text{mA}$

At $t = 0^+$:

$$R_e = 15\,\text{k}\|60\,\text{k}\|8\,\text{k} = 4800\,\Omega$$

$$v_1(0^+) = (5 \times 10^{-3})(4800) = 24\,\text{V}$$

$$i_1(0^+) = \frac{v_1(0^+)}{60{,}000} + \frac{v_1(0^+)}{8000} = 0.4\,\text{m} + 3\,\text{m} = 3.4\text{mA} \quad \text{(checks)}$$

P 7.49 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \dfrac{V_o}{R}\right)e^{-t/RC}$

$$\therefore\ \ I_sR = 40, \qquad V_o - I_sR = -24$$

$$\therefore\ \ V_o = 16\,\text{V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$$

$$\therefore\ \ I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$$

$$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$$

[b] $v(\infty) = 40\,\text{V}$

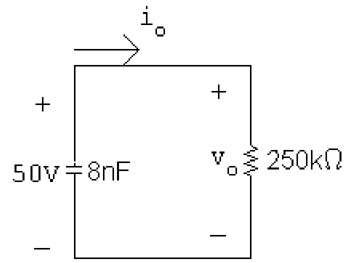
$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$$

$$0.81w(\infty) = 32.4\,\mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$$

$$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore\ \ t_o = 716.70\,\mu\text{s}$$

P 7.50 **[a]** For $t > 0$:



$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{v_o}{250{,}000} = \dfrac{50e^{-500t}}{250{,}000} = 200e^{-500t}\,\mu\text{A}$

$$v_1 = \frac{-1}{40 \times 10^{-9}} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 7.51 **[a]** $w = \dfrac{1}{2}C_{\text{eq}}v_o^2 = \dfrac{1}{2}(8 \times 10^{-9})(50^2) = 10\,\mu\text{J}$

[b] $w_{\text{trapped}} = \dfrac{1}{2}(40)^2(50 \times 10^{-9}) = 40\,\mu\text{J}$

[c] $w(0) = \dfrac{1}{2}(40 \times 10^{-9})(50^2) = 50\,\mu\text{J}$

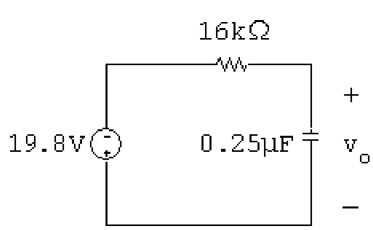
P 7.52 For $t > 0$

$$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$$

$$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$



$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

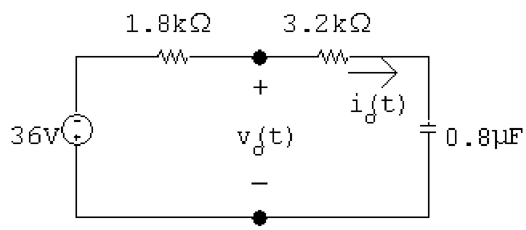
$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\text{ms}$$

P 7.53 **[a]**



$$i_o(0^+) = \frac{-36}{5000} = -7.2\,\text{mA}$$

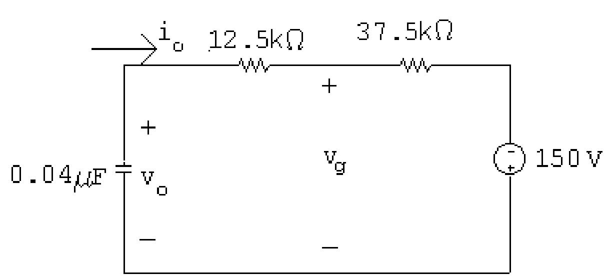
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (5000)(0.8 \times 10^{-6}) = 4\,\text{ms}$

[d] $i_o = 0 + (-7.2)e^{-250t} = -7.2e^{-250t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = -[36 + 1800(-7.2 \times 10^{-3}e^{-250t})] = -36 + 12.96e^{-250t}\,\text{V}, \qquad t \geq 0^+$

P 7.54 **[a]** $v_o(0^-) = v_o(0^+) = 120\,\text{V}$



$$v_o(\infty) = -150\,\text{V}; \qquad \tau = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = -150 + (120 - (-150))e^{-500t}$$

$$v_o = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.04 \times 10^{-6}(-500)[270e^{-500t}] = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 12.5 \times 10^3 i_o = -150 + 202.5e^{-500t}\,\text{V}$

[d] $v_g(0^+) = -150 + 202.5 = 52.5\,\text{V}$

Checks:

$$v_g(0^+) = i_o(0^+)[37.5 \times 10^3] - 150 = 202.5 - 150 = 52.5\,\text{V}$$

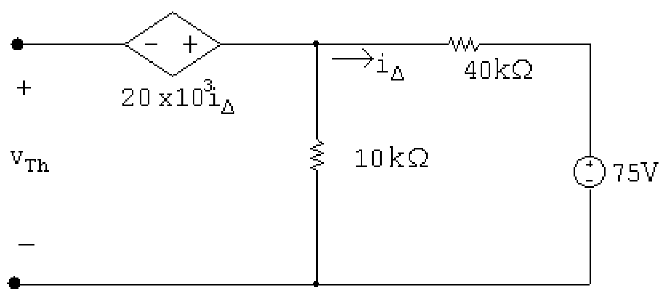
$$i_{50k} = \frac{v_g}{50\text{k}} = -3 + 4.05e^{-500t}\,\text{mA}$$

$$i_{150k} = \frac{v_g}{150\text{k}} = -1 + 1.35e^{-500t}\,\text{mA}$$

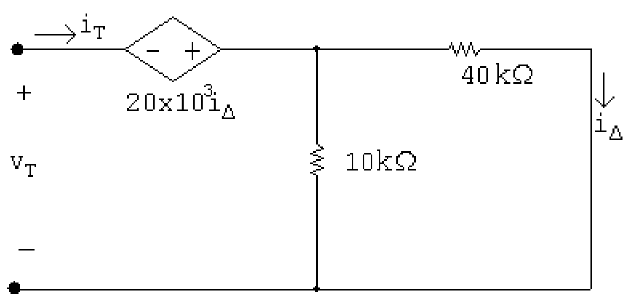
$-i_o + i_{50k} + i_{150k} + 4 = 0$ (ok)

P 7.55 For $t < 0$, $v_o(0) = (-3\,\text{m})(15\,\text{k}) = -45\,\text{V}$

$t > 0$:



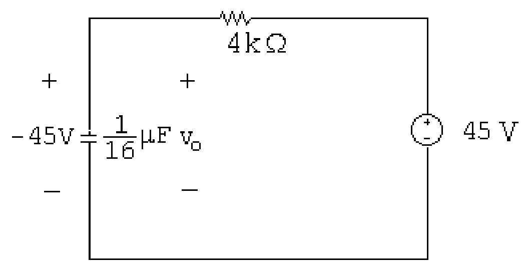
$$V_{\text{Th}} = -20 \times 10^3 i_\Delta + \frac{10}{50}(75) = -20 \times 10^3 \left(\frac{-75}{50 \times 10^3}\right) + 15 = 45\,\text{V}$$



$$v_T = -20 \times 10^3 i_\Delta + 8 \times 10^3 i_T = -20 \times 10^3 (0.2) i_T + 8 \times 10^3 i_T = 4 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 4\,\text{k}\Omega$$

$t > 0$



$$v_o = 45 + (-45 - 45)e^{-t/\tau}$$

$$\tau = RC = (4000)\left(\frac{1}{16}\times 10^{-6}\right) = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$v_o = 45 - 90e^{-4000t}\,\text{V}, \quad t \geq 0$$

P 7.56 $v_o(0) = 45\,\text{V}; \qquad v_o(\infty) = -45\,\text{V}$

$$R_{\text{Th}} = 20\,\text{k}\Omega$$

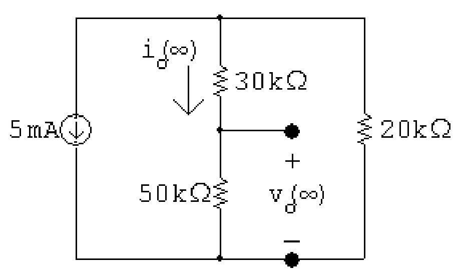
$$\tau = (20\times 10^3)\left(\frac{1}{16}\times 10^{-6}\right) = 1.25\times 10^{-3}; \qquad \frac{1}{\tau} = 800$$

$$v = -45 + (45+45)e^{-800t} = -45 + 90e^{-800t}\,\text{V}, \quad t \geq 0$$

P 7.57 $t < 0;$

$$i_o(0^-) = \frac{20}{100}(10\times 10^{-3}) = 2\,\text{mA}; \qquad v_o(0^-) = (2\times 10^{-3})(50{,}000) = 100\,\text{V}$$

$t = \infty$:

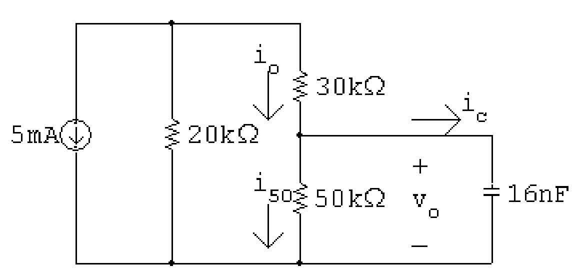


$$i_o(\infty) = -5\times 10^{-3}\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\text{V}$$

$$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$$

$$\tau = (25{,}000)(16\times 10^{-9}) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$$

$$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$$



$$i_c = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{50k} = \frac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \ge 0^+$$

$$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \ge 0^+$$

P 7.58 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

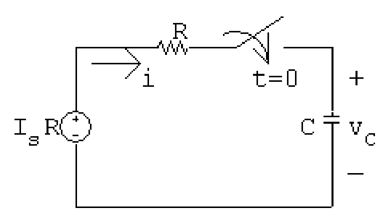
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\Big|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 7.59 **[a]**



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^t i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\dfrac{di}{dt} = -\dfrac{i}{RC}; \qquad \dfrac{di}{i} = -\dfrac{dt}{RC}$

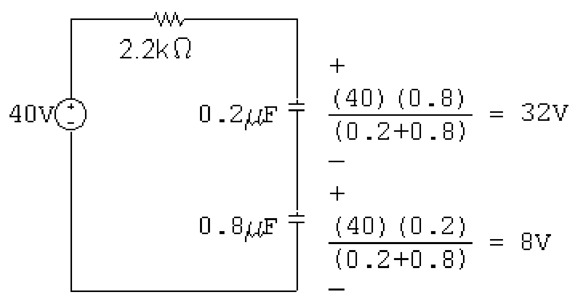
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

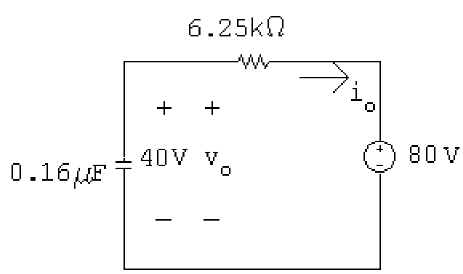
$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.60 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16\times10^{-6})(6.25\times10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \ge 0$$

[b] $i_o = -C\dfrac{dv_o}{dt} = -0.16\times10^{-6}[40{,}000e^{-1000t}]$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \ge 0^+$$

[c] $v_1 = \dfrac{-1}{0.2 \times 10^{-6}} \displaystyle\int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 32$

$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$

[d] $v_2 = \dfrac{-1}{0.8 \times 10^{-6}} \displaystyle\int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 8$

$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(0.2 \times 10^{-6})(64)^2 + \dfrac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$

P 7.61 **[a]** $v_c(0^+) = 50\,\text{V}$

[b] Use voltage division to find the final value of voltage:

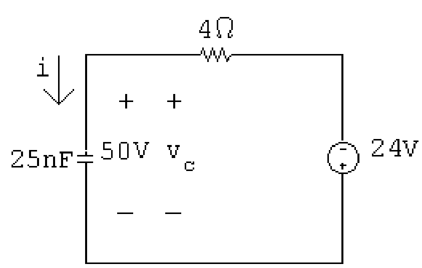
$$v_c(\infty) = \frac{20}{20+5}(-30) = -24\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$V_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$

Therefore $\tau = R_{\text{eq}}C = 4(25 \times 10^{-9}) = 0.1\,\mu\text{s}$

The simplified circuit for $t > 0$ is:



[d] $i(0^+) = \dfrac{-24 - 50}{4} = -18.5\,\text{A}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$= -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7 t}\,\text{V}, \qquad t \geq 0$

[f] $i = C\dfrac{dv_c}{dt} = (25 \times 10^{-9})(-10^7)(74e^{-10^7 t}) = -18.5e^{-10^7 t}\,\text{A}, \qquad t \geq 0^+$

P 7.62 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} = 1000\,\mu\text{s}$$

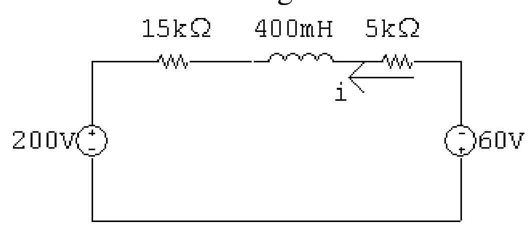
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

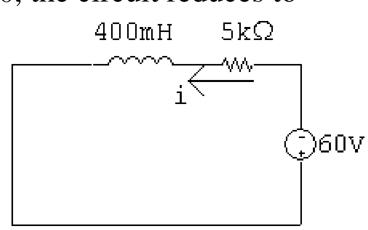
P 7.63 **[a]** For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 400-mH inductor. We get



$$i(0^-) = \frac{-60 - 200}{15\,\text{k} + 5\,\text{k}} = -13\,\text{mA}$$

$$i(0^-) = i(0^+) = -13\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



Therefore $\quad i(\infty) = -60/5{,}000 = -12\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{400 \times 10^{-3}}{5000} = 80\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= -12 + [-13 + 12]e^{-12{,}500t} = -12 - e^{-12{,}500t}\,\text{mA}, \qquad t \geq 0$$

P 7.64 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{36 - 16}{20 - 8} = \frac{5}{3}\,\text{H}$$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{(5/3)}{(50/3)} = \frac{1}{10}$$

$$i_o = \frac{100}{(50/3)} - \frac{100}{(50/3)}e^{-10t} = 6 - 6e^{-10t}\,\text{A} \quad t \geq 0$$

[b] $v_o = 100 - \frac{50}{3}i_o = 100 - \frac{50}{3}(6 - 6e^{-10t}) = 100e^{-10t}\,\text{V}, \quad t \geq 0^+$

[c] $v_o = 2\frac{di_1}{dt} + 4\frac{di_2}{dt}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di_2}{dt} = \frac{di_o}{dt} - \frac{di_1}{dt} = 60e^{-10t} - \frac{di_1}{dt}$$

$$\therefore \quad 100e^{-10t} = 2\frac{di_1}{dt} + 4\left(60e^{-10t} - \frac{di_1}{dt}\right)$$

$$\therefore \quad \frac{di_1}{dt} = 70e^{-10t}$$

$$di_1 = 70e^{-10t}\,dt$$

$$\int_0^{i_1} dx = 70\int_0^t e^{-10y}\,dy$$

$$\therefore \quad i_1 = 70\frac{e^{-10y}}{-10}\Big|_0^t = 7 - 7e^{-10t}\,\text{A}, \quad t \geq 0$$

[d]
$$\begin{aligned} i_2 &= i_o - i_1 \\ &= 6 - 6e^{-10t} - 7 + 7e^{-10t} \\ &= -1 + e^{-10t}\,\text{A}, \quad t \geq 0 \end{aligned}$$

[e]
$$\begin{aligned} v_o &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \\ &= 18(-10e^{-10t}) + 4(70e^{-10t}) \\ &= 100e^{-10t}\,\text{V}, \quad t \geq 0^+ \quad \text{(checks)} \end{aligned}$$

Also,

$$\begin{aligned} v_o &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ &= 2(70e^{-10t}) + 4(-10e^{-10t}) \\ &= 100e^{-10t}\,\text{V}, \quad t \geq 0^+ \quad \text{CHECKS} \end{aligned}$$

$i_1(0) = 7 - 7 = 0$; agrees with initial conditions;
$i_2(0) = -1 + 1 = 0$; agrees with initial conditions;
The final values of i_o, i_1, and i_2 can be checked via the conservation of Wb-turns:

$$i_o(\infty)L_{\text{eq}} = 6 \times (5/3) = 10 \text{ Wb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = 7(2) - 1(4) = 10 \text{ Wb-turns}$$

$$i_2(\infty)L_2 + i_1(\infty)M = -1(18) + 7(4) = 10 \text{ Wb-turns}$$

Thus our solutions make sense in terms of known circuit behavior.

P 7.65 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore \quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore \quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.66 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\dfrac{di_1}{dt} - 5\dfrac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t} \text{ A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V},$ which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.67 **[a]** $L_{\text{eq}} = 5 + 10 - 2.5(2) = 10\,\text{H}$

$$\tau = \frac{L}{R} = \frac{10}{40} = \frac{1}{4}; \qquad \frac{1}{\tau} = 4$$

$i = 2 - 2e^{-4t}\,\text{A}, \quad t \geq 0$

[b] $v_1(t) = 5\frac{di_1}{dt} - 2.5\frac{di}{dt} = 2.5\frac{di}{dt} = 2.5(8e^{-4t}) = 20e^{-4t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} - 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(8e^{-4t}) = 60e^{-4t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 2 - 2 = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-4t}) + 20e^{-4t} + 60e^{-4t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.68 **[a]** $L_{\text{eq}} = 5 + 10 + 2.5(2) = 20\,\text{H}$

$$\tau = \frac{L}{R} = \frac{20}{40} = \frac{1}{2}; \qquad \frac{1}{\tau} = 2$$

$i = 2 - 2e^{-2t}\,\text{A}, \quad t \geq 0$

[b] $v_1(t) = 5\frac{di_1}{dt} + 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(4e^{-2t}) = 30e^{-2t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} + 2.5\frac{di}{dt} = 12.5\frac{di}{dt} = 12.5(4e^{-2t}) = 50e^{-2t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-2t}) + 30e^{-2t} + 50e^{-2t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.69 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40+60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

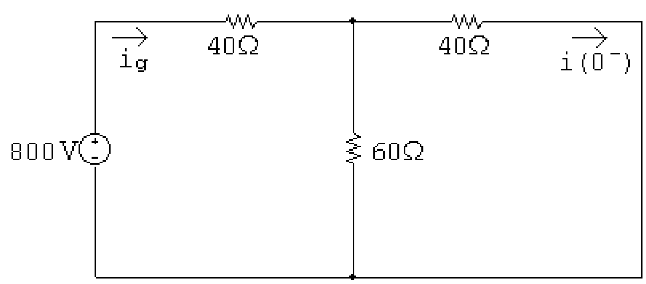
$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

$$\tau = RC = (20\times 10^3)(250\times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

$$= -100 + (30+100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.70 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \leq t \leq 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

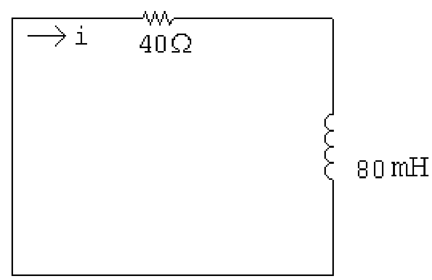
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80\times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times 10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t < \infty$



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

$$i = 2.7591e^{-500(t-0.001)}$$

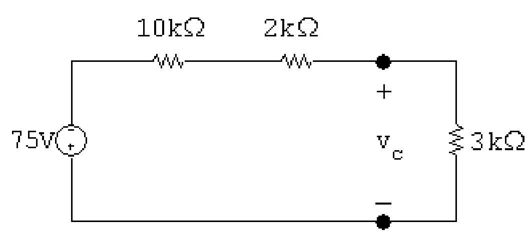
$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.591e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

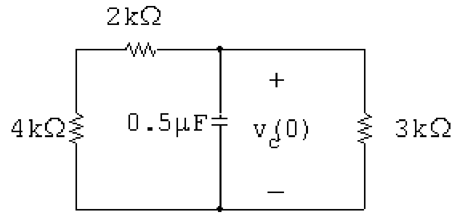
P 7.71 Note that for $t > 0$, $v_o = (4/6)v_c$, where v_c is the voltage across the $0.5\,\mu$F capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{3}{15}(-75) = -15\,\text{V}$$

$0 \le t \le 800\,\mu$s:



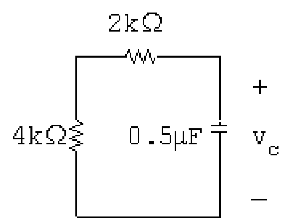
$$\tau = R_eC, \qquad R_e = \frac{(6000)(3000)}{9000} = 2\,\text{k}\Omega$$

$$\tau = (2\times 10^3)(0.5\times 10^{-6}) = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_c = -15e^{-1000t}\,\text{V}, \qquad t \ge 0$$

$$v_c(800\,\mu\text{s}) = -15e^{-0.8} = -6.74\,\text{V}$$

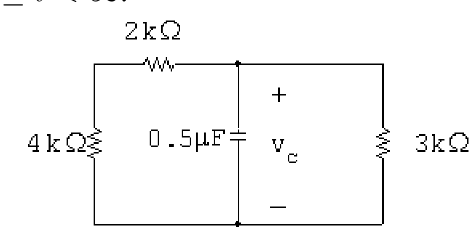
$800\,\mu$s $\le t \le 1.1$ ms:



$$\tau = (6\times 10^3)(0.5\times 10^{-6}) = 3\,\text{ms}, \qquad \frac{1}{\tau} = 333.33$$

$$v_c = -6.74e^{-333.33(t-800\times 10^{-6})}\,\text{V}$$

1.1 ms $\le t < \infty$:



$$\tau = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_c(1.1\text{ms}) = -6.74e^{-333.33(1100-800)10^{-6}} = -6.74e^{-0.1} = -6.1\,\text{V}$$

$$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\mathrm{V}$$

$$v_c(1.5\mathrm{ms}) = -6.1e^{-1000(1.5-1.1)10^{-3}} = -6.1e^{-0.4} = -4.09\,\mathrm{V}$$

$$v_o = (4/6)(-4.09) = -2.73\,\mathrm{V}$$

P 7.72 $w(0) = \frac{1}{2}(0.5\times10^{-6})(-15)^2 = 56.25\,\mu\mathrm{J}$

$0 \le t \le 800\,\mu$s:

$$v_c = -15e^{-1000t}; \qquad v_c^2 = 225e^{-2000t}$$

$$p_{3k} = 75e^{-2000t}\,\mathrm{mW}$$

$$w_{3k} = \int_0^{800\times10^{-6}} 75\times10^{-3}e^{-2000t}\,dt$$
$$= 75\times10^{-3}\frac{e^{-2000t}}{-2000}\Big|_0^{800\times10^{-6}}$$
$$= -37.5\times10^{-6}(e^{-1.6}-1) = 29.93\,\mu\mathrm{J}$$

$1.1\,\mathrm{ms} \le t \le \infty$:

$$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\mathrm{V}; \qquad v_c^2 = 37.19e^{-2000(t-1.1\times10^{-3})}$$

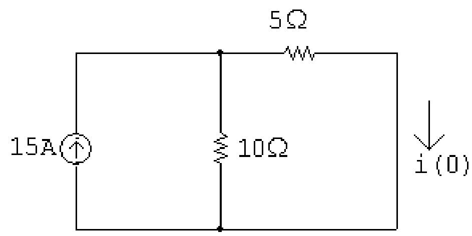
$$p_{3k} = 12.4e^{-2000(t-1.1\times10^{-3})}\,\mathrm{mW}$$

$$w_{3k} = \int_{1.1\times10^{-3}}^{\infty} 12.4\times10^{-3}e^{-2000(t-1.1\times10^{-3})}\,dt$$
$$= 12.4\times10^{-3}\frac{e^{-2000(t-1.1\times10^{-3})}}{-2000}\Big|_{1.1\times10^{-3}}^{\infty}$$
$$= -6.2\times10^{-6}(0-1) = 6.2\,\mu\mathrm{J}$$

$$w_{3k} = 29.93 + 6.2 = 36.13\,\mu\mathrm{J}$$

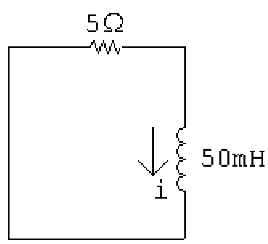
$$\% = \frac{36.13}{56.25}(100) = 64.23\%$$

P 7.73 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

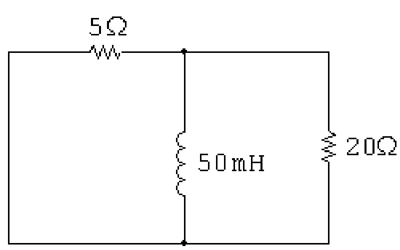
$0 \leq t \leq 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \leq t \leq 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \leq t \leq \infty$:

$$i(20\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$i = 1.65e^{-100(t-0.02)}$ A

$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$

$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$

$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$

$\quad = -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^+\,\text{ms}$

$v_o(25\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5\,\text{V}$

P 7.74 From the solution to Problem 7.73, the initial energy is

$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$

$0.04w(0) = 0.1\,\text{J}$

$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

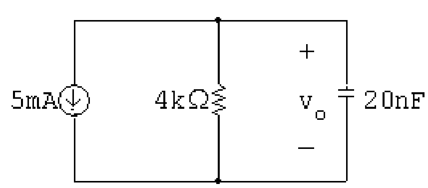
$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$

For $10\,\text{ms} \le t \le 20\,\text{ms}$:

$i = 3.68e^{-80(t-0.01)} = 2$

$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$

P 7.75 $0 \le t \le 10\,\mu\text{s}$:

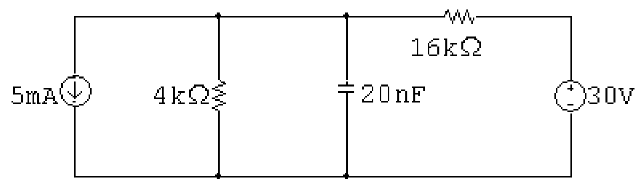


$\tau = RC = (4 \times 10^3)(20 \times 10^{-9}) = 80\,\mu\text{s}; \qquad 1/\tau = 12{,}500$

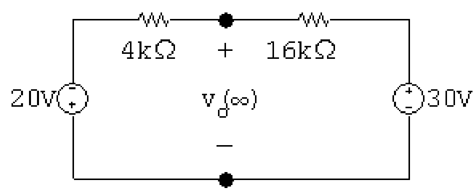
$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$

$v_o = -20 + 20e^{-12{,}500t}\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$

$10\,\mu\text{s} \le t \le \infty$:



$t = \infty$:



$$i = \frac{-50\,\text{V}}{20\,\text{k}\Omega} = -2.5\,\text{mA}$$

$v_o(\infty) = (-2.5 \times 10^{-3})(16{,}000) + 30 = -10\,\text{V}$

$v_o(10\,\mu\text{s}) = -20 + 20^{-0.125} = -2.35\,\text{V}$

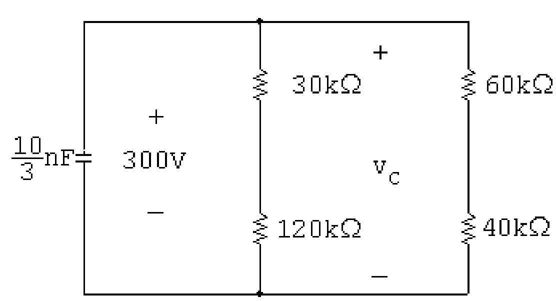
$v_o = -10 + (-2.35 + 10)e^{-(t\,-\,10\times 10^{-6})/\tau}$

$R_{\text{Th}} = 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 3.2\,\text{k}\Omega$

$\tau = (3200)(20 \times 10^{-9}) = 64\,\mu\text{s}; \qquad 1/\tau = 15{,}625$

$v_o = -10 + 7.65e^{-15{,}625(t\,-\,10\times 10^{-6})} \qquad 10\,\mu\text{s} \le t \le \infty$

P 7.76 $0 \le t \le 200\,\mu\text{s}$;

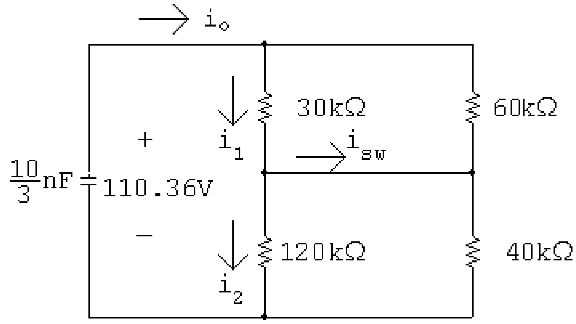


$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$

$v_c = 300e^{-5000t}\,\text{V}$

$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$

$200\,\mu\text{s} \leq t \leq \infty$:



$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$

$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$

$v_c = 110.36e^{-6000(t\,-\,200\,\mu s)}\,\text{V}$

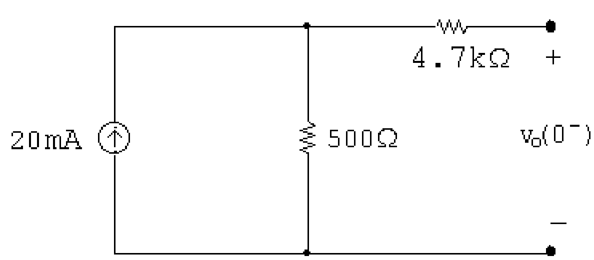
$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$

$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$

$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$

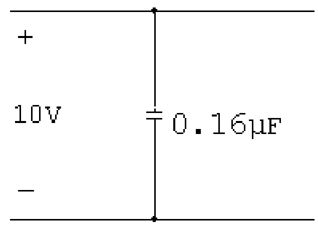
$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$

P 7.77 $t < 0$:



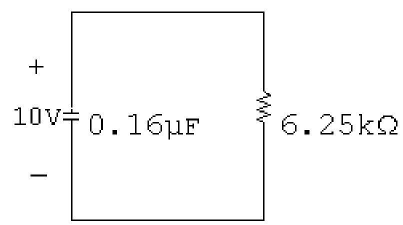
$v_c(0^-) = (20 \times 10^{-3})(500) = 10\,\text{V} = v_c(0^+)$

$0 \le t \le 50\,\text{ms}$:



$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = 10e^{-0} = 10\,\text{V}$

$50\,\text{ms} \le t \le \infty$:



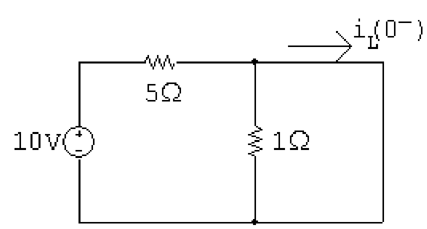
$\tau = (6.25\,\text{k})(0.16\,\mu) = 1\,\text{ms}; \qquad 1/\tau = 1000; \qquad v_o = 10e^{-1000(t\,-\,0.05)}\,\text{V}$

Summary:

$v_o = 10\,\text{V}, \qquad 0 \le t \le 50\,\text{ms}$

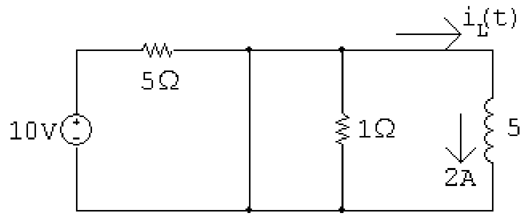
$v_o = 10e^{-1000(t\,-\,0.05)}\,\text{V}, \qquad 50\,\text{ms} \le t \le \infty$

P 7.78 $t < 0$:



$i_L(0^-) = 10\,\text{V}/5\,\Omega = 2\,\text{A} = i_L(0^+)$

$0 \leq t \leq 5$:

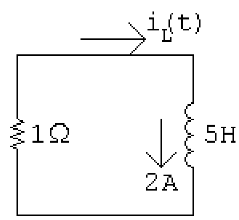


$$\tau = 5/0 = \infty$$

$$i_L(t) = 2e^{-t/\infty} = 2e^{-0} = 2$$

$$i_L(t) = 2\,\text{A}, \quad 0 \leq t \leq 5\,\text{s}$$

$5 \leq t \leq \infty$:



$$\tau = \frac{5}{1} = 5\,\text{s}; \qquad 1/\tau = 0.2$$

$$i_L(t) = 2e^{-0.2(t-5)}\,\text{A}, \quad t \geq 5\,\text{s}$$

P 7.79 **[a]** $0 \leq t \leq 2.5\,\text{ms}$

$$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$$

$$\tau = \frac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \leq t \leq 2.5\,\text{ms}$$

$$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$$

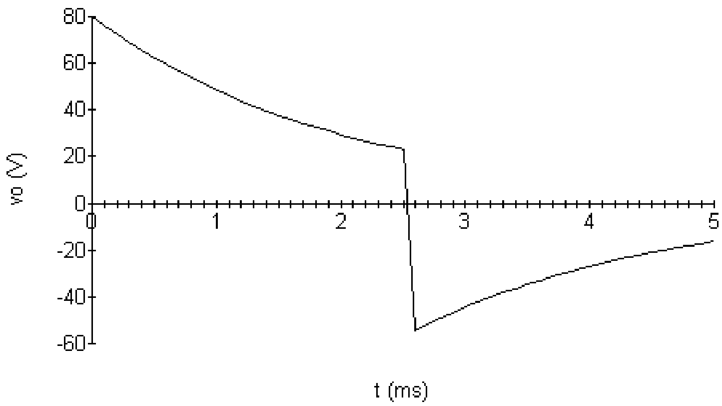
$$i_o(2.5^-\,\text{ms}) = \frac{(80 - 22.92)}{20} = 2.85\,\text{A}$$

$$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$$

$$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = -57.08e^{-500(t-0.0025)}\,\text{V} \qquad 2.5^+\,\text{ms} \leq t \leq \infty$$

[b]



[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$$i_o = \frac{+16.35}{20} = 817.68\,\text{mA}$$

P 7.80 **[a]** $i_o(0) = 0; \qquad i_o(\infty) = 25\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{2000}{250} \times 10^3 = 8000$$

$$i_o = (25 - 25e^{-8000t})\,\text{mA}, \qquad 0 \le t \le 75\,\mu\text{s}$$

$$v_o = 0.25\frac{di_o}{dt} = 50e^{-8000t}\,\text{V}, \qquad 0^+ \le t \le 75^-\,\mu s$$

$75^+\,\mu\text{s} \le t \le \infty$:

$$i_o(75\mu\text{s}) = 25 - 25e^{-0.6} = 11.28\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 11.28e^{-8000(t-75\times10^{-6})}\,\text{mA}$$

$$v_o = (0.25)\frac{di_o}{dt} = -22.56e^{-8000(t-75\mu s)}$$

$$\therefore \quad t < 0: \qquad v_o = 0$$

$$0^+ \le t \le 75^-\,\mu\text{s}: \qquad v_o = 50e^{-8000t}\,\text{V}$$

$$75^+\,\mu\text{s} \le t \le \infty: \qquad v_o = -22.56e^{-8000(t-75\mu s)}$$

[b] $v_o(75^-\mu\text{s}) = 50e^{-0.6} = 27.44\,\text{V}$

$$v_o(75^+\mu\text{s}) = -22.56\,\text{V}$$

[c] $i_o(75^-\mu\text{s}) = i_o(75^+\mu\text{s}) = 11.28\,\text{mA}$

P 7.81 **[a]** $0 \le t < 1\,\text{ms}$:

$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$

$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$

$v_c = 50 - 50e^{-250t}$

$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$

$1\,\text{ms} < t \le \infty$:

$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$

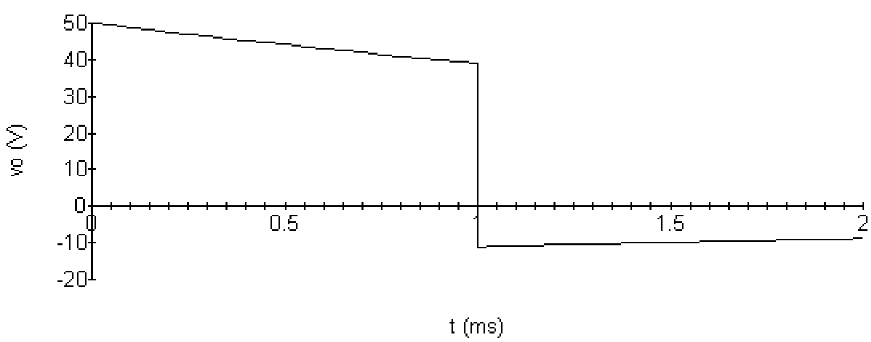
$v_c(\infty) = 0\,\text{V}$

$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$

$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad 1\,\text{ms} < t \le \infty$

[b]



P 7.82 **[a]** $t < 0; \qquad v_o = 0$

$0 \le t \le 4\,\text{ms}$:

$\tau = (200 \times 10^3)(0.025 \times 10^{-6}) = 5\,\text{ms}; \qquad 1/\tau = 200$

$v_o = 100 - 100e^{-200t}\,\text{V}, \qquad 0 \le t \le 4\,\text{ms}$

$v_o(4\,\text{ms}) = 100(1 - e^{-0.8}) = 55.07\,\text{V}$

$4\,\text{ms} \le t \le 8\,\text{ms}$:

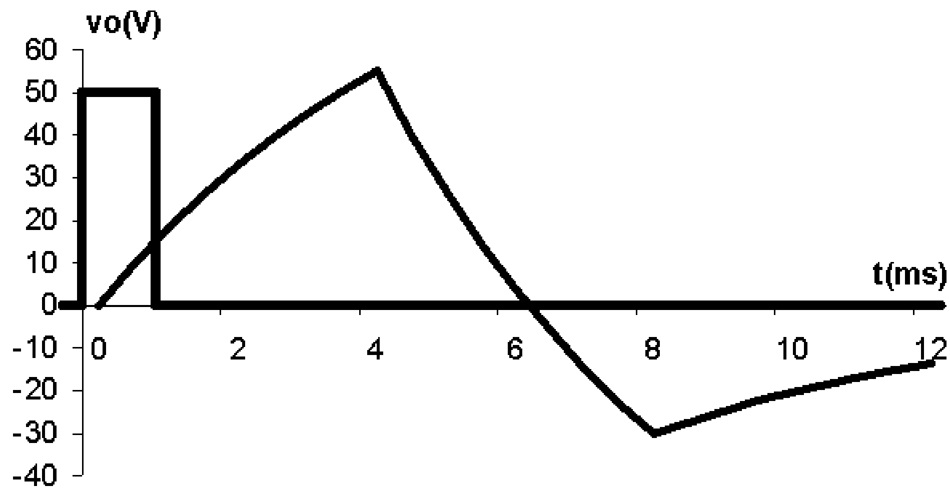
$v_o = -100 + 155.07e^{-200(t-0.004)}\,\text{V}$

$v_o(8\,\text{ms}) = -100 + 155.07e^{-0.8} = -30.32\,\text{V}$

$8\,\text{ms} \le t \le \infty$:

$v_o = -30.32e^{-200(t-0.008)}\,\text{V}$

[b]



[c] $t \leq 0:\qquad v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$$\tau = (50\times 10^3)(0.025\times 10^{-6}) = 1.25\,\text{ms} \quad 1/\tau = 800$$

$$v_o = 100 - 100e^{-800t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100 - 100e^{-3.2} = 95.92\,\text{V}$$

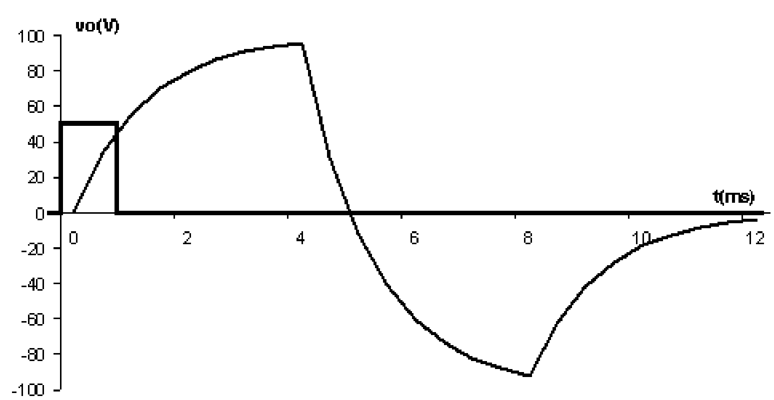
$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

$$v_o = -100 + 195.92e^{-800(t-0.004)}\,\text{V}, \qquad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 195.92e^{-3.2} = -92.01\,\text{V}$$

$8\,\text{ms} \leq t \leq \infty$:

$$v_o = -92.01e^{-800(t-0.008)}\,\text{V}, \qquad 8\,\text{ms} \leq t \leq \infty$$



P 7.83 **[a]** $\tau = RC = (20{,}000)(0.2 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$i_o = v_o = 0 \qquad t < 0$

$i_o(0^+) = 20\left(\frac{16}{20}\right) = 16\,\text{mA}, \qquad i_o(\infty) = 0$

$\therefore \quad i_o = 16e^{-250t}\,\text{mA} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$

$i_{16k\Omega} = 20 - 16e^{-250t}\,\text{mA}$

$\therefore \quad v_o = 320 - 256e^{-250t}\,\text{V} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$

$v_c = v_o - 4 \times 10^3 i_o = 320 - 320e^{-250t}\,\text{V} \qquad 0 \leq t \leq 2\,\text{ms}$

$v_c(2\,\text{ms}) = 320 - 320e^{-0.5} = 125.91\,\text{V}$

$\therefore \quad i_o(2^+\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$

$i_o(\infty) = 0$

$v_c = 125.91e^{-250(t-0.002)}, \quad 2^+\,\text{ms} \leq t \leq \infty$

$$i_o = C\frac{dv_c}{dt} = (0.2 \times 10^{-6})(-250)(125.91)e^{-250(t-0.002)}$$

$$= -6.3e^{-250(t-0.002)}\,\text{mA}, \quad 2^+\,\text{ms} \leq t \leq \infty$$

$v_o = 4000i_o + v_c = 100.73e^{-250(t-0.002)}\,\text{V} \qquad 2^+\,\text{ms} \leq t \leq \infty$

Summary part (a)

$i_o = 0 \qquad t < 0$

$i_o = 16e^{-250t}\,\text{mA} \qquad (0^+ \leq t \leq 2^-\,\text{ms})$

$i_o = -6.3e^{-250(t-0.002)}\,\text{mA} \qquad 2^+\,\text{ms} \leq t \leq \infty$

$v_o = 0 \qquad t < 0$

$v_o = 320 - 256e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 2^-\,\text{ms}$

$v_o = 100.73e^{-250(t-0.002)}\,\text{V}, \qquad 2^+\,\text{ms} \leq t \leq \infty$

[b] $i_o(0^-) = 0$

$i_o(0^+) = 16\,\text{mA}$

$i_o(2^-\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$

$i_o(2^+\,\text{ms}) = -6.3\,\text{mA}$

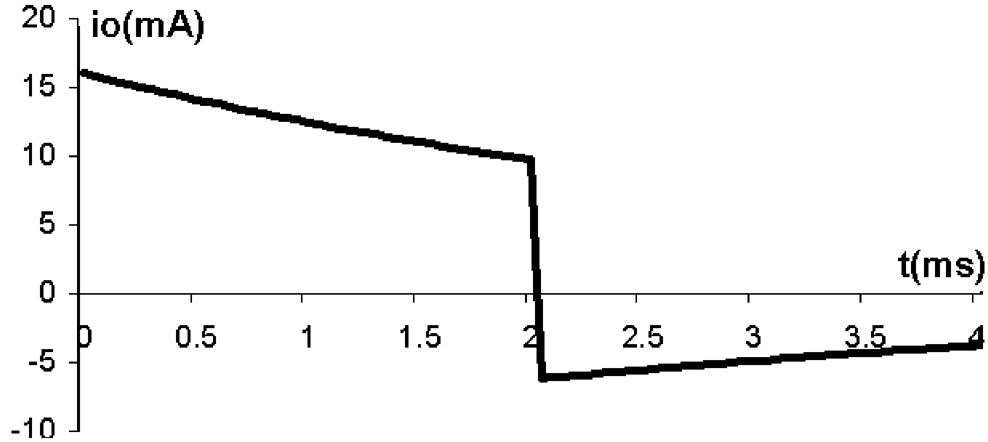
[c] $v_o(0^-) = 0$

$v_o(0^+) = 64\,\text{V}$

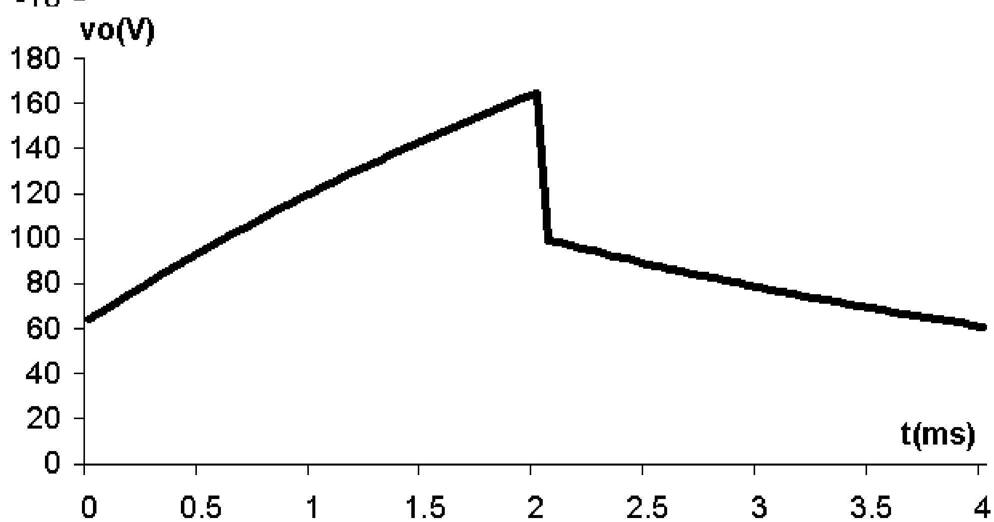
$v_o(2^-\,\text{ms}) = 320 - 256e^{-0.5} = 164.73\,\text{V}$

$v_o(2^+\,\text{ms}) = 100.73$

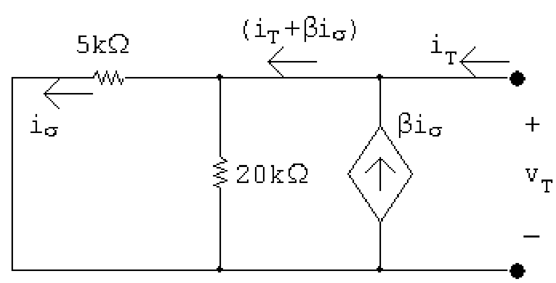
[d]



[e]



P 7.84 **[a]**



Using Ohm's law,

$v_T = 5000i_\sigma$

Using current division,

$$i_\sigma = \frac{20{,}000}{20{,}000 + 5000}(i_T + \beta i_\sigma) = 0.8i_T + 0.8\beta i_\sigma$$

Solve for i_σ:

$$i_\sigma(1-0.8\beta) = 0.8i_T$$

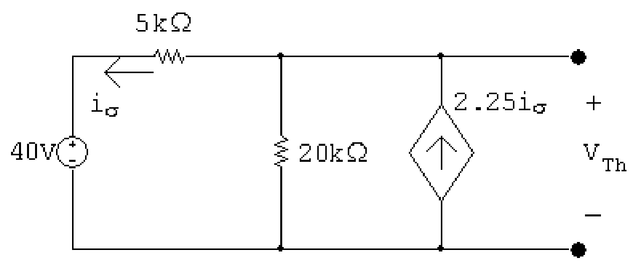
$$i_\sigma = \frac{0.8i_T}{1-0.8\beta}; \qquad v_T = 5000i_\sigma = \frac{4000i_T}{(1-0.8\beta)}$$

Find β such that $R_{\rm Th} = -5\,{\rm k}\Omega$:

$$R_{\rm Th} = \frac{v_T}{i_T} = \frac{4000}{1-0.8\beta} = -5000$$

$$1-0.8\beta = -0.8 \qquad \therefore \quad \beta = 2.25$$

[b] Find $V_{\rm Th}$;



Write a KCL equation at the top node:

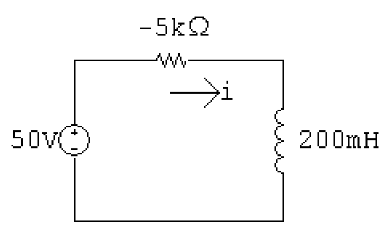
$$\frac{V_{\rm Th}-40}{5000} + \frac{V_{\rm Th}}{20{,}000} - 2.25i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\rm Th}-40)}{5000} = 0$$

Solving,

$$V_{\rm Th} = 50\,{\rm V}$$



Write a KVL equation around the loop:

$$50 = -5000i + 0.2\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 250 + 25{,}000i = 25{,}000(i+0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i+0.01} = 25{,}000\,dt$$

$$\int_0^i \frac{dx}{x+0.01} = \int_0^t 25{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{25{,}000t}\ \text{mA}$$

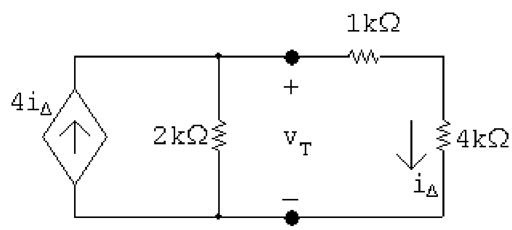
$$\frac{di}{dt} = (10\times 10^{-3})(25{,}000)e^{25{,}000t} = 250e^{25{,}000t}$$

Solve for the arc time:

$$v = 0.2\frac{di}{dt} = 50e^{25{,}000t} = 45{,}000; \qquad e^{25{,}000t} = 900$$

$$\therefore \quad t = \frac{\ln 900}{25{,}000} = 272.1\,\mu\text{s}$$

P 7.85 Find the Thévenin equivalent with respect to the terminals of the capacitor.
R_{Th} calculation:

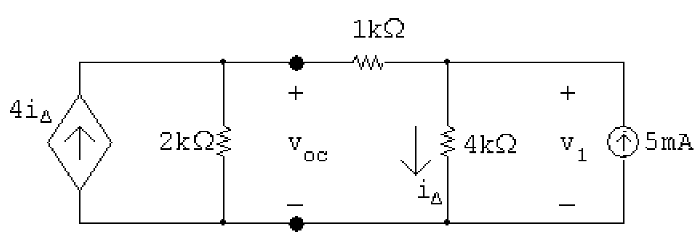


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

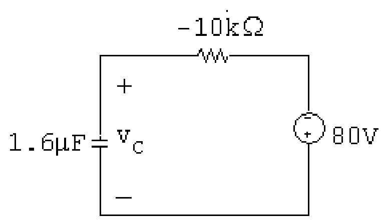
$$\frac{v_{\rm oc}}{2000} + \frac{v_{\rm oc} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\rm oc}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\rm oc} = -80\,\mathrm{V}, \quad v_1 = -60\ \mathrm{V}$



$$v_c(0) = 0; \qquad v_c(\infty) = -80\,\mathrm{V}$$

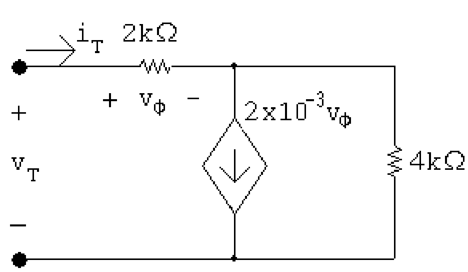
$$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\mathrm{ms}; \qquad \frac{1}{\tau} = -62.5$$

$$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$$

Solve for the time of the maximum voltage rating:

$$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.18\,\mathrm{ms}$$

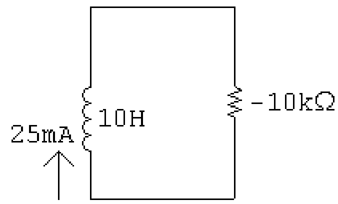
P 7.86



$$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

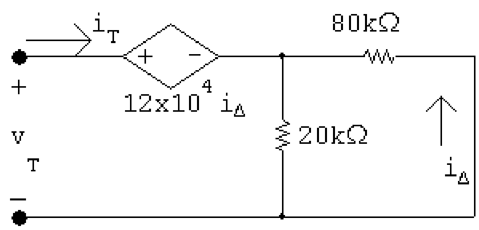


$$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$$

$$i = 25e^{1000t}\,\text{mA}$$

$$\therefore \quad 25e^{1000t} \times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$$

P 7.87 $t > 0$:



$$v_T = 12 \times 10^4 i_\Delta + 16 \times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$$

$$\therefore \quad v_T = -24 \times 10^3 i_T + 16 \times 10^3 i_T$$

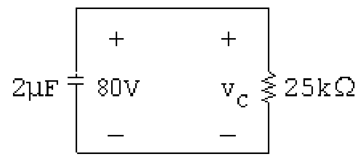
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8 \times 10^3)(2.5 \times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.88 **[a]**



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

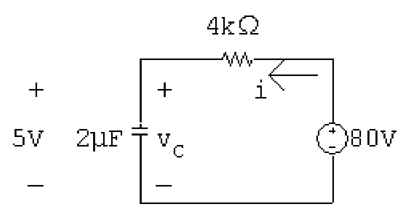
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[b] $0^+ < t < 138.63\,\text{ms}$:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$138.63^+\,\text{ms} < t \leq \infty$:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad 138.63^+\,\text{ms} \leq t \leq \infty$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad 138.63^+\,\text{ms} \leq t \leq \infty$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{12.5} \cong 14.66\,\text{ms}$$

P 7.89 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36-14}{80{,}000} + 2.5\times 10^{-6}\frac{d}{dt}(-36-v_o) = 0$$

$$\therefore \qquad 2.5\times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \qquad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \qquad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \qquad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.90 The equation for an integrating amplifier:

$$v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy + v_o(0)$$

Find the values and substitute them into the equation:

$$RC = (100\times 10^3)(0.05\times 10^{-6}) = 5\,\text{ms}$$

$$\frac{1}{RC} = 200; \qquad v_\text{b} - v_\text{a} = -15 - (-7) = -8\,\text{V}$$

$$v_o(0) = -4 + 12 = 8\,\text{V}$$

$$v_o = 200\int_0^t -8\,dx + 8 = (-1600t + 8)\,\text{V}, \quad 0 \le t \le t_\text{sat}$$

RC circuit analysis for v_2:

$$v_2(0^+) = -4\,\text{V}; \qquad v_2(\infty) = -15\,\text{V}; \qquad \tau = RC = (100\,\text{k})(0.05\,\mu) = 5\,\text{ms}$$

$$v_2 = v_2(\infty) + [v_2(0^+) - v_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

$$v_f + v_2 = v_o \quad \therefore \quad v_f = v_o - v_2 = 23 - 1600t - 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

Note that

$$-1600t_{\text{sat}} + 8 = -20 \quad \therefore \quad t_{\text{sat}} = \frac{-28}{-1600} = 17.5\,\text{ms}$$

so the op amp operates in its linear region until it saturates at 17.5 ms.

P 7.91 $$v_o = -\frac{1}{R(0.5 \times 10^{-6})}\int_0^t 4\,dx + 0 = \frac{-4t}{R(0.5 \times 10^{-6})}$$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore \quad R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12\,\text{k}\Omega$$

P 7.92 $$v_o = \frac{-4t}{R(0.5 \times 10^{-6})} + 6 = \frac{-4(40 \times 10^{-3})}{R(0.5 \times 10^{-6})} + 6 = -10$$

$$\therefore \quad R = \frac{-4(40 \times 10^{-3})}{-16(0.5 \times 10^{-6})} = 20\,\text{k}\Omega$$

P 7.93 **[a]** $$\frac{C\,dv_p}{dt} + \frac{v_p - v_{\text{b}}}{R} = 0; \qquad \text{therefore} \quad \frac{dv_p}{dt} + \frac{1}{RC}v_p = \frac{v_{\text{b}}}{RC}$$

$$\frac{v_n - v_{\text{a}}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

therefore $$\frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_{\text{a}}}{RC}$$

But $v_n = v_p$

Therefore $$\frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_{\text{b}}}{RC}$$

Therefore $$\frac{dv_o}{dt} = \frac{1}{RC}(v_{\text{b}} - v_{\text{a}}); \qquad v_o = \frac{1}{RC}\int_0^t (v_{\text{b}} - v_{\text{a}})\,dy$$

[b] The output is the integral of the difference between v_{b} and v_{a} and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_\mathrm{b} - v_\mathrm{a})\,dx$

$RC = (50\times10^3)(10\times10^{-9}) = 0.5\,\mathrm{ms}$

$v_\mathrm{b} - v_\mathrm{a} = -25\,\mathrm{mV}$

$v_o = \dfrac{1}{0.0005}\displaystyle\int_0^t -25\times10^{-3}dx = -50t$

$-50t_\mathrm{sat} = -6; \qquad t_\mathrm{sat} = 120\,\mathrm{ms}$

P 7.94 **[a]** $RC = (25\times10^3)(0.4\times10^{-6}) = 10\,\mathrm{ms}; \qquad \dfrac{1}{RC} = 100$

$v_o = 0, \quad t < 0$

[b] $0 \le t \le 250\,\mathrm{ms}:$

$v_o = -100\displaystyle\int_0^t -0.20\,dx = 20t\,\mathrm{V}$

[c] $250\,\mathrm{ms} \le t \le 500\,\mathrm{ms};$

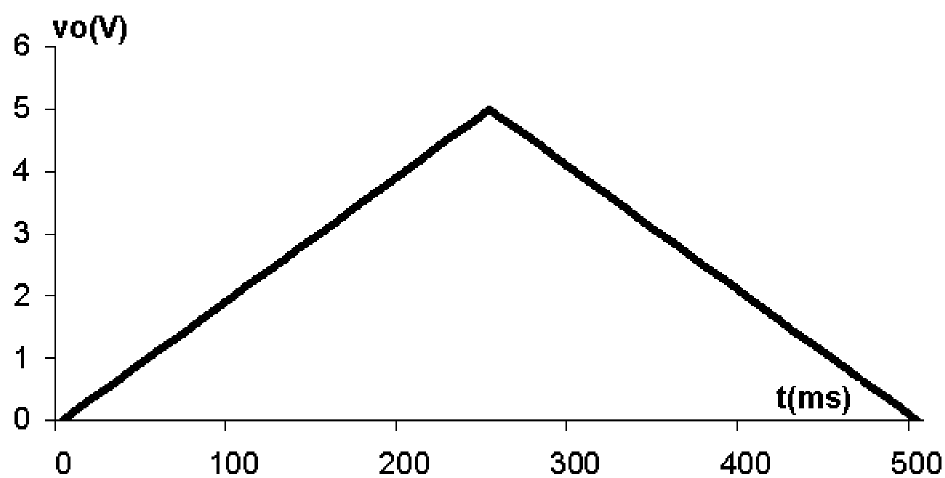
$v_o(0.25) = 20(0.25) = 5\,\mathrm{V}$

$v_o(t) = -100\displaystyle\int_{0.25}^t 0.20\,dx + 5 = -20(t-0.25)+5 = -20t+10\,\mathrm{V}$

[d] $500\,\mathrm{ms} \le t \le \infty:$

$v_o(0.5) = -10 + 10 = 0\,\mathrm{V}$

$v_o(t) = 0\,\mathrm{V}$



P 7.95 **[a]** $v_o = 0, \qquad t < 0$

$$RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms} \quad \frac{1}{RC} = 100$$

[b] $R_fC_f = (5 \times 10^6)(0.4 \times 10^{-6}) = 2; \qquad \frac{1}{R_fC_f} = 0.5$

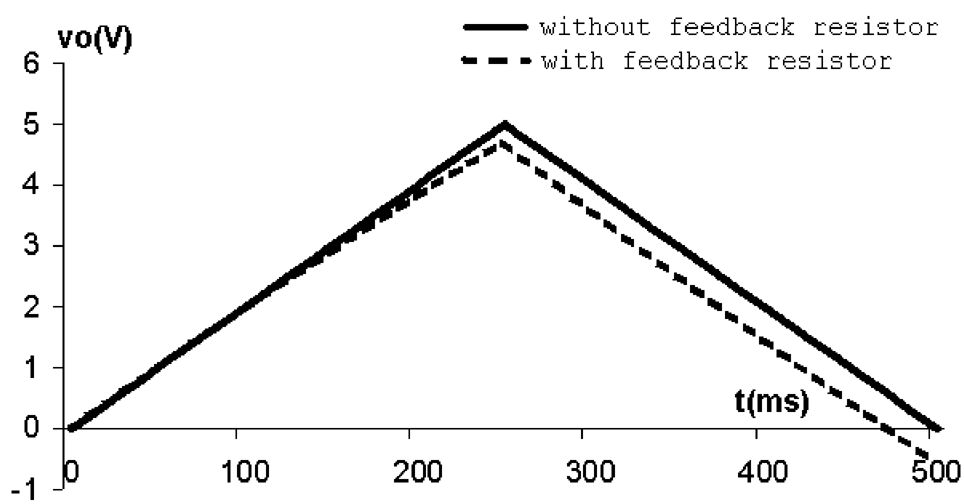
$$v_o = \frac{-5 \times 10^6}{25 \times 10^3}(-0.2)[1 - e^{-0.5t}] = 40(1 - e^{-0.5t})\,\text{V}, \qquad 0 \le t \le 250\,\text{ms}$$

[c] $v_o(0.25) = 40(1 - e^{-0.125}) \cong 4.70\,\text{V}$

$$\begin{aligned} v_o &= \frac{-V_mR_f}{R_s} + \frac{V_mR_f}{R_s}(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 40(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 44.70e^{-0.5(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \le t \le 500\,\text{ms} \end{aligned}$$

[d] $v_o(0.5) = -40 + 44.70e^{-0.125} \cong -0.55\,\text{V}$

$$v_o = -0.55e^{-0.5(t-0.5)}\,\text{V}, \qquad 500\,\text{ms} \le t \le \infty$$



P 7.96 **[a]** $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \qquad \frac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu\text{s}$:

$$v_g = 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -1.25 \times 10^6 \int_0^t 2 \times 10^6 x\,dx + 0 \\ &= -2.5 \times 10^{12}\frac{x^2}{2}\Big|_0^t = -125 \times 10^{10}t^2\,\text{V}, \quad 0 \le t \le 1\,\mu\text{s} \end{aligned}$$

$$v_o(1\,\mu\text{s}) = -125 \times 10^{10}(1 \times 10^{-6})^2 = -1.25\,\text{V}$$

$1\,\mu\text{s} \le t \le 3\,\mu\text{s}$:

$$v_g = 4 - 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -125 \times 10^4 \int_{1\times10^{-6}}^{t} (4 - 2 \times 10^6 x)\, dx - 1.25 \\ &= -125 \times 10^4 \left[4x \Big|_{1\times10^{-6}}^{t} - 2 \times 10^6 \frac{x^2}{2} \Big|_{1\times10^{-6}}^{t} \right] - 1.25 \\ &= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25 \\ &= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5\,\text{V}, \quad 1\,\mu\text{s} \le t \le 3\,\mu\text{s} \end{aligned}$$

$$v_o(3\,\mu\text{s}) = 125 \times 10^{10}(3 \times 10^{-6})^2 - 5 \times 10^6(3 \times 10^{-6}) + 2.5$$

$$= -1.25$$

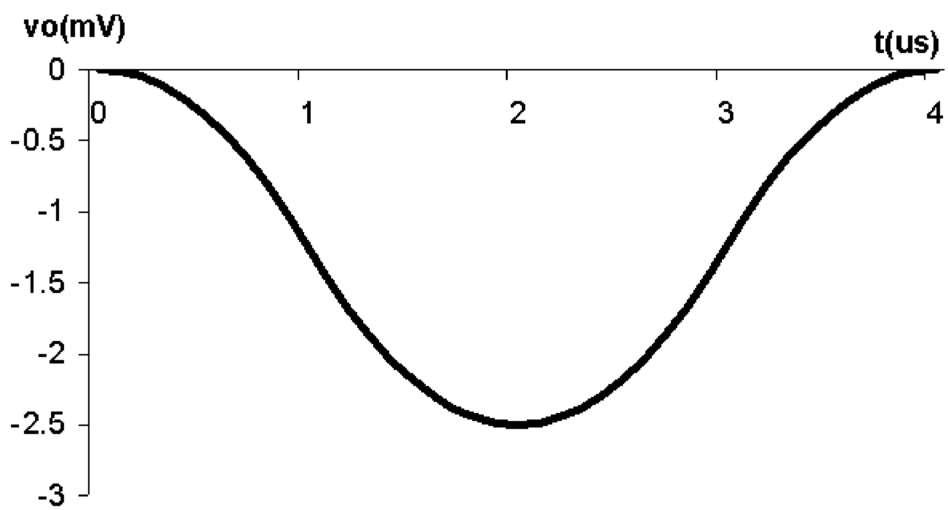
$3\,\mu\text{s} \le t \le 4\,\mu\text{s}$:

$$v_g = -8 + 2 \times 10^6 t$$

$$\begin{aligned} v_o &= -125 \times 10^4 \int_{3\times10^{-6}}^{t} (-8 + 2 \times 10^6 x)\, dx - 1.25 \\ &= -125 \times 10^4 \left[-8x \Big|_{3\times10^{-6}}^{t} + 2 \times 10^6 \frac{x^2}{2} \Big|_{3\times10^{-6}}^{t} \right] - 1.25 \\ &= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25 \\ &= -125 \times 10^{10} t^2 + 10^7 t - 20\,\text{V}, \quad 3\,\mu\text{s} \le t \le 4\,\mu\text{s} \end{aligned}$$

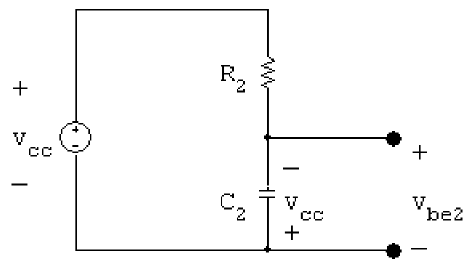
$$v_o(4\,\mu\text{s}) = -125 \times 10^{10}(4 \times 10^{-6})^2 + 10^7(4 \times 10^{-6}) - 20 = 0$$

[b]



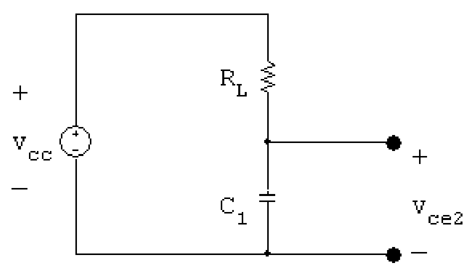
[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu\text{s}$ the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu\text{s}$ as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 **[a]** While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\rm be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, $v_{\rm be2}$ is zero, T_2 turns ON. This makes $v_{\rm be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\rm be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage $v_{\rm ce2}$ is the same as the voltage across C_1, thus



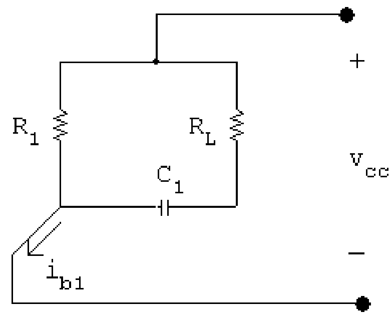
It follows that $v_{\rm ce2} = V_{CC} - V_{CC}e^{-t/R_{\rm L}C_1}$.

[c] T_2 will be OFF until $v_{\rm be2}$ reaches zero. As soon as $v_{\rm be2}$ is zero, $i_{\rm b2}$ will become positive and turn T_2 ON. $v_{\rm be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{\rm ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_{\rm L}C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, $i_{\rm b1}$ is zero. At the instant T_1 turns ON, we have



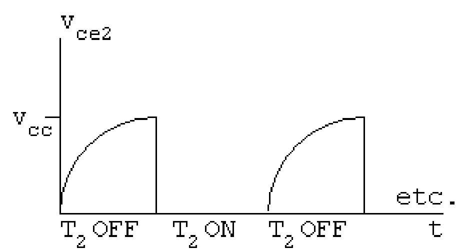
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L} e^{-t/R_L C_1}$$

[f] At the instant T_2 turns back ON, $t = R_2 C_2 \ln 2$; therefore

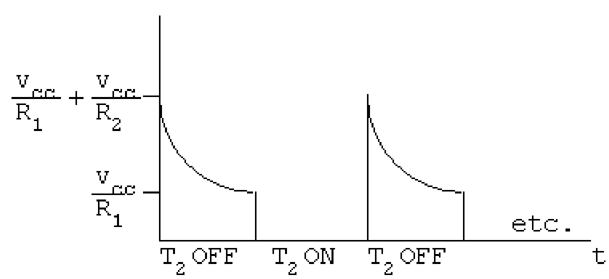
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L} e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

When T_2 turns OFF, i_{b1} drops to zero instantaneously.

[g]



[h]



P 7.98 **[a]** $t_{OFF2} = R_2 C_2 \ln 2 = 14.43 \times 10^3 (1 \times 10^{-9}) \ln 2 \cong 10\,\mu s$

[b] $t_{ON2} = R_1 C_1 \ln 2 \cong 10\,\mu s$

[c] $t_{OFF1} = R_1 C_1 \ln 2 \cong 10\,\mu s$

[d] $t_{ON1} = R_2 C_2 \ln 2 \cong 10\,\mu s$

[e] $i_{b1} = \dfrac{10}{1000} + \dfrac{10}{14{,}430} \cong 10.69\,\text{mA}$

[f] $i_{b1} = \dfrac{10}{14{,}430} + \dfrac{10}{1000} e^{-10} \cong 0.693\,\text{mA}$

[g] $v_{\text{ce2}} = 10 - 10e^{-10} \cong 10\,\text{V}$

P 7.99 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = (14.43 \times 10^3)(0.8 \times 10^{-9}) \ln 2 \cong 8\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 10\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 10\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 8\,\mu\text{s}$

[e] $i_{\text{b1}} = 10.69\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{10}{14{,}430} + \dfrac{10}{1000}e^{-8} \cong 0.693\,\text{mA}$

[g] $v_{\text{ce2}} = 10 - 10e^{-8} \cong 10\,\text{V}$

Note in this circuit T_2 is OFF $8\,\mu$s and ON $10\,\mu$s of every cycle, whereas T_1 is ON $8\,\mu$s and OFF $10\,\mu$s every cycle.

P 7.100 If $R_1 = R_2 = 50R_{\text{L}} = 100\,\text{k}\Omega,$ then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_{\text{L}} = 12\,\text{k}\Omega,$ then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \leq C_1 \leq 5.77\,\text{nF}$ and $519.37\,\text{pF} \leq C_2 \leq 4.33\,\text{nF}$

P 7.101 **[a]** T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{\text{b2}} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{\text{ce2}} = 0$, therefore $i_{\text{b1}} = 0$. When $i_{\text{b1}} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

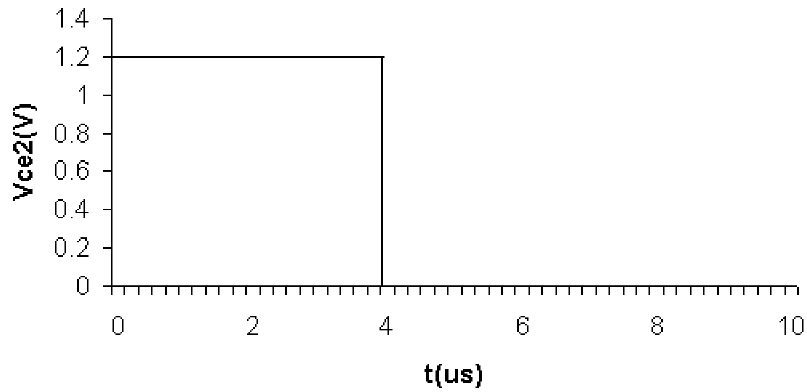
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_{\text{L}})$ and i_{b1} jumps to $V_{CC}/(R_1 + R_{\text{L}})$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{\text{be2}} = 0$. The equation for v_{be2} is $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{\text{be2}} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

P 7.102 **[a]** For $t < 0$, $v_{\text{ce2}} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{\text{ce2}} = \left(\frac{V_{CC}}{R_1 + R_{\text{L}}}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu$s.

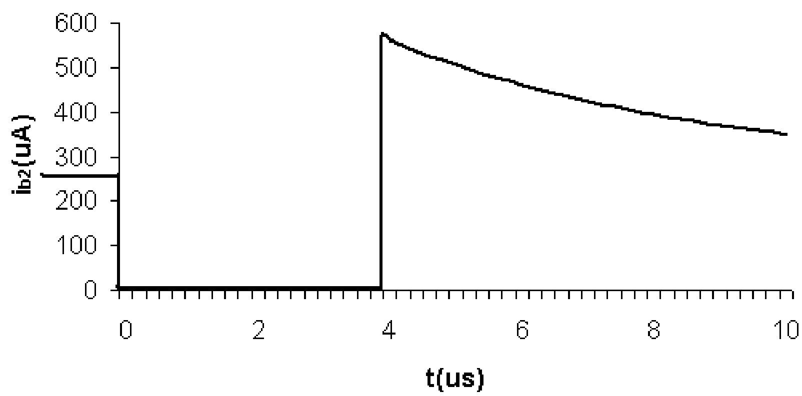


[b] $i_{b2} = \frac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \leq t \leq 0\,\mu\text{s}$

$i_{b2} = 0, \qquad 0 < t < RC \ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L} e^{-(t-RC\ln 2)/R_LC}$$

$$= 259.93 + 300e^{-0.2\times10^6(t-4\times10^{-6})}\,\mu\text{A}, \qquad RC \ln 2 < t$$



P 7.103 **[a]** We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10 \times 10^{-6}) \ln \frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\ \text{M}\Omega$$

[b] When the lamp is conducting

$$V_{Th} = \frac{20 \times 10^3}{20 \times 10^3 + 1.091 \times 10^6}(6) = 0.108\,\text{V}$$

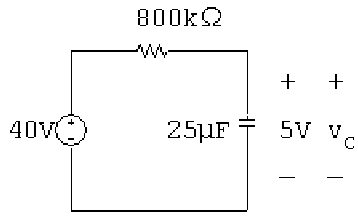
$$R_{Th} = 20\ \text{k}||1.091\ \text{M} = 19{,}640\ \Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10 \times 10^{-6}) \ln \frac{4 - 0.108}{1 - 0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 **[a]** At $t = 0$ we have



$\tau = (800)(25) \times 10^{-3} = 20$ sec; $\qquad 1/\tau = 0.05$

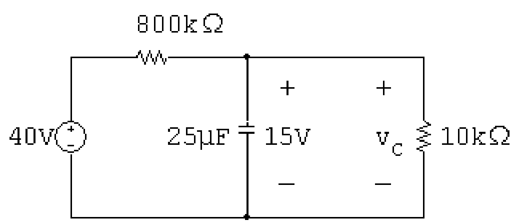
$v_c(\infty) = 40\,\text{V}; \qquad v_c(0) = 5\,\text{V}$

$v_c = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$

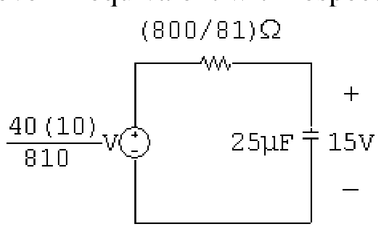
$40 - 35e^{-0.05t_o} = 15; \qquad \therefore \quad e^{0.05t_o} = 1.4$

$t_o = 20\,\ln 1.4\,\text{s} = 6.73\,\text{s}$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right)e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)}$$

$$\therefore \quad \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

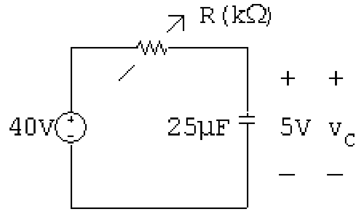
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\,\ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



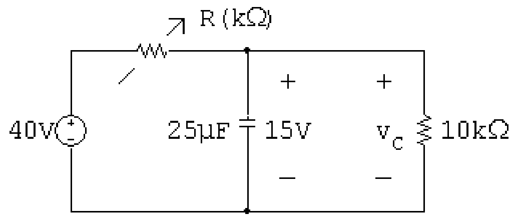
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$$v_c = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore \quad t_o = \frac{R}{40}\ln 1.4, \qquad R \quad \text{in} \quad \text{k}\Omega$$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore \quad \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or} \quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \quad \underbrace{\frac{R}{40} \ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ \ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$. Second trial set $(R/40)\ \ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $\quad R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

The procedure converges to $R = 559.3\ \text{k}\Omega$

P 7.105 **[a]** $t_o = RC \ln\left(\frac{V_{\text{min}} - V_s}{V_{\text{max}} - V_s}\right) = (3700)(250 \times 10^{-6}) \ln\left(\frac{-700}{-100}\right)$

$$= 1.80\,\text{s}$$

$$t_c - t_o = \frac{RCR_{\text{L}}}{R + R_{\text{L}}} \ln\left(\frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}\right)$$

$$\frac{R_{\text{L}}}{R + R_{\text{L}}} = \frac{1.3}{1.3 + 3.7} = 0.26 \quad RC = (3700)(250 \times 10^{-6}) = 0.925\ \text{s}$$

$$V_{\text{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\text{V} \quad R_{\text{Th}} = 3.7\ \text{k}||1.3\ \text{k} = 962\ \Omega$$

$$\therefore \quad t_c - t_o = (0.925)(0.26) \ln(640/40) = 0.67\,\text{s}$$

$$\therefore \quad t_c = 1.8 + 0.67 = 2.47\,\text{s}$$

$$\text{flashes/min} = \frac{60}{2.47} = 24.32$$

[b] $0 \le t \le t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\text{s}$$

$t_o \le t \le t_c$:

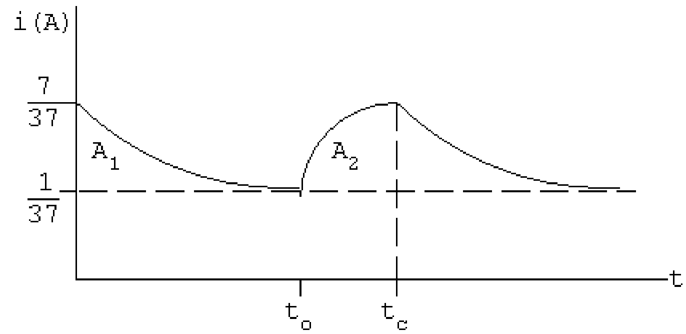
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\text{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\text{s}$$

$$0 \le t \le t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\text{ A}$$

$$t_o \le t \le t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\text{avg}} = \frac{A_1 + A_2}{t_c}$$

$$\begin{aligned} A_1 &= \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt \\ &= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15\text{ A–s} \\ A_2 &= \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt \\ &= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1) \\ &= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16}) \\ &= 0.09436\text{ A–s} \end{aligned}$$

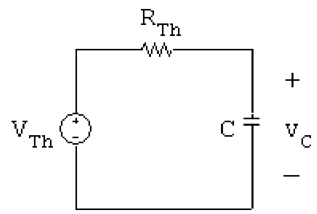
$$i_{\text{avg}} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\text{mA}$$

[c] $P_{\text{avg}} = (1000)(99.06 \times 10^{-3}) = 99.06\,\text{W}$

$$\text{No. of kw hrs/yr} = \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} = (867.77)(0.05) = 43.39\text{ dollars/year}$$

P 7.106 **[a]** Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\rm Th} = R \| R_{\rm L} = \frac{RR_{\rm L}}{R + R_{\rm L}}; \qquad V_{\rm Th} = \frac{R_{\rm L}}{R + R_{\rm L}} V_s$$

From this circuit,

$$v_{\rm C}(\infty) = V_{\rm Th}; \qquad v_{\rm C}(0) = V_{\max}; \qquad \tau = R_{\rm Th}C$$

Thus,

$$v_{\rm C}(t) = V_{\rm Th} + (V_{\max} - V_{\rm Th})e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\rm L}C}{R + R_{\rm L}}$$

[b] Now, set $v_{\rm C}(t_c) = V_{\min}$ and solve for $(t_c - t_o)$:

$$V_{\rm Th} + (V_{\max} - V_{\rm Th})e^{-(t_c-t_o)/\tau} = V_{\min}$$

$$e^{-(t_c-t_o)/\tau} = \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}}$$

$$\frac{-(t_c - t_o)}{\tau} = \ln \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}}$$

$$(t_c - t_o) = -\frac{RR_{\rm L}C}{R + R_{\rm L}} \ln \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}}$$

$$(t_c - t_o) = \frac{RR_{\rm L}C}{R + R_{\rm L}} \ln \frac{V_{\max} - V_{\rm Th}}{V_{\min} - V_{\rm Th}}$$

P 7.107 **[a]** $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right)e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31\ln 1.25}{60} \cong 0.10\,\text{s}$$

8

Natural and Step Responses of RLC Circuits

Assessment Problems

AP 8.1 **[a]** $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}$, therefore $C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2

$$\begin{aligned}
i_{\text{L}} &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20\left\{ \frac{-14e^{-5000x}}{-5000}\bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\bigg|_0^t \right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0$, therefore $i_{\text{R}}(0^+) = 0$

[b] $i_{\rm C}(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4,\qquad$ therefore $\quad\dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V},\qquad t\geq 0^+$

$$v(0^+) = A_1 + A_2,\qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $\quad A_1 + A_2 = 0,\qquad -A_1 - 4A_2 = 40{,}000;\qquad A_1 = 40{,}000/3\,\text{V}$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V},\qquad t\geq 0$

AP 8.4 **[a]** $\dfrac{1}{2RC} = 8000,\qquad$ therefore $\quad R = 62.5\,\Omega$

[b] $i_{\rm R}(0^+) = \dfrac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_{\rm C}(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\quad\dfrac{dv(0^+)}{dt} = \dfrac{-240\,\text{m}}{C} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V},\qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $\quad 6000B_2 - 8000B_1 = -240{,}000,\qquad B_2 = (-80/3)\,\text{V}$

[d] $i_{\rm L} = -(i_{\rm R} + i_{\rm C});\qquad i_{\rm R} = v/R;\qquad i_{\rm C} = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $\quad i_{\rm R} = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_{\rm C} = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_{\rm L} = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA},\qquad t\geq 0$$

AP 8.5 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4},\qquad$ therefore $\quad\dfrac{1}{2RC} = 500,\quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3},\qquad$ therefore $\quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3},\qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$$i_{\rm R}(0^+) = \frac{50}{100} = 500\,\text{mA}$$

Therefore $\quad i_{\rm C}(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\quad \frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \frac{1}{2RC} = 500, \quad D_1 = -50{,}000$ V/s

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$$i_{\rm R} = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$$

AP 8.6 **[a]** $i_{\rm R}(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08\,\text{A}$

[b] $i_{\rm C}(0^+) = I - i_{\rm R}(0^+) - i_{\rm L}(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\frac{di_{\rm L}(0^+)}{dt} = \frac{V_o}{L} = \frac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750$ rad/s

[e] $i_{\rm L} = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_{\rm L}(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $\quad B_1' = 1.5\,\text{A}$

$\frac{di_{\rm L}(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $\quad B_2' = (25/12)\,\text{A}$

Therefore $\quad i_{\rm L}(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \frac{Ł di_{\rm L}}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_{\rm C}(0^-) = \left(\frac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\frac{di(0^+)}{dt} = 100, \qquad \frac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \frac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000$ rad/s

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \frac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos\omega_d t + B_2' \sin\omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50\cos 6000t + 66.67\sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(1000)(2\times 10^{-6})} = 250$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(12.5)(2\times 10^{-6})} = 40{,}000$$

$$s_{1,2} = -250 \pm \sqrt{250^2 - 40{,}000} = -250 \pm 150$$

$$s_1 = -100 \text{ rad/s}$$

$$s_2 = -400 \text{ rad/s}$$

[b] overdamped

[c] Note — we want $\omega_d = 120$ rad/s:

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 40{,}000 - (120)^2 = 25{,}600$$

$$\alpha = 160$$

$$\frac{1}{2RC} = 160; \qquad \therefore \quad R = \frac{1}{2(160)(2\times 10^{-6})} = 1562.5\,\Omega$$

[d] $s_1,\ s_2 = -160 \pm \sqrt{160^2 - 40{,}000} = -160 \pm j120$ rad/s

[e] $\alpha = \sqrt{40{,}000} = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2(200)(2\times 10^{-6})} = 1250\,\Omega$

P 8.2 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -250$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1000$$

Adding the above equations, $\quad -2\alpha = -1250$

$$\alpha = 625\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{1}{2R(0.1 \times 10^{-6})} = 625$$

$$R = 8\,\text{k}\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 750$$

$$4(\alpha^2 - \omega_o^2) = 562{,}500$$

$$\therefore \quad \omega_o = 500\,\text{rad/s}$$

$$\omega_o^2 = 25 \times 10^4 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{(25 \times 10^4)(0.1 \times 10^{-6})} = 40\,\text{H}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -1e^{-250t} + 4e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}, \qquad t \geq 0$$

P 8.3 **[a]** $i_\text{R}(0) = \dfrac{15}{200} = 75\text{mA}$

$$i_\text{L}(0) = -45\text{mA}$$

$$i_\text{C}(0) = -i_\text{L}(0) - i_\text{R}(0) = 45 - 75 = -30\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000\,\text{rad/s}; \qquad s_2 = -20{,}000\,\text{rad/s}$$

$$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$$

$$v(0) = A_1 + A_2 = 15$$

$$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4 \text{V/s}$$

Solving, $A_1 = 10; \quad A_2 = 5$

$$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $$i_C = C\frac{dv}{dt}$$

$$= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}]$$

$$= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}$$

$$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$$

$$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.4 $$\frac{1}{2RC} = \frac{1}{2(312.5)(0.2 \times 10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -8000 \pm \sqrt{8000^2 - 10^8} = -8000 \pm j6000\,\text{rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1e^{-8000t}\cos 6000t + B_2e^{-8000t}\sin 6000t$$

$$v(0^+) = 15\,\text{V} = B_1; \qquad i_R(0^+) = \frac{15}{312.5} = 48\,\text{mA}$$

$$i_C(0^+) = [-i_L(0^+) + i_R(0^+)] = -[-45 + 48] = -3\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{-3 \times 10^{-3}}{0.2 \times 10^{-6}} = -15{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -8000B_1 + 6000B_2 = -15{,}000$$

$$6000B_2 = 8000(15) - 15{,}000; \qquad \therefore \quad B_2 = 17.5\,\text{V}$$

$$v(t) = 15e^{-8000t}\cos 6000t + 17.5e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.5 $\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$

$\alpha^2 = 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$

Critical damping:

$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

$i_R(0^+) = \frac{15}{250} = 60\,\text{mA}$

$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-45 + 60] = -15\,\text{mA}$

$v(0) = D_2 = 15$

$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$

$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_\text{C}(0)}{C} = \frac{-15 \times 10^{-3}}{0.2 \times 10^{-6}} = -75{,}000$

$D_1 = \alpha D_2 - 75{,}000 = (10^4)(15) - 75{,}000 = 75{,}000$

$v = (75{,}000t + 15)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.6 $\alpha = 1000/2 = 500$

$R = \frac{1}{2\alpha C} = \frac{1}{2(500)(2.5 \times 10^{-6})} = 400\,\Omega$

$v(0^+) = 3(1 + 1) = 6\,\text{V}$

$i_\text{R}(0^+) = \frac{6}{400} = 15\,\text{mA}$

$\frac{dv}{dt} = -300e^{-100t} - 2700e^{-900t}$

$\frac{dv(0^+)}{dt} = -300 - 2700 = -3000\,\text{V/s}$

$i_\text{C}(0^+) = 2.5 \times 10^{-6}(-3000) = -7.5\,\text{mA}$

$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[15 - 7.5] = -7.5\,\text{mA}$

P 8.7 **[a]** $\alpha = 20{,}000; \qquad \omega_d = 15{,}000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 225 \times 10^6 + 400 \times 10^6 = 625 \times 10^6$$

$$\frac{1}{LC} = 625 \times 10^6$$

$$L = \frac{1}{(625 \times 10^6)(40 \times 10^{-9})} = 40\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{2(20{,}000)(40 \times 10^{-9})} = 625\,\Omega$$

[c] $V_o = v(0) = 100\,\text{V}$

[d] $I_o = i_\text{L}(0) = -i_\text{R}(0) - i_\text{C}(0)$

$$i_\text{R}(0) = \frac{V_o}{R} = \frac{100}{625} = 160\,\text{mA}$$

$$i_\text{C}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 100\{e^{-20{,}000t}[-15{,}000 \sin 15{,}000t - 30{,}000 \cos 15{,}000t] -$$

$$20{,}000e^{-20{,}000t}[\cos 15{,}000t - 2 \sin 15{,}000t]$$

$$\frac{dv}{dt}(0) = 100\{1(-30{,}000) - 20{,}000\} = -500 \times 10^4$$

$$C\frac{dv}{dt}(0) = -500 \times 10^4(40 \times 10^{-9}) = -200\,\text{mA}$$

$$\therefore \quad I_o = 200 - 160 = 40\,\text{mA}$$

[e]

$$\frac{dv}{dt} = 100e^{-20{,}000t}[25{,}000 \sin 15{,}000t - 50{,}000 \cos 15{,}000t]$$

$$= 25 \times 10^5 e^{-20{,}000t}[\sin 15{,}000t - 2 \cos 15{,}000t]$$

$$C\frac{dv}{dt} = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2 \cos 15{,}000t)$$

$$i_\text{C}(t) = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2 \cos 15{,}000t)\,\text{A}$$

$$i_\text{R}(t) = 0.16e^{-20{,}000t}(\cos 15{,}000t - 2 \sin 15{,}000t)\,\text{A}$$

$$i_\text{L}(t) = -i_\text{R}(t) - i_\text{C}(t)$$

$$= e^{-20{,}000t}(40 \cos 15{,}000t + 220 \sin 15{,}000t)\,\text{mA}, \quad t \geq 0$$

P 8.8 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4 \times 10^{-6})} = 1.5625\,\text{H}$$

$$i_{\text{C}}(0^+) = A_1 + A_2 = 45\,\text{mA}$$

$$\frac{di_{\text{C}}}{dt} + \frac{di_{\text{L}}}{dt} + \frac{di_{\text{R}}}{dt} = 0$$

$$\frac{di_{\text{C}}(0)}{dt} = -\frac{di_{\text{L}}(0)}{dt} - \frac{di_{\text{R}}(0)}{dt}$$

$$\frac{di_{\text{L}}(0)}{dt} = \frac{15}{1.5625} = 9.6\,\text{A/s}$$

$$\frac{di_{\text{R}}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_{\text{C}}(0)}{C} = \frac{45 \times 10^{-3}}{(250)(4 \times 10^{-6})} = 45\,\text{A/s}$$

$$\therefore \quad \frac{di_{\text{C}}(0)}{dt} = -9.6 - 45 = -54.6\,\text{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 54.6 \quad A_1 + A_2 = 0.045$$

Solving, $A_1 = -31\,\text{mA}; \qquad A_2 = 76\,\text{mA}$

$$\therefore \quad i_{\text{C}} = -31e^{-200t} + 76e^{-800t}\,\text{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3 e^{-200t} + A_4 e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 15$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 38.75\,\text{V}; \qquad A_4 = -23.75\,\text{V}$$

$$v = 38.75e^{-200t} - 23.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

[c] $i_{\text{R}}(t) = \dfrac{v}{250} = 155e^{-200t} - 95e^{-800t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_{\text{L}} = -i_{\text{R}} - i_{\text{C}}$

$$i_{\text{L}} = -124e^{-200t} + 19e^{-800t}\,\text{mA}, \qquad t \geq 0$$

P 8.9 **[a]** $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (500)^2$

$\therefore \quad C = \frac{1}{(500)^2(4)} = 1\,\mu\text{F}$

$\frac{1}{2RC} = 500$

$\therefore \quad R = \frac{1}{2(500)(10^{-6})} = 1\,\text{k}\Omega$

$v(0) = D_2 = 8\,\text{V}$

$i_{\text{R}}(0) = \frac{8}{1000} = 8\text{mA}$

$i_{\text{C}}(0) = -8 + 10 = 2\,\text{mA}$

$\frac{dv}{dt}(0) = D_1 - 500D_2 = \frac{2 \times 10^{-3}}{10^{-6}} = 2000\,\text{V/s}$

$\therefore \quad D_1 = 2000 + 500(8) = 6000$ V/s

[b] $v = 6000te^{-500t} + 8e^{-500t}\,\text{V}, \quad t \geq 0$

$\frac{dv}{dt} = [-3 \times 10^6 t + 2000]e^{-500t}$

$i_{\text{C}} = C\frac{dv}{dt} = (-3000t + 2)e^{-500t}\,\text{mA}, \quad t \geq 0^+$

P 8.10 **[a]** $\alpha = \frac{1}{2RC} = 0.5$ rad/s

$\omega_o^2 = \frac{1}{LC} = 25.25$

$\omega_d = \sqrt{25.25 - (0.5)^2} = 5$ rad/s

$\therefore \quad v = B_1 e^{-t/2}\cos 5t + B_2 e^{-t/2}\sin 5t$

$v(0) = B_1 = 0; \qquad v = B_2 e^{-t/2}\sin 5t$

$i_R(0^+) = 0\,\text{A}; \qquad i_C(0^+) = 4\,\text{A}; \qquad \frac{dv}{dt}(0^+) = \frac{4}{0.08} = 50$ V/s

$50 = -\alpha B_1 + \omega_d B_2 = -0.5(0) + 5B_2$

$\therefore \quad B_2 = 10$

$\therefore \quad v = 10e^{-t/2}\sin 5t\,\text{V}, \qquad t \geq 0$

[b] $\frac{dv}{dt} = -5e^{-t/2}\sin 5t + 10e^{-t/2}(5\cos 5t)$

$\frac{dv}{dt} = 0$ when $10\cos 5t = \sin 5t$ or $\tan 5t = 10$

$\therefore\quad 5t_1 = 1.47, \qquad t_1 = 294.23\,\text{ms}$

$5t_2 = 1.47 + \pi, \qquad t_2 = 922.54\,\text{ms}$

$5t_3 = 1.47 + 2\pi, \qquad t_3 = 1550.86\,\text{ms}$

[c] $t_3 - t_1 = 1256.6\,\text{ms}; \qquad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{5} = 1256.6\,\text{ms}$

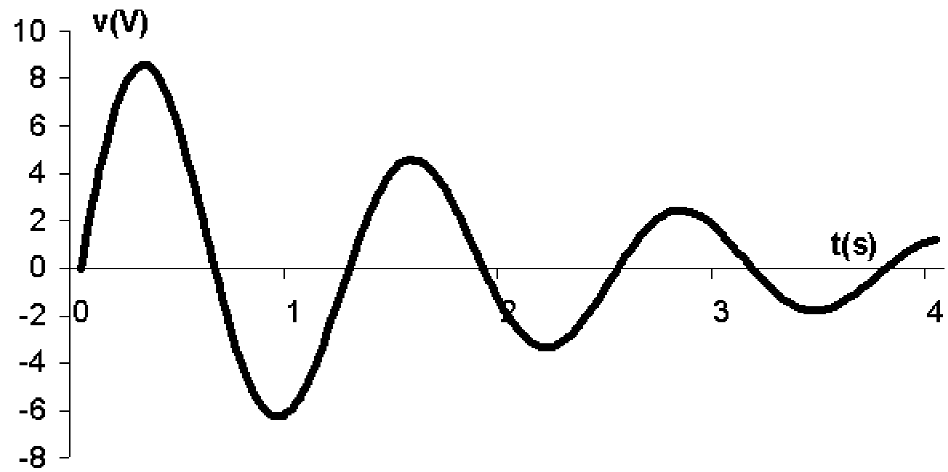
[d] $t_2 - t_1 = 628.3\,\text{ms}; \qquad \frac{T_d}{2} = \frac{1256.6}{2} = 628.3\,\text{ms}$

[e] $v(t_1) = 10e^{-(0.147115)}\sin 5(0.29423) = 8.59\,\text{V}$

$v(t_2) = 10e^{-(0.46127)}\sin 5(0.92254) = -6.27\,\text{V}$

$v(t_3) = 10e^{-(0.77543)}\sin 5(1.55086) = 4.58\,\text{V}$

[f]



P 8.11 **[a]** $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{25.25} = 5.02\,\text{rad/s}$

$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2\sin\omega_o t$

$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = 4$

$50 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{25.25}B_2$

$\therefore\quad B_2 = 50/\sqrt{25.25} = 9.95\,\text{V}$

$v = 9.95\sin 5.02t\,\text{V}, \qquad t \geq 0$

[b] $2\pi f = 5.02; \qquad f = \frac{5.02}{2\pi} \cong 0.80\,\text{Hz}$

[c] $9.95\,\text{V}$

P 8.12 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(12.5)(3.2\times10^{-9})} = 25\times10^6$

$\omega_o = 5000$ rad/s

$\dfrac{1}{2RC} = 5000; \qquad R = \dfrac{1}{2(5000)(3.2\times10^{-9})} = 31.25\,\text{k}\Omega$

[b] $v(t) = D_1te^{-5000t} + D_2e^{-5000t}$

$v(0) = 100\,\text{V} = D_2$

$\dfrac{dv}{dt} = (D_1t+100)(-5000e^{-5000t}) + D_1e^{-5000t}$

$\dfrac{dv}{dt}(0) = -500\times10^3 + D_1 = \dfrac{i_\text{C}(0)}{C}$

$i_\text{C}(0) = -i_\text{R}(0) - i_\text{L}(0)$

$i_\text{R}(0) = \dfrac{100}{31{,}500} = 3.2\,\text{mA}$

$\therefore \quad i_\text{C}(0) = -(3.2+6.4) = -9.6\,\text{mA}$

$\therefore \quad \dfrac{dv}{dt}(0) = -\dfrac{9.6\times10^{-3}}{3.2\times10^{-9}} = -3\times10^6$

$\therefore \quad -500\times10^3 + D_1 = -3\times10^6$

$D_1 = -25\times10^5\text{V/s}$

$\therefore \quad v(t) = (-25\times10^5t + 100)e^{-5000t}\,\text{V}, \qquad t\geq0$

[c] $i_\text{C}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$\dfrac{dv}{dt} = (-25\times10^5t+100)(-5000)e^{-5000t} + e^{-5000t}(-25\times10^5)$

$= (125\times10^8t - 30\times10^5)e^{-5000t}$

$\dfrac{dv}{dt} = 0$ when $125\times10^8t_1 = 3\times10^6; \qquad \therefore \quad t_1 = 240\,\mu\text{s}$

$v(240\mu\text{s}) = e^{-1.2}[(-25\times10^5)(240\times10^{-6})+100] = -150.6\,\text{V}$

[d] $i_L(240\mu s) = -i_R(240\mu s) = \dfrac{-150.6}{31{,}250} = -4.82\,\text{mA}$

$$\omega_C(240\mu s) = \frac{1}{2}(3.2 \times 10^{-9})(-150.6)^2 = 36.29\,\mu J$$

$$\omega_L(240\mu s) = \frac{1}{2}(12.5)(-4.82 \times 10^{-3})^2 = 145.2\,\mu J$$

$$\omega(240\mu s) = \omega_C + \omega_L = 181.49\,\mu J$$

$$\omega(0) = \frac{1}{2}(12.5)(6.4 \times 10^{-3})^2 + \frac{1}{2}(3.2 \times 10^{-9})(100)^2 = 272\,\mu J$$

$$\%\text{ remaining } = \frac{181.49}{272}(100) = 66.72\%$$

P 8.13 **[a]** $\alpha = \dfrac{1}{2RC} = 1250, \qquad \omega_o = 10^3,$ therefore overdamped

$$s_1 = -500, \qquad s_2 = -2000$$

therefore $\quad v = A_1e^{-500t} + A_2e^{-2000t}$

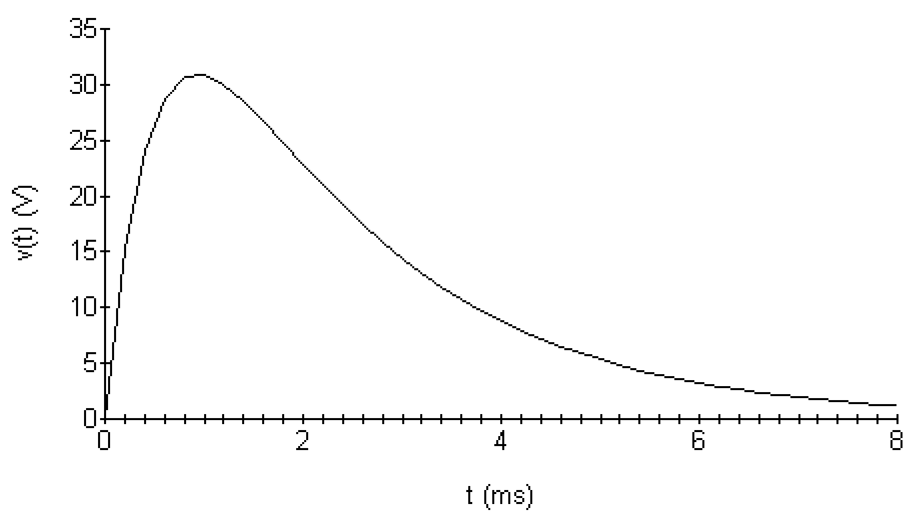
$$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_C(0^+)}{C} = 98{,}000\,\text{V/s}$$

Therefore $\quad -500A_1 - 2000A_2 = 98{,}000$

$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$$

[b]



Example 8.4: $\quad v_{\max} \cong 74.1\,\text{V} \quad$ at 1.4 ms

Example 8.5: $v_{\max} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.13: $v_{\max} \cong 30.9$ at 0.92 ms

P 8.14 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $$N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$$

It follows that $$A_1 = \frac{N_1}{\Delta} = \frac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$$

and $$A_2 = \frac{N_2}{\Delta} = \frac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$$

We see from these expressions that $A_1 = A_2^*$

P 8.15 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.14 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.14 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

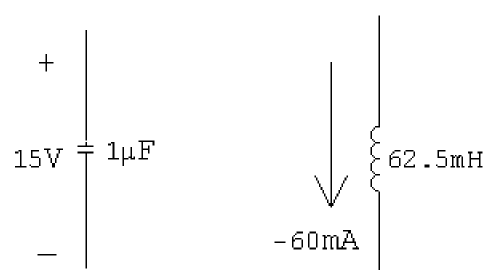
$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1$$

Thus we have

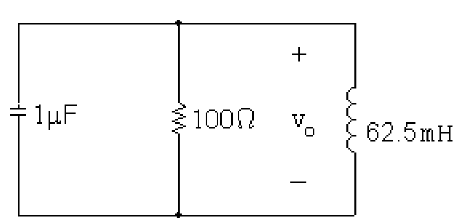
$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.16 $t < 0: \qquad V_o = 15\,\text{V}, \qquad I_o = -60\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{15}{100} = 150\,\text{mA}; \qquad i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -150 - (-60) = -90\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(10^{-6})} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 15$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-90 \times 10^{-3}}{10^{-6}} = -90{,}000$$

Solving, $\qquad A_1 = 5\,\text{V}, \qquad A_2 = 10\,\text{V}$

$$\therefore \quad v_o = 5e^{-2000t} + 10e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.17 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(250)(10^{-6})} = 2500$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 16 \times 10^6} = -2500 \pm j3122.5 \text{rad/s}$$

$$v_o(t) = B_1 e^{-2500t} \cos 3122.5t + B_2 e^{-2500t} \sin 3122.5t$$

$$v_o(0) = B_1 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{200} = 75\,\text{mA}$$

$$i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -15\,\text{mA} \quad \therefore \quad \frac{i_{\text{C}}(0)}{C} = -15{,}000 \text{ V/s}$$

$$\frac{dv_o}{dt}(0) = -2500B_1 + 3122.5B_2 = -15{,}000 \text{ V/s}$$

$$\therefore \quad 3122.5B_2 = 2500(15) - 15{,}000 \quad \therefore \quad B_2 = 7.21 \text{ V}$$

$$v_o(t) = 15e^{-2500t} \cos 3122.5t + 7.21e^{-2500t} \sin 3122.5t\,\text{V}, \qquad t \geq 0$$

P 8.18 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$

$\alpha = \frac{1}{2RC} = \frac{1}{2(125)(10^{-6})} = 4000$

$\therefore \quad \alpha^2 = \omega_o^2$ (critical damping)

$v_o(t) = D_1 te^{-4000t} + D_2 e^{-4000t}$

$v_o(0) = D_2 = 15\,\text{V}$

$i_R(0) = \frac{15}{125} = 120\,\text{mA}$

$i_{\text{L}}(0) = -60\,\text{mA}$

$i_{\text{C}}(0) = -60\,\text{mA}$

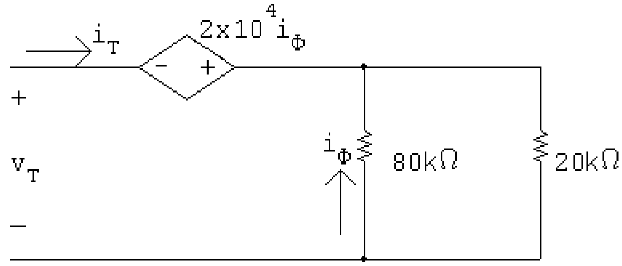
$\frac{dv_o}{dt}(0) = -4000D_2 + D_1$

$\frac{i_{\text{C}}(0)}{C} = \frac{-60 \times 10^{-3}}{10^{-6}} = -60{,}000$

$D_1 - 4000D_2 = -60{,}000; \qquad D_1 = 0$

$v_o(t) = 15e^{-4000t}\,\text{V}, \qquad t \geq 0$

P 8.19



$v_T = -2 \times 10^4 i_\phi + 16 \times 10^3 i_T; \qquad i_\phi = \frac{20}{100}(-i_T)$

$= 4000i_t + 16{,}000i_T = 20{,}000i_T$

$\frac{v_T}{i_T} = 20\,\text{k}\Omega$

$$V_o = \frac{3000}{5000}(50) = 30\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{30}{20{,}000} = -1.5\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-1.5\times 10^{-3}}{0.25\times 10^{-6}} = -6000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(40)(0.25\times 10^{-6})} = 10^5$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(20\times 10^3)(0.25\times 10^{-6})} = 100\ \text{rad/s}$$

$$\omega_d = \sqrt{10^5 - 100^2} = 300\ \text{rad/s}$$

$$v_o = B_1 e^{-100t}\cos 300t + B_2 e^{-100t}\sin 300t$$

$$v_o(0) = B_1 = 30\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 300B_2 - 100B_1 = -6000$$

$$\therefore\quad 300B_2 = 100(30) - 6000; \qquad \therefore\quad B_2 = -10\,\text{V}$$

$$v_o = 30e^{-100t}\cos 300t - 10e^{-100t}\sin 300t\,\text{V}, \qquad t \geq 0$$

P 8.20 **[a]** $v = L\left(\frac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_R = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.21 **[a]** $v = L\left(\frac{di_L}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

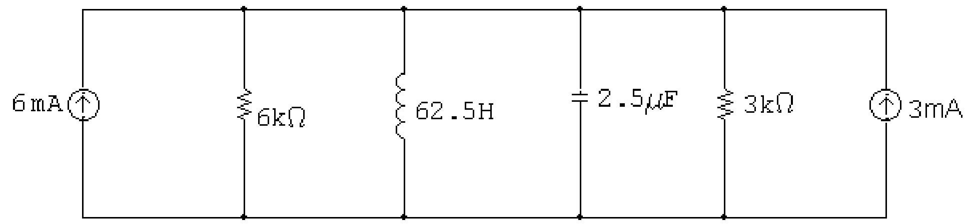
[b] $i_C(t) = I - i_R - i_L = 24\times 10^{-3} - \frac{v}{625} - i_L$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

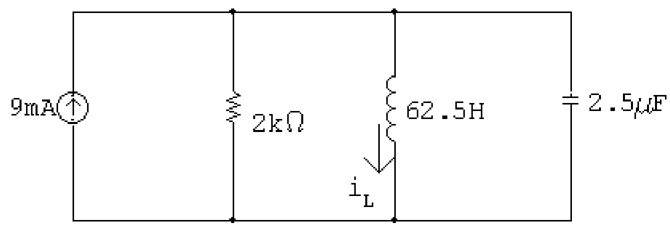
P 8.22 $v = L\left(\frac{di_L}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.23 $t < 0$: $i_L = 9/3000 = 3\,\text{mA}$

$t > 0$:



$6\,\text{k}\|3\,\text{k} = 2\,\text{k}\Omega$



$i_L(0) = 3\,\text{mA}, \qquad i_L(\infty) = 9\,\text{mA}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5)(2.5\times10^{-6})} = 6400; \qquad \omega_o = 80 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2000)(2.5\times10^{-6})} = 100; \qquad \alpha^2 = 10^4$$

$\alpha^2 - \omega_o^2 = 10^4 - 6400 = 3600$

$s_{1,2} = -100 \pm 60$ rad/s

$s_1 = -40$ rad/s; $\qquad s_2 = -160$ rad/s

$i_L = I_f + A_1' e^{-40t} + A_2' e^{-160t}$

$i_L(\infty) = I_f = 9\text{mA}$

$i_L(0) = A_1' + A_2' + I_f = 3\,\text{mA}$

$\therefore \quad A_1' + A_2' + 9\,\text{m} = 3\,\text{m} \quad \text{so} \quad A_1' + A_2' = -6\,\text{mA}$

$$\frac{di_L}{dt}(0) = 0 = -40A_1' - 160A_2'$$

Solving, $\quad A_1' = -8\,\text{mA}, \qquad A_2' = 2\,\text{mA}$

$i_L = 9 - 8e^{-40t} + 2e^{-160t}\,\text{mA}, \qquad t \geq 0$

P 8.24 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8; \qquad \omega_o = 10^4$ rad/s

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 12{,}500 \text{ rad/s} \qquad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500 \text{ rad/s}$$

$$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$$

$$I_f = 60\,\text{mA}$$

$$i_\text{L} = 60\times10^{-3} + A_1'e^{-5000t} + A_2'e^{-20{,}000t}$$

$$\therefore \quad -45\times10^{-3} = 60\times10^{-3} + A_1' + A_2'; \qquad A_1' + A_2' = -105\times10^{-3}$$

$$\frac{di_\text{L}}{dt} = -5000A_1' - 20{,}000A_2' = \frac{15}{0.05} = 300$$

Solving, $\quad A_1' = -120\,\text{mA}; \qquad A_2' = 15\,\text{mA}$

$$i_\text{L} = 60 - 120e^{-5000t} + 15e^{-20{,}000t}\,\text{mA}, \quad t \ge 0$$

P 8.25 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(312.5)(0.2\times10^{-6})} = 8000; \qquad \alpha^2 = 64\times10^6$

$\omega_o = 10^4 \qquad$ underdamped

$$s_{1,2} = -8000 \pm j\sqrt{8000^2 - 10^8} = -8000 \pm j6000 \text{ rad/s}$$

$$i_\text{L} = 60\times10^{-3} + B_1'e^{-8000t}\cos 6000t + B_2'e^{-8000t}\sin 6000t$$

$$-45\times10^{-3} = 60\times10^{-3} + B_1' \qquad \therefore \quad B_1' = -105\,\text{mA}$$

$$\frac{di_\text{L}}{dt}(0) = -8000B_1' + 6000B_2' = 300$$

$$\therefore \quad B_2' = -90\,\text{mA}$$

$$i_\text{L} = 60 - 105e^{-8000t}\cos 6000t - 90e^{-8000t}\sin 6000t\,\text{mA}, \quad t \ge 0$$

P 8.26 $\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2\times 10^{-6})} = 10^4$

$\alpha^2 = 10^8 = \omega_o^2$ critical damping

$i_L = I_f + D_1' te^{-10^4 t} + D_2' e^{-10^4 t} = 60\times 10^{-3} + D_1' te^{-10^4 t} + D_2' e^{-10^4 t}$

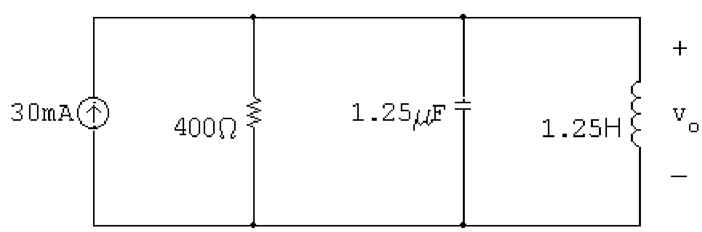
$i_L(0) = -45\times 10^{-3} = 60\times 10^{-3} + D_2';\qquad \therefore\quad D_2' = -105\,\text{mA}$

$\frac{di_L}{dt}(0) = -10^4 D_2' + D_1' = 300\,\text{A/s}$

$\therefore\quad D_1' = 300 + 10^4(-105\times 10^{-3}) = -750\,\text{A/s}$

$i_L = 60 - 750{,}000te^{-10^4 t} - 105e^{-10^4 t}\,\text{mA},\quad t \geq 0$

P 8.27 For $t > 0$



$\alpha = \frac{1}{2RC} = 1000;\qquad \frac{1}{LC} = 64\times 10^4$

$s_{1,2} = -1000 \pm 600$ rad/s

$s_1 = -400$ rad/s; $\quad s_2 = -1600$ rad/s

$v_o = V_f + A_1' e^{-400t} + A_2' e^{-1600t}$

$V_f = 0;\qquad v_o(0^+) = 0;\qquad i_C(0^+) = 30\,\text{mA}$

$\therefore\quad A_1' + A_2' = 0$

$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{1.25\times 10^{-6}} = 24{,}000\,\text{V/s}$

$\frac{dv_o(0^+)}{dt} = -400A_1' - 1600A_2' = 24{,}000$

Solving,

$A_1' = 20\,\text{V};\qquad A_2' = -20\,\text{V}$

$v_o = 20e^{-400t} - 20e^{-1600t}\,\text{V},\qquad t \geq 0$

P 8.28 **[a]** From the solution to Prob. 8.27 $s_1 = -400$ rad/s and $s_2 = -1600$ rad/s, therefore

$$i_o = I_f + A_1'e^{-400t} + A_2'e^{-1600t}$$

$$I_f = 30\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 30 \times 10^{-3} + A_1' + A_2'; \qquad -400A_1' - 1600A_2' = 0$$

Solving

$$A_1' = -40\,\text{mA}; \qquad A_2' = 10\,\text{mA}$$

$$\therefore \quad i_o = 30 - 40e^{-400t} + 10e^{-1600t}\,\text{mA}, \qquad t \geq 0$$

[b] $$\frac{di_o}{dt} = 16e^{-400t} - 16e^{-1600t}$$

$$v_o = L\frac{di_o}{dt} = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

This agrees with the solution to Problem 8.27

P 8.29 $$\alpha = \frac{1}{2RC} = \frac{1}{2(400)(1.25 \times 10^{-6})} = 1000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1.25 \times 10^{-6})(1.25)} = 64 \times 10^4$$

$$s_{1,2} = -1000 \pm \sqrt{1000^2 - 64 \times 10^4} = -1000 \pm 600 \text{ rad/s}$$

$$s_1 = -400 \text{ rad/s}; \qquad s_2 = -1600 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore \quad v_o = A_1'e^{-400t} + A_2'e^{-1600t}$$

$$v_o(0) = 12 = A_1' + A_2'$$

Note: $\qquad i_C(0^+) = 0$

$$\therefore \quad \frac{dv_o}{dt}(0) = 0 = -400A_1' - 1600A_2'$$

Solving, $\qquad A_1' = 16\,\text{V}, \qquad A_2' = -4\,\text{V}$

$$v_o(t) = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \qquad t > 0$$

P 8.30 **[a]** $i_o = I_f + A_1'e^{-400t} + A_2'e^{-1600t}$

$$I_f = \frac{12}{400} = 30\text{mA}; \qquad i_o(0) = 0$$

$$0 = 30 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -30 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{12}{1.25} = -400A_1' - 1600A_2'$$

Solving, $\quad A_1' = -32\,\text{mA}; \qquad A_2' = 2\,\text{mA}$

$$i_o = 30 - 32e^{-400t} + 2e^{-1600t}\,\text{mA}, \quad t \geq 0$$

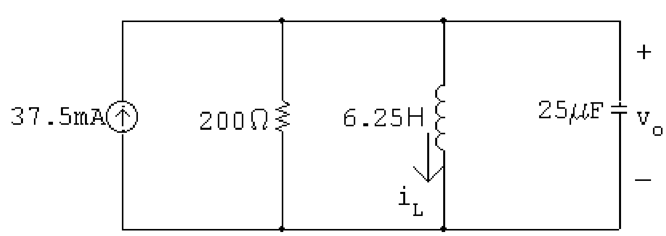
[b] $\dfrac{di_o}{dt} = [12.8e^{-400t} - 3.2e^{-1600t}]$

$$v_o = L\frac{di_o}{dt} = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 8.29.

P 8.31 $i_{\rm L}(0^-) = i_{\rm L}(0^+) = 37.5\,\text{mA}$

For $t > 0$



$$i_{\rm L}(0^-) = i_{\rm L}(0^+) = 37.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 6400$$

$$s_1 = -40\,\text{rad/s} \qquad s_2 = -160\,\text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$v_o = A_1'e^{-40t} + A_2'e^{-160t}$$

$$i_{\rm C}(0^+) = -37.5 + 37.5 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -40A_1' - 160A_2'$$

$$\therefore \quad A_1' + 4A_2' = 0; \qquad A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 0; \qquad A_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \frac{dv_o(0)}{dt} = 0$

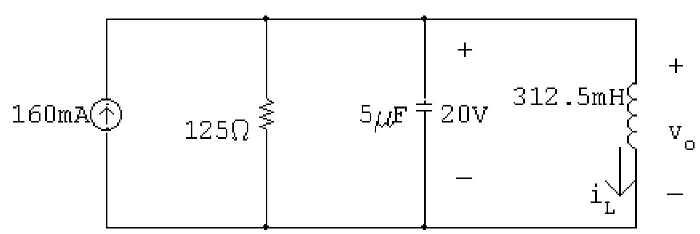
Hence the 37.5 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 7.5 V source sustains a current of 37.5 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.32 $\quad t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{625}{781.25}(25) = 20\,\text{V}$$

$$i_\text{L}(0^-) = i_\text{L}(0^+) = 0$$

$t > 0$



$$-160 \times 10^{-3} + \frac{20}{125} + i_\text{C}(0^+) + 0 = 0; \qquad \therefore \quad i_\text{C}(0^+) = 0$$

$$\frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(312.5 \times 10^{-3})(5 \times 10^{-6})} = 64 \times 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[a] $v_o = V_f + D_1' te^{-800t} + D_2' e^{-800t}$

$$V_f = 0$$

$$\frac{dv_o(0)}{dt} = -800D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 800D_2' = 16{,}000\,\text{V/s}$$

$$\therefore \quad v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}, \quad t \geq 0^+$$

[b] $i_L = I_f + D_3' te^{-800t} + D_4' e^{-800t}$

$$i_L(0^+) = 0; \qquad I_f = 160\,\text{mA}; \qquad \frac{di_L(0^+)}{dt} = \frac{20}{312.5 \times 10^{-3}} = 64\ \text{A/s}$$

$$\therefore \quad 0 = 160 + D_4'; \qquad D_4' = -160\,\text{mA};$$

$$-800D_4' + D_3' = 64; \qquad D_3' = -64\,\text{A/s}$$

$$\therefore \quad i_L = 160 - 64{,}000te^{-800t} - 160e^{-800t}\,\text{mA} \qquad t \geq 0$$

P 8.33 **[a]** $w_L = \int_0^\infty p dt = \int_0^\infty v_o i_L\, dt$

$$v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$$

$$i_L = 0.16 - 64te^{-800t} - 0.16e^{-800t}\,\text{A}$$

$$p = 3.2e^{-800t} + 2560te^{-800t} - 3840te^{-1600t}$$
$$-1{,}024{,}000t^2e^{-1600t} - 3.2e^{-1600t}\,\text{W}$$

$$w_L = 3.2\int_0^\infty e^{-800t}\,dt + 2560\int_0^\infty te^{-800t}\,dt - 3480\int_0^\infty te^{-1600t}\,dt$$
$$-1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 3.2\int_0^\infty e^{-1600t}\,dt$$
$$= 3.2\frac{e^{-800t}}{-800}\bigg|_0^\infty + \frac{2560}{(800)^2}e^{-800t}(-800t - 1)\bigg|_0^\infty$$
$$- \frac{3840}{(1600)^2}e^{-1600t}(-1600t - 1)\bigg|_0^\infty$$
$$- \frac{1{,}024{,}000}{(-1600)^3}e^{-1600t}(1600^2t^2 + 3200t + 2)\bigg|_0^\infty$$
$$- 3.2\frac{e^{-1600t}}{(-1600)}\bigg|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_L = \frac{3.2}{800} + \frac{2560}{800^2} - \frac{3840}{1600^2} - \frac{(1{,}024{,}000)(2)}{1600^3} - \frac{3.2}{1600} = 4\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(312.5 \times 10^{-3})(0.16)^2 = 4\,\text{mJ}.$$

[b] $v = 16{,}000te^{-800t} + 20e^{-800t}$ V

$$i_R = \frac{v}{125} = 128te^{-800t} + 0.16e^{-800t}\ \text{A}$$

$$p_R = vi_R = 2{,}048{,}000t^2e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t}$$

$$w_R = \int_0^\infty p_R\,dt$$

$$= 2{,}048{,}000\int_0^\infty t^2e^{-1600t}\,dt + 5120\int_0^\infty te^{-1600t}\,dt + 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= \frac{2{,}048{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty +$$

$$\frac{5120e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty + \frac{3.2e^{-1600t}}{(-1600)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_R = \frac{2{,}048{,}000(2)}{1600^3} + \frac{5120}{1600^2} + \frac{3.2}{1600} = 5\,\text{mJ}$$

[c] $160 = i_R + i_C + i_L \quad \text{(mA)}$

$$i_R + i_L = 160 + 64{,}000te^{-800t}\ \text{mA}$$

$$\therefore\quad i_C = 160 - (i_R + i_L) = -64{,}000te^{-800t}\ \text{mA} = -64te^{-800t}\ \text{A}$$

$$p_C = vi_C = [16{,}000te^{-800t} + 20e^{-800t}][-64te^{-800t}]$$

$$= -1{,}024{,}000t^2e^{-1600t} - 1280e^{-1600t}$$

$$w_C = -1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 1280\int_0^\infty te^{-1600t}\,dt$$

$$w_C = \frac{-1{,}024{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty - \frac{1280e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-1{,}024{,}000(2)}{1600^3} - \frac{1280(1)}{1600^2} = -1\,\text{mJ}$$

Note this 1 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(5 \times 10^{-6})(20)^2 = 1\,\text{mJ}.$$

Thus $w_C(\infty) = 0$ mJ which agrees with the final value of $v = 0$.

[d] $i_s = 160\,\text{mA}$

$$\begin{aligned} p_s(\text{del}) &= 160v\,\text{mW} \\ &= 0.16[16{,}000te^{-800t} + 20e^{-800t}] \\ &= 3.2e^{-800t} + 2560te^{-800t}\,\text{W} \\ w_s &= 3.2\int_0^\infty e^{-800t}\,dt + \int_0^\infty 2560te^{-800t}\,dt \\ &= \frac{3.2e^{-800t}}{-800}\Big|_0^\infty + \frac{2560e^{-800t}}{800^2}(-800t - 1)\Big|_0^\infty \\ &= \frac{3.2}{800} + \frac{2560}{800} = 8\,\text{mJ} \end{aligned}$$

[e] $w_L = 4\,\text{mJ}$ (absorbed)

$w_R = 5\,\text{mJ}$ (absorbed)

$w_C = 1\,\text{mJ}$ (delivered)

$w_S = 8\,\text{mJ}$ (delivered)

$\sum w_{\text{del}} = w_{\text{abs}} = 9\,\text{mJ}.$

P 8.34 $$v_C(0^+) = \frac{3.75 \times 10^3}{11.25 \times 10^3}(150) = 50\,\text{V}$$

$$i_L(0^+) = 100\,\text{mA}; \qquad i_L(\infty) = \frac{150}{7500} = 20\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2500)(0.25 \times 10^{-6})} = 800$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(4)(0.25 \times 10^{-6})} = 10^6$$

$$\alpha^2 = 64 \times 10^4; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -800 \pm j\sqrt{800^2 - 10^6} = -800 \pm j600\ \text{rad/s}$$

$$i_{\rm L} = I_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$
$$= 20 + B_1' e^{-800t}\cos 600t + B_2' e^{-800t}\sin 600t$$

$$i_{\rm L}(0) = 20\times 10^{-3} + B_1'; \qquad B_1' = 100\,\text{m} - 20\,\text{m} = 80\,\text{mA}$$

$$\frac{di_{\rm L}}{dt}(0) = 600B_2' - 800B_1' = \frac{50}{4} = 12.5$$

$$\therefore \quad 600B_2 = 800(80\times 10^{-3}) + 12.5; \qquad B_2' = 127.5\,\text{mA}$$

$$\therefore \quad i_{\rm L} = 20 + 80e^{-800t}\cos 600t + 127.5e^{-800t}\sin 600t\,\text{mA}, \qquad t \geq 0$$

P 8.35 **[a]** $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$$\sqrt{\alpha^2 - \omega_o^2} = 1500; \qquad \omega_o^2 = 4\times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$$

$$\omega_o^2 = \frac{1}{LC} = 4\times 10^6; \qquad L = \frac{10^9}{4\times 10^6(50)} = 5\text{H}$$

$$R = 25{,}000\,\Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50)\times 10^{-9} v_c^2(0) = 90\times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$\dfrac{di}{dt} = 0$ when $16e^{-4000t} = 4e^{-1000t}$

or $e^{3000t} = 4$

$\therefore \quad t = \dfrac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.36 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

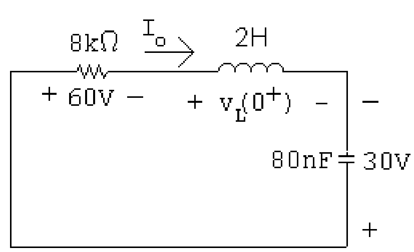
$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 2500\,\text{rad/s}$

$\alpha = \dfrac{R}{2L} = 2000; \qquad R = 4000L$

$\dfrac{1}{LC} = 625 \times 10^4; \qquad L = \dfrac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$

$\therefore \quad R = 8\,\text{k}\Omega$

$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad$ at $t = 0^+$



$60 + v_{\text{L}}(0^+) - 30 = 0; \qquad \therefore \qquad v_{\text{L}}(0^+) = -30\,\text{V}$

$\dfrac{di(0^+)}{dt} = \dfrac{-30}{2} = -15\,\text{A/s}$

$\therefore \quad \dfrac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$

$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\text{A}$

$\therefore \quad i = 7.5e^{-2000t}\sin 1500t\,\text{mA}, \quad t \geq 0$

P 8.37 From Prob. 8.36 we know v_c will be of the form

$$v_c = B_3 e^{-2000t}\cos 1500t + B_4 e^{-2000t}\sin 1500t$$

From Prob. 8.36 we have

$$v_c(0) = -30\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_\text{C}(0)}{C} = \frac{7.5\times10^{-3}}{80\times10^{-9}} = 93.75\times10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore\quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\text{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\text{V} \qquad t \geq 0$$

P 8.38 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(80\times10^{-3})(0.5\times10^{-6})} = 25\times10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore\quad R = (5000)(2)L = 800\,\Omega$$

[b] $i(0) = i_\text{L}(0) = 30\,\text{mA}$

$$v_c(0) = 800i(0) + 80\times10^{-3}\frac{di(0)}{dt}$$

$$\frac{20 - 800(30\times10^{-3})}{80\times10^{-3}} = \frac{di(0)}{dt}$$

$$\therefore\quad \frac{di(0)}{dt} = -50\,\text{A/s}$$

[c] $v_\text{C} = D_1 te^{-5000t} + D_2 e^{-5000t}$

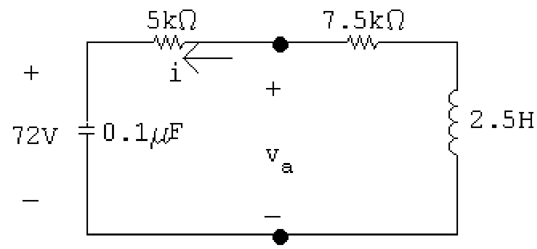
$$v_\text{C}(0) = D_2 = 20\,\text{V}$$

$$\frac{dv_\text{C}}{dt}(0) = D_1 - 5000D_2 = \frac{i_\text{C}(0)}{C} = \frac{-i_\text{L}(0)}{C}$$

$$D_1 - 100{,}000 = -\frac{30\times10^{-3}}{0.5\times10^{-6}} = -60{,}000 \quad \therefore \quad D_1 = 40{,}000\,\text{V/s}$$

$$v_\text{C} = 40{,}000te^{-5000t} + 20e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.39 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 72\,\text{V}$

[b] $v_a = 5000i + \dfrac{1}{0.1 \times 10^{-6}} \displaystyle\int_0^t i\,dx + 72$

$$\frac{dv_a}{dt} = 5000\frac{di}{dt} + 10 \times 10^6 i$$

$$\frac{dv_a(0^+)}{dt} = 5000\frac{di(0^+)}{dt} + 10 \times 10^6 i(0^+) = 5000\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 72$$

$$\frac{di(0^+)}{dt} = -\frac{72}{2.5} = -28.8\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -144{,}000\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{12{,}500}{2(2.5)} = 2500\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5)(0.1 \times 10^{-6})} = 4 \times 10^6$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 4 \times 10^6} = -2500 \pm 1500\,\text{rad/s}$$

Overdamped:

$$v_a = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$v_a(0) = 72 = A_1 + A_2$$

$$\frac{dv_a(0)}{dt} = -144{,}000 = -1000A_1 - 4000A_2$$

Solving, $\quad A_1 = 48; \quad A_2 = 24$

$$v_a = 48e^{-1000t} + 24e^{-4000t}\,\text{V}, \quad t \geq 0^+$$

P 8.40 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(10)(4 \times 10^{-3})} = 25$

$$\alpha = \frac{R}{2L} = \frac{80}{2(10)} = 4; \qquad \alpha^2 = 16$$

$\alpha^2 < \omega_o^2 \quad \therefore \quad$ underdamped

$s_{1,2} = -4 \pm j\sqrt{9} = -4 \pm j3$ rad/s

$$i = B_1 e^{-4t} \cos 3t + B_2 e^{-4t} \sin 3t$$

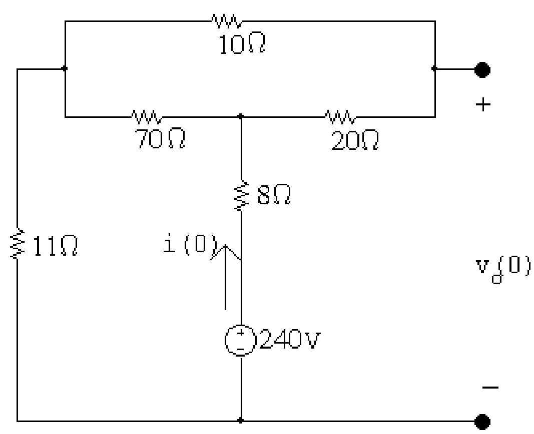
$$i(0) = B_1 = -240/100 = -2.4\,\text{A}$$

$$\frac{di}{dt}(0) = 3B_2 - 4B_1 = 0$$

$$\therefore \quad B_2 = -3.2\,\text{A}$$

$$i = -2.4e^{-4t} \cos 3t - 3.2 \sin 3t\,\text{A}, \qquad t \geq 0$$

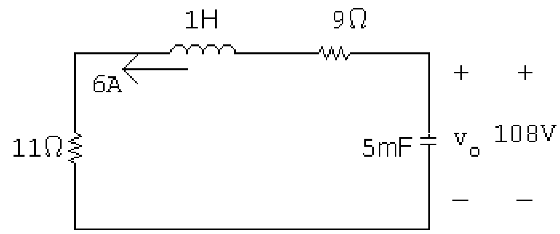
P 8.41 $t < 0$:



$$i(0) = \frac{240}{8 + 30\|70 + 11} = \frac{240}{40} = 6\,\text{A}$$

$$v_o(0) = 240 - 8(6) - \frac{70}{100}(6)(20) = 108\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{2(1)} = 10, \qquad \alpha^2 = 100$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(5 \times 10^{-3})} = 200$$

$\omega_o^2 > \alpha^2$ underdamped

$$s_{1,2} = -100 \pm \sqrt{100 - 200} = -10 \pm j10 \text{ rad/s}$$

$$v_o = B_1 e^{-10t} \cos 10t + B_2 e^{-10t} \sin 10t$$

$$v_o(0) = B_1 = 108\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -6, \qquad \frac{dv_o}{dt} = \frac{-6}{5 \times 10^{-3}} = -1200\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -10B_1 + 10B_2 = -1200$$

$$10B_2 = -1200 + 10B_1 = -1200 + 1080; \qquad B_2 = -120/10 = -12\,\text{V}$$

$$\therefore \quad v_o = 108e^{-10t} \cos 10t - 12e^{-10t} \sin 10t\,\text{V}, \qquad t \geq 0$$

P 8.42 **[a]** $t < 0$:

$$i_o = \frac{80}{800} = 100\,\text{mA}; \qquad v_o = 500i_o = (500)(0.01) = 50\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(2.5 \times 10^{-3})} = 10^5 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(40 \times 10^{-9})} = 100 \times 10^8$$

$\alpha^2 = \omega_o^2 \quad \therefore \quad$ critically damped

$$\therefore \quad i_o(t) = D_1 te^{-10^5 t} + D_2 e^{-10^5 t}$$

$$i_o(0) = D_2 = 100\,\text{mA}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0$$

$$\therefore \qquad D_1 = 10^5(100 \times 10^{-3}) = 10{,}000$$

$$i_o(t) = 10{,}000te^{-10^5 t} + 0.1e^{-10^5 t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o(t) = D_3 te^{-10^5 t} + D_4 e^{-10^5 t}$

$$v_o(0) = D_4 = 50$$

$$C\frac{dv_o}{dt}(0) = -0.1$$

$$\frac{dv_o}{dt}(0) = \frac{-0.1}{40 \times 10^{-9}} = -25 \times 10^5\,\text{V/s} = -\alpha D_4 + D_3$$

$$\therefore \qquad D_3 = 10^5(50) - 25 \times 10^5 = 25 \times 10^5$$

$$v_o(t) = 25 \times 10^5 te^{-10^5 t} + 50e^{-10^5 t}\,\text{V}, \quad t \geq 0$$

P 8.43 $\alpha = \dfrac{R}{2L} = \dfrac{8000}{2(1)} = 4000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(50 \times 10^{-9})} = 20 \times 10^6$$

$$s_{1,2} = -4000 \pm \sqrt{4000^2 - 20 \times 10^6} = -4000 \pm j2000\,\text{rad/s}$$

$$v_o = V_f + B_1' e^{-4000t}\cos 2000t + B_2' e^{-4000t}\sin 2000t$$

$$v_o(0) = 0 = V_f + B_1'$$

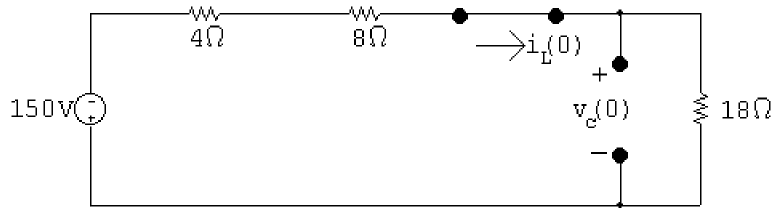
$$v_o(\infty) = 80\,\text{V}; \qquad \therefore \quad B_1' = -80\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 2000B_2' - 4000B_1'$$

$$\therefore \quad 2000B_2' = 4000(-80) \quad \therefore \qquad B_2' = -160\,\text{V}$$

$$v_o = 80 - 80e^{-4000t}\cos 2000t - 160e^{-4000t}\sin 2000t\,\text{V}, \quad t \geq 0$$

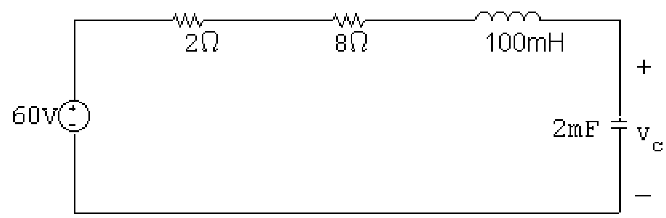
P 8.44 $t < 0$:



$$i_L(0) = \frac{-150}{30} = -5\,\text{A}$$

$$v_C(0) = 18 i_L(0) = -90\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50\text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2\times 10^{-3})} = 5000$$

$$\omega_o > \alpha^2 \quad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t}\cos 50t + B_2' e^{-50t}\sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \quad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2\times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \quad B_2' = -200$$

$$v_c = 60 - 150e^{-50t}\cos 50t - 200e^{-50t}\sin 50t\,\text{V}, \quad t \geq 0$$

P 8.45 $i_C(0) = 0; \qquad v_o(0) = 50\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{8000}{2(160 \times 10^{-3})} = 25{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(160 \times 10^{-3})(10 \times 10^{-9})} = 625 \times 10^6$$

$\therefore \quad \alpha^2 = \omega_o^2; \qquad$ critical damping

$$v_o(t) = V_f + D_1' te^{-25{,}000t} + D_2' e^{-25{,}000t}$$

$$V_f = 250\,\text{V}$$

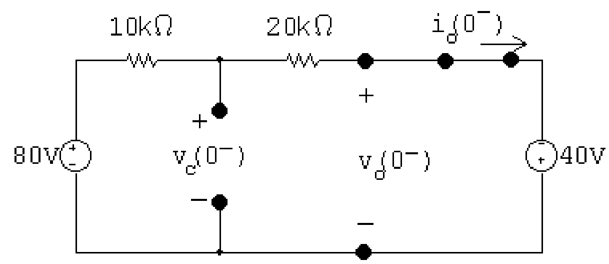
$$v_o(0) = 250 + D_2' = 50; \qquad D_2' = -200\,V$$

$$\frac{dv_o}{dt}(0) = -25{,}000 D_2' + D_1' = 0$$

$$D_1' = 25{,}000 D_2' - 5 \times 10^6 \text{ V/s}$$

$$v_o = 250 - 5 \times 10^6 te^{-25{,}000t} - 200e^{-25{,}000t}\,\text{V}, \quad t \geq 0$$

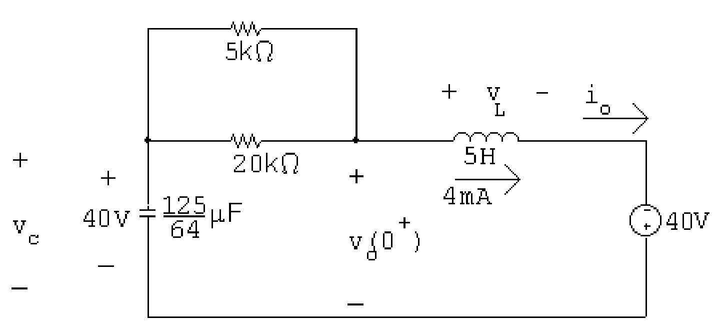
P 8.46 **[a]** $t < 0$:



$$i_o(0^-) = \frac{120}{30{,}000} = 4\,\text{mA}$$

$$v_C(0^-) = 80 - (10{,}000)(0.004) = 40\,\text{V}$$

$t = 0^+$:



$$5\,\text{k}\Omega \| 20\,\text{k}\Omega = 4\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = -(0.004)(4000) + 40 = 40 - 16 = 24\,\text{V}$$

[b] $v_o(t) = v_{\text{c}} - 4000 i_o$

$$\frac{dv_o}{dt}(0^+) = \frac{dv_c}{dt}(0^+) - 4000\frac{di_o}{dt}(0^+)$$

$$\frac{dv_c}{dt}(0^+) = \frac{-4 \times 10^{-3}}{(125/64) \times 10^{-6}} = -2048\,\text{V/s}$$

$$-v_{\text{L}}(0^+) + v_o(0^+) + 40 = 0 \quad v_{\text{L}} = 64\text{ V}$$

$$\frac{di_o}{dt}(0^+) = \frac{64}{5} = 12.8\,\text{A/s}$$

$$\frac{dv_o}{dt}(0^+) = -2048 - 4000(12.8) = -53{,}248\,\text{V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5)[(125/64) \times 10^{-6}]} = 10.24 \times 10^4$

$$\alpha = \frac{R}{2L} = \frac{4000}{2(5)} = 400 \text{ rad/s}; \qquad \alpha^2 = 16 \times 10^4$$

$\alpha^2 > \omega_o^2$ overdamped

$$s_{1,2} = -400 \pm 240 \text{ rad/s}$$

$$v_o(t) = V_f + A_1' e^{-160t} + A_2' e^{-640t}$$

$$V_f = v_o(\infty) = -40\,\text{V}$$

$$-40 + A_1' + A_2' = 24$$

$$-160A_1' - 640A_2' = -53{,}248$$

Solving, $A_1' = -25.6;$ $A_2' = 89.6$

$$\therefore \quad v_o(t) = -40 - 25.6e^{-160t} + 89.6e^{-640t}\,\text{V}, \qquad t \geq 0^+$$

P 8.47 **[a]** $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$v_c(0^+) = 0$ and $\dfrac{dv_c(0^+)}{dt} = 0$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin \omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t} \sin \omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin \omega_d t = 0,$ or $\omega_d t = n\pi$

where $n = 0, 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d},$ $\cos \omega_d t_n = \cos n\pi = (-1)^n$

and $\sin \omega_d t = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$v(t_1) = V + V e^{-(\alpha\pi/\omega_d)}$ and $v_c(t_3) = V + V e^{-(3\alpha\pi/\omega_d)}$

Therefore $\dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d,$ thus $\alpha = \dfrac{1}{T_d} \ln \dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 8.48 $\alpha = \dfrac{1}{T_d} \ln \left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\};$ $T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}$ ms

$$\alpha = \frac{7000}{2\pi} \ln \left[\frac{63.84}{26.02}\right] \approx 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000 \text{ rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49 \times 10^6 + 10^6 = 50 \times 10^6$$

$$L = \frac{1}{(50 \times 10^6)(0.1 \times 10^{-6})} = 200 \text{ mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.49 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\, e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $\dfrac{dv_o}{dt} = -\dfrac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.50 **[a]** From Problem 8.49 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{4800}{2(64\times10^{-3})} = 37{,}500\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(64\times10^{-3})(4\times10^{-9})} = 3906.25\times10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 50\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-72)}{(4800)(4\times 10^{-9})(50\times 10^3)} = 75$$

$$\therefore\quad v_o = 75e^{-37{,}500t}\sin 50{,}000t\,\text{V},\qquad t\geq 0$$

[b] From Problem 8.49

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{50{,}000}\tan^{-1}\left(\frac{50{,}000}{37{,}500}\right)$$

$$t_d = 18.55\,\mu\text{s}$$

[c] $v_{\text{max}} = 75e^{-0.0375(18.55)}\sin[(0.05)(18.55)] = 29.93\,\text{V}$

[d] $R = 480\,\Omega;\qquad \alpha = 3750\,\text{rad/s}$

$$\omega_d = 62{,}387.4\,\text{rad/s}$$

$$v_o = 601.08e^{-3750t}\sin 62{,}387.4t\,\text{V},\qquad t\geq 0$$

$$t_d = 24.22\,\mu\text{s}$$

$$v_{\text{max}} = 547.92\,\text{V}$$

P 8.51 **[a]** $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$$\therefore\quad \frac{d^2v_o}{dt^2} = 250v_g$$

$0\leq t\leq 0.5^-$:

$$v_g = 80\,\text{mV}$$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 20$ or $dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t,\qquad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_o(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^6} = 2\,\mu\text{A}$$

$$\frac{dv_o}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^t -10y\,dy + \int_{0.5^+}^t 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

Summary:

$0 \le t \le 0.5^-\text{s}: \qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^+\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$$

P 8.52 $\quad \tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{20}{2} = 10\,\text{V}$$

$$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' = -20, \qquad A_2' = 10\,\text{V}$$

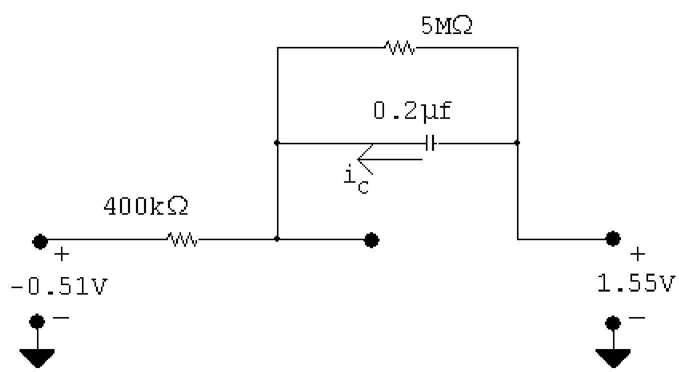
$$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \quad v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$$

$$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$$

$$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$$

At $t = 0.5\,\text{s}$



$$i_C = \frac{0 + 0.51}{400 \times 10^3} - \frac{1.55 - 0}{5 \times 10^6} = 0.954\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 0.954\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{0.954}{0.2} = 4.773\ \text{V/s}$$

$t \ge 0.5\,\text{s}$

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = -10$$

$$v_o(\infty) = -5$$

$$\therefore \quad v_o = -5 + A_1'e^{-(t-0.5)} + A_2'e^{-2(t-0.5)}$$

$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 4.773 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 4.773$$

Solving,

$$A_1' = 17.87\text{ V}; \qquad A_2' = -11.32\text{ V}$$

$$\therefore \quad v_o = -5 + 17.87e^{-(t-0.5)} - 11.32e^{-2(t-0.5)}\text{ V}, \qquad t \geq 0.5\text{ s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

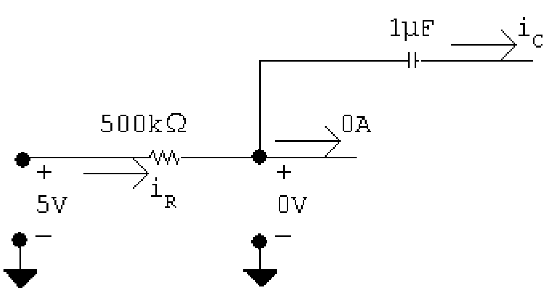
$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\text{ V}, \qquad t \geq 0.5\text{ s}$$

P 8.53 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500\,\text{k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1\,\mu$F capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.54 **[a]** From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

therefore $\dfrac{dg(t)}{dt} = 2, \quad g(t) = \dfrac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_{\text{R}} = \frac{5}{500} \times 10^{-3} = 1\,\mu\text{A} = i_{\text{C}} = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-1 \times 10^{-6}}{1 \times 10^{-6}} = -1 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 1$$

$$dv_o = 2t\,dt - \,dt$$

$$v_o - v_o(0) = t^2 - t; \qquad v_o(0) = 8\text{ V}$$

$$v_o = t^2 - t + 8, \qquad 0 \leq t \leq t_{\text{sat}}$$

[b] $t^2 - t + 8 = 9$

$t^2 - t - 1 = 0$

$t = (1/2) \pm (\sqrt{5}/2) \cong 1.62\,\text{s}, \quad t_{\text{sat}} \cong 1.62\text{ s}$

(Negative value has no physical significance.)

P 8.55 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_\text{a} = 100\,\text{k}\Omega; \qquad C_1 = 0.1\,\mu\text{F}; \qquad R_\text{b} = 25\,\text{k}\Omega; \qquad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_\text{a}C_1}\right)\left(\frac{1}{R_\text{b}C_2}\right)v_g; \qquad \frac{1}{R_\text{a}C_1} = 100 \quad \frac{1}{R_\text{b}C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$ V

The second op-amp will saturate when

$v_o = 6\,\text{V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_\text{a}C_1}v_g = -25$

[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

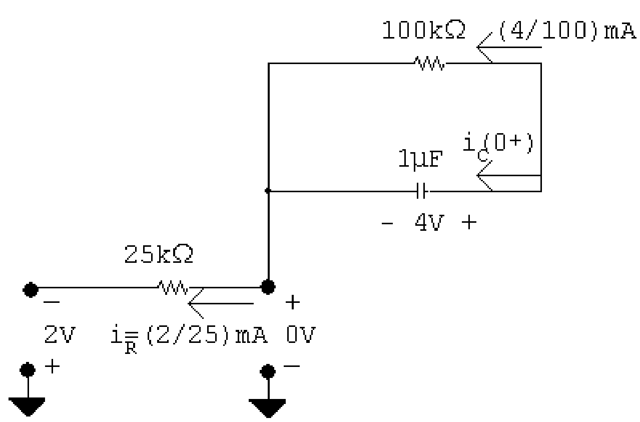
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \leq t \leq 0.1095$ s. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2$ V and $v_o(0) = 4$ V:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04\times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}, \quad t \geq 0$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

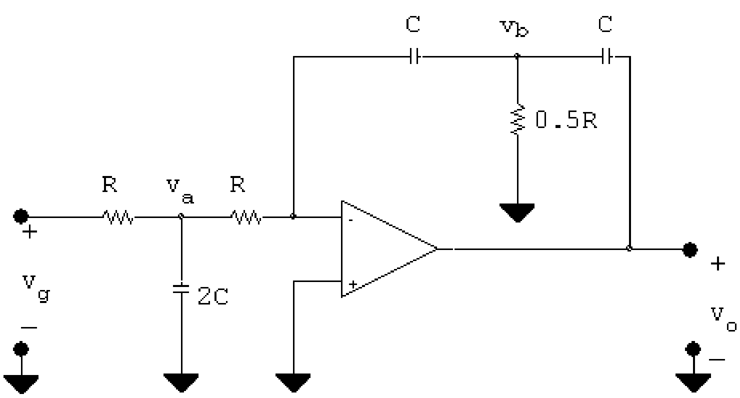
[d] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V}$; $\quad v_1(0) = -2\,\text{V}$ (given)

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}, \quad t \geq 0$$

P 8.56 **[a]**



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$; $\qquad \dfrac{0 - v_a}{R} + C\dfrac{d(0 - v_b)}{dt} = 0$

(2) Therefore $\dfrac{dv_b}{dt} + \dfrac{v_a}{RC} = 0$, $\quad v_a = -RC\dfrac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\quad \frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2}\frac{dv_o}{dt}$

From (2) we have $\quad \frac{dv_a}{dt} = -RC\frac{d^2v_b}{dt^2} \quad$ and $\quad v_a = -RC\frac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $\; -RC\frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\; \frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\; \frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $\quad R_1C_1 = R_2C_2 = RC: \quad \frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.57 **[a]** $f(t)$ = inertial force + frictional force + spring force

$\quad = M[d^2x/dt^2] + D[dx/dt] + Kx$

[b] $\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$

Given $\quad v_A = \frac{d^2x}{dt^2}, \quad$ then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right] v_C = \left[\frac{R_5 + R_6}{R_6}\right] \cdot \frac{1}{R_1 R_2 C_1 C_2} \cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right] f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $\dfrac{d^2x}{dt^2} = \left[\dfrac{R_8}{R_7}\right] f(t) - \left[\dfrac{R_3}{R_4 R_1 C_1}\right] \dfrac{dx}{dt} - \left[\dfrac{R_5 + R_6}{R_6 R_1 R_2 C_1 C_2}\right] x$

Therefore $M = \dfrac{R_7}{R_8}, \qquad D = \dfrac{R_3 R_7}{R_8 R_4 R_1 C_1}$ and $K = \dfrac{R_7(R_5 + R_6)}{R_8 R_6 R_1 R_2 C_1 C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and scaling
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.58 **[a]** Given that the current response is underdamped we know i will be of the form

$$i = I_f + [B_1' \cos \omega_d t + B_2' \sin \omega_d t] e^{-\alpha t}$$

where $\qquad \alpha = \dfrac{R}{2L}$

and $\qquad \omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\dfrac{1}{LC} - \alpha^2}$

The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$ therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is zero hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1') \cos \omega_d t - (\omega_d B_1' + \alpha B_2') \sin \omega_d t] e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d} B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R} \left[\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t\right] e^{-\alpha t}$$

[b] Since $\omega_d B_1' - \alpha B_1' = 0$ it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\ \sin\omega_d t$$

But $\quad \alpha B_2' = \frac{\alpha^2 V_{dc}}{\omega_d R} \quad$ and $\quad \omega_d B_1' = \frac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\quad \omega_d^2 + \alpha^2 = \omega_o^2 = \frac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $\quad v_1 = L\frac{di}{dt} \quad$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\ \sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\ \sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\frac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\ \sin\omega_d t\right)e^{-\alpha t}$$

$$v_c = V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\ \sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right]$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right]$$

P 8.59 $\quad v_{sp} = V_{dc}\left[1 - \frac{a}{\omega_d RC}e^{-\alpha t}\ \sin\omega_d t\right]$

$$\frac{dv_{sp}}{dt} = \frac{-aV_{dc}}{\omega_d RC}\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t]$$

$$= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d\ \cos\omega_d t e^{-\alpha t}]$$

$$= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\ \sin\omega_d t - \omega_d\ \cos\omega_d t]$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha \sin\omega_d t = \omega_d \cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore \quad t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

P 8.60 **[a]** $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\sin\omega_d t - \cos\omega_d t) +$$
$$e^{-\alpha t}[\omega_d K\cos\omega_d t + \omega_d\sin\omega_d t]\}$$
$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\sin\omega_d t + (\alpha + \omega_d K)\cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\sin\omega_d t = -(\alpha + \omega_d K)\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4 \times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$$t_{c\,\max} = 55.23\,\mu\text{s}$$

[b] $v_c(t_{c\,\max}) = 12[1 - e^{-\alpha t_{c\,\max}}\cos\omega_d t_{c\,\max} + Ke^{-\alpha t_{c\,\max}}\sin\omega_d t_{c\,\max}]$

$$= 262.42\,\text{V}$$

[c] From the text example the voltage across the spark plug reaches its maximum value in $53.63\,\mu$s. If the spark plug does not fire the capacitor voltage peaks in $55.23\,\mu$s. When v_{sp} is maximum the voltage across the capacitor is 262.15 V. If the spark plug does not fire the capacitor voltage reaches 262.42 V.

P 8.61 **[a]** $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16) \times 10^{-3} = 40\,\text{mJ}$

[b] $\alpha = \frac{R}{2L} = \frac{3 \times 10^3}{10} = 300\,\text{rad/s}$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{RC} = \frac{10^6}{0.75} = \frac{4 \times 10^6}{3}$$

$$t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\,(t_{\max}) = 12 - \frac{12(50)(4 \times 10^6)}{3(28{,}282.68)}e^{-\alpha t_{\max}}\sin\omega_d t_{\max} = -27{,}808.04\,\text{V}$$

[c] $v_c\,(t_{\max}) = 12[1 - e^{-\alpha t_{\max}}\cos\omega_d t_{\max} + Ke^{-\alpha t_{\max}}\sin\omega_d t_{\max}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$$v_c\,(t_{\max}) = 568.15\,\text{V}$$

9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\angle{-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\angle{-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\angle{36.87^\circ} + 10\angle{-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\angle{-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\angle{45^\circ} - 100\angle{-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\angle{61.51^\circ}\,\text{mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\angle{45^\circ} - 50\angle{-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\angle{126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\angle{15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\angle{97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 **[a]** $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A},$ therefore $i_4 = 0\,\text{A}$

AP 9.6 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore\ \theta_Z = 45^\circ$

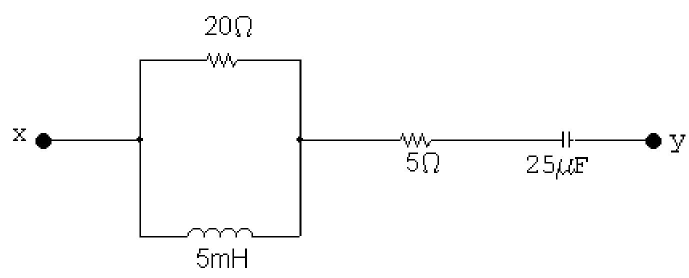
$Z = 90 + j160 + jX_C$

$\therefore\ X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore\ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore\ |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 **[a]**



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$Z_{xy} = 20\|j10 + 5 + j20 = \dfrac{20(j10)}{(20 + j10)} + 5 - j20$

$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,$rad/s.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

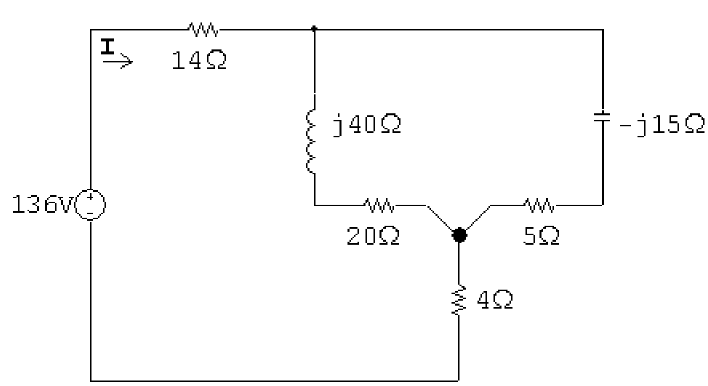
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$

Therefore $\mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\mathrm{A}$

AP 9.10 $\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\mathrm{V}$

$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\mathrm{V}$

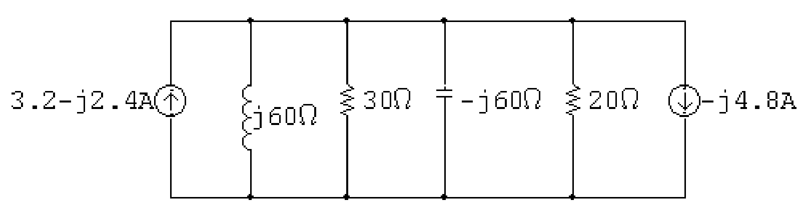
$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform source transformations:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\mathrm{A}$$

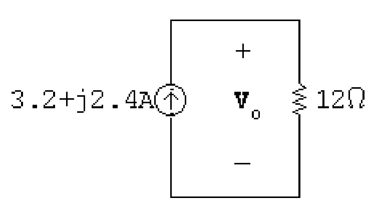
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\mathrm{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\mathrm{V} = 48\underline{/36.87^\circ}\,\mathrm{V}$

$v_o = 48\cos(4000t + 36.87^\circ)\,\mathrm{V}$

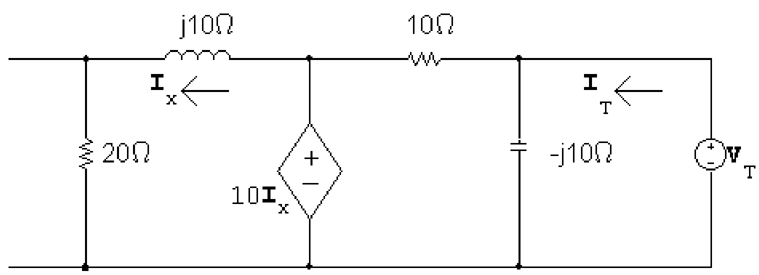
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

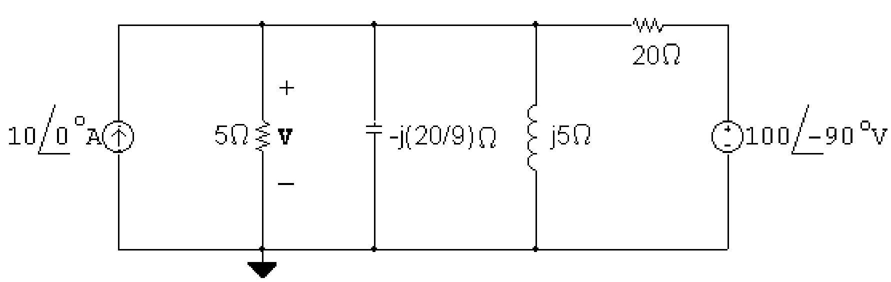
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_\text{a}$, $\mathbf{I}_\text{b}$, and $\mathbf{I}_\text{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\text{a} + (3 - j5)(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\text{b} - \mathbf{I}_\text{a}) + 2(\mathbf{I}_\text{b} - \mathbf{I}_\text{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b}),$$

therefore

$$\mathbf{I}_\text{c} = -0.75[-j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\text{a} = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_r = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $$\mathbf{I}_1 = \frac{245.20}{184 + 100 + j400 + Z_r} = 0.50\underline{/-53.13^\circ}\,\text{A}$$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $$\mathbf{I}_2 = \left(\frac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \frac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$$

$$i_2 = 80\cos 800t\,\text{mA}$$

AP 9.15 $\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + Z_2/a^2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$

$= 4 + j3 = 5\underline{/36.87^\circ}$ A

$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$

$= 37{,}000 - j28{,}500$

$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}$ V

$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/-143.13^\circ}$ A

Also, $I_2 = -25I_1$

Problems

P 9.1 **[a]** $\omega = 2\pi f = 3769.91\,\text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 600\,\text{Hz}$

[b] $T = 1/f = 1.67\,\text{ms}$

[c] $V_m = 10\,\text{V}$

[d] $v(0) = 10\cos(-53.13^\circ) = 6\,\text{V}$

[e] $\phi = -53.13^\circ; \qquad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273\ \text{rad}$

[f] $V = 0$ when $3769.91t - 53.13^\circ = 90^\circ$. Now resolve the units:

$$(3769.91\ \text{rad/s})t = \frac{143.13^\circ}{(180^\circ/\pi)} = 2.498\,\text{rad}, \qquad t = 662.64\,\mu\text{s}$$

[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$$

$$\text{or} \qquad 3769.91t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.9273\,\text{rad}$$

Therefore $t = 245.97\,\mu\text{s}$

P 9.2 $$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2\frac{2\pi}{T}t\,dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\,dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

P 9.3 **[a]** $40\,\text{V}$

[b] $2\pi f = 100\pi; \qquad f = 50\text{Hz}$

[c] $\omega = 100\pi = 314.159\ \text{rad/s}$

[d] $\theta(\text{rad}) = \dfrac{2\pi}{360^\circ}(60^\circ) = \dfrac{\pi}{3} = 1.05\ \text{rad}$

[e] $\theta = 60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{50} = 20\,\text{ms}$

[g] $v = -40$ when

$$100\pi t + \frac{\pi}{3} = \pi; \qquad \therefore \quad t = 6.67\,\text{ms}$$

[h] $v = 40\cos\left[100\pi\left(t - \frac{0.01}{3}\right) + \frac{\pi}{3}\right]$

$= 40\cos[100\pi t - (\pi/3) + (\pi/3)]$

$= 40\cos 100\pi t$ V

[i] $100\pi(t - t_o) + (\pi/3) = 100\pi t - (\pi/2)$

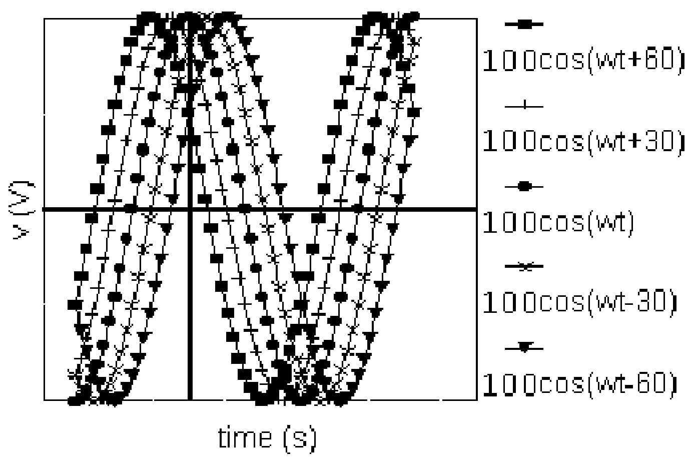
$\therefore \quad 100\pi t_o = \frac{5\pi}{6}; \qquad t_o = 8.33\,\text{ms}$

[j] $100\pi(t + t_o) + (\pi/3) = 100\pi t + 2\pi$

$\therefore \quad 100\pi t_o = \frac{5\pi}{3}; \qquad t_o = 16.67\,\text{ms}$

16.67 ms to the left

P 9.4



[a] Left as ϕ becomes more positive

[b] Left

P 9.5 **[a]** By hypothesis

$v = 80\cos(\omega t + \theta)$

$\frac{dv}{dt} = -80\omega\sin(\omega t + \theta)$

$\therefore \quad 80\omega = 80{,}000; \qquad \omega = 1000\,\text{rad/s}$

[b] $f = \frac{\omega}{2\pi} = 159.155$ Hz; $\qquad T = \frac{1}{f} = 6.28\,\text{ms}$

$\frac{-2\pi/3}{6.28} = -0.3333, \qquad \therefore \quad \theta = -90 - (-0.3333)(360) = 30^\circ$

$\therefore \quad v = 80\cos(1000t + 30^\circ)$ V

P 9.6 **[a]** $\frac{T}{2} = 8 + 2 = 10\,\text{ms}; \qquad T = 20\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50\text{Hz}$$

[b] $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$100\pi(-2 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{5} \text{ rad} = 36^\circ$$

$$v = V_m \sin[100\pi t + 36^\circ]$$

$$80.9 = V_m \sin 36^\circ; \qquad V_m = 137.64\,\text{V}$$

$$v = 137.64 \sin[100\pi t + 36^\circ] = 137.64 \cos[100\pi t - 54^\circ]\,\text{V}$$

P 9.7

$$\begin{aligned} u &= \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\,dt \\ &= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\,dt \\ &= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\,dt\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\ \Big|_{t_o}^{t_o+T}\right]\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\} \\ &= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right) \end{aligned}$$

P 9.8 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\,\text{V}$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$$V_m = 20, \qquad R/L = 1066.67, \qquad \omega L = 60$$

$$\sqrt{R^2 + \omega^2 L^2} = 100$$

$$\phi = 25^\circ, \qquad \theta = \tan^{-1} 60/80, \qquad \theta = 36.87^\circ$$

Substitute these values into Equation 9.9:

$$i = \left[-195.72e^{-1066.67t} + 200\cos(800t - 11.87^\circ)\right] \text{ mA}, \qquad t \geq 0$$

[b] Transient component $= -195.72e^{-1066.67t}$ mA
Steady-state component $= 200\cos(800t - 11.87^\circ)$ mA

[c] By direct substitution into Eq 9.9 in part (a), $i(1.875\,\text{ms}) = 28.39\,\text{mA}$

[d] $200\,\text{mA}, \quad 800\,\text{rad/s}, \quad -11.87^\circ$

[e] The current lags the voltage by $36.87°$.

P 9.10 **[a]** From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t+\phi)$$

At $t=0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2-\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right] = V_m\cos(\omega t+\phi)$$

P 9.11 **[a]** $\mathbf{Y} = 50\underline{/60°} + 100\underline{/-30°} = 111.8\underline{/-3.43°}$

$$y = 111.8\cos(500t - 3.43°)$$

[b] $\mathbf{Y} = 200\underline{/50°} - 100\underline{/60°} = 102.99\underline{/40.29°}$

$$y = 102.99\cos(377t + 40.29°)$$

[c] $\mathbf{Y} = 80\underline{/30°} - 100\underline{/-225°} + 50\underline{/-90°} = 161.59\underline{/-29.96°}$

$$y = 161.59\cos(100t - 29.96°)$$

[d] $\mathbf{Y} = 250\underline{/0^\circ} + 250\underline{/120^\circ} + 250\underline{/-120^\circ} = 0$

$y = 0$

P 9.12 **[a]** 1000Hz

[b] $\theta_v = 0^\circ$

[c] $\mathbf{I} = \dfrac{200\underline{/0^\circ}}{j\omega L} = \dfrac{200}{\omega L}\underline{/-90^\circ} = 25\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$

[d] $\dfrac{200}{\omega L} = 25; \qquad \omega L = \dfrac{200}{25} = 8\,\Omega$

[e] $L = \dfrac{8}{2\pi(1000)} = 1.27\,\text{mH}$

[f] $Z_L = j\omega L = j8\,\Omega$

P 9.13 **[a]** $\omega = 2\pi f = 314{,}159.27\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{10 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(10 \times 10^{-3})\underline{/0^\circ} = 10 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32 \times 10^{-6} = 10 \times 10^{-3}\,\omega C$

$\dfrac{1}{\omega C} = \dfrac{10 \times 10^{-3}}{628.32 \times 10^{-6}} = 15.92\,\Omega, \quad \therefore \quad X_C = -15.92\,\Omega$

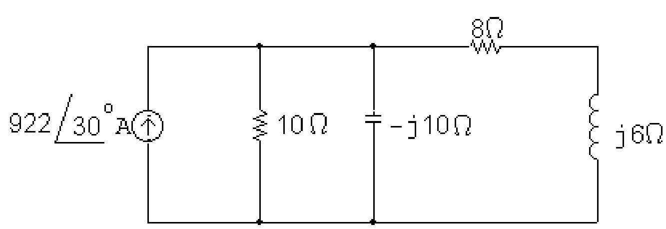
[d] $C = \dfrac{1}{15.92(\omega)} = \dfrac{1}{(15.92)(100\pi \times 10^3)}$

$C = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j15.92\,\Omega$

P 9.14 **[a]** $j\omega L = j(2 \times 10^4)(300 \times 10^{-6}) = j6\,\Omega$

$\dfrac{1}{j\omega C} = -j\dfrac{1}{(2 \times 10^4)(5 \times 10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\,\text{A}$



[b] $\mathbf{V}_o = 922\underline{/30^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8 + j6}$$

$$Y_e = 0.18 + j0.04\,\text{S}$$

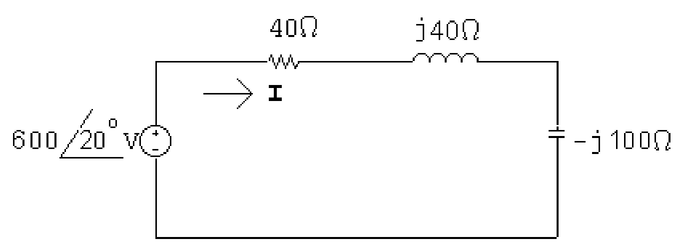
$$Z_e = \frac{1}{0.18 + j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\,\text{V}$$

[c] $v_o = 5000.25\cos(2 \times 10^4 t + 17.47^\circ)\,\text{V}$

P 9.15 **[a]** $Z_L = j(8000)(5 \times 10^{-3}) = j40\,\Omega$

$$Z_C = \frac{-j}{(8000)(1.25 \times 10^{-6})} = -j100\,\Omega$$



[b] $\mathbf{I} = \dfrac{600\underline{/20^\circ}}{40 + j40 - j100} = 8.32\underline{/76.31^\circ}\,\text{A}$

[c] $i = 8.32\cos(8000t + 76.31^\circ)\,\text{A}$

P 9.16 $Z = 4 + j(50)(0.24) - j\dfrac{1}{(50)(0.0025)} = 4 + j4 = 5.66\underline{/45^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{0.1\underline{/-90^\circ}}{5.66\underline{/45^\circ}} = 17.68\underline{/-135^\circ}\,\text{mA}$$

$$i_o(t) = 17.68\cos(50t - 135^\circ)\,\text{mA}$$

P 9.17 **[a]** $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$$

[b] $G = 160\,\text{mS}$

[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\angle 0^\circ\,\text{A}, \qquad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\angle 36.87^\circ} = 40\angle -36.87^\circ\,\text{V}$

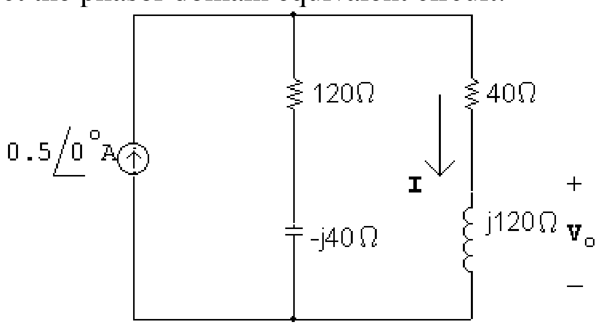
$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\angle -36.87^\circ}{4\angle -90^\circ} = 10\angle 53.13^\circ\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

P 9.18 $Z_L = j(2000)(60\times 10^{-3}) = j120\,\Omega$

$$Z_C = \frac{-j}{(2000)(12.5\times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\,\text{A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\angle 45^\circ\,\text{V}$$

$$v_o = 42.43\cos(2000t + 45^\circ)\,\text{V}$$

P 9.19 **[a]** $\mathbf{V}_g = 300\angle 78^\circ; \qquad \mathbf{I}_g = 6\angle 33^\circ$

$$\therefore\quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\angle 78^\circ}{6\angle 33^\circ} = 50\angle 45^\circ\,\Omega$$

[b] i_g lags v_g by 45°:

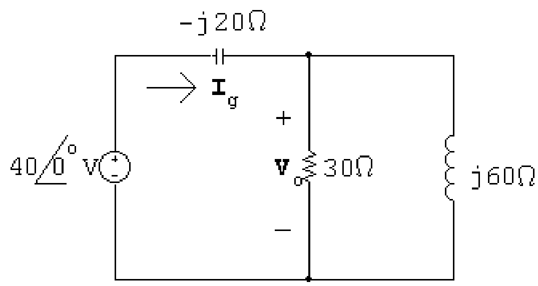
$$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore\quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.20 $\frac{1}{j\omega C} = \frac{1}{(1 \times 10^{-6})(50 \times 10^3)} = -j20\,\Omega$

$j\omega L = j50 \times 10^3(1.2 \times 10^{-3}) = j60\,\Omega$

$\mathbf{V}_g = 40\underline{/0^\circ}\text{ V}$



$Z_e = -j20 + 30\|j60 = 24 - j8\,\Omega$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{24 - j8} = 1.5 + j0.5\,\text{mA}$$

$$\mathbf{V}_o = (30\|j60)\mathbf{I}_g = \frac{30(j60)}{30 + j60}(1.5 + j0.5) = 30 + j30 = 42.43\underline{/45^\circ}\text{ V}$$

$v_o = 42.43\cos(50{,}000t + 45^\circ)\text{ V}$

P 9.21 **[a]** $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$Z_1 = Z_2$ when $R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2}$ and

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \frac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2} = 200\,\Omega$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5\,\text{nF}$$

P 9.22 **[a]** $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \frac{1 + (50 \times 10^3)^2 (1000)^2 (40 \times 10^{-9})^2}{(50 \times 10^3)^2 (1000)(40 \times 10^{-9})^2} = 1250\,\Omega$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2 (1000)^2 (40 \times 10^{-9})^2} = 8\,\text{nF}$$

P 9.23 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \frac{(4000)^2 (1.25)^2 (5000)}{5000^2 + 4000^2 (1.25)^2} = 2500\,\Omega$

$$L_1 = \frac{(5000)^2 (1.25)}{5000^2 + 4000^2 (1.25)^2} = 625\,\text{mH}$$

P 9.24 **[a]** $Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \frac{8000^2 + 1000^2 (4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2 (4)^2}{1000^2 (4)} = 20\,\text{H}$$

P 9.25 $\mathbf{V}_g = 500\underline{/30^\circ}\,\text{V}; \qquad \mathbf{I}_g = 0.1\underline{/83.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5000\underline{/-53.13^\circ}\,\Omega = 3000 - j4000\,\Omega$$

$$z = 3000 + j\left(\omega - \frac{32 \times 10^3}{\omega}\right)$$

$$\omega - \frac{32 \times 10^3}{\omega} = -4000$$

$$\omega^2 + 4000\omega - 32 \times 10^3 = 0$$

$$\omega = 7.984 \text{ rad/s}$$

P 9.26 **[a]** $Z_{\text{eq}} = \frac{50{,}000}{3} + \frac{-j20 \times 10^6}{\omega}\|(1200 + j0.2\omega)$

$$= \frac{50{,}000}{3} + \frac{-j20 \times 10^6}{\omega}\frac{(1200 + j0.2\omega)}{1200 + j[0.2\omega - \frac{20\times10^6}{\omega}]}$$

$$= \frac{50{,}000}{3} + \frac{\frac{-j20\times10^6}{\omega}(1200 + j0.2\omega)\left[1200 - j\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right]}{1200^2 + \left(0.2\omega - \frac{20\times10^6}{\omega}\right)^2}$$

$$\mathbf{Im}(Z_{\text{eq}}) = -\frac{20 \times 10^6}{\omega}(1200)^2 - \frac{20 \times 10^6}{\omega}\left[0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)\right] = 0$$

$$-20 \times 10^6(1200)^2 - 20 \times 10^6\left[0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)\right] = 0$$

$$-(1200)^2 = 0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)$$

$$0.2^2\omega^2 - 0.2(20 \times 10^6) + 1200^2 = 0$$

$$\omega^2 = 64 \times 10^6 \qquad \therefore \qquad \omega = 8000\,\text{rad/s}$$

$$\therefore \qquad f = 1273.24\,\text{Hz}$$

[b] $Z_{\text{eq}} = \frac{50{,}000}{3} + -j2500\|(1200 + j1600)$

$$= \frac{50{,}000}{3} + \frac{(-j2500)(1200 + j1600)}{1200 - j900} = 20{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{30\underline{/0^\circ}}{20{,}000} = 1.5\underline{/0^\circ}\,\text{mA}$$

$$i_g(t) = 1.5\cos 8000t\,\text{mA}$$

P 9.27 **[a]** Find the equivalent impedance seen by the source, as a function of L, and set the imaginary part of the equivalent impedance to 0, solving for L:

$$Z_C = \frac{-j}{(500)(2 \times 10^{-6})} = -j1000\,\Omega$$

$$Z_{\text{eq}} = -j1000 + j500L \| 2000 = -j1000 + \frac{2000(j500L)}{2000 + j500L}$$

$$= -j1000 + \frac{2000(j500L)(2000 - j500L)}{2000^2 + (500L)^2}$$

$$\mathbf{Im}(Z_{\text{eq}}) = -1000 + \frac{2000^2(500L)}{2000^2 + (500L)^2} = 0$$

$$\therefore \qquad \frac{2000^2(500L)}{2000^2 + (500L)^2} = 1000$$

$$\therefore \qquad 500^2L^2 - \frac{1}{2}2000^2L + 2000^2 = 0$$

Solving the quadratic equation, $L = 4\,\text{H}$

[b] $$\mathbf{I}_g = \frac{100\underline{/0^\circ}}{-j1000 + j2000\|2000} = \frac{100\underline{/0^\circ}}{1000} = 0.1\underline{/0^\circ}\ \text{A}$$

$$i_g(t) = 0.1\cos 500t\ \text{A}$$

P 9.28 **[a]** $$j\omega L + R\|(-j/\omega C) = j\omega L + \frac{-jR/\omega C}{R - j/\omega C}$$

$$= j\omega L + \frac{-jR}{\omega CR - j1}$$

$$= j\omega L + \frac{-jR(\omega CR + j1)}{\omega^2C^2R^2 + 1}$$

$$\mathbf{Im}(Z_{\text{ab}}) = \omega L - \frac{\omega CR^2}{\omega^2C^2R^2 + 1} = 0$$

$$\therefore \qquad L = \frac{CR^2}{\omega^2C^2R^2 + 1}$$

$$\therefore \qquad \omega^2C^2R^2 + 1 = \frac{CR^2}{L}$$

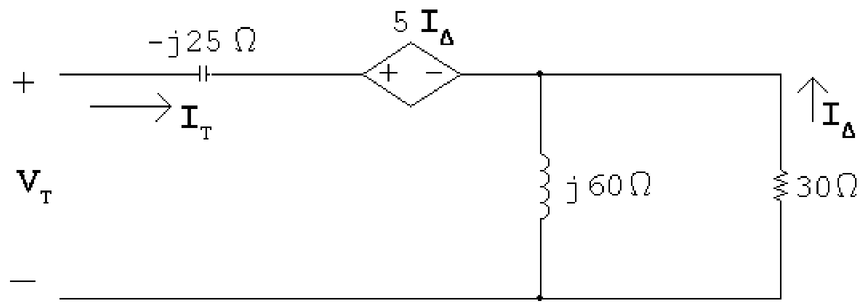
$$\therefore \qquad \omega^2 = \frac{(CR^2/L) - 1}{C^2R^2} = \frac{\frac{(25\times10^{-9})(100)^2}{160\times10^{-6}} - 1}{(25 \times 10^{-9})^2(100)^2} = 900 \times 10^8$$

$$\omega = 300\,\text{krad/s}$$

[b] $Z_{\rm ab}(300 \times 10^3) = j48 + \frac{(100)(-j133.33)}{100 - j133.33} = 64\,\Omega$

P 9.29 $j\omega L = j100 \times 10^3(0.6 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25\,\Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

$$\mathbf{I}_\Delta = \frac{-j60}{30 + j60}\mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30 + j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\rm ab} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.30 **[a]** $Z_1 = 400 - j\frac{10^6}{500(2.5)} = 400 - j800\,\Omega$

$$Z_2 = 2000 \| j500L = \frac{j10^6 L}{2000 + j500L}$$

$$Z_T = Z_1 + Z_2 = 400 - j800 + \frac{j10^6 L}{2000 + j500L}$$

$$= 400 + \frac{500 \times 10^6 L^2}{2000^2 + 500^2 L^2} - j800 + j\frac{2 \times 10^9 L}{2000^2 + 500^2 L^2}$$

Z_T is resistive when

$$\frac{2 \times 10^9 L}{2000^2 + 500^2 L^2} = 800 \quad \text{or} \quad 500^2 L^2 - 25 \times 10^5 L + 2000^2 = 0$$

Solving, $L_1 = 8$ H and $L_2 = 2$ H.

[b] When $L = 8$ H:

$$Z_T = 400 + \frac{500 \times 10^6 (8)^2}{2000^2 + 500^2 (8)^2} = 2000\,\Omega$$

$$\mathbf{I}_g = \frac{200\underline{/0^\circ}}{2000} = 100\underline{/0^\circ}\text{ mA}$$

$$i_g = 100 \cos 500t \text{ mA}$$

When $L = 2$ H:

$$Z_T = 400 + \frac{500 \times 10^6 (2)^2}{2000^2 + 500(2)^2} = 800\,\Omega$$

$$\mathbf{I}_g = \frac{200\underline{/0^\circ}}{800} = 250\underline{/0^\circ}\text{ mA}$$

$$i_g = 250 \cos 500t \text{ mA}$$

P 9.31 **[a]** $Y_1 = \dfrac{11}{2500 \times 10^3} = 4.4 \times 10^{-6}$ S

$$Y_2 = \frac{1}{14{,}000 + j5\omega}$$

$$= \frac{14{,}000}{196 \times 10^6 + 25\omega^2} - j\frac{5\omega}{196 \times 10^6 + 25\omega^2}$$

$$Y_3 = j\omega 2 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 2 \times 10^{-9} = \frac{5\omega}{196 \times 10^6 + 25\omega^2}$$

or

$$25\omega^2 + 196 \times 10^6 = \frac{5}{2 \times 10^{-9}}$$

$$\therefore \quad 25\omega^2 = 2304 \times 10^6 \qquad \therefore \quad \omega = 9600 \text{ rad/s}$$

[b] $Y_T = 4.4 \times 10^{-6} + \dfrac{14{,}000}{196 \times 10^6 + 25(9600)^2} = 10 \times 10^{-6}$ S

$$\therefore \quad Z_T = 100\,\text{k}\Omega$$

$$\mathbf{V}_o = (0.25 \times 10^{-3}\underline{/0^\circ})(100 \times 10^3) = 25\underline{/0^\circ}\text{ V}$$

$$v_o = 25 \cos 9600t \text{ V}$$

P 9.32 **[a]** $Z_g = 500 - j\frac{10^6}{\omega} + \frac{10^3(j0.5\omega)}{10^3 + j0.5\omega}$

$$= 500 - j\frac{10^6}{\omega} + \frac{500j\omega(1000 - j0.5\omega)}{10^6 + 0.25\omega^2}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{250\omega^2}{10^6 + 0.25\omega^2} + j\frac{5\times 10^5\omega}{10^6 + 0.25\omega^2}$$

$\therefore$ If Z_g is purely real, $\frac{10^6}{\omega} = \frac{5\times 10^5\omega}{10^6 + 0.25\omega^2}$

$2(10^6 + 0.25\omega^2) = \omega^2 \quad \therefore \quad 4\times 10^6 = \omega^2$

$\therefore \quad \omega = 2000$ rad/s

[b] When $\omega = 2000$ rad/s

$Z_g = 500 - j500 + (j1000\|1000) = 1000\,\Omega$

$\therefore \quad \mathbf{I}_g = \frac{20\underline{/0^\circ}}{1000} = 20\underline{/0^\circ}$ mA

$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$

$Z_1 = 500 - j500\,\Omega$

$\mathbf{V}_o = 20\underline{/0^\circ} - (0.02\underline{/0^\circ})(500 - j500) = 10 + j10 = 14.14\underline{/45^\circ}$ V

$v_o = 14.14\cos(2000t + 45^\circ)$ V

P 9.33 $Z_{\rm ab} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.971\underline{/45^\circ}\ \Omega$$

P 9.34 First find the admittance of the parallel branches

$$Y_p = \frac{1}{2 - j6} + \frac{1}{12 + j4} + \frac{1}{2} + \frac{1}{j0.5} = 0.625 - j1.875\,\mathrm{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.625 - j1.875} = 0.16 + j0.48\,\Omega$$

$$Z_{\rm ab} = -j4.48 + 0.16 + j0.48 + 2.84 = 3 - j4\,\Omega$$

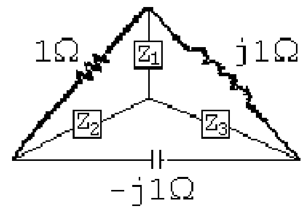
$$Y_{\rm ab} = \frac{1}{Z_{\rm ab}} = \frac{1}{3 - j4} = 120 + j160\,\mathrm{mS}$$

$$= 200\underline{/53.13^\circ}\,\mathrm{mS}$$

P 9.35 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

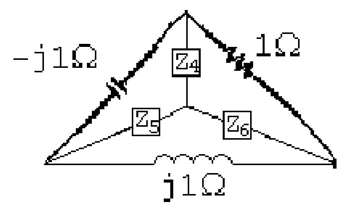


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

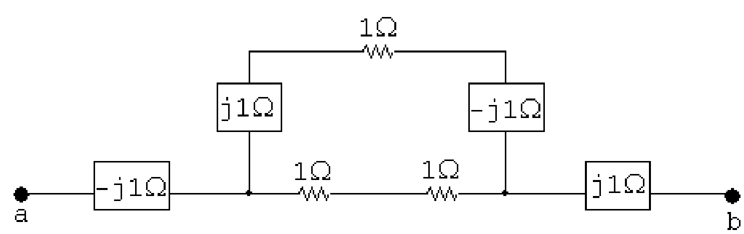


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$

$Z_{\text{ab}} = -j1 + 2/3 + j1 = 2/3\,\Omega$

P 9.36 $\mathbf{V}_o = \mathbf{V}_g \frac{Z_o}{Z_T} = \frac{500 - j1000}{300 + j1600 + 500 - j1000}(100\underline{/0^\circ}) = 111.8\underline{/-100.3^\circ}\,\text{V}$

$v_o = 111.8\cos(8000t - 100.3^\circ)\,\text{V}$

P 9.37 $\frac{1}{j\omega C} = -j400\,\Omega$

$j\omega L = j1200\,\Omega$

Let $Z_1 = 200 - j400\,\Omega$; $Z_2 = 600 + j1200\,\Omega$

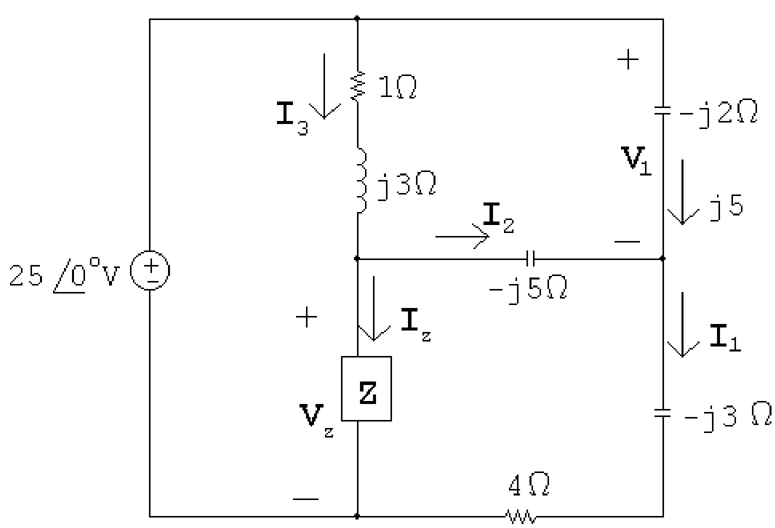
$\mathbf{I}_g = 400\underline{/0^\circ}\,\text{mA}$

$\mathbf{I}_o = \frac{Z_2}{Z_1 + Z_2}\mathbf{I}_g = \frac{600 + j1200}{800 + j800}(0.4\underline{/0^\circ})$

$= 450 + j150\,\text{mA} = 474.34\underline{/18.43^\circ}\,\text{mA}$

$i_o = 474.34\cos(20{,}000t + 18.43^\circ)\,\text{mA}$

P 9.38



$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$

$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$

$\mathbf{I}_2 = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \qquad \therefore \qquad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

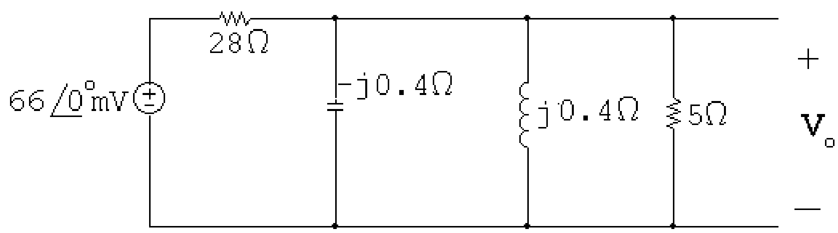
$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.39 $\mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = -j0.4\,\Omega$$

$$j\omega L = j0.4\,\Omega$$

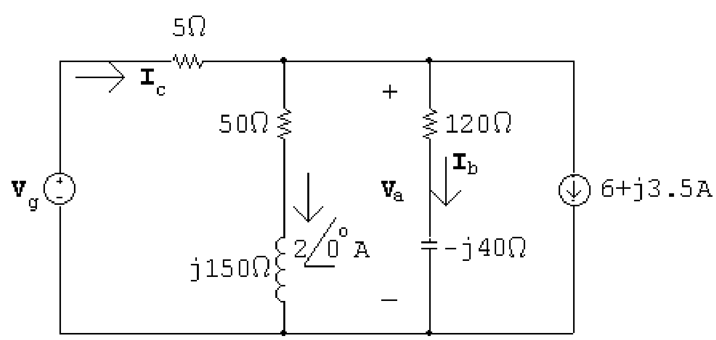
After source transformation we have



$$\mathbf{V}_o = \frac{-j0.4\|j0.4\|5}{28 + -j0.4\|j0.4\|5}(66 \times 10^{-3}) = 10\,\text{mV}$$

$$v_o = 10\cos 200t\,\text{mV}$$

P 9.40 **[a]**



$$\mathbf{V}_\text{a} = (50 + j150)(2\underline{/0^\circ}) = 100 + j300\,\text{V}$$

$$\mathbf{I}_\text{b} = \frac{100 + j300}{120 - j40} = j2.5\,\text{A} = 2.5\underline{/90^\circ}\,\text{A}$$

$$\mathbf{I}_\text{c} = 2\underline{/0^\circ} + j2.5 + 6 + j3.5 = 8 + j6\,\text{A} = 10\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_\text{c} + \mathbf{V}_\text{a} = 5(8 + j6) + 100 + j300 = 140 + j330\,\text{V} = 358.47\underline{/67.01^\circ}\,\text{V}$$

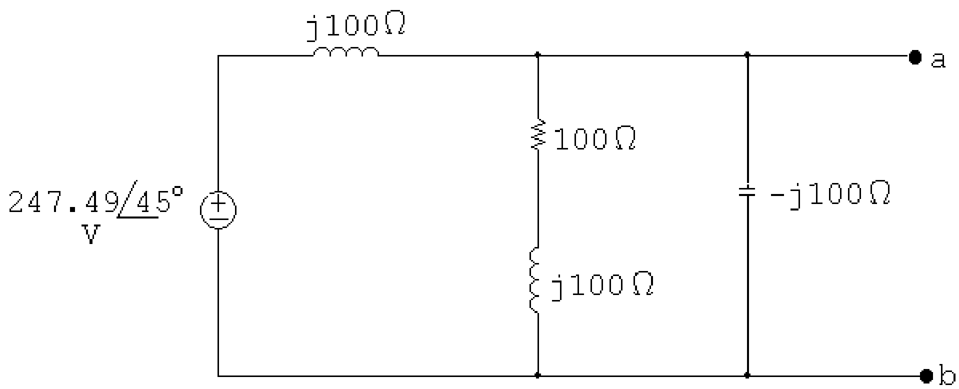
[b] $i_b = 2.5\cos(800t + 90^\circ)$ A

$i_c = 10\cos(800t + 36.87^\circ)$ A

$v_g = 358.47\cos(800t + 67.01^\circ)$ V

P 9.41 **[a]** $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\text{ab}} = \frac{(100 + j100)\|(-j100)}{j100 + (100 + j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

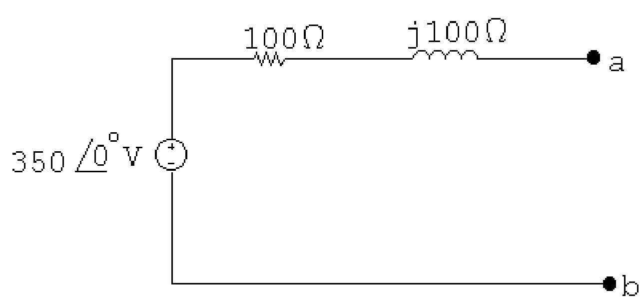
$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{ab}} = 350\underline{/0^\circ}\text{ V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\text{Th}} = Z_{\text{ab}}$:

$$Y_{\text{ab}} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{\text{Th}} = Z_{\text{ab}} = \frac{1}{Y_{\text{ab}}} = 100 + j100\,\Omega$$

[c]



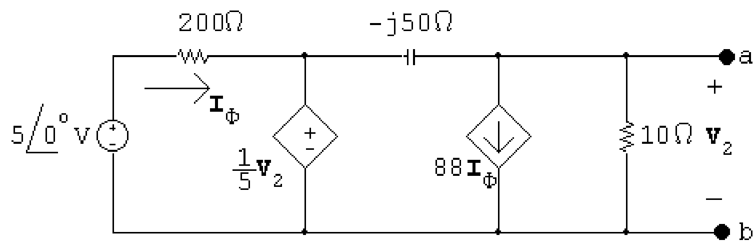
P 9.42 Using voltage division:

$$\mathbf{V}_{\text{Th}} = \frac{36}{36 + j60 - j48}(240) = 216 - j72 = 227.68\underline{/-18.43^\circ}\text{ V}$$

Remove the source and combine impedances in series and in parallel:

$$Z_{\text{Th}} = 36\|(j60 - j48) = 3.6 + j10.8\,\Omega$$

P 9.43 Open circuit voltage:



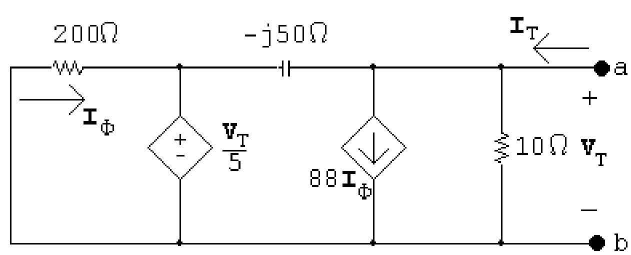
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\text{ V} = \mathbf{V}_{\text{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

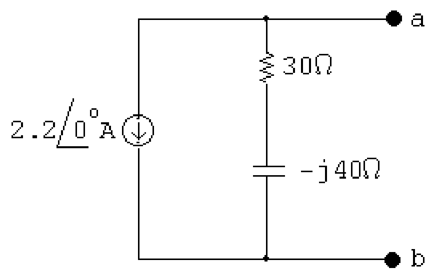
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{1/5}{200} + \frac{0.8}{-j50}\right)$$

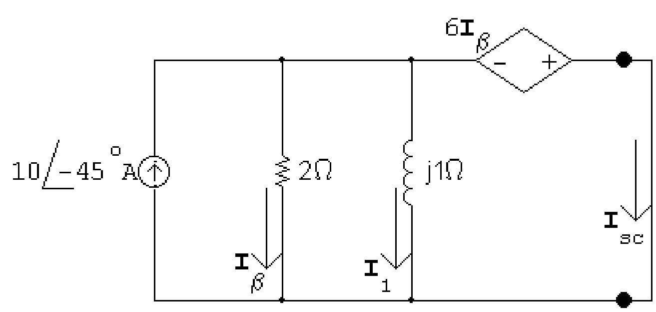
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A} = 2.2\underline{/180^\circ}\ \text{A}$$

The Norton equivalent circuit:



P 9.44 Short circuit current

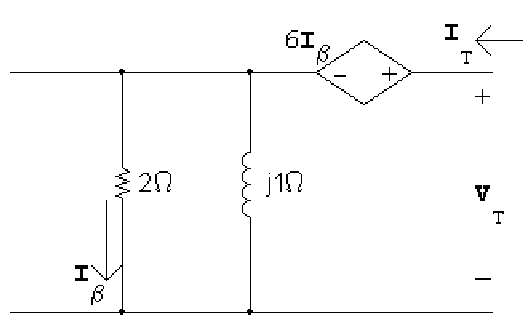


$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 10\underline{/-45^\circ}\,\text{A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Find it using a test source



$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2 + j1}\mathbf{I}_T$$

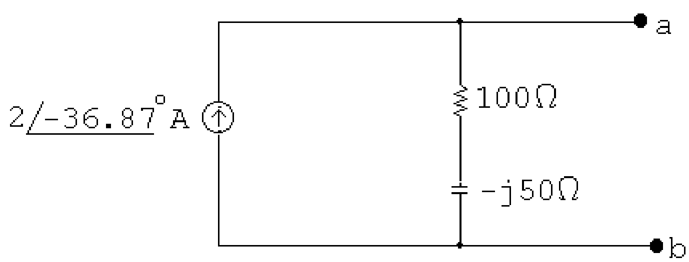
$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

P 9.45 Using current division:

$$\mathbf{I}_{\text{N}} = \mathbf{I}_{\text{sc}} = \frac{50}{80+j60}(4) = 1.6 - j1.2 = 2\underline{/-36.87^\circ}\,\text{A}$$

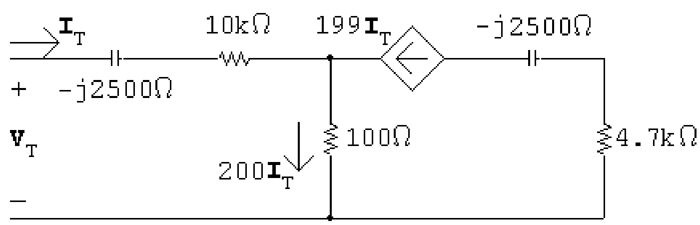
$$Z_{\text{N}} = -j100\|(80+j60) = 100 - j50\,\Omega$$

The Norton equivalent circuit:



P 9.46 $\omega = 2\pi(200/\pi) = 400$ rad/s

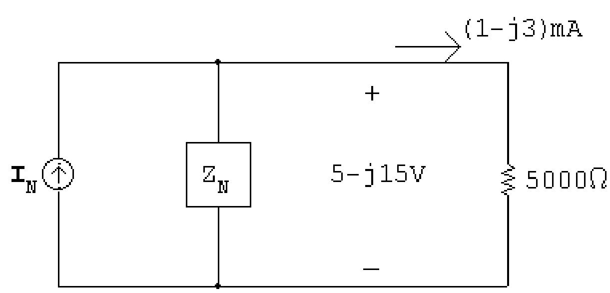
$$Z_c = \frac{-j}{400(10^{-6})} = -j2500\,\Omega$$



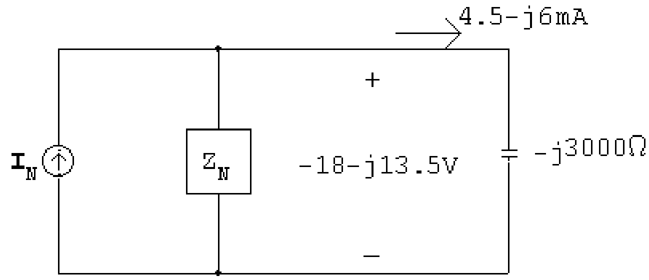
$$\mathbf{V}_T = (10{,}000 - j2500)\mathbf{I}_T + 100(200)\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j2.5\,\text{k}\Omega$$

P 9.47



$$\mathbf{I}_N = \frac{5-j15}{Z_{\text{N}}} + (1-j3)\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

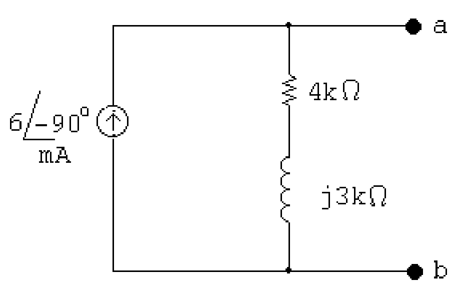


$$\mathbf{I}_N = \frac{-18 - j13.5}{Z_N} + 4.5 - j6\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

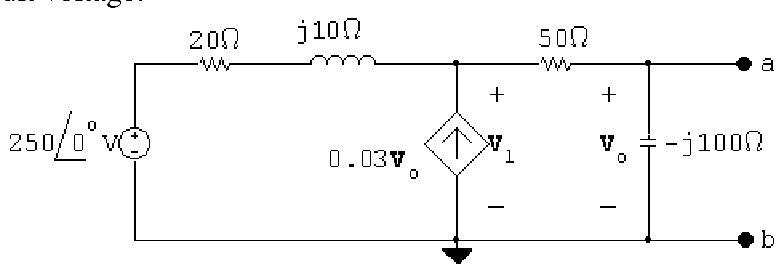
$$\frac{5 - j15}{Z_N} + 1 - j3 = \frac{-18 - j13.5}{Z_N} + (4.5 - j6)$$

$$\frac{23 - j1.5}{Z_N} = 3.5 - j3 \quad \therefore \quad Z_N = 4 + j3\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{5 - j15}{4 + j3} + 1 - j3 = -j6\,\text{mA} = 6\underline{/-90^\circ}\ \text{mA}$$



P 9.48 Open circuit voltage:



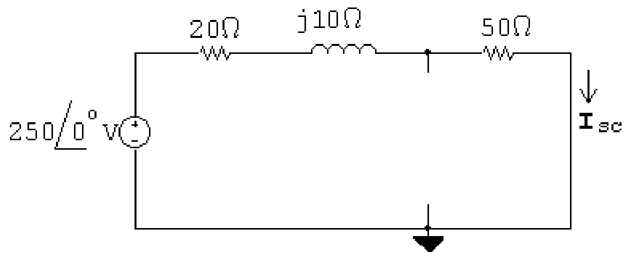
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20+j10} + \frac{j3\mathbf{V}_1}{50-j100} + \frac{\mathbf{V}_1}{50-j100} = \frac{250}{20+j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}} = 500\underline{/-53.13^\circ}\,\text{V}$$

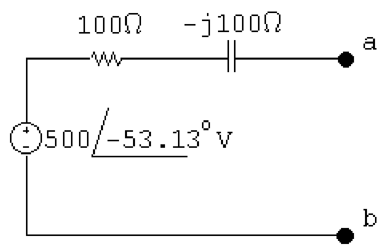
Short circuit current:



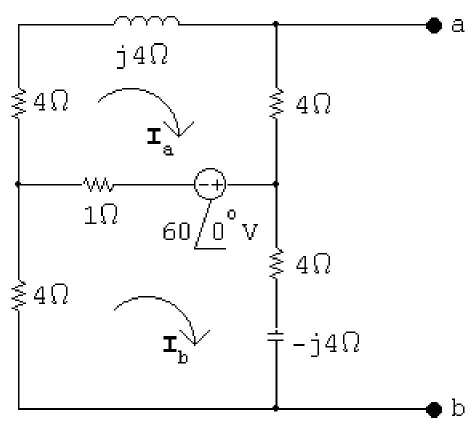
$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70+j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300-j400}{3.5-j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.49 Open circuit voltage:



$$(9+j4)\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}} = -60\underline{/0^\circ}$$

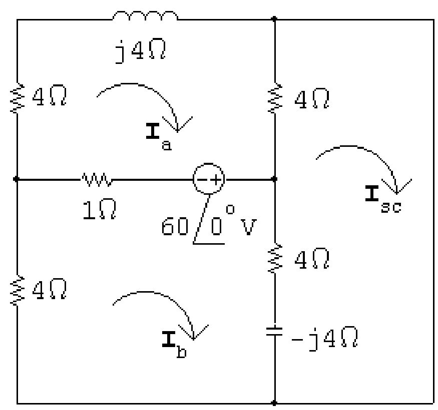
$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b = 60\angle 0^\circ$$

Solving,

$$\mathbf{I}_a = -5 + j2.5\,\text{A}; \qquad \mathbf{I}_b = 5 + j2.5\,\text{A}$$

$$\mathbf{V}_{Th} = 4\mathbf{I}_a + (4 - j4)\mathbf{I}_b = 10\angle 0^\circ\,\text{V}$$

Short circuit current:



$$(9 + j4)\mathbf{I}_a - 1\mathbf{I}_b - 4\mathbf{I}_{sc} = -60$$

$$-1\mathbf{I}_a + (9 - j4)\mathbf{I}_b - (4 - j4)\mathbf{I}_{sc} = 60$$

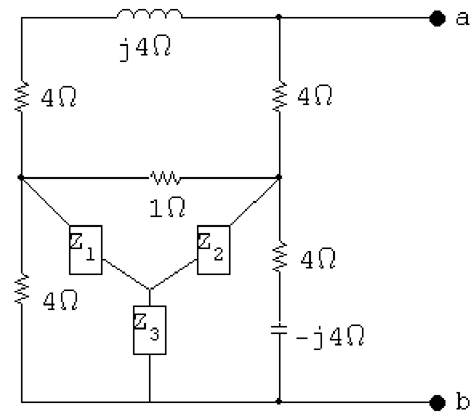
$$-4\mathbf{I}_a - (4 - j4)\mathbf{I}_b + (8 - j4)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 2.07\angle 0^\circ$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{10\angle 0^\circ}{2.07\angle 0^\circ} = 4.83\,\Omega$$

Alternate calculation for Z_{Th}:

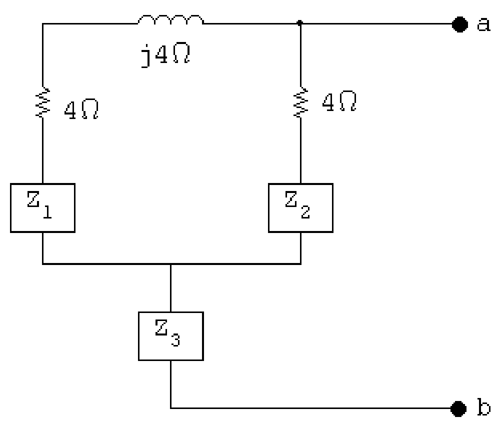


$$\sum Z = 4 + 1 + 4 - j4 = 9 - j4$$

$$Z_1 = \frac{4}{9 - j4}$$

$$Z_2 = \frac{4 - j4}{9 - j4}$$

$$Z_3 = \frac{16 - j16}{9 - j4}$$



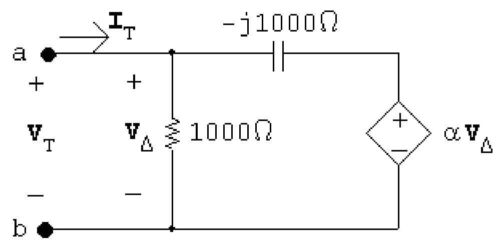
$$Z_{\text{a}} = 4 + j4 + \frac{4}{9 - j4} = \frac{56 + j20}{9 - j4}$$

$$Z_{\text{b}} = 4 + \frac{4 - j4}{9 - j4} = \frac{40 - j20}{9 - j4}$$

$$Z_{\text{a}} \| Z_{\text{b}} = \frac{2640 - j320}{864 - j384}$$

$$Z_3 + Z_{\text{a}} \| Z_{\text{b}} = \frac{16 - j16}{9 - j4} + \frac{2640 - j320}{864 - j384} = \frac{4176 - j1856}{864 - j384} = 4.83\,\Omega$$

P 9.50 **[a]**



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j - 1 + \alpha}{j1000}$$

$$\therefore \quad Z_{\rm Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

$Z_{\rm Th}$ is real when $\alpha = 1$.

[b] $Z_{\rm Th} = 1000\,\Omega$

[c] $Z_{\rm Th} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2+1} + j\frac{1000(\alpha-1)}{(\alpha-1)^2+1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha-1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

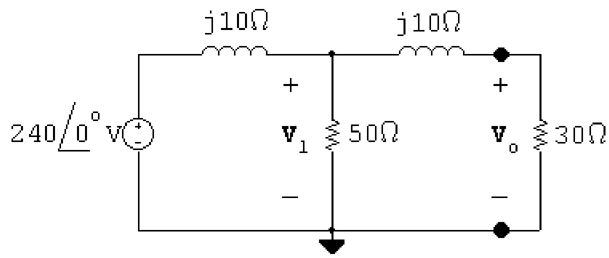
$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

Thus, $\alpha = 0$.

[d] $Z_{\rm Th} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

For $\mathbf{Im}(Z_{\rm Th}) > 0$, α must be greater than 1. So $Z_{\rm Th}$ is inductive for $1 < \alpha \leq 10$.

P 9.51



$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

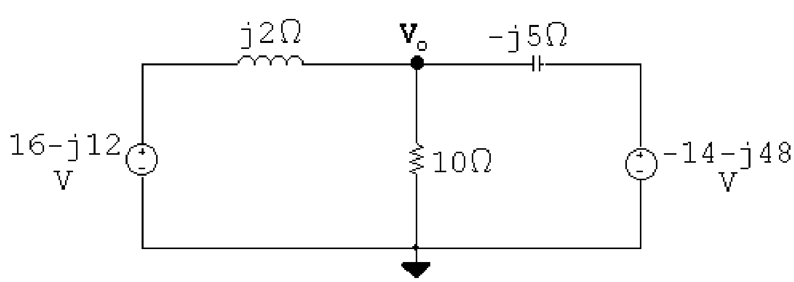
$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

P 9.52 $j\omega L = j(2000)(1 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(2000)(100)} = -j5\,\Omega$$

$$\mathbf{V}_{g1} = 20\underline{/-36.87^\circ} = 16 - j12\,\text{V}$$

$$\mathbf{V}_{g2} = 50\underline{/-106.26^\circ} = -14 - j48\,\text{V}$$



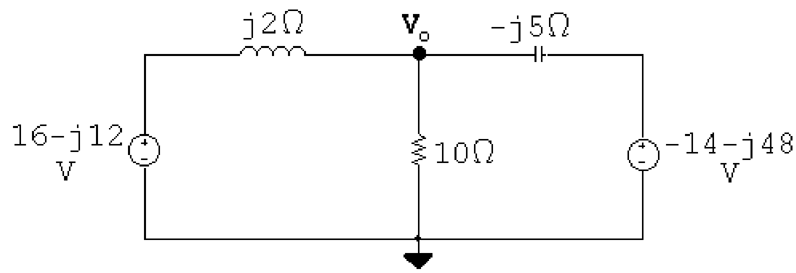
$$\frac{\mathbf{V}_o - (16 - j12)}{j2} + \frac{\mathbf{V}_o}{10} + \frac{\mathbf{V}_o - (-14 - j48)}{-j5} = 0$$

Solving,

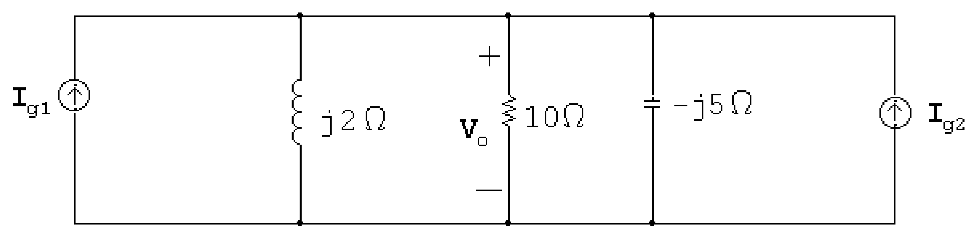
$$\mathbf{V}_o = 36\underline{/0^\circ}\,\text{V}$$

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.53 From the solution to Problem 9.52 the phasor-domain circuit is



Making two source transformations yields



$$\mathbf{I}_{g1} = \frac{16 - j12}{j2} = -6 - j8\,\text{A}$$

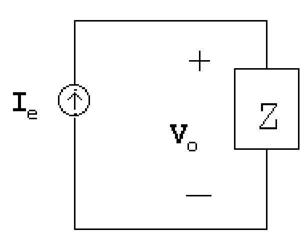
$$\mathbf{I}_{g2} = \frac{-14 - j48}{-j5} = 9.6 - j2.8\,\text{A}$$

$$Y = \frac{1}{j2} + \frac{1}{10} + \frac{1}{-j5} = (0.1 - j0.3)\,\text{S}$$

$$Z = \frac{1}{Y} = 1 + j3\,\Omega$$

$$\mathbf{I}_e = \mathbf{I}_{g1} + \mathbf{I}_{g2} = 3.6 - j10.8\,\text{A}$$

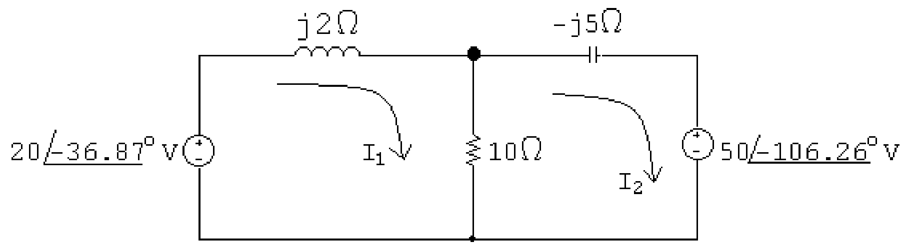
Hence the circuit reduces to



$$\mathbf{V}_o = Z\mathbf{I}_e = (1 + j3)(3.6 - j10.8) = 36\underline{/0^\circ}\text{ V}$$

$$\therefore \quad v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.54 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-20\underline{/-36.87^\circ} + j2\mathbf{I}_1 + 10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$50\underline{/-106.26^\circ} + 10(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(10 + j2) + \mathbf{I}_2(-10) = 20\underline{/-36.87^\circ}$$

$$\mathbf{I}_1(-10) + \mathbf{I}_2(10 - j5) = -50\underline{/-106.26^\circ} = 50\underline{/73.74^\circ}$$

Solving on a calculator yields:

$\mathbf{I}_1 = -6 + j10\text{A};$ $\mathbf{I}_2 = -9.6 + j10\text{A}$

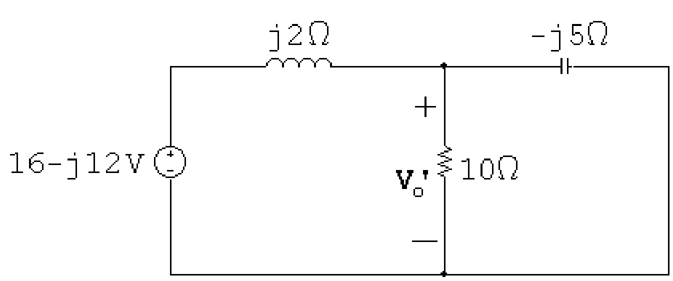
Thus,

$$\mathbf{V}_o = 10(\mathbf{I}_1 - \mathbf{I}_2) = 36\text{V}$$

and

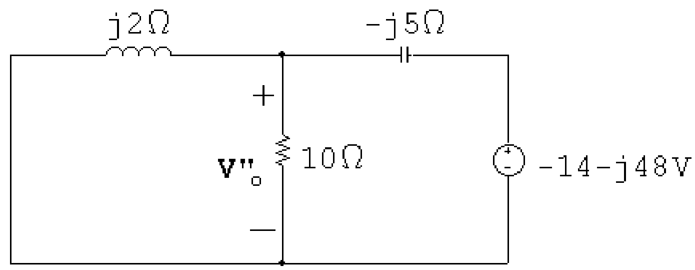
$$v_o(t) = 36\cos 2000t\text{V}$$

P 9.55 From the solution to Problem 9.52 the phasor-domain circuit with the right-hand source removed is



$$\mathbf{V}_o' = \frac{10\| - j5}{j2 + 10\| - j5}(16 - j12) = 18 - j26\,\text{V}$$

With the left hand source removed



$$\mathbf{V}_o'' = \frac{10\|j2}{-j5 + 10\|j2}(-14 - j48) = 18 + j26\,\mathrm{V}$$

$$\mathbf{V}_o = \mathbf{V}_o' + \mathbf{V}_o'' = 18 - j26 + 18 + j26 = 36\,\mathrm{V}$$

$$v_o(t) = 36\cos 2000t\,\mathrm{V}$$

P 9.56 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

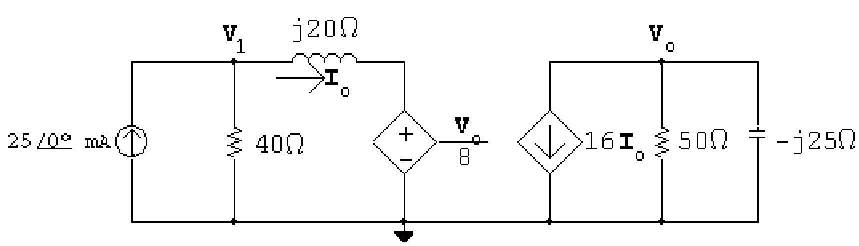
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\mathrm{V}$$

P 9.57



Write node voltage equations:

Left Node:

$$\frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1 - \mathbf{V}_o/8}{j20} = 0.025\underline{/0^\circ}$$

Right Node:

$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{j25} + 16\mathbf{I}_o = 0$$

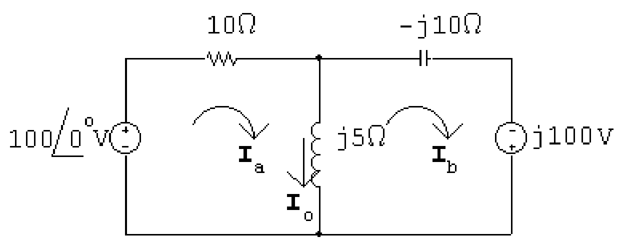
The constraint equation is

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_o/8}{j20}$$

Solution:

$$\mathbf{V}_o = (4 + j4) = 5.66\underline{/45^\circ}\text{ V}$$
$$\mathbf{V}_1 = (0.8 + j0.6) = 1.0\underline{/36.87^\circ}\text{ V}$$
$$\mathbf{I}_o = (5 - j15) = 15.81\underline{/-71.57^\circ}\text{ mA}$$

P 9.58



$$(10 + j5)\mathbf{I}_\mathrm{a} - j5\mathbf{I}_\mathrm{b} = 100\underline{/0^\circ}$$

$$-j5\mathbf{I}_\mathrm{a} - j5\mathbf{I}_\mathrm{b} = j100$$

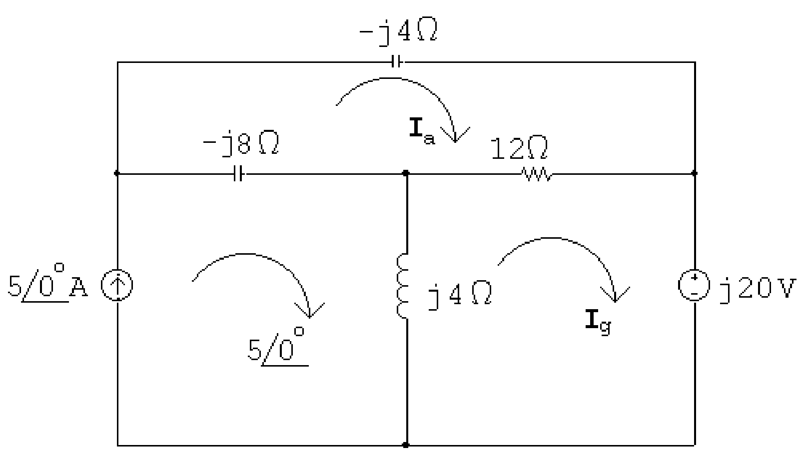
Solving,

$$\mathbf{I}_\mathrm{a} = -j10\text{ A}; \qquad \mathbf{I}_\mathrm{b} = -20 + j10\text{ A}$$

$$\mathbf{I}_o = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 20 - j20 = 28.28\underline{/-45^\circ}\text{ A}$$

$$i_o(t) = 28.28\cos(50{,}000t - 45^\circ)\text{ A}$$

P 9.59



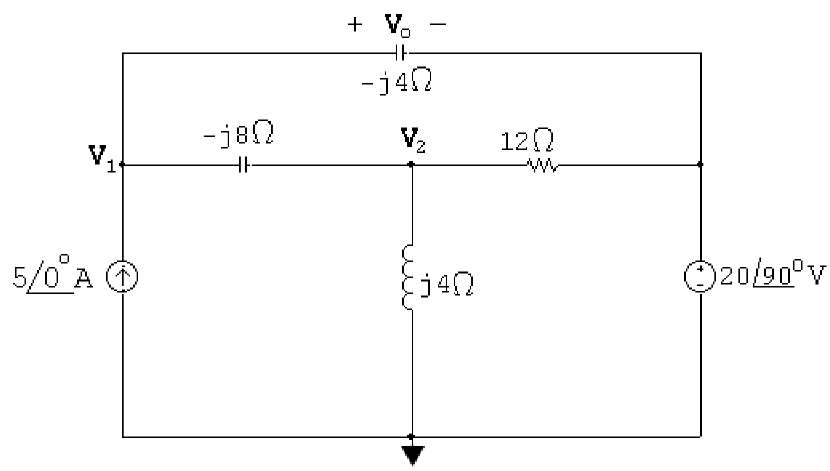
$$(12 - j12)\mathbf{I}_\mathrm{a} - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\text{ A}$$

P 9.60 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

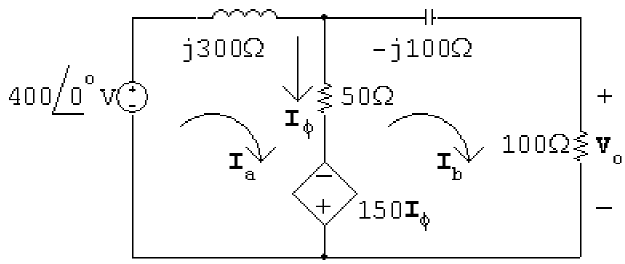
$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3}\text{ V} \qquad \mathbf{V}_2 = -8 + j4\text{ V}$$

Thus

$$\mathbf{V}_0 = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3} = 18.86\underline{/-98.13^\circ}\text{ V}$$

P 9.61 $j\omega L = j5000(60 \times 10^{-3}) = j300\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(2 \times 10^{-6})} = -j100\,\Omega$$



$$-400\underline{/0^\circ} + (50 + j300)\mathbf{I}_\mathrm{a} - 50\mathbf{I}_\mathrm{b} - 150(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}) = 0$$

$$(150 - j100)\mathbf{I}_\mathrm{b} - 50\mathbf{I}_\mathrm{a} + 150(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}) = 0$$

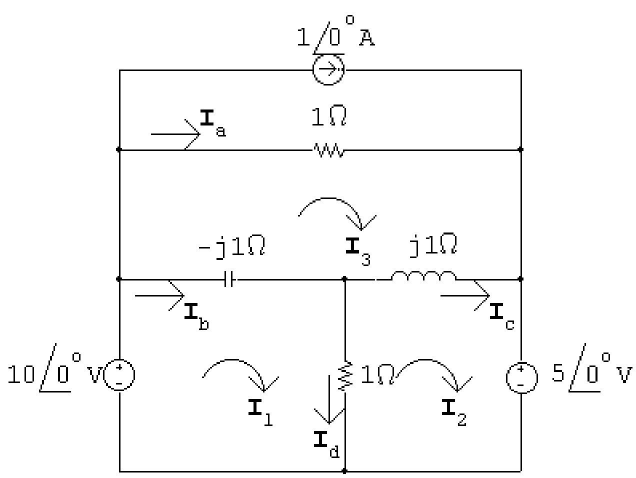
Solving,

$$\mathbf{I}_\mathrm{a} = -0.8 - j1.6\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{b} = -1.6 + j0.8\,\mathrm{A}$$

$$\mathbf{V}_o = 100\mathbf{I}_\mathrm{b} = -160 + j80 = 178.89\underline{/153.43^\circ}\ \mathrm{V}$$

$$v_o = 178.89\cos(5000t + 153.43^\circ)\ \mathrm{V}$$

P 9.62



$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 11 + j10\,\text{A}; \qquad \mathbf{I}_2 = 11 + j5\,\text{A}; \qquad \mathbf{I}_3 = 6\,\text{A}$$

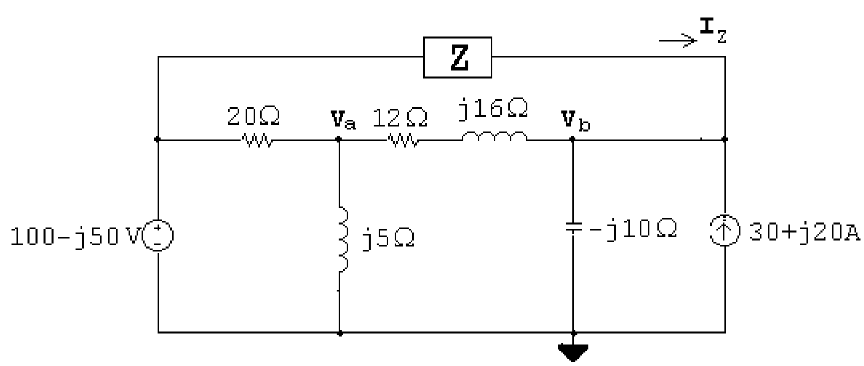
$$\mathbf{I}_\text{a} = \mathbf{I}_3 - 1 = 5\,\text{A} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A} = 11.18\underline{/63.43^\circ}\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A} = 7.07\underline{/45^\circ}\,\text{A}$$

$$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A} = 5\underline{/90^\circ}\,\text{A}$$

P 9.63



$$\frac{\mathbf{V}_\text{a} - (100 - j50)}{20} + \frac{\mathbf{V}_\text{a}}{j5} + \frac{\mathbf{V}_\text{a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_\text{a} = 40 + j30\,\text{V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\text{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

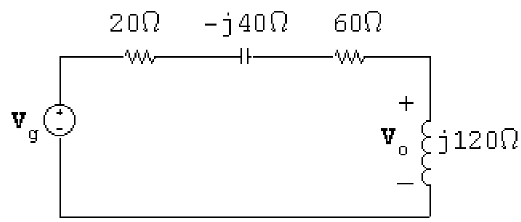
P 9.64 **[a]** $\frac{1}{j\omega C} = -j50\,\Omega$

$j\omega L = j120\,\Omega$

$Z_e = 100\| - j50 = 20 - j40\,\Omega$

$\mathbf{I}_g = 2\underline{/0^\circ}$

$\mathbf{V}_g = \mathbf{I}_g Z_e = 2(20 - j40) = 40 - j80\,\mathrm{V}$



$\mathbf{V}_o = \frac{j120}{80 + j80}(40 - j80) = 90 - j30 = 94.87\underline{/-18.43^\circ}\,\mathrm{V}$

$v_o = 94.87\cos(16 \times 10^5 t - 18.43^\circ)\,\mathrm{V}$

[b] $\omega = 2\pi f = 16 \times 10^5; \qquad f = \frac{8 \times 10^5}{\pi}$

$T = \frac{1}{f} = \frac{\pi}{8 \times 10^5} = 1.25\pi\,\mu\mathrm{s}$

$\therefore \quad \frac{18.43}{360}(1.25\pi\,\mu\mathrm{s}) = 201.09\,\mathrm{ns}$

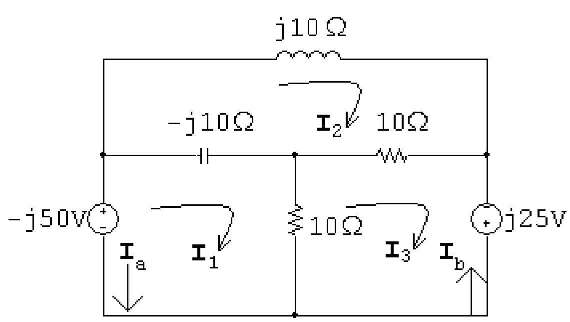
$\therefore \quad v_o$ lags i_g by 201.09 ns

P 9.65 $j\omega L = j10^6(10 \times 10^{-6}) = j10\,\Omega$

$\frac{1}{j\omega C} = \frac{-j}{(10^6)(0.1 \times 10^{-6})} = -j10\,\Omega$

$\mathbf{V}_\mathrm{a} = 50\underline{/-90^\circ} = -j50\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 25\underline{/90^\circ} = j25\,\mathrm{V}$



$(10 - j10)\mathbf{I}_1 + j10\mathbf{I}_2 - 10\mathbf{I}_3 = -j50$

$j10\mathbf{I}_1 + 10\mathbf{I}_2 - 10\mathbf{I}_3 = 0$

$-10\mathbf{I}_1 - 10\mathbf{I}_2 + 20\mathbf{I}_3 = j25$

Solving,

$\mathbf{I}_1 = 0.5 - j1.5\,\text{A}; \qquad \mathbf{I}_3 = -1 + j0.5\,\text{A} \qquad \mathbf{I}_2 = -2.5\,\text{A}$

$\mathbf{I}_\text{a} = -\mathbf{I}_1 = -0.5 + j1.5 = 1.58\underline{/108.43^\circ}\,\text{A}$

$\mathbf{I}_\text{b} = -\mathbf{I}_3 = 1 - j0.5 = 1.12\underline{/-26.57^\circ}\,\text{A}$

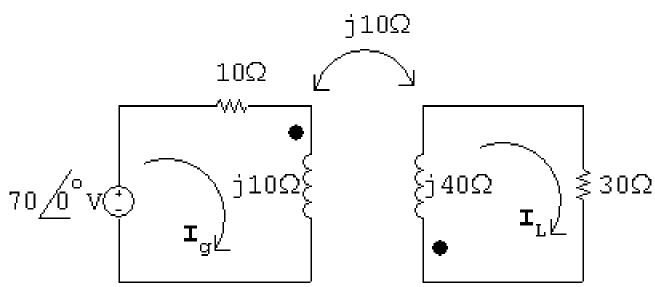
$i_\text{a} = 1.58\cos(10^6 t + 108.43^\circ)\,\text{A}$

$i_\text{b} = 1.12\cos(10^6 t - 26.57^\circ)\,\text{A}$

P 9.66 **[a]** $j\omega L_1 = j(5000)(2 \times 10^{-3}) = j10\,\Omega$

$j\omega L_2 = j(5000)(8 \times 10^{-3}) = j40\,\Omega$

$j\omega M = j10\,\Omega$



$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$

$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$

Solving,

$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$

$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$

$i_\text{L} = 1\cos(5000t - 180^\circ)\,\text{A}$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu$s,

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\,\text{A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2\times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu$s,

$$5000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(200\pi\mu s) = 5\cos(180^\circ - 36.87^\circ) = -4\,\text{A}$$

$$i_L(200\pi\mu s) = 1\cos(180^\circ - 180^\circ) = 1\,\text{A}$$

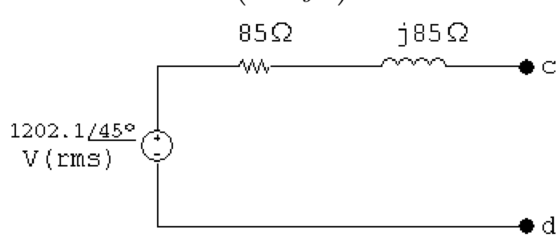
$$w = \frac{1}{2}(2\times 10^{-3})(16) + \frac{1}{2}(8\times 10^{-3})(1) + 2\times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

P 9.67 Remove the voltage source to find the equivalent impedance:

$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5+j5|}\right)^2 (5 - j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5+j5}\right) = 850 + j850\,\text{V} = 1202.1\underline{/45^\circ}\ \text{V}$$



P 9.68 **[a]** $j\omega L_1 = j(200\times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200\times 10^3)(4\times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200\times 10^3)(12.5\times 10^{-9})} = -j400\,\Omega$$

$$\therefore\quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore\quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k\times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\rm in} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\rm in}| = [(200 + 192k^2)^2 + (200 - 256k^2)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\rm in}|}{dk} = \frac{1}{2}[(200 + 192k^2)^2 + (200 - 256k^2)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\rm in}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b] $Z_{\rm in}$ (min) $= 200 + 192(0.125) + j[200 - 0.125(256)]$

$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1 \text{ (max)} = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1 \text{ (peak)} = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\rm in}|/dk = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{\rm in} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=1} > |Z_{\rm in}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$$Z_{\rm in} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=0} > |Z_{\rm in}|_{k=\sqrt{0.125}}$$

P 9.69 $j\omega L_1 = j50\,\Omega$

$$j\omega L_2 = j32\,\Omega$$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4 \times 10^3)k\sqrt{(12.5)(8)} \times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5 + j12|}\right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$$

$$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$$

P 9.70 **[a]** $j\omega L_L = j100\,\Omega$

$$j\omega L_2 = j500\,\Omega$$

$$Z_{22} = 300 + 500 + j100 + j500 = 800 + j600\,\Omega$$

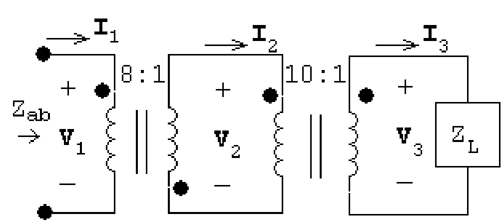
$$Z_{22}^* = 800 - j600\,\Omega$$

$$\omega M = 270\,\Omega$$

$$Z_r = \left(\frac{270}{1000}\right)^2 [800 - j600] = 58.32 - j43.74\,\Omega$$

[b] $Z_{\text{ab}} = R_1 + j\omega L_1 + Z_r = 41.68 + j180 + 58.32 - j43.74 = 100 + j136.26\,\Omega$

P 9.71



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3} = 80\underline{/60^\circ}\,\Omega$$

$$\frac{\mathbf{V}_2}{10} = \frac{\mathbf{V}_3}{1}; \qquad 10\mathbf{I}_2 = 1\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{8} = -\frac{\mathbf{V}_2}{1}; \qquad 8\mathbf{I}_1 = -1\mathbf{I}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{-8\mathbf{V}_2}{-\mathbf{I}_2/8} = \frac{8^2\mathbf{V}_2}{\mathbf{I}_2}$$

$$= \frac{8^2(10\mathbf{V}_3)}{\mathbf{I}_3/10} = \frac{(8)^2(10)^2\mathbf{V}_3}{\mathbf{I}_3} = (8)^2(10)^2 Z_L = 512,000\underline{/60^\circ}\ \Omega$$

P 9.72 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write X_{ab} as

$$X_{\text{ab}} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{ 1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2} \right\}$$

For X_{ab} to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

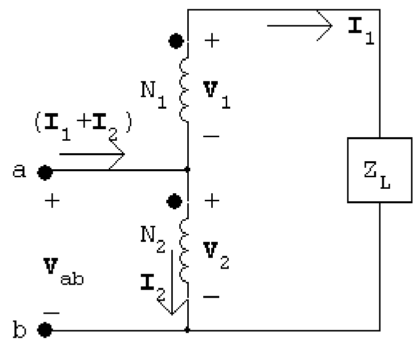
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1 - k^2) + \omega L_2 \omega L_L(2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$ hence it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

P 9.73 **[a]**



$$Z_{\rm ab} = \frac{\mathbf{V}_{\rm ab}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\rm ab} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\rm ab} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

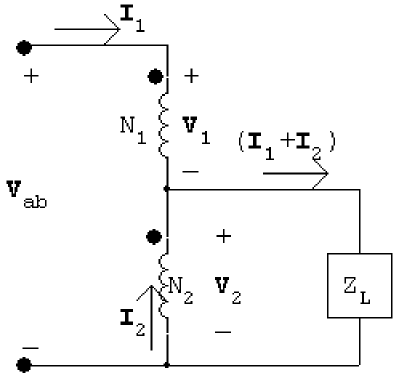
As before

$$Z_{\rm ab} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\rm ab} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\rm ab} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.74 **[a]**



$$Z_{\rm ab} = \frac{\mathbf{V}_{\rm ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\rm ab} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\rm ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{\rm ab} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{\rm ab} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\rm ab} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.75 **[a]** $\mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5)\,\text{A}$

$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\,\text{V}$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$

$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$

[c] Let $\mathbf{I}_c$ denote the magnitude of the current in the capacitor branch. Then

$\mathbf{I} = (10 - j7.5 + j\mathbf{I}_c) = 10 + j(\mathbf{I}_c - 7.5)\,\text{A}$

$$\begin{aligned}\mathbf{V}_s &= 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(\mathbf{I}_c - 7.5)]\\ &= (247 - 0.8\mathbf{I}_c) + j(7.25 + 0.1\mathbf{I}_c)\end{aligned}$$

It follows that

$240\cos\alpha = (247 - 0.8\mathbf{I}_c)$ and $240\sin\alpha = (7.25 + 0.1\mathbf{I}_c)$

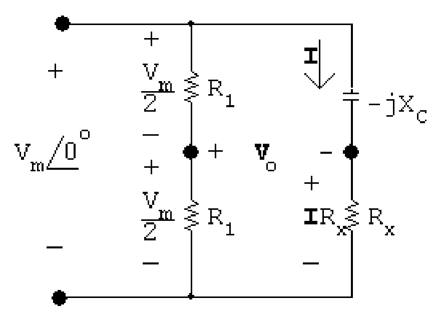
Now square each term and then add to generate the quadratic equation

$\mathbf{I}_c^2 - 605.77\mathbf{I}_c + 5325.48 = 0; \qquad \mathbf{I}_c = 302.88 \pm 293.96$

Therefore

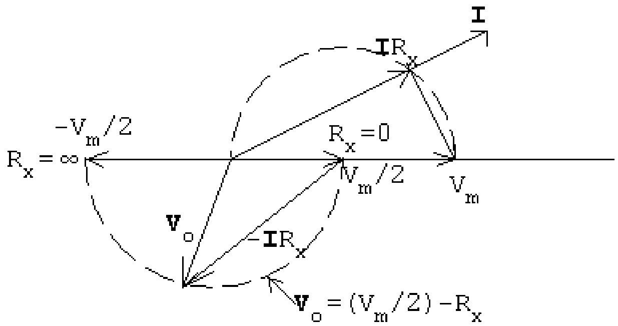
$\mathbf{I}_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

P 9.76 The phasor domain equivalent circuit is

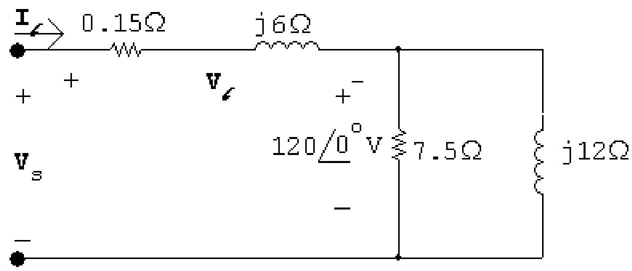


$$V_o = \frac{V_m\underline{/0^\circ}}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m\underline{/0^\circ}}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle decreases from 0° to -180°, as shown in the following phasor diagram:



P 9.77 **[a]**

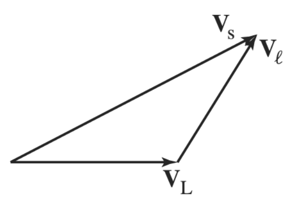


$$\mathbf{I}_\ell = \frac{120}{7.5} + \frac{120}{j12} = 16 - j10\,\text{A}$$

$$\mathbf{V}_\ell = (0.15 + j6)(16 - j10) = 62.4 + j94.5 = 113.24\angle 56.56^\circ\,\text{V}$$

$$\text{V}_\text{s} = 120\angle 0^\circ + \mathbf{V}_\ell = 205.43\angle 27.39^\circ\,\text{V}$$

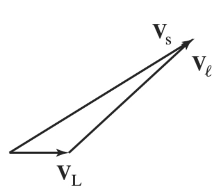
[b]



[c] $\mathbf{I}_\ell = \dfrac{120}{2.5} + \dfrac{120}{j4} = 48 - j30\,\text{A}$

$$\mathbf{V}_\ell = (0.15 + j6)(48 - j30) = 339.73\angle 56.56^\circ\,\text{V}$$

$$\mathbf{V}_\text{s} = 120 + \mathbf{V}_\ell = 418.02\angle 42.7^\circ\,\text{V}$$

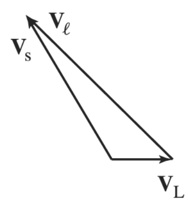


The amplitude of $\mathbf{V}_s$ must be increased from 205.43 V to 418.02 V (more than doubled) to maintain the load voltage at 120 V.

[d] $\mathbf{I}_\ell = \frac{120}{2.5} + \frac{120}{j4} + \frac{120}{-j2} = 48 + j30\,\text{A}$

$\mathbf{V}_\ell = (0.15 + j6)(48 + j30) = 339.73\underline{/120.57^\circ}$ V

$\mathbf{V}_\text{s} = 120 + \mathbf{V}_\ell = 297.23\underline{/100.23^\circ}$ V



The amplitude of $\mathbf{V}_s$ must be increased from 205.43 V to 297.23 V to maintain the load voltage at 120 V.

P 9.78 $\mathbf{V}_g = 4\underline{/0^\circ}$ V; $\frac{1}{j\omega C} = -j20\,\text{k}\Omega$

Let $\mathbf{V}_\text{a}$ = voltage across the capacitor, positive at upper terminal
Then:

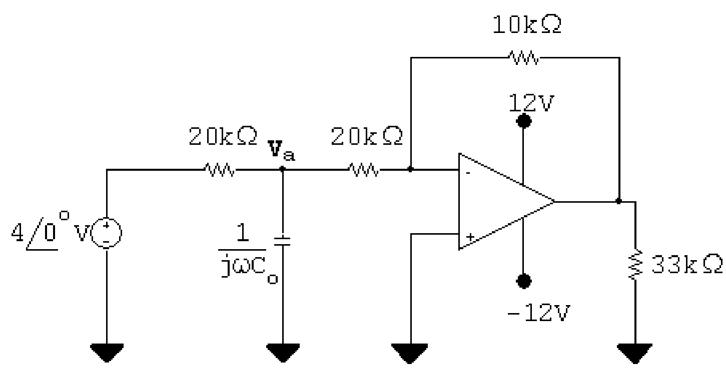
$$\frac{\mathbf{V}_\text{a} - 4\underline{/0^\circ}}{20{,}000} + \frac{\mathbf{V}_\text{a}}{-j20{,}000} + \frac{\mathbf{V}_\text{a}}{20{,}000} = 0; \qquad \therefore \quad \mathbf{V}_\text{a} = (1.6 - j0.8)\,\text{V}$$

$$\frac{0 - \mathbf{V}_\text{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{10{,}000} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\text{a}}{2}$$

$$\therefore \quad \mathbf{V}_o = -0.8 + j0.4 = 0.89\underline{/153.43^\circ}\,\text{V}$$

$$v_o = 0.89\cos(200t + 153.43^\circ)\,\text{V}$$

P 9.79 **[a]**



$$\frac{\mathbf{V}_\text{a} - 4\underline{/0^\circ}}{20{,}000} + j\omega C_o\mathbf{V}_\text{a} + \frac{\mathbf{V}_\text{a}}{20{,}000} = 0$$

$$\mathbf{V}_\text{a} = \frac{4}{2 + j20{,}000\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_\text{a}}{2} \qquad \text{(see solution to Prob. 9.78)}$$

$$\mathbf{V}_o = \frac{-2}{2 + j4\times 10^6 C_o} = \frac{2\underline{/180^\circ}}{2 + j4\times 10^6 C_o}$$

$\therefore$ denominator angle $= 45^\circ$

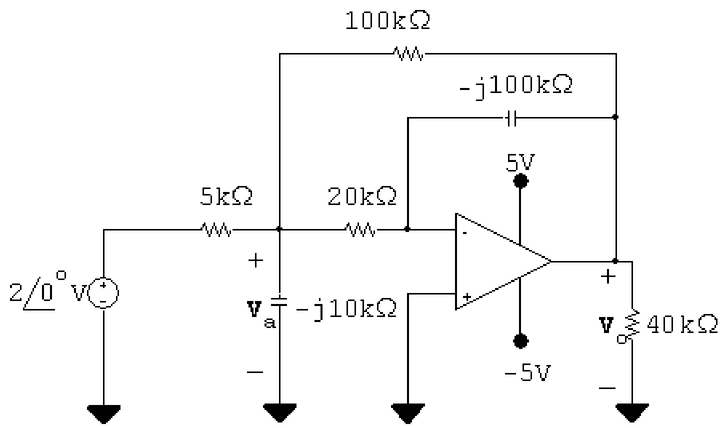
so $4\times 10^6 C_o = 2 \quad \therefore \quad C_o = 0.5\,\mu\text{F}$

[b] $\mathbf{V}_o = \dfrac{2\underline{/180^\circ}}{2 + j2} = 0.707\underline{/135^\circ}\,\text{V}$

$v_o = 0.707\cos(200t + 135^\circ)\,\text{V}$

P 9.80 $\dfrac{1}{j\omega C_1} = -j10\,\text{k}\Omega$

$\dfrac{1}{j\omega C_2} = -j100\,\text{k}\Omega$



$$\frac{\mathbf{V}_a - 2}{5000} + \frac{\mathbf{V}_a}{-j10{,}000} + \frac{\mathbf{V}_a}{20{,}000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{100{,}000} = 0$$

$$20\mathbf{V}_a - 40 + j10\mathbf{V}_a + 5\mathbf{V}_a + \mathbf{V}_a - \mathbf{V}_o = 0$$

$$\therefore \qquad (26 + j10)\mathbf{V}_a - \mathbf{V}_o = 40$$

$$\frac{0 - \mathbf{V}_a}{20{,}000} + \frac{0 - \mathbf{V}_o}{-j100{,}000} = 0$$

$$j5\mathbf{V}_a - \mathbf{V}_o = 0$$

Solving,

$$\mathbf{V}_o = 1.43 + j7.42 = 7.55\underline{/79.11^\circ}\,\text{V}$$

$$v_o(t) = 7.55\cos(10^6 t + 79.11^\circ)\,\text{V}$$

P 9.81 **[a]** $\mathbf{V}_g = 25\underline{/0^\circ}\,\text{V}$

$$\mathbf{V}_\text{p} = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_\text{n} = \mathbf{V}_\text{p} = 5\underline{/0^\circ}\,\text{V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_\text{p}} = 0$$

$$Z_\text{p} = -j80{,}000\|40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_\text{p}}{80{,}000} + 5 = 7 - j1 = 7.07\underline{/-8.13^\circ}\,\text{V}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\,\text{V}$$

[b] $\mathbf{V}_\text{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\text{n} = \mathbf{V}_\text{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = 0.2V_m(1.4 - j0.2)$$

$$\therefore \quad |0.2V_m(1.4 - j0.2)| \leq 10$$

$$\therefore \quad V_m \leq 35.36\,\text{V}$$

P 9.82 **[a]** $\dfrac{1}{j\omega C} = -j20\,\Omega$

$$\frac{\mathbf{V}_\text{n}}{20} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j20} = 0$$

$$\frac{\mathbf{V}_o}{-j20} = \frac{\mathbf{V}_\text{n}}{20} + \frac{\mathbf{V}_\text{n}}{-j20}$$

$$\mathbf{V}_o = -j1\mathbf{V}_\text{n} + \mathbf{V}_\text{n} = (1 - j1)\mathbf{V}_\text{n}$$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{5 + (1/j\omega C_o)} = \frac{\mathbf{V}_g}{1 + j(5)(10^5)C_o}$$

$$\mathbf{V}_g = 6\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_\text{p} = \frac{6\underline{/0^\circ}}{1 + j5 \times 10^5 C_o} = \mathbf{V}_\text{n}$$

$$\therefore \quad \mathbf{V}_o = \frac{(1 - j1)6\underline{/0^\circ}}{1 + j5 \times 10^5 C_o}$$

$$|\mathbf{V}_o| = \frac{\sqrt{2}(6)}{\sqrt{1 + 25 \times 10^{10}C_o^2}} = 6$$

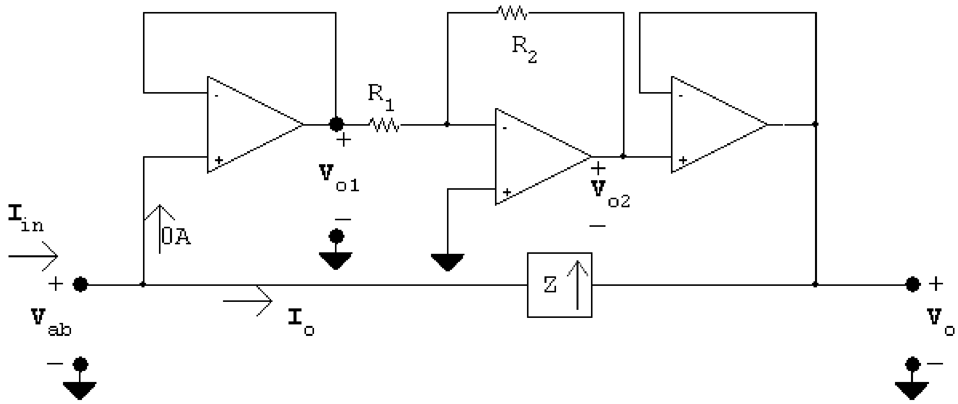
Solving,

$$C_o = 2\,\mu\text{F}$$

[b] $\mathbf{V}_o = \dfrac{6(1 - j1)}{1 + j1} = -j6\,\text{V}$

$$v_o = 6\cos(10^5 t - 90^\circ)\,\text{V}$$

P 9.83 **[a]**



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

$$\therefore \quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \dfrac{1}{j\omega C}; \qquad Z_{\text{ab}} = \dfrac{1}{j\omega C(1+K)}; \qquad \therefore \quad C_{\text{ab}} = C(1+K)$

P 9.84 **[a]** $\mathbf{I}_1 = \dfrac{120}{24} + \dfrac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[b]

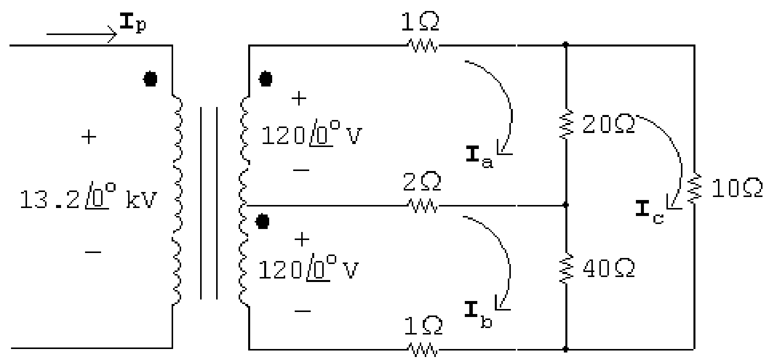
$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15\,\text{A}$	$\mathbf{I}_5 = 10\,\text{A}$
$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$	$\mathbf{I}_4 = -5\,\text{A}$	$\mathbf{I}_6 = 5\,\text{A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to $5\,\text{A}$, well below its normal running value of $22.86\,\text{A}$.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.85 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$ $\quad\mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$ $\quad\mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\,\text{A}$$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

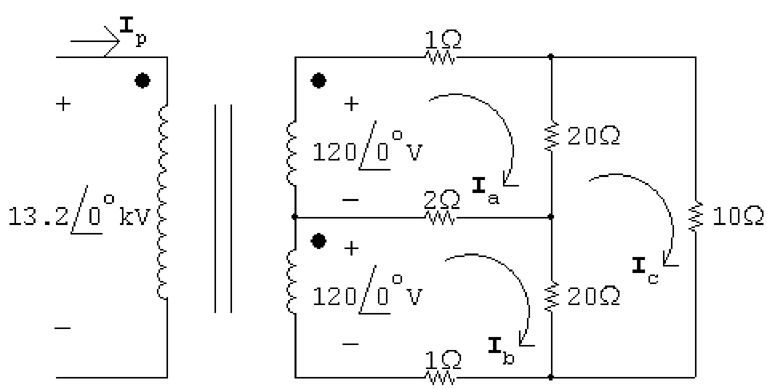
$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\text{ A}$$

Check voltages —

$$\mathbf{V}_4 = 10\text{I}_4 = 194\underline{/0^\circ}\text{ V}$$
$$\mathbf{V}_5 = 20\text{I}_5 = 92\underline{/0^\circ}\text{ V}$$
$$\mathbf{V}_6 = 40\text{I}_6 = 102\underline{/0^\circ}\text{ V}$$

All of these voltages are low for a reasonable distribution circuit.

P 9.86 **[a]**



The three mesh current equations are

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_b = 24\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_c = 19.2\underline{/0^\circ}\text{ A}$$

$$\therefore \quad \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0\text{ A}$$

[b] $\mathbf{I}_p = \dfrac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \dfrac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b$

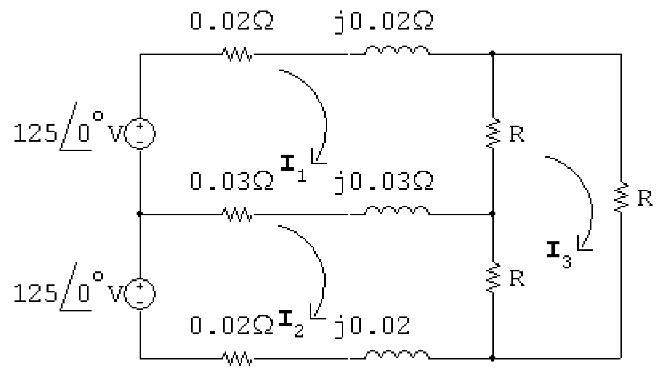
$$= \frac{1}{110}(24 + 24) = 0.436\underline{/0^\circ}\text{ A}$$

[c] Check voltages —

$$\mathbf{V}_4 = 10\mathrm{I}_4 = 10\mathrm{I}_c = 192\underline{/0^\circ}\ \mathrm{V}$$
$$\mathbf{V}_5 = 20\mathrm{I}_5 = 20(\mathrm{I}_a - \mathrm{I}_c) = 96\underline{/0^\circ}\ \mathrm{V}$$
$$\mathbf{V}_6 = 40\mathrm{I}_6 = 20(\mathrm{I}_b - \mathrm{I}_c) = 96\underline{/0^\circ}\ \mathrm{V}$$

Where the two loads are equal, the current in the neutral conductor (I_2) is zero, and the voltages V_5 and V_6 are equal. The voltages V_4, V_5, and V_6 are too low for a reasonable dirtribution circuit.

P 9.87 **[a]**



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

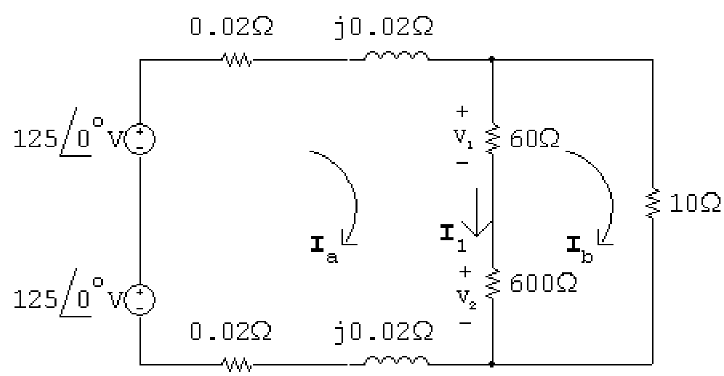
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore\quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_\mathrm{n} = \mathbf{I}_1 - \mathbf{I}_2 = 0\,\mathrm{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3);\qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$250 = (660.04 + j0.04)\mathbf{I}_a - 660\mathbf{I}_b$

$0 = -660\mathbf{I}_a + 670\mathbf{I}_b$

Solving,

$\mathbf{I}_a = 25.28\underline{/-0.23^\circ} = 25.28 - j0.10\,\text{A}$

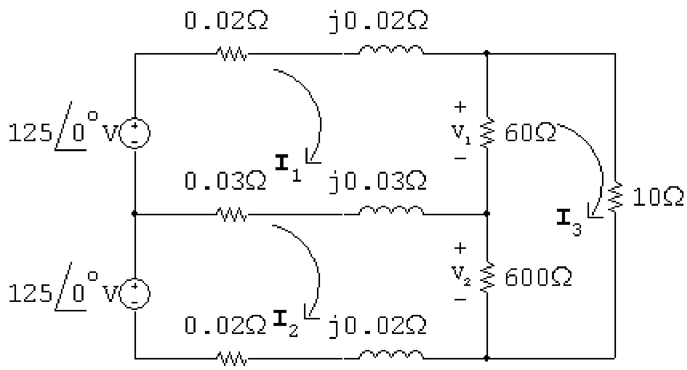
$\mathbf{I}_b = 24.90\underline{/-0.23^\circ} = 24.90 - j0.10\,\text{A}$

$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.377 - j0.00153\,\text{A}$

$\mathbf{V}_1 = 60\mathbf{I}_1 = 22.63 - j0.0195 = 22.64\underline{/-0.23^\circ}\,\text{V}$

$\mathbf{V}_2 = 600\mathbf{I}_1 = 226.3 - j0.915 = 226.4\underline{/-0.23^\circ}\,\text{V}$

[d]



$125 = (60.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 60\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (600.05 + j0.05)\mathbf{I}_2 - 600\mathbf{I}_3$

$0 = -60\mathbf{I}_1 - 600\mathbf{I}_2 + 670\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 26.97\underline{/-0.24^\circ} = 26.97 - j0.113\,\text{A}$

$\mathbf{I}_2 = 25.10\underline{/-0.24^\circ} = 25,10 - j0.104\,\text{A}$

$\mathbf{I}_3 = 24.90\underline{/-0.24^\circ} = 24.90 - j0.104\,\text{A}$

$\mathbf{V}_1 = 60(\mathbf{I}_1 - \mathbf{I}_3) = 124.4\underline{/-0.27^\circ}\,\text{V}$

$\mathbf{V}_2 = 600(\mathbf{I}_2 - \mathbf{I}_3) = 124.6\underline{/-0.20^\circ}\,\text{V}$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.88 **[a]** Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$$\mathbf{I}_\mathrm{p} = 2a\mathbf{I}_\mathrm{a}$$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{2N_2\mathbf{I}_\mathrm{a}}{N_1} = \frac{1}{56}\mathbf{I}_\mathrm{a}$$

$$= \frac{1}{56}(25.28 - j0.10)$$

$$\mathbf{I}_\mathrm{p} = 451.4 - j1.8\,\mathrm{mA} = 451.4\underline{/-0.23^\circ}\ \mathrm{mA}$$

In part d),

$$\mathbf{I}_\mathrm{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(26.97 - j0.11 + 25.10 - j0.10)$$

$$= \frac{1}{112}(52.07 - j0.22)$$

$$\mathbf{I}_\mathrm{p} = 464.9 - j1.9\,\mathrm{mA} = 464.9\underline{/-0.24^\circ}\ \mathrm{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 **[a]** $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2 pf $= \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5$ leading

rf $= \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$

AP 10.3 From Ex. 9.4 $I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}$ A

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

$$\text{Therefore}\quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A(rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\ \text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\ \text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\ \text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18$ VAR and

$1129.09 + 23.52 = 1152.62$ W

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore}\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A(rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$

Parallel circuit derivation:

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6 $S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$

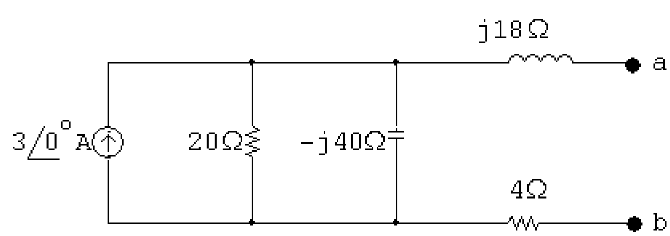
$$S_2 = 6000(0.8) + j6000(0.6) = 4800 - j3600\,\text{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

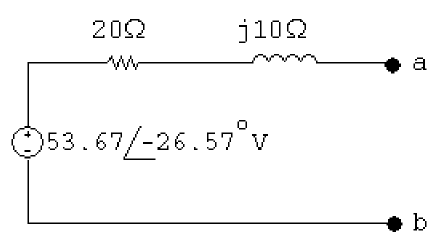
$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V(rms)}$$

AP 10.7 **[a]** The phasor domain equivalent circuit and the Thévenin equivalent are shown below:
Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$

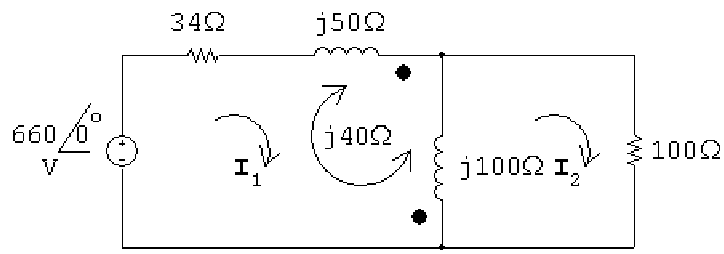
Therefore $P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 18\,\text{W}$

[c] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$

Therefore $P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

AP 10.8



Mesh current equations:

$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$

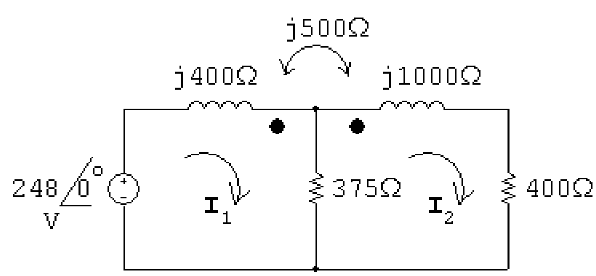
$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$

Solving,

$\mathbf{I}_1 = 3.536\underline{/-45^\circ}$ A,

$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore \quad P = \dfrac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$

AP 10.9 **[a]**



$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$

Solving,

$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}\,\text{A}$

$\therefore \quad P = \dfrac{1}{2}(0.25)(400) = 50\,\text{W}$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \dfrac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\text{max}} = \left(\dfrac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V(rms)} = 584\underline{/180^\circ}\,\text{V(rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\dfrac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 **[a]** $P = \frac{1}{2}(100)(10)\cos(50-15) = 500\cos 35^\circ = 409.58\,\text{W} \quad \text{(abs)}$

$Q = 500\sin 35^\circ = 286.79\,\text{VAR} \quad \text{(abs)}$

[b] $P = \frac{1}{2}(40)(20)\cos(-15-60) = 400\cos(-75^\circ) = 103.53\,\text{W} \quad \text{(abs)}$

$Q = 400\sin(-75^\circ) = -386.37\,\text{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(400)(10)\cos(30-150) = 2000\cos(-120^\circ) = -1000\,\text{W} \quad \text{(del)}$

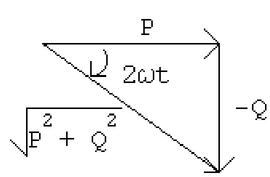
$Q = 2000\sin(-120^\circ) = -1732.05\,\text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(200)(5)\cos(160-40) = 500\cos(120^\circ) = -250\,\text{W} \quad \text{(del)}$

$Q = 500\sin(120^\circ) = 433.01\,\text{VAR} \quad \text{(abs)}$

P 10.2 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$\frac{dp}{dt} = 0$ when $-2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t$ or $\tan 2\omega t = -\frac{Q}{P}$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $p_{\max} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $p_{\min} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.3 **[a]** hair dryer $= 600\,\text{W}$ vacuum $= 630\,\text{W}$

sun lamp $= 279\,\text{W}$ air conditioner $= 860\,\text{W}$

television $= 240\,\text{W}$ $\sum P = 2609\,\text{W}$

Therefore $I_{\text{eff}} = \dfrac{2609}{120} = 21.74\,\text{A}$

Yes, the breaker will trip.

[b] $\sum P = 2609 - 909 = 1700\,\text{W}; \qquad I_{\text{eff}} = \dfrac{1700}{120} = 14.17\,\text{A}$

Yes, the breaker will not trip if the current is reduced to $14.17\,\text{A}$.

P 10.4 **[a]** $I_{\text{eff}} = 40/115 \cong 0.35\,\text{A}$; [b] $I_{\text{eff}} = 130/115 \cong 1.13\,\text{A}$

P 10.5 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T; \qquad W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.6 **[a]** Area under one cycle of v_g^2:

$$A = (5^2)(2)(30 \times 10^{-6}) + 2^2(2)(37.5 \times 10^{-6})$$
$$= 1800 \times 10^{-6}$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{200 \times 10^{-6}} = \frac{1800 \times 10^{-6}}{200 \times 10^{-6}} = 9$$

$$\therefore \quad V_{\text{rms}} = \sqrt{9} = 3\,\text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{3^2}{2.25} = 4\,\text{W}$

P 10.7 $i(t) = 200t \qquad 0 \le t \le 75\,\text{ms}$

$i(t) = 60 - 600t \qquad 75\,\text{ms} \le t \le 100\,\text{ms}$

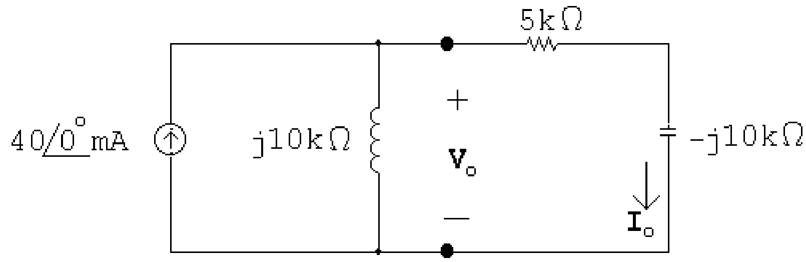
$$I_{\text{rms}} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.075} (200)^2 t^2\,dt + \int_{0.075}^{0.1} (60 - 600t)^2\,dt\right\}}$$

$$= \sqrt{10(5.625) + 10(1.875)} = \sqrt{75} = 8.66\,\text{A(rms)}$$

P 10.8 $P = I_{\rm rms}^2 R \qquad \therefore \quad R = \dfrac{3\times 10^3}{75} = 40\,\Omega$

P 10.9 $\mathbf{I}_g = 40\underline{/0^\circ}$ mA

$$j\omega L = j10{,}000\,\Omega; \qquad \frac{1}{j\omega C} = -j10{,}000\,\Omega$$



$$\mathbf{I}_o = \frac{j10{,}000}{5000}(40\underline{/0^\circ}) = 80\underline{/90^\circ}\text{ mA}$$

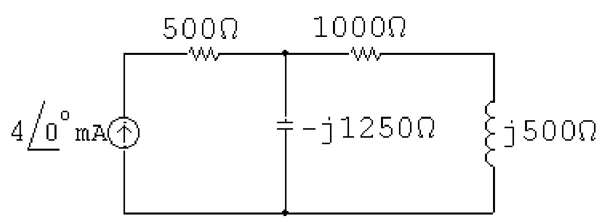
$$P = \frac{1}{2}|\mathbf{I}_o|^2(5000) = \frac{1}{2}(0.08)^2(5000) = 16\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(-10{,}000) = -32\,\text{VAR}$$

$$S = P + jQ = 16 - j32\,\text{VA}$$

$$|S| = 35.78\,\text{VA}$$

P 10.10 $\mathbf{I}_g = 4\underline{/0^\circ}$ mA; $\qquad \dfrac{1}{j\omega C} = -j1250\,\Omega; \qquad j\omega L = j500\,\Omega$

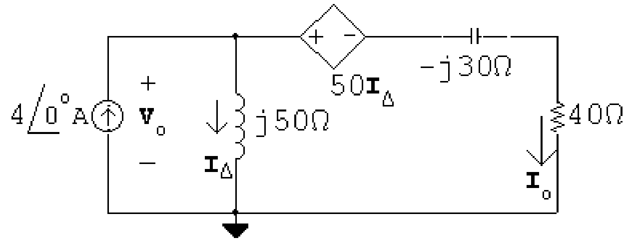


$$Z_{\rm eq} = 500 + [-j1250 \| (1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2\text{Re}\{Z_{\rm eq}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

P 10.11 $j\omega L = j10^5(0.5 \times 10^{-3}) = j50\,\Omega;\qquad \frac{1}{j\omega C} = \frac{1}{j10^5[(1/3) \times 10^{-6}]} = -j30\,\Omega$



$$-4 + \frac{\mathbf{V}_o}{j50} + \frac{\mathbf{V}_o - 50\mathbf{I}_\Delta}{40 - j30} = 0$$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j50}$$

Place the equations in standard form:

$$\mathbf{V}_o\left(\frac{1}{j50} + \frac{1}{40 - j30}\right) + \mathbf{I}_\Delta\left(\frac{-50}{40 - j30}\right) = 4$$

$$\mathbf{V}_o\left(\frac{1}{j50}\right) + \mathbf{I}_\Delta(-1) = 0$$

Solving,

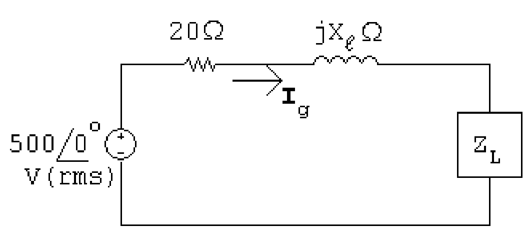
$$\mathbf{V}_o = 200 - j400 \text{ V};\qquad \mathbf{I}_\Delta = -8 - j4 \text{ A}$$

$$\mathbf{I}_o = 4 - (-8 - j4) = 12 + j4 \text{ A}$$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(40) = \frac{1}{2}(160)(40) = 3200\,\text{W}$$

P 10.12 **[a]** line loss $= 7500 - 2500 = 5\,\text{kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore\quad |\mathbf{I}_g|^2 = 250$

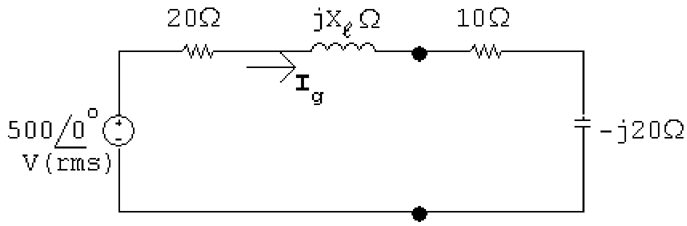


$$|\mathbf{I}_g| = \sqrt{250}\,\text{A}$$

$|\mathbf{I}_g|^2 R_\mathrm{L} = 2500 \qquad \therefore \quad R_\mathrm{L} = 10\,\Omega$

$|\mathbf{I}_g|^2 X_\mathrm{L} = -5000 \qquad \therefore \quad X_\mathrm{L} = -20\,\Omega$

Thus,



$|Z| = \sqrt{(30)^2 + (X_\ell - 20)^2} \qquad |\mathbf{I}_g| = \dfrac{500}{\sqrt{900 + (X_\ell - 20)^2}}$

$$\therefore \quad 900 + (X_\ell - 20)^2 = \frac{25 \times 10^4}{250} = 1000$$

Solving, $\qquad (X_\ell - 20) = \pm 10.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 30\,\Omega$

[b] If $X_\ell = 30\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 + j10} = 15 - j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 - j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and 2500 magnetizing vars.

$Q_{j30} = |\mathbf{I}_g|^2 X_\ell = 250(30) = 7500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 7500 magnetizing vars.

$Q_{-j20} = |\mathbf{I}_g|^2 X_\mathrm{L} = 250(-20) = -5000\,\mathrm{VAR}$

Therefore the load reactance is generating 5000 magnetizing vars.

$\sum Q_\mathrm{gen} = 7500\,\mathrm{VAR} = \sum Q_\mathrm{abs}$

If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 - j10} = 15 + j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 + j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and absorbing 2500 magnetizing vars.

$Q_{j10} = |\mathbf{I}_g|^2(10) = 250(10) = 2500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 2500 magnetizing vars. The load continues to generate 5000 magnetizing vars.

$\sum Q_\mathrm{gen} = 5000\,\mathrm{VAR} = \sum Q_\mathrm{abs}$

P 10.13 $Z_\mathrm{f} = -j10{,}000 \| 20{,}000 = 4000 - j8000\,\Omega$

$$Z_\mathrm{i} = 2000 - j2000\,\Omega$$

$$\therefore \quad \frac{Z_\mathrm{f}}{Z_\mathrm{i}} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_\mathrm{f}}{Z_\mathrm{i}}\mathbf{V}_g; \qquad \mathbf{V}_g = 1\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_o = (3 - j1)(1) = 3 - j1 = 3.16\underline{/-18.43^\circ}\,\mathrm{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5 \times 10^{-3} = 5\,\mathrm{mW}$$

P 10.14 **[a]** $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\mathrm{W}$

$$-\frac{1}{\omega C} = \frac{-9 \times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\mathrm{VAR}$$

$$p_{\max} = P + \sqrt{P^2 + Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160\,\mathrm{W(del)}$$

[b] $p_{\min} = 60 - \sqrt{60^2 + 80^2} = -40\,\mathrm{W(abs)}$

[c] $P = 60\,\mathrm{W}$ from (a)

[d] $Q = -80\,\mathrm{VAR}$ from (a)

[e] generate, because $Q < 0$

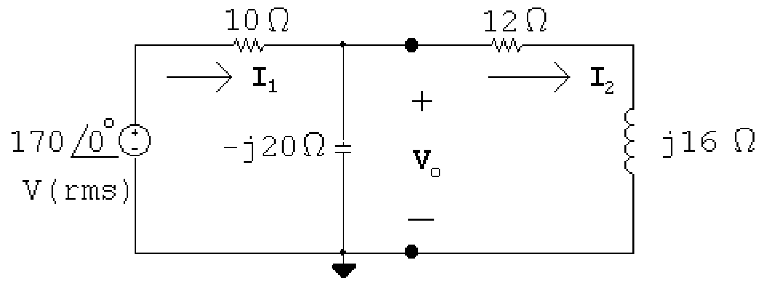
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\mathrm{A}$$

$\therefore$ pf $= \cos(0 - 53.13^\circ) = 0.6$ leading

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 10.15 **[a]**



The mesh equations are:

$(10 - j20)\mathbf{I}_1 + (j20)\mathbf{I}_2 = 170$

$(j20)\mathbf{I}_1 + (12 - j4)\mathbf{I}_2 = 0$

Solving,

$\mathbf{I}_1 = 4 + j1\,\text{A}; \qquad \mathbf{I}_2 = 3.5 - j5.5\,\text{A}$

$S = -\mathbf{V}_g\mathbf{I}_1^* = -(170)(4 - j1) = -680 + j170\,\text{VA}$

[b] Source is delivering 680 W.

[c] Source is absorbing 170 magnetizing VAR.

[d] $P_{10\Omega} = (\sqrt{17})^2(10) = 170\,\text{W}$

$P_{12\Omega} = (\sqrt{42.5})^2(12) = 510\,\text{W} \qquad (\mathbf{I}_1 - \mathbf{I}_2) = 0.5 + j6.5\text{ A}$

$Q_{-j20\Omega} = (\sqrt{42.5})^2(20) = -850\,\text{VAR} \qquad |\mathbf{I}_1 - \mathbf{I}_2| = \sqrt{42.5}$

$Q_{j16\Omega} = (\sqrt{42.5})^2(16) = 680\,\text{VAR}$

[e] $\sum P_{\text{del}} = 680\,\text{W}$

$\sum P_{\text{diss}} = 170 + 510 = 680\,\text{W}$

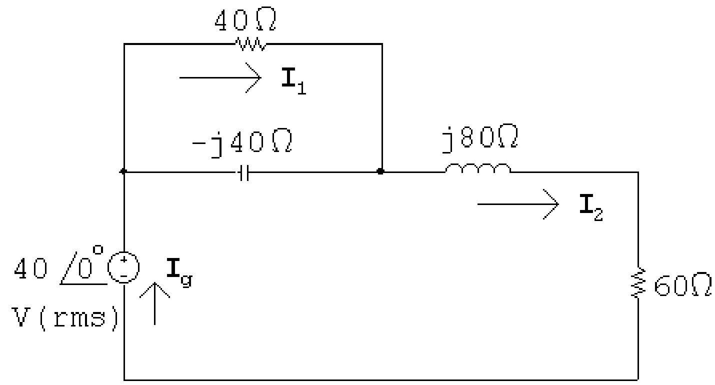
$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 680\,\text{W}$

[f] $\sum Q_{\text{abs}} = 170 + 680 = 850\,\text{VAR}$

$\sum Q_{\text{dev}} = 850\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 850$

P 10.16 **[a]** $\frac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$



$$Z_{\text{eq}} = 40\| - j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W(del)}; \qquad Q = 4.8\,\text{VAR(del)}$$

$$|S| = |S_g| = 8\,\text{VA}$$

[b] $\mathbf{I}_1 = \frac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR(del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR(abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.17 **[a]** $Z_1 = 240 + j70 = 250/16.26^\circ\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.80 \text{ leading}$$

$$\text{rf} = \sin(-36.87^\circ) = -0.60$$

$$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

$$\text{rf} = \sin(-53.13^\circ) = -0.8$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.18 **[a]** $S_1 = 16 + j18\,\text{kVA}; \qquad S_2 = 6 - j8\,\text{kVA}; \qquad S_3 = 8 + j0\,\text{kVA}$

$$S_{\text{T}} = S_1 + S_2 + S_3 = 30 + j10\,\text{kVA}$$

$$250\mathbf{I}^* = (30 + j10) \times 10^3; \qquad \therefore \quad \mathbf{I} = 120 - j40\,\text{A}$$

$$Z = \frac{250}{120 - j40} = 1.875 + j0.625\,\Omega = 1.98\underline{/18.43^\circ}\,\Omega$$

[b] $\text{pf} = \cos(18.43^\circ) = 0.9487$ lagging

P 10.19 **[a]** From the solution to Problem 10.18 we have

$$\mathbf{I}_{\text{L}} = 120 - j40\,\text{A(rms)}$$

$$\therefore \quad \mathbf{V}_s = 250\underline{/0^\circ} + (120 - j40)(0.01 + j0.08) = 254.4 + j9.2$$
$$= 254.57\underline{/2.07^\circ}\,\text{V(rms)}$$

[b] $|\mathbf{I}_{\text{L}}| = \sqrt{16{,}000}$

$$P_\ell = (16{,}000)(0.01) = 160\,\text{W} \qquad Q_\ell = (16{,}000)(0.08) = 1280\,\text{VAR}$$

[c] $P_s = 30{,}000 + 160 = 30.16\,\text{kW} \qquad Q_s = 10{,}000 + 1280 = 11.28\,\text{kVAR}$

[d] $\eta = \dfrac{30}{30.16}(100) = 99.47\%$

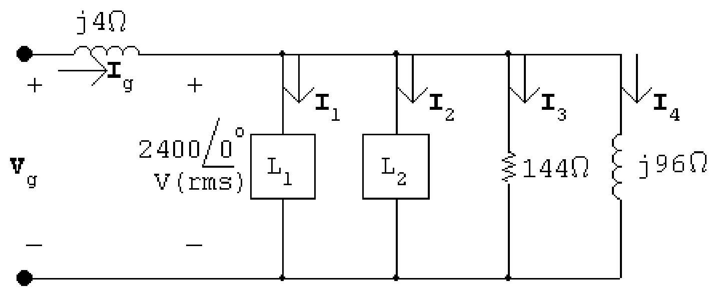
P 10.20 $S_{\rm T} = 4500 - j\dfrac{4500}{0.96}(0.28) = 4500 - j1312.5\,\text{VA}$

$$S_1 = \frac{2700}{0.8}(0.8 + j0.6) = 2700 + j2025\,\text{VA}$$

$$S_2 = S_{\rm T} - S_1 = 1800 - j3337.5 = 3791.95\underline{/-61.66^\circ}\,\text{VA}$$

$$\text{pf} = \cos(-61.66^\circ) = 0.4747 \text{ leading}$$

P 10.21



$$2400\mathbf{I}_1^* = 60{,}000 + j40{,}000$$

$$\mathbf{I}_1^* = 25 + j16.67; \qquad \therefore \quad \mathbf{I}_1 = 25 - j16.67\,\text{A(rms)}$$

$$2400\mathbf{I}_2^* = 20{,}000 - j10{,}000$$

$$\mathbf{I}_2^* = 8.33 - j4,167; \qquad \therefore \quad \mathbf{I}_2 = 8.33 + j4.167\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\underline{/0^\circ}}{144} = 16.67 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{2400\underline{/0^\circ}}{j96} = 0 - j25\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 50 - j37.5\,\text{A}$$

$$\mathbf{V}_g = 2400 + (j4)(50 - j37.5) = 2550 + j200 = 2557.83\underline{/4.48^\circ}\,\text{V(rms)}$$

P 10.22 **[a]** $S_1 = 60{,}000 - j70{,}000\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_{\rm L}|^2}{Z_2^*} = \frac{(2500)^2}{24 - j7} = 240{,}000 + j70{,}000\,\text{VA}$$

$$S_1 + S_2 = 300{,}000\,\text{VA}$$

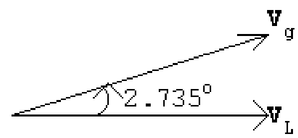
$$2500\mathbf{I}_{\rm L}^* = 300{,}000; \qquad \therefore \quad \mathbf{I}_{\rm L} = 120\underline{/0^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_g = \mathbf{V}_{\rm L} + \mathbf{I}_{\rm L}(0.1 + j1) = 2500 + (120)(0.1 + j1)$$
$$= 2512 + j120 = 2514.86\underline{/2.735^\circ}\,\text{V(rms)}$$

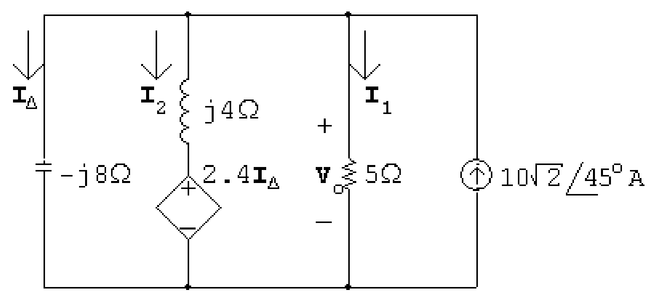
[b] $T = \frac{1}{f} = \frac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.735^\circ}{360^\circ} = \frac{t}{16.67\text{ ms}}; \qquad \therefore \quad t = 126.62\,\mu\text{s}$$

[c] $\mathbf{V}_\text{L}$ lags $\mathbf{V}_g$ by 2.735° or $126.62\,\mu$s



P 10.23 **[a]** From the solution to Problem 9.56 we have:



$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\text{ V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(j80)(10 - j10) = -400 - j400\,\text{VA}$$

Therefore, the independent current source is delivering 400 W and 400 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{5} = j16\,\text{A}$$

$$P_{5\Omega} = \frac{1}{2}(16)^2(5) = 640\,\text{W}$$

Therefore, the $8\,\Omega$ resistor is absorbing 640 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8} = -10\,\text{A}$$

$$Q_\text{cap} = \frac{1}{2}(10)^2(-8) = -400\,\text{VAR}$$

Therefore, the $-j8\,\Omega$ capacitor is developing 400 magnetizing vars.

$$2.4\mathbf{I}_\Delta = -24\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} = \frac{j80 + 24}{j4}$$

$$= 20 - j6\,\text{A} = 20.88\underline{/-16.7^\circ}\,\text{A}$$

$$Q_{j4} = \frac{1}{2}|\mathbf{I}_2|^2(4) = 872\,\text{VAR}$$

Therefore, the $j4\,\Omega$ inductor is absorbing 872 magnetizing vars.

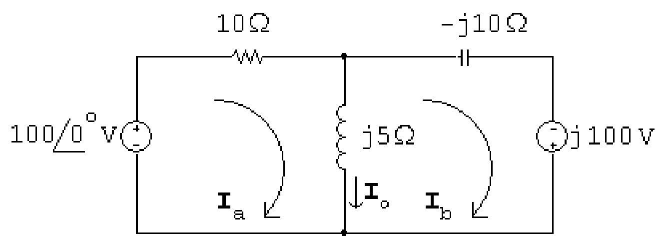
$$S_{\text{d.s.}} = \tfrac{1}{2}(2.4\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24)(20 + j6)$$
$$= -240 - j72\,\text{VA}$$

Thus the dependent source is delivering 240 W and 72 magnetizing vars.

[b] $\sum P_{\text{gen}} = 400 + 240 = 640\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 400 + 400 + 72 = 872\,\text{VAR} = \sum Q_{\text{abs}}$

P 10.24 **[a]** From the solution to Problem 9.58 we have



$\mathbf{I}_a = -j10\,\text{A}$; $\mathbf{I}_b = -20 + j10\,\text{A}$; $\mathbf{I}_o = 20 - j20$ A

$$S_{100V} = -\frac{1}{2}(100)\mathbf{I}_a^* = -50(j10) = -j500\,\text{VA}$$

Thus, the 100 V source is developing 500 magnetizing vars.

$$S_{j100V} = -\tfrac{1}{2}(j100)\mathbf{I}_b^* = -j50(-20 - j10)$$
$$= -500 + j1000\,\text{VA}$$

Thus, the $j100$ V source is developing 500 W and absorbing 1000 magnetizing vars.

$$P_{10\Omega} = \frac{1}{2}|\mathbf{I}_a|^2(10) = 500\,\text{W}$$

Thus the $10\,\Omega$ resistor is absorbing 500 W.

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_b|^2(-10) = -2500\,\text{VAR}$$

Thus the $-j10\,\Omega$ capacitor is developing 2500 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 2000\,\text{VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 2000 magnetizing vars.

[b] $\sum P_{\text{dev}} = 500\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 500 + 2500 = 3000\,\text{VAR}$

$$\sum Q_{\text{abs}} = 1000 + 2000 = 3000\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.25 **[a]** $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{124 + j93} = 2.4 - j1.8 = 3\underline{/-36.87^\circ}\,\text{A(rms)}$

$$P = (3)^2(4) = 36\,\text{W}$$

[b] $Y_{\text{L}} = \dfrac{1}{120 + j90} = 5.33 - j4\ \text{mS}$

$$\therefore \quad X_{\text{C}} = \frac{1}{-4 \times 10^{-3}} = -250\,\Omega$$

[c] $Z_{\text{L}} = \dfrac{1}{5.33 \times 10^{-3}} = 187.5\,\Omega$

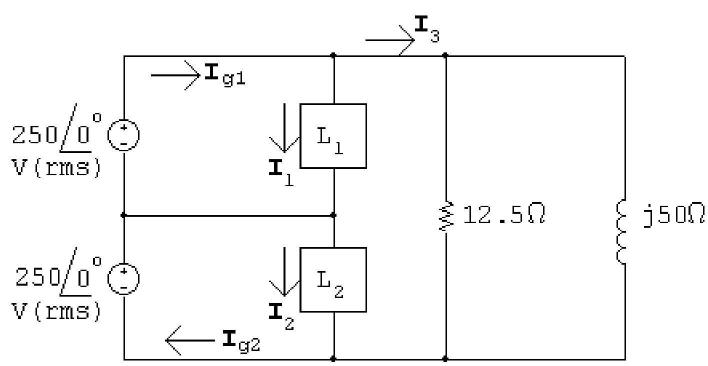
[d] $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{191.5 + j3} = 2.43\underline{/-0.9^\circ}\,\text{A}$

$$P = (2.43)^2(4) = 23.58\,\text{W}$$

[e] $\% = \dfrac{23.58}{36}(100) = 65.5\%$

Thus the power loss after the capacitor is added is 65.6% of the power loss before the capacitor is added.

P 10.26 **[a]**



$$250\mathbf{I}_1^* = 7500 + j2500; \qquad \therefore \quad \mathbf{I}_1 = 30 - j10\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 2800 - j9600; \qquad \therefore \quad \mathbf{I}_2 = 11.2 + j38.4\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{500}{12.5} + \frac{500}{j50} = 40 - j10\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 70 - j20\,\text{A}$$

$$S_{g1} = 250(70 + j20) = 17{,}500 + j5000\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 17.5 kW and 5000 magnetizing vars.

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 51.2 + j28.4\,\text{A(rms)}$

$S_{g2} = 250(51.2 - j28.4) = 12{,}800 - j7100\,\text{VA}$

Thus the $\mathbf{V}_{g2}$ source is delivering 12.8 kW and absorbing 7100 magnetizing vars.

[b] $\sum P_{\text{gen}} = 17.5 + 12.8 = 30.3\,\text{kW}$

$$\sum P_{\text{abs}} = 7500 + 2800 + \frac{(500)^2}{12.5} = 30.3\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 9600 + 5000 = 14.6\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 2500 + 7100 + \frac{(500)^2}{50} = 14.6\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.27 $S_1 = 1200 + 1196 = 2396 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{2396}{120} = 19.97\,\text{A}$$

$S_2 = 860 + 600 + 240 = 1700 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_2 = \frac{1700}{120} = 14.167\,\text{A}$$

$S_3 = 4474 + 12{,}200 = 16{,}674 + j0\,\text{VA}$

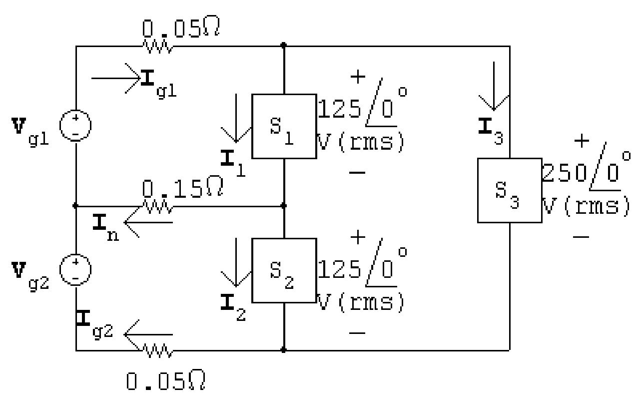
$$\therefore \quad \mathbf{I}_3 = \frac{16{,}674}{240} = 69.48\,\text{A}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 89.44\,\text{A}$

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 83.64\,\text{A}$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.28 **[a]**



$$\mathbf{I}_1 = \frac{4000 - j1000}{125} = 32 - j8\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{10{,}000 + j0}{250} = 40 + j0\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 72 - j8\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -8 + j8\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 80 - j16\,\text{A(rms)}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + 0.15\mathbf{I}_n = 127.4 + j0.8\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.15\mathbf{I}_n + 125 + 0.05\mathbf{I}_{g2} = 130.2 - j2\,\text{V(rms)}$$

$$S_{g1} = [(127.4 + j0.8)(72 + j8)] = [9166.4 + j1076.8]\,\text{VA}$$

$$S_{g2} = [(130.2 - j2)(80 + j16)] = [10{,}448 + j1923.2]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 262.4\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.15) = 19.2\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 332.8\,\text{W}$$

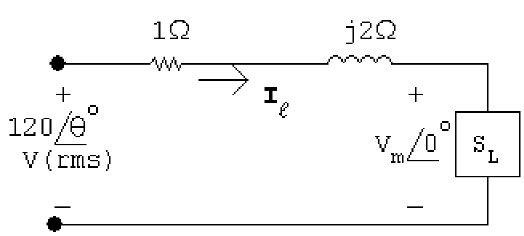
$$\sum P_{\text{dis}} = 262.4 + 19.2 + 332.8 + 4000 + 5000 + 10{,}000 = 19{,}614.4\,\text{W}$$

$$\sum P_{\text{dev}} = 9166.4 + 10{,}448 = 19{,}614.4\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 1000 + 2000 = 3000\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1076.8 + 1923.2 = 3000\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.29 **[a]** Let $\mathbf{V}_\text{L} = V_m\underline{/0^\circ}$:



$$S_\text{L} = 600(0.8 + j0.6) = 480 + j360\,\text{VA}$$

$$\mathbf{I}_\ell^* = \frac{480}{V_m} + j\frac{360}{V_m}; \qquad \mathbf{I}_\ell = \frac{480}{V_m} - j\frac{360}{V_m}$$

$$120\underline{/\theta} = V_m + \left(\frac{480}{V_m} - j\frac{360}{V_m}\right)(1 + j2)$$

$$120V_m\underline{/\theta} = V_m^2 + (480 - j360)(1 + j2) = V_m^2 + 1200 + j600$$

$$120V_m \cos\theta = V_m^2 + 1200; \qquad 120V_m \sin\theta = 600$$

$$(120)^2V_m^2 = (V_m^2 + 1200)^2 + 600^2$$

$$14{,}400V_m^2 = V_m^4 + 2400V_m^2 + 18 \times 10^5$$

or

$$V_m^4 - 12{,}000V_m^2 + 18 \times 10^5 = 0$$

Solving,

$V_m = 108.85$ V and $V_m = 12.326$ V

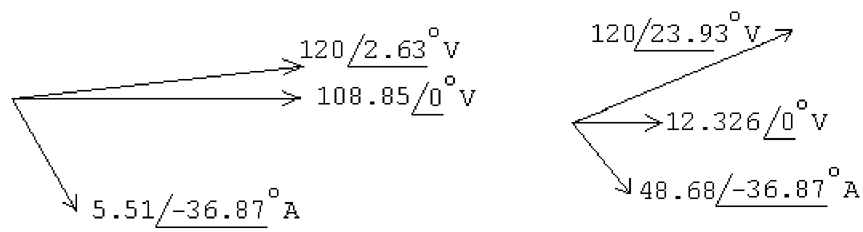
If $V_m = 108.85$ V:

$$\sin\theta = \frac{600}{(108.85)(120)} = 0.045935; \qquad \therefore \quad \theta = 2.63^\circ$$

If $V_m = 12.326$ V:

$$\sin\theta = \frac{600}{(12.326)(120)} = 0.405647; \qquad \therefore \quad \theta = 23.93^\circ$$

[b]



P 10.30 **[a]** $S_L = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65$ VA

$$125\mathbf{I}_L^* = (17{,}000 + j10{,}535.65); \quad \mathbf{I}_L^* = 136 + j84.29 \text{ A(rms)}$$

$$\therefore \quad \mathbf{I}_L = 136 - j84.29 \text{ A(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$

$$= 133.48\underline{/4.31^\circ} \text{ V(rms)}$$

$$|\mathbf{V}_s| = 133.48 \text{ V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256$ W

[c] $\frac{(125)^2}{X_C} = -10{,}535.65; \qquad X_C = -1.483\,\Omega$

$$-\frac{1}{\omega C} = -1.48; \qquad C = \frac{1}{(1.48)(120\pi)} = 1788.59\,\mu\text{F}$$

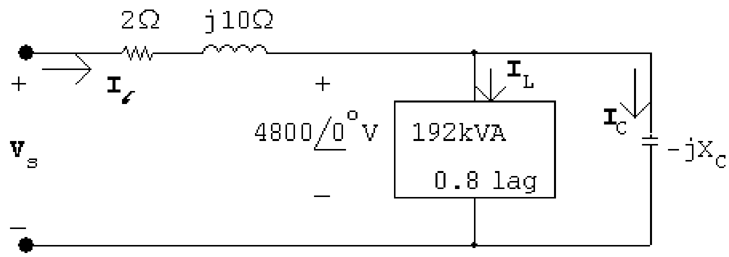
[d] $\mathbf{I}_\ell = 136 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$
$$= 126.83\underline{/4.92^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\text{V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\text{W}$

P 10.31



$$\mathbf{I}_L = \frac{153{,}600 - j115{,}200}{4800} = 32 - j24\,\text{A(rms)}$$

$$\mathbf{I}_C = \frac{4800}{-jX_C} = j\frac{4800}{X_C} = jI_C$$

$$\mathbf{I}_\ell = 32 - j24 + jI_C = 32 + j(I_C - 24)$$

$$\mathbf{V}_s = 4800 + (2 + j10)[32 + j(I_C - 24)]$$
$$= (5104 - 10I_C) + j(272 + 2I_C)$$

$$|\mathbf{V}_s|^2 = (5104 - 10I_C)^2 + (272 + 2I_C)^2 = (4800)^2$$

$$\therefore \quad 104I_C^2 - 100{,}992I_C + 3{,}084{,}800 = 0$$

Solving, $\quad \mathbf{I}_C = 31.57\,\text{A(rms)}; \quad \mathbf{I}_C = 939.51\,\text{A(rms)}$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_C = -\frac{4800}{31.57} = -152.04$$

$$\therefore \quad C = \frac{1}{(152.04)(120\pi)} = 17.45\,\mu\text{F}$$

P 10.32 $Z_L = |Z_L|\underline{/\theta^\circ} = |Z_L|\cos\theta^\circ + j|Z_L|\sin\theta^\circ$

Thus $|\mathbf{I}| = \dfrac{|\mathbf{V}_{Th}|}{\sqrt{(R_{Th} + |Z_L|\cos\theta)^2 + (X_{Th} + |Z_L|\sin\theta)^2}}$

Therefore $P = \dfrac{0.5|\mathbf{V}_{Th}|^2|Z_L|\cos\theta}{(R_{Th} + |Z_L|\cos\theta)^2 + (X_{Th} + |Z_L|\sin\theta)^2}$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_L|} = \frac{(0.5|\mathbf{V}_{Th}|^2\cos\theta)(D\cdot 1 - |Z_L|dD/d|Z_L|)}{D^2}$$

$$\frac{dD}{d|Z_L|} = 2(R_{Th} + |Z_L|\cos\theta)\cos\theta + 2(X_{Th} + |Z_L|\sin\theta)\sin\theta$$

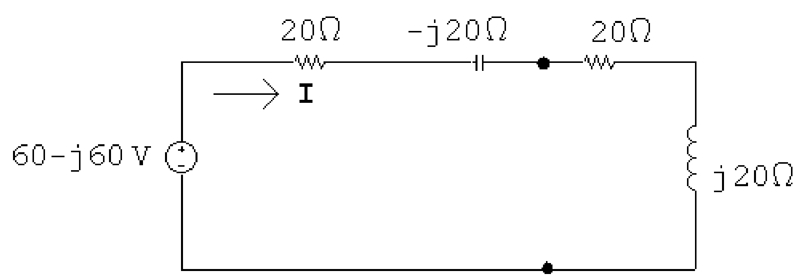
$$\frac{dP}{d|Z_L|} = 0 \quad \text{when} \quad D = |Z_L|\left(\frac{dD}{d|Z_L|}\right)$$

Substituting the expressions for D and $(dD/d|Z_L|)$ into this equation gives us the relationship $R_{Th}^2 + X_{Th}^2 = |Z_L|^2$ or $|Z_{Th}| = |Z_L|$.

P 10.33 **[a]** $Z_{Th} = j40\|40 - j40 = 20 - j20$

$\therefore \quad Z_L = Z_{Th}^* = 20 + j20\,\Omega$

[b] $\mathbf{V}_{Th} = \dfrac{40}{40 + j40}(120) = 60 - j60\,\text{V}$



$$\mathbf{I} = \frac{60 - j60}{40} = 1.5 - j1.5\,\text{A}$$

$$P_{load} = \frac{1}{2}|\mathbf{I}|^2(20) = 45\,\text{W}$$

P 10.34 **[a]** $\dfrac{115.2 + j33.6 - 240}{Z_{Th}} + \dfrac{115.2 + j33.6}{80 - j60} = 0$

$\therefore \quad Z_{Th} = 40 - j100\,\Omega$

$\therefore \quad Z_L = 40 + j100\,\Omega$

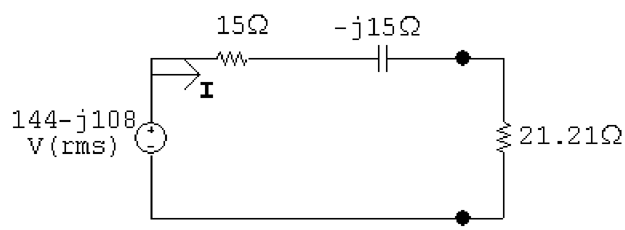
[b] $\mathbf{I} = \dfrac{240}{80} = 3\,\text{A(rms)}$

$P = (3)^2(40) = 360\,\text{W}$

P 10.35 **[a]** $Z_{\text{Th}} = [(3 + j4)\| - j8] + 7.32 - j17.24 = 15 - j15\,\Omega$

$\therefore \quad R = |Z_{\text{Th}}| = 21.21\,\Omega$

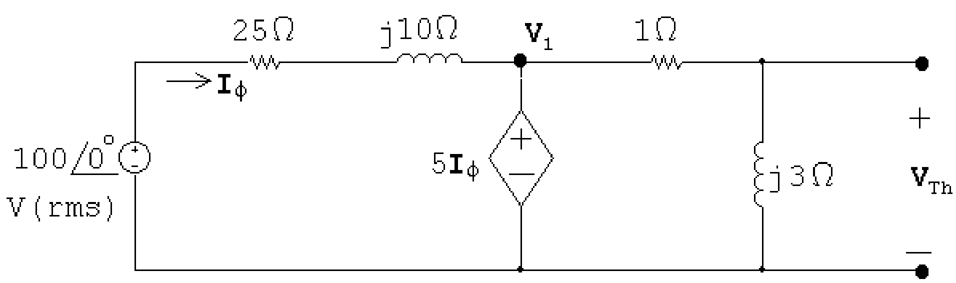
[b] $\mathbf{V}_{\text{Th}} = \dfrac{-j8}{3 - j4}(112.5) = 144 - j108\,\text{V(rms)}$



$$\mathbf{I} = \frac{144 - j108}{35.21 - j15} = 4.45 - j1.14$$

$$P = |\mathbf{I}|^2(21.21) = 447.35\,\text{W}$$

P 10.36 **[a]** Open circuit voltage:



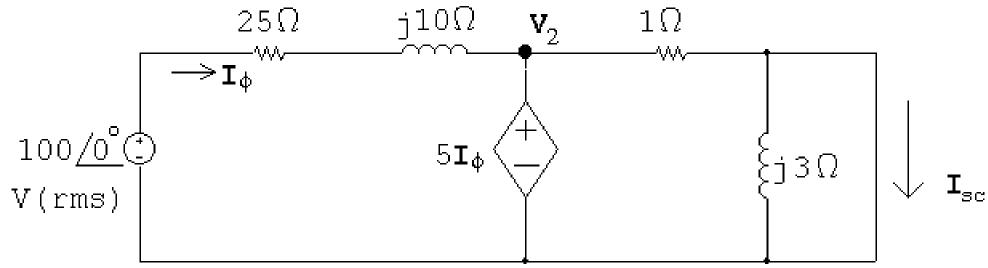
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j1\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = \frac{j3}{1 + j3}(5\mathbf{I}_\phi) = 15\underline{/0^\circ}\,\text{V}$$

Short circuit current:



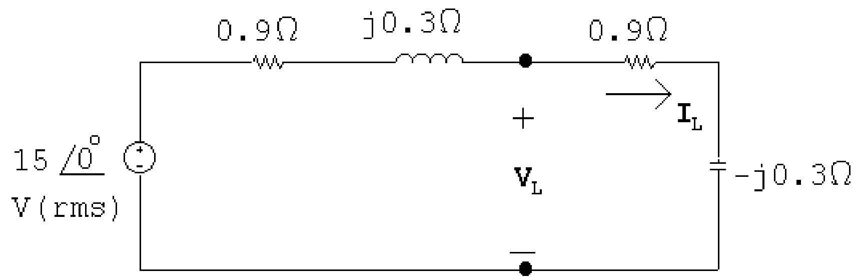
$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$\mathbf{I}_\phi = 3 - j1\,\text{A}$$

$$\mathbf{I}_{\rm sc} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\text{A}$$

$$Z_{\rm Th} = \frac{15}{15 - j5} = 0.9 + j0.3\,\Omega$$

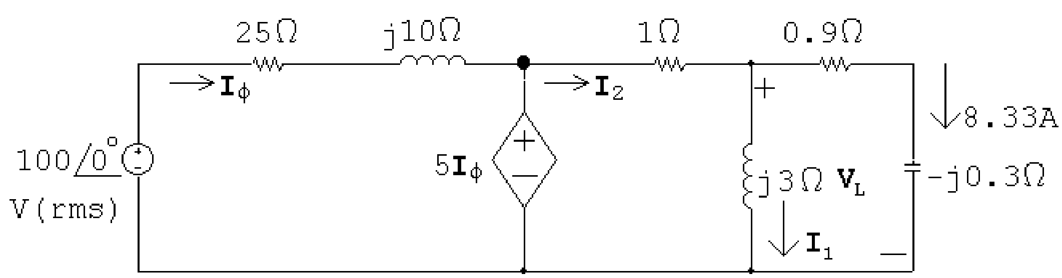
$$Z_L = Z_{\rm Th}^* = 0.9 - j0.3\,\Omega$$



$$\mathbf{I}_{\rm L} = \frac{15}{1.8} = 8.33\,\text{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\text{W}$$

[b] $\mathbf{V}_{\rm L} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\text{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\rm L}}{j3} = -0.833 - j2.5\,\text{A(rms)}$$

$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_\mathrm{L} = 7.5 - j2.5\,\mathrm{A(rms)}$

$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1\,\mathrm{A}$

$\mathbf{I}_{\mathrm{d.s.}} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\mathrm{A}$

$S_g = -100(3 + j1) = -300 - j100\,\mathrm{VA}$

$S_{\mathrm{d.s.}} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\mathrm{VA}$

$P_{\mathrm{dev}} = 300 + 75 = 375\,\mathrm{W}$

$\%\ \text{developed} = \frac{62.5}{375}(100) = 16.67\%$

Checks:

$P_{25\Omega} = (10)(25) = 250\,\mathrm{W}$

$P_{1\Omega} = (62.5)(1) = 62.5\,\mathrm{W}$

$P_{0.9\Omega} = 62.5\,\mathrm{W}$

$\sum P_{\mathrm{abs}} = 250 + 62.5 + 62.5 = 375\ \mathrm{W} = \sum P_{\mathrm{dev}}$

$Q_{j10} = (10)(10) = 100\,\mathrm{VAR}$

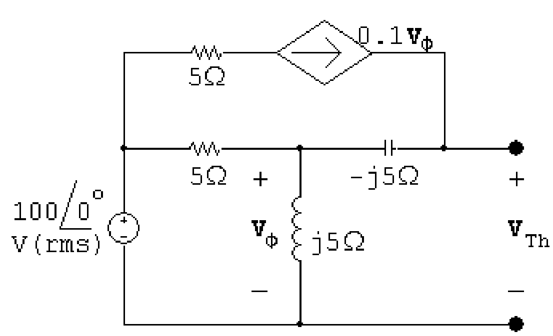
$Q_{j3} = (6.94)(3) = 20.83\,\mathrm{VAR}$

$Q_{-j0.3} = (69.4)(-0.3) = -20.83\,\mathrm{VAR}$

$Q_{\mathrm{source}} = -100\,\mathrm{VAR}$

$\sum Q = 100 + 20.83 - 20.83 - 100 = 0$

P 10.37 **[a]** Open circuit voltage:

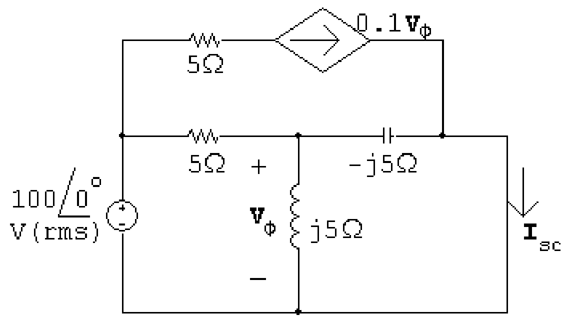


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\text{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\text{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

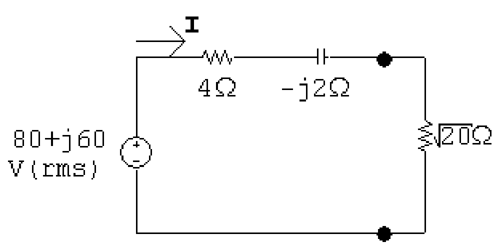
$$\therefore \quad \mathbf{V}_\phi = 100\,\text{V(rms)}$$

$$\mathbf{I}_{\text{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\text{A(rms)}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\text{Th}}| = 4.47\,\Omega$$

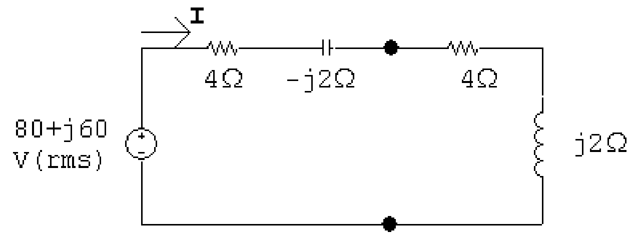
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\text{A(rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\text{W}$$

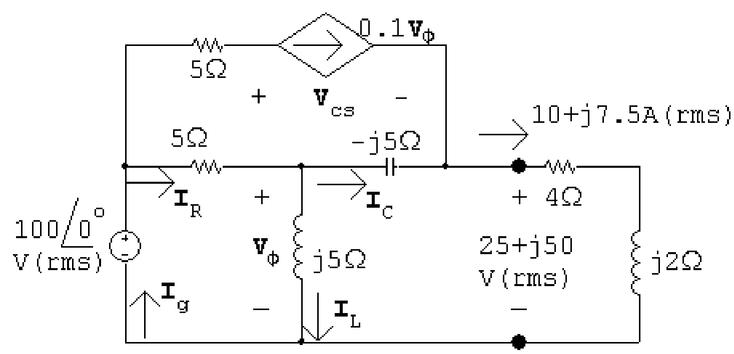
[c]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\text{A(rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\text{W}$$

[d]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi - (25 + j50)}{-j5} = 0$$

$$\mathbf{V}_\phi = 50 + j25\,\text{V(rms)}$$

$$0.1\mathbf{V}_\phi = 5 + j2.5$$

$$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$$

$$\mathbf{I}_C = 5 + j5\,\text{A(rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A(rms)}$$

$$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A(rms)}$$

$$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A(rms)}$$

$$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$$

$$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V(rms)}$$

$$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$$

Thus,

$$\sum P_{\text{dev}} = 1500$$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

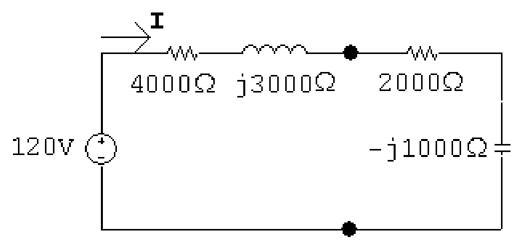
P 10.38 **[a]** First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\rm Th} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\rm Th} = \frac{12{,}000}{6000+12{,}000}(180) = 120\underline{/0^\circ}\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$



$$\mathbf{I} = \frac{120}{6000+j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

[b] Set $C_o = 0.1\,\mu$F so $-j/\omega C = -j2000\,\Omega$ $\quad j3000 - j2000 = j1000\,\Omega$
Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + 1000^2} = 4123.1\,\Omega$$

$$\therefore \quad R_o = 4000\,\Omega$$

[c] $\mathbf{I} = \dfrac{120}{8000+j1000} = 14.77 - j1.85\text{ mA}$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $\quad 443.1\,\text{mW} > 360\,\text{mW}$

[d] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[e] $R_o = 4000\,\Omega$; $\quad C_o = 66.67\,\text{nF}$

[f] Yes; $\quad 450\,\text{mW} > 443.1\,\text{mW}$

P 10.39 **[a]** Set $C_o = 0.1\,\mu$F, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\text{ mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

[b] Yes; $443.18\,\text{mW} > 360\,\text{mW}$

[c] Yes; $443.18\,\text{mW} < 450\,\text{mW}$

P 10.40 **[a]** $\dfrac{1}{\omega C} = 100\,\Omega; \qquad C = \dfrac{1}{(100)(120\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{I}_{\text{wo}} = \dfrac{13{,}800}{300} + \dfrac{13{,}800}{j100} = 46 - j138\,\text{A(rms)}$

$$\mathbf{V}_{\text{swo}} = 13{,}800 + (46 - j138)(1.5 + j12) = 15{,}525 + j345$$

$$= 15{,}528.83\underline{/1.27^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_{\text{w}} = \frac{13{,}800}{300} = 46\,\text{A(rms)}$$

$$\mathbf{V}_{\text{sw}} = 13{,}800 + 46(1.5 + j12) = 13{,}869 + j552 = 13{,}879.98\underline{/2.28^\circ}\,\text{V(rms)}$$

$$\%\text{ increase } = \left(\frac{15{,}528.82}{13{,}879.98} - 1\right)(100) = 11.88\%$$

[c] $P_{\ell\text{wo}} = |46 - j138|^2 1.5 = 31.74\,\text{kW}$

$$P_{\ell\text{w}} = 46^2(1.5) = 3174\,\text{W}$$

$$\%\text{ increase } = \left(\frac{31{,}740}{3174} - 1\right)(100) = 900\%$$

P 10.41 **[a]** $S_o = \text{ original load } = 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$$

$$\therefore\quad Q_{\text{added}} = 560 - 1200 = -640\,\text{kVAR}$$

[b] deliver

[c] $S_{\text{a}} = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-63.43) = 0.4472 \text{ leading}$$

[d] $\mathbf{I}_L^* = \dfrac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\text{A}$

$\mathbf{I}_L = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\text{A(rms)}$

$|\mathbf{I}_L| = 833.33\,\text{A(rms)}$

[e] $\mathbf{I}_L^* = \dfrac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

$\mathbf{I}_L = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\text{A(rms)}$

$|\mathbf{I}_L| = 833.33\,\text{A(rms)}$

P 10.42 **[a]** $P_{\text{before}} = P_{\text{after}} = (833.33)^2(0.05) = 34{,}722.22\,\text{W}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.05 + j0.4)$

$= 2633.33 + j241.67 = 2644.4\underline{/5.24^\circ}\,\text{V(rms)}$

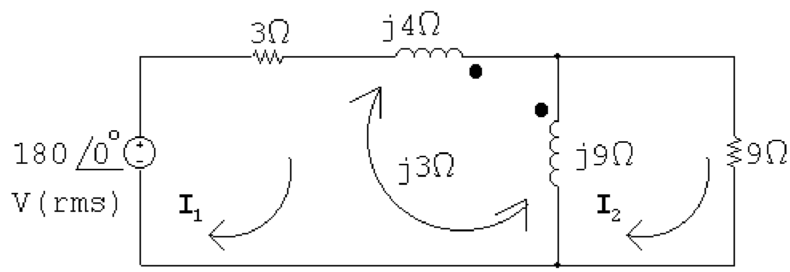
$|\mathbf{V}_s(\text{before})| = 2644.4\,\text{V(rms)}$

$\mathbf{V}_s(\text{after}) = 2400 + (800 + j233.33)(0.05 + j0.4)$

$= 2346.67 + j331.67 = 2369.99\underline{/8.04^\circ}\,\text{V(rms)}$

$|\mathbf{V}_s(\text{after})| = 2369.99\,\text{V(rms)}$

P 10.43 **[a]**



$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_2 - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1$

$0 = 9\mathbf{I}_2 + j9(\mathbf{I}_2 - \mathbf{I}_1) + j3\mathbf{I}_1$

Solving,

$\mathbf{I}_1 = 18 - j18\,\text{A(rms)}; \qquad \mathbf{I}_2 = 12\underline{/0^\circ}\,\text{A(rms)}$

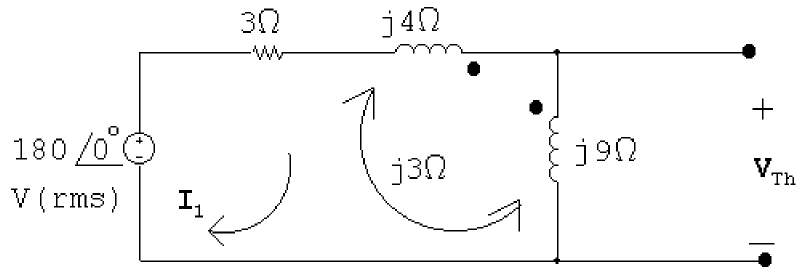
$\therefore \quad \mathbf{V}_o = (12)(9) = 108\underline{/0^\circ}\,\text{V(rms)}$

[b] $P = (12)^2(9) = 1296\,\text{W}$

[c] $S_g = -(180)(18 + j18) = -3240 - j3240\,\text{VA} \quad \therefore \quad P_g = -3240\,\text{W}$

$\%\ \text{delivered} = \dfrac{1296}{3240}(100) = 40\%$

P 10.44 **[a]** Open circuit voltage:

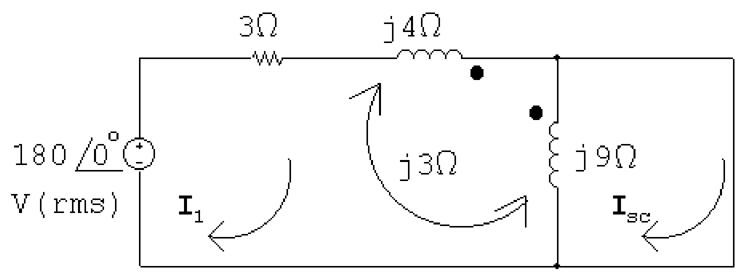


$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 - j3\mathbf{I}_1 + j9\mathbf{I}_1 - j3\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = \frac{180}{3 + j7} = 9.31 - j21.72\,\text{A(rms)}$$

$$\mathbf{V}_{\text{Th}} = j9\mathbf{I}_1 - j3\mathbf{I}_1 = j6\mathbf{I}_1 = 130.34 + j55.86\,\text{V} = 141.81\underline{/23.20^\circ}\ \text{V(rms)}$$

Short circuit current:



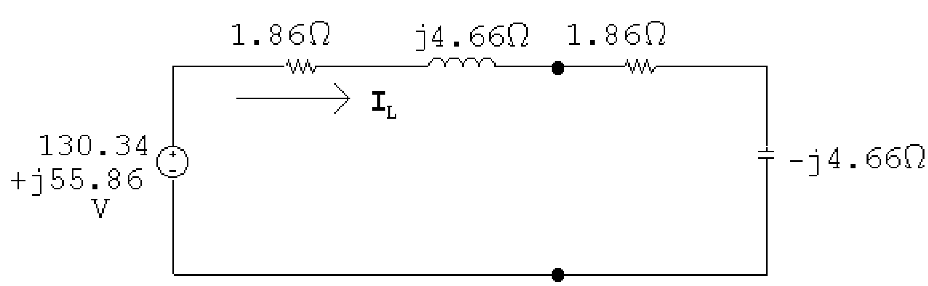
$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) - j3\mathbf{I}_1$$

$$0 = j9(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j3\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\text{sc}} = 20 - j20\,\text{A} \quad \mathbf{I}_1 = 30 - j20\ \text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{130.34 + j55.86}{20 - j20} = 1.86 + j4.66\,\Omega$$



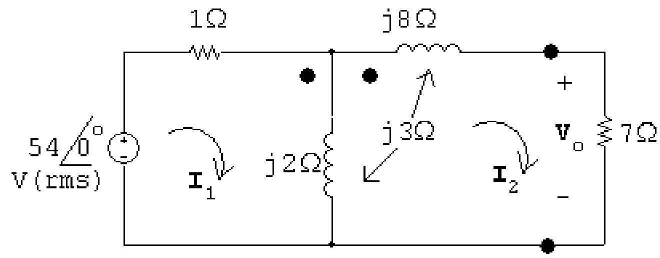
$$\mathbf{I}_{\text{L}} = \frac{130.34 + j55.86}{3.72} = 35 + j15 = 38.08\underline{/23.20^\circ}\,\text{A}$$

$$P_{\rm L} = (38.12)^2(1.86) = 2700\,\text{W}$$

[b] $\mathbf{I}_1 = \dfrac{Z_o + j9}{j6}\mathbf{I}_2 = \dfrac{1.86 - j4.66 + j9}{j6}(35 + j15) = 30\underline{/0^\circ}\,\text{A(rms)}$

$$P_{\rm dev} = (180)(30) = 5400\,\text{W}$$

P 10.45 **[a]**



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$\mathbf{I}_1 = 12 - j21\,\text{A(rms)};\qquad \mathbf{I}_2 = -3\,\text{A(rms)}$

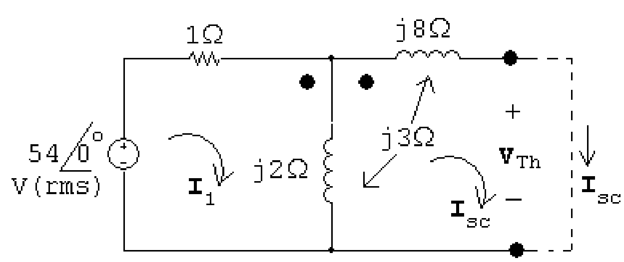
$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/180^\circ}\,\text{V(rms)}$

[b] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[c] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.46 **[a]**



Open circuit:

$$\mathbf{V}_{\rm Th} = -j3\mathbf{I}_1 + j_2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6\ \text{A}$$

$$\mathbf{V}_{\rm Th} = -21.6 - j10.8\,\text{V}$$

Short circuit:

$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{sc}) + j3\mathbf{I}_{sc}$

$0 = j2(\mathbf{I}_{sc} - \mathbf{I}_1) - j3\mathbf{I}_{sc} + j8\mathbf{I}_{sc} + j3(\mathbf{I}_1 - \mathbf{I}_{sc})$

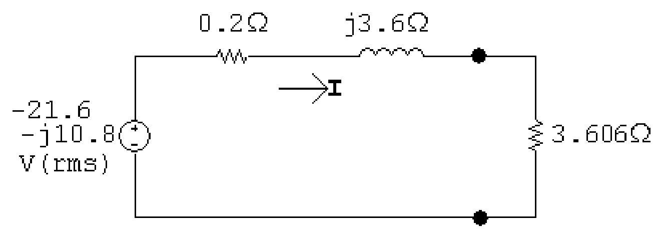
Solving,

$\mathbf{I}_{sc} = -3.32 + j5.82$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + 3.6j = 3.6\underline{/86.82^\circ}\,\Omega$$

$\therefore \quad R_L = |Z_{\text{Th}}| = 3.606\,\Omega$

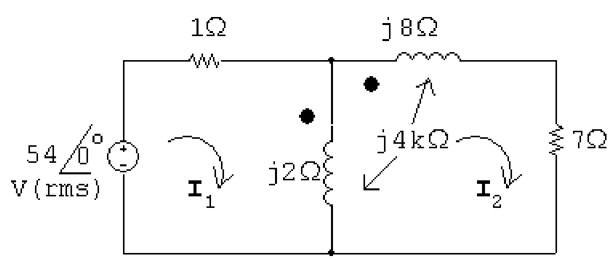
[b]



$$\mathbf{I} = \frac{-21.6 - j10.8}{3.806 + j3.6} = 4.610\underline{/163.2^\circ}\text{ A}$$

$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}$, which is greater than when $R_L = 7\,\Omega$

P 10.47 **[a]**



$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$

$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$

Place the equations in standard form:

$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$

$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = -\frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) + (4k - 2)^2}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[b] When $\mathbf{I}_2 = 0$

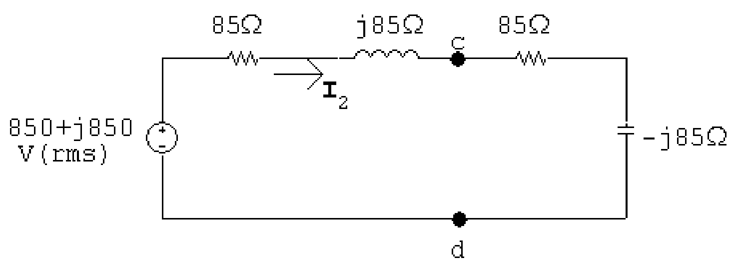
$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

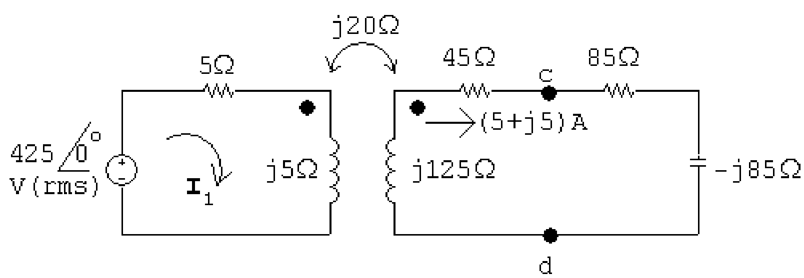
Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

P 10.48 **[a]** From Problem 9.67, $Z_{\text{Th}} = 85 + j85\,\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\,\text{V}$. Thus, for maximum power transfer, $Z_{\text{L}} = Z_{\text{Th}}^{*} = 85 - j85\,\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\text{A}$$



$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\ \text{loss in transformer} = \frac{18{,}062.5 - 13{,}812.5}{18{,}062.5}(100) = 23.53\%$$

P 10.49 **[a]** From Problem 9.70,

$Z_{\text{ab}} = 100 + j136.26$ so

$$\mathbf{I}_1 = \frac{50}{100 + j13.74 + 100 + 136.26} = \frac{50}{200 + j150} = 160 - j120\,\text{mA}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j270}{800 + j600}(0.16 - j0.12) = 51.84 + j15.12\,\text{mA}$$

$$\mathbf{V}_L = (300 + j100)(51.84 + j15.12)10^3 = 14.04 + j9.72\text{ V}$$

$$|\mathbf{V}_L| = 17.08\,\text{V}$$

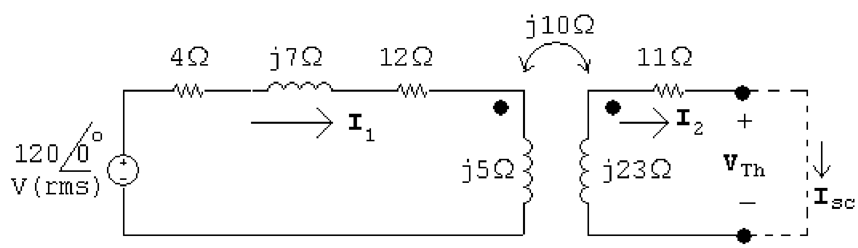
[b] $P_g(\text{ideal}) = 50(0.16) = 8\,\text{W}$

$$P_g(\text{practical}) = 8 - |\mathbf{I}_1|^2(100) = 4\,\text{W}$$

$$P_{\text{L}} = |\mathbf{I}_2|^2(300) = 874.8\text{ mW}$$

$$\%\text{ delivered } = \frac{0.8748}{4}(100) = 21.87\%$$

P 10.50 **[a]**



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.4\underline{/0^\circ}\text{ A}$$

$$Z_{\text{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore\quad Z_{\text{L}} = Z^*_{\text{Th}} = 15 - j20\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)} = 2.0\underline{/53.13^\circ}\text{ A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(15) = 60\,\text{W}$$

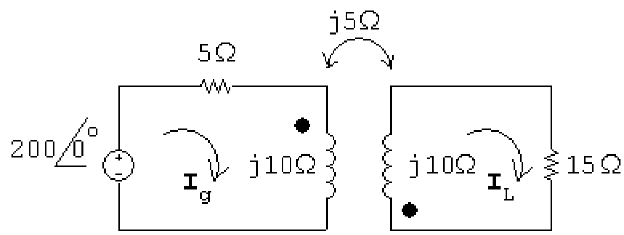
[b] $\mathbf{I}_1 = \dfrac{Z_{22}\mathbf{I}_2}{j\omega M} = \dfrac{26 + j3}{j10}(1.2 + j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A)rms)}$

$$P_{\text{transformer}} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\text{ delivered } = \frac{60}{432.8}(100) = 13.86\%$$

P 10.51 **[a]** $j\omega L_1 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_\text{L}$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_\text{L}$$

Solving,

$$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_\text{L} = -5\,\text{A}$$

Thus,

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_\text{L} = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{0.5}{\sqrt{1}} = 0.5$

[c] When $t = 50\pi\,\mu\text{s}$:

$$10{,}000t = (10{,}000)(50\pi) \times 10^{-6} = 0.5\pi \text{ rad } = 90^\circ$$

$$i_g(50\pi\,\mu\text{s}) = 18.03\cos(90^\circ - 56.31^\circ) = 15\,\text{A}$$

$$i_\text{L}(50\pi\,\mu\text{s}) = 5\cos(90^\circ - 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = \frac{1}{2}(10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\,\mu\text{s}$:

$$10{,}000t = (10^4)(100\pi) \times 10^{-6} = \pi = 180^\circ$$

$$i_g(100\pi\,\mu\text{s}) = 18.03\cos(180 - 56.31^\circ) = -10\,\text{A}$$

$$i_\text{L}(100\pi\,\mu\text{s}) = 5\cos(180 - 180^\circ) = 5\,\text{A}$$

$$w = \frac{1}{2}(10^{-3})(10)^2 + \frac{1}{2}(10^{-3})(5)^2 + 0.5 \times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

[d] From (a), $I_m = 5$ A,

$$\therefore \quad P = \frac{1}{2}(5)^2(15) = 187.5\,\text{W}$$

[e] Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{200}{5 + j10}(-j5) = -80 - j40\,\text{V}$$

Short circuit:

$$200 = (5 + j10)\mathbf{I}_1 + j5\mathbf{I}_{\text{sc}}$$

$$0 = j10\mathbf{I}_{\text{sc}} + j5\mathbf{I}_1$$

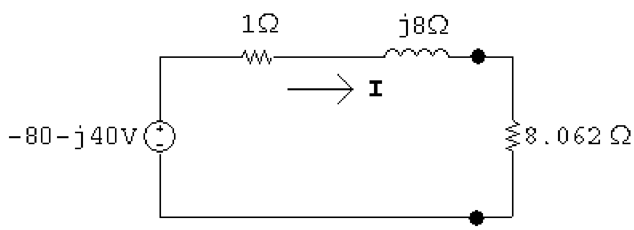
Solving,

$$\mathbf{I}_{\text{sc}} = -11.094\underline{/123.69^\circ}\ \text{A}; \quad \mathbf{I}_1 = 22.188\underline{/-56.31^\circ}\ \text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-80 - j40}{11.094\underline{/123.69^\circ}} = 1 + j8\,\Omega$$

$$\therefore \quad R_{\text{L}} = 8.962\,\Omega$$

[f]



$$\mathbf{I} = \frac{-80 - j40}{9.062 + j8} = 7.399\underline{/165.13^\circ}\ \text{A}$$

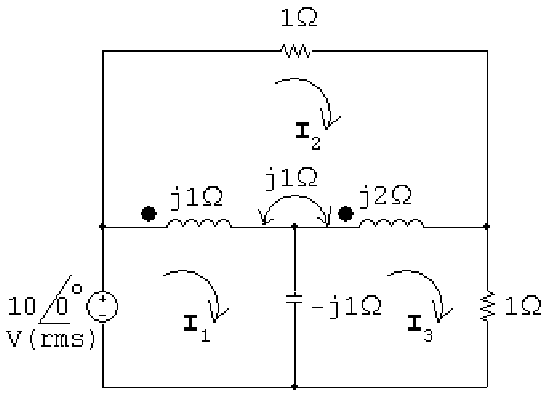
$$P = \frac{1}{2}(7.399)^2(8.062) = 220.70\ \text{W}$$

[g] $Z_{\text{L}} = Z_{\text{Th}}^* = 1 - j8\,\Omega$

[h] $\mathbf{I} = \dfrac{-80 - j40}{2} = 44.72\underline{/-153.43^\circ}\ \text{A}$

$$P = \frac{1}{2}(44.72)^2(1) = 1000\,\text{W}$$

P 10.52 **[a]**



$$10 = j1(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_3 - j1(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) + j1(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

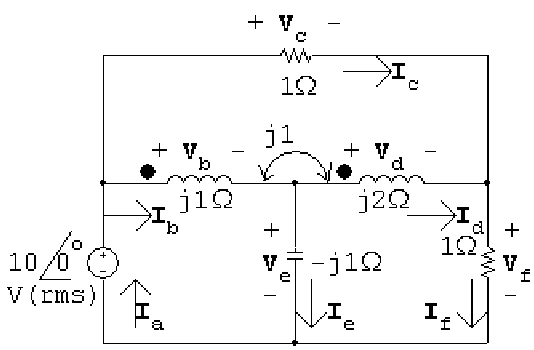
$\mathbf{I}_1 = 6.25 + j7.5\,\text{A(rms)}$; $\mathbf{I}_2 = 5 + j2.5\,\text{A(rms)}$; $\mathbf{I}_3 = 5 - j2.5\,\text{A(rms)}$

$\mathbf{I}_\text{a} = \mathbf{I}_1 = 6.25 + j7.5\,\text{A}$ $\quad\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25 + j5\,\text{A}$

$\mathbf{I}_\text{c} = \mathbf{I}_2 = 5 + j2.5\,\text{A}$ $\quad\mathbf{I}_\text{d} = \mathbf{I}_3 - \mathbf{I}_2 = -j5\,\text{A}$

$\mathbf{I}_\text{e} = \mathbf{I}_1 - \mathbf{I}_3 = 1.25 + j10\,\text{A}$ $\quad\mathbf{I}_\text{f} = \mathbf{I}_3 = 5 - j2.5\,\text{A}$

[b]



$\mathbf{V}_\text{a} = 10\,\text{V}$ $\quad\mathbf{V}_\text{b} = j1\mathbf{I}_\text{b} + j1\mathbf{I}_\text{d} = j1.25\,\text{V}$

$\mathbf{V}_\text{c} = 1\mathbf{I}_\text{c} = 5 + j2.5\,\text{V}$ $\quad\mathbf{V}_\text{d} = j2\mathbf{I}_\text{d} + j1\mathbf{I}_\text{b} = 5 + j1.25\,\text{V}$

$\mathbf{V}_\text{e} = -j1\mathbf{I}_\text{e} = 10 - j1.25\,\text{V}$ $\quad\mathbf{V}_\text{f} = 1\mathbf{I}_\text{f} = 5 - j2.5\,\text{V}$

$$S_\text{a} = -10\mathbf{I}_\text{a}^* = -62.5 + j75\,\text{VA}$$

$$S_\text{b} = \mathbf{V}_\text{b}\mathbf{I}_\text{b}^* = 6.25 + j1.5625\,\text{VA}$$

$S_c = \mathbf{V}_c\mathbf{I}_c^* = 31.25 + j0\,\text{VA}$

$S_d = \mathbf{V}_d\mathbf{I}_d^* = -6.25 + j25\,\text{VA}$

$S_e = \mathbf{V}_e\mathbf{I}_e^* = 0 - j101.5625\,\text{VA}$

$S_f = \mathbf{V}_f\mathbf{I}_f^* = 31.25\,\text{VA}$

[c] $\sum P_{dev} = 62.5\,\text{W}$

$\sum P_{abs} = 6.25 + 31.25 - 6.25 + 31.25 = 62.5\,\text{W}$

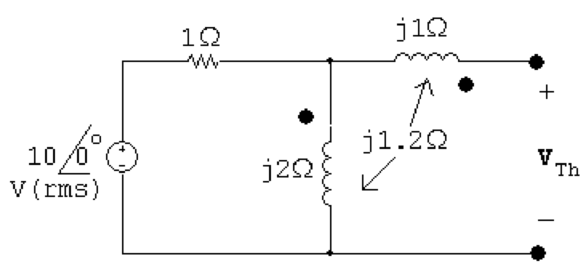
Note that the total power absorbed by the coupled coils is zero:
$6.25 - 6.25 = 0 = P_b + P_d$

[d] $\sum Q_{dev} = 101.5625\,\text{VAR}$
The capacitor is developing magnetizing vars.

$\sum Q_{abs} = 75 + 1.5625 + 25 = 101.5625\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_b + Q_d = 26.5625\,\text{VAR}$

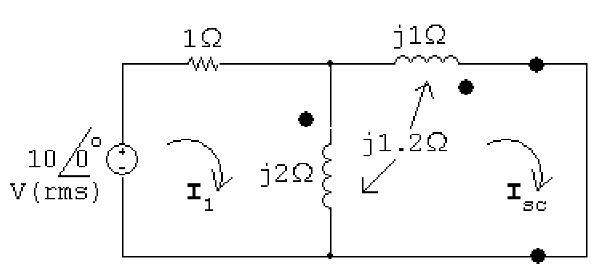
P 10.53 Open circuit voltage:



$$\mathbf{I}_1 = \frac{10\angle 0^\circ}{1 + j2} = 2 - j4\,\text{A}$$

$$\mathbf{V}_{Th} = j2\mathbf{I}_1 + j1.2\mathbf{I}_1 = j3.2\mathbf{I}_1 = 12.8 + j6.4 = 14.31\angle 26.57^\circ\ \text{V}$$

Short circuit current:



$$10\angle 0^\circ = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_{sc}$$

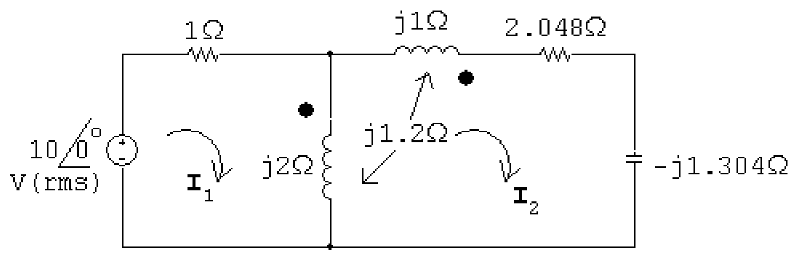
$0 = -j3.2\mathbf{I}_1 + j5.4\mathbf{I}_{\rm sc}$

Solving,

$\mathbf{I}_{\rm sc} = 5.89\underline{/-5.92^\circ}\,\text{A}$

$$Z_{\rm Th} = \frac{14.31\underline{/26.57^\circ}}{5.89\underline{/-5.92^\circ}} = 2.43\underline{/32.49^\circ} = 2.048 + j1.304\,\Omega$$

$$\therefore\quad \mathbf{I}_2 = \frac{14.31\underline{/26.57^\circ}}{4.096} = 3.49\underline{/26.57^\circ}\,\text{A}$$

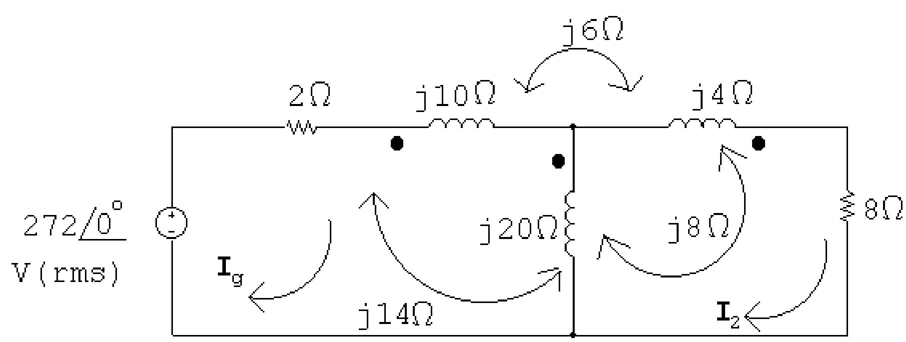


$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_2$

$$\therefore\qquad \mathbf{I}_1 = \frac{10 + j3.2\mathbf{I}_2}{1 + j2} = \frac{10 + j3.2(3.49\underline{/26.57^\circ})}{1 + j2} = 5\underline{/0^\circ}\,\text{A}$$

$$Z_g = \frac{10\underline{/0^\circ}}{5\underline{/0^\circ}} = 2 + j0 = 2\underline{/0^\circ}\,\Omega$$

P 10.54 **[a]**



$$\begin{aligned} 272\underline{/0^\circ} &= 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2 \\ &\quad + j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2) \\ 0 &= j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2 \\ &\quad + j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2 \end{aligned}$$

Solving,

$\mathbf{I}_g = 20 - j4\,\text{A(rms)}; \qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\text{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\text{W}$

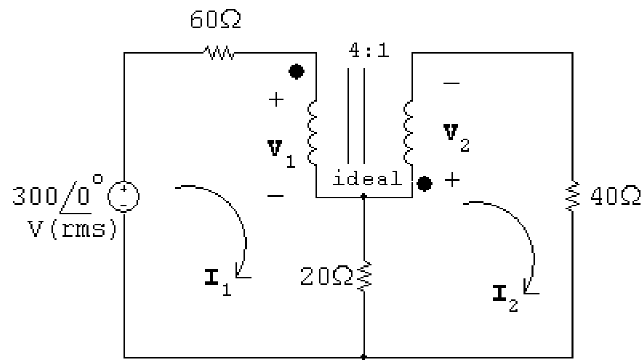
[b] $P_g(\text{developed}) = (272)(20) = 5440\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |\mathbf{I}_g|^2(2) = 832\,\text{W}$

$\sum P_{\text{diss}} = 832 + 4608 = 5440\,\text{W} = \sum P_{\text{dev}}$

P 10.55 **[a]**



$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$

$\mathbf{V}_2 = \dfrac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$

Solving,

$\mathbf{V}_1 = 260\,\text{V(rms)}; \qquad \mathbf{V}_2 = 65\,\text{V(rms)}$

$\mathbf{I}_1 = 0.25\,\text{A(rms)}; \qquad \mathbf{I}_2 = -1.0\,\text{A(rms)}$

$\mathbf{V}_{5\text{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\text{V(rms)}$

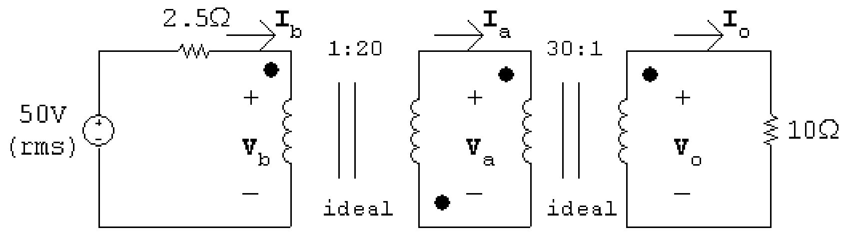
$\therefore \quad P = -(285)(5) = -1425\,\text{W}$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\text{A(rms)}$

$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\text{W}$

P 10.56



$$30\mathbf{V}_o = \mathbf{V}_a; \qquad \frac{\mathbf{I}_o}{30} = \mathbf{I}_a; \qquad \mathbf{V}_o = 10\mathbf{I}_o \qquad \text{therefore} \quad \frac{\mathbf{V}_a}{\mathbf{I}_a} = 9\,\text{k}\Omega$$

$$\frac{\mathbf{V}_b}{1} = \frac{-\mathbf{V}_a}{20}; \qquad \mathbf{I}_b = -20\mathbf{I}_a; \qquad \text{therefore} \quad \frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{9000}{400} = 22.5\,\Omega$$

Therefore $\mathbf{I}_b = [50/(2.5 + 22.5)] = 2$ A (rms); since the ideal transformers are lossless, $P_{10\Omega} = P_{22.5\Omega}$, and the power delivered to the $22.5\,\Omega$ resistor is $2^2(22.5)$ or 90 W.

P 10.57 **[a]** $\dfrac{\mathbf{V}_b}{\mathbf{I}_b} = \dfrac{a^2 10}{400} = 2.5\,\Omega; \qquad \text{therefore} \quad a^2 = 100, \qquad a = 10$

[b] $\mathbf{I}_b = \dfrac{50}{5} = 10\,\text{A}; \qquad P = (100)(2.5) = 250\,\text{W}$

P 10.58 **[a]** $Z_{\text{Th}} = 720 + j1500 + \left(\dfrac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

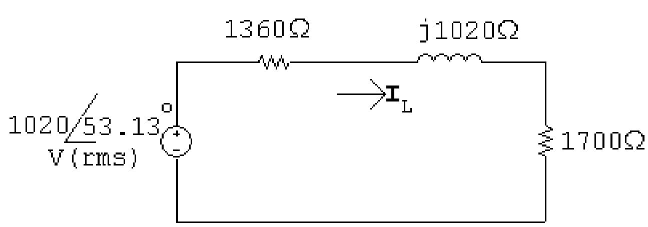
$$\therefore \quad Z_{ab} = 1700\,\Omega$$

$$Z_{ab} = \frac{Z_L}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

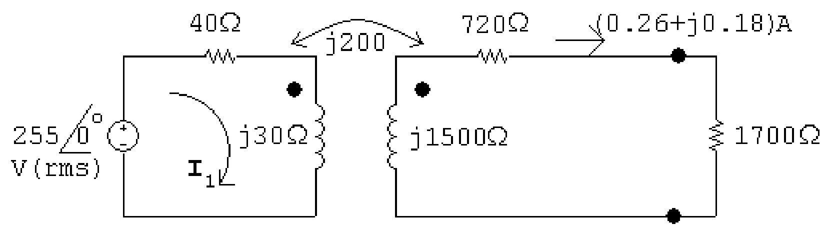


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$$P = |\mathbf{I}_L|^2(1700) = 170\,\text{W}$$

[c]



$$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

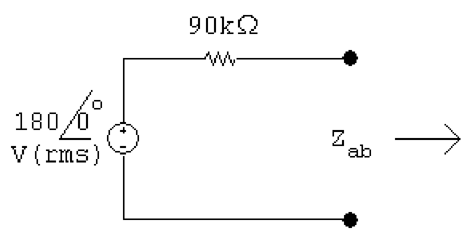
$$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$$

$$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$$

$$P_{\text{trans}} = 1053 - 170 = 883\,\text{W}$$

$$\%\ \text{transmitted} = \frac{883}{1053}(100) = 83.85\%$$

P 10.59 **[a]**



For maximum power transfer, $Z_{\text{ab}} = 90\,\text{k}\Omega$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$$

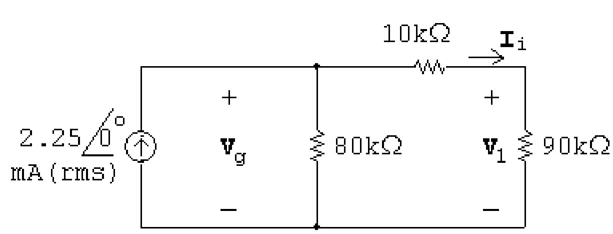
$$\therefore \quad \left(1 + \frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1 + \frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15 - 1 = 14$$

[b] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\dfrac{180}{180{,}000}\right)^2(90{,}000) = 90\,\text{mW}$

[c] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\dfrac{180}{180{,}000}\right) = 90\,\text{V}$

[d]



$$\mathbf{V}_g = (2.25 \times 10^{-3})(100{,}000\|80{,}000) = 100\,\text{V}$$

$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225\,\text{mW}$$

$$\%\ \text{delivered} = \frac{90}{225}(100) = 40\%$$

P 10.60 **[a]** $Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 (1 - j2) = 25 - j50\,\Omega$

$$\therefore \quad \mathbf{I}_1 = \frac{100\underline{/0^\circ}}{15 + j50 + 25 - j50} = 2.5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/0^\circ}\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{L}} = \mathbf{I}_1 + \mathbf{I}_2 = 12.5\underline{/0^\circ}\,\text{A(rms)}$$

$$P_{1\Omega} = (12.5)^2(1) = 156.25\,\text{W}$$

$$P_{15\Omega} = (2.5)^2(15) = 93.75\,\text{W}$$

[b] $P_g = -100(2.5\underline{/0^\circ}) = -250\,\text{W}$

$$\sum P_{\text{abs}} = 156.25 + 93.75 = 250\,\text{W} = \sum P_{\text{dev}}$$

P 10.61 **[a]** $25a_1^2 + 4a_2^2 = 500$

$$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$$

$$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$$

$$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$$

$$\therefore \qquad 100a_1^2 = 4a_2^2$$

$$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$$

$$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$$

[b] $\mathbf{I} = \frac{2000\underline{/0^\circ}}{500 + 500} = 2\underline{/0^\circ}\,\text{A(rms)}$

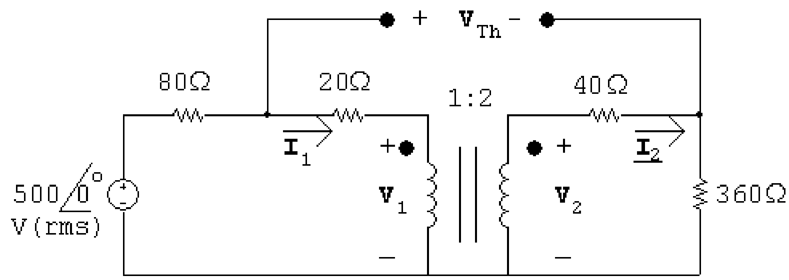
$$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$$

$$P_{25\Omega} = (16)(25) = 400\,\text{W}$$

[c] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$$\mathbf{V}_4 = (20)(4) = 80\underline{/0^\circ}\,\text{V(rms)}$$

P 10.62 **[a]** Open circuit voltage:



$500 = 100\mathbf{I}_1 + \mathbf{V}_1$

$\mathbf{V}_2 = 400\mathbf{I}_2$

$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \quad \therefore \quad \mathbf{V}_2 = 2\mathbf{V}_1$

$\mathbf{I}_1 = 2\mathbf{I}_2$

Substitute and solve:

$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \quad \therefore \quad \mathbf{V}_1 = 100\mathbf{I}_1$

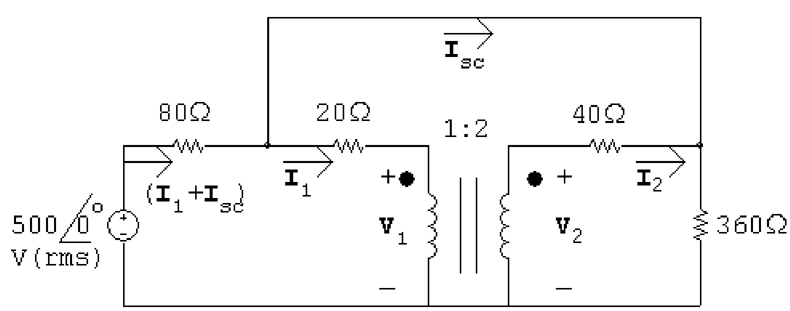
$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \quad \therefore \quad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$

$\therefore \quad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$

$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$

$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$

Short circuit current:



$500 = 80(\mathbf{I}_{\text{sc}} + \mathbf{I}_1) + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$

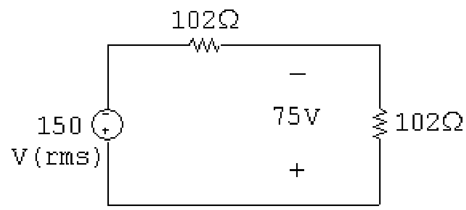
$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$

$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\text{sc}}) + 20\mathbf{I}_1 + \mathbf{V}_1$

Solving,

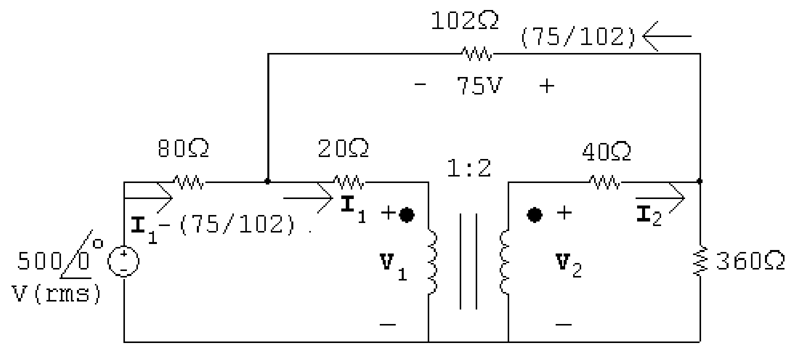
$\mathbf{I}_{sc} = -1.47\,\text{A}; \quad \mathbf{I}_1 = 4.41\ \text{A}; \quad \mathbf{V}_1 = 176.47\ \text{V}$

$R_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{-150}{-1.47} = 102\,\Omega$



$P = \frac{75^2}{102} = 55.15\,\text{W}$

[b]



$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$

$575 + \frac{6000}{102} + \frac{27{,}000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_1$

$\therefore \qquad \mathbf{I}_1 = 3.456\,\text{A}$

$P_{source} = (500)[3.456 - (75/102)] = 1360.29\,\text{W}$

$\%\ \text{delivered} = \frac{55.15}{1360.29}(100) = 4.05\%$

[c] $P_{80\Omega} = 80\left(\mathbf{I}_1 - \frac{75}{102}\right)^2 = 592.13\,\text{W}$

$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$

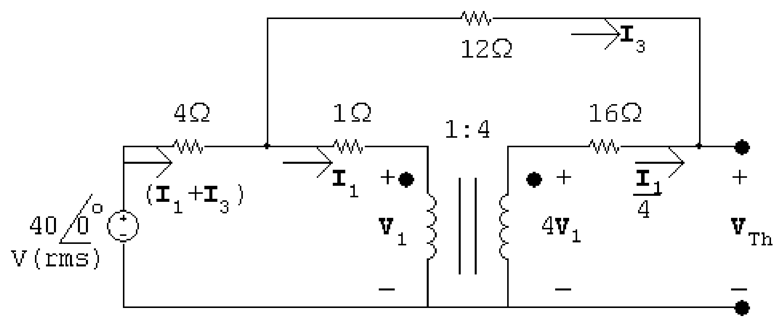
$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$

$P_{102\Omega} = \frac{75^2}{102} = 55.15\,\text{W}$

$P_{360\Omega} = 360\left(\mathbf{I}_2 - \frac{75}{102}\right)^2 = 354.73\,\text{W}$

$\sum P_{abs} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{dev}$

P 10.63 **[a]** Open circuit voltage:



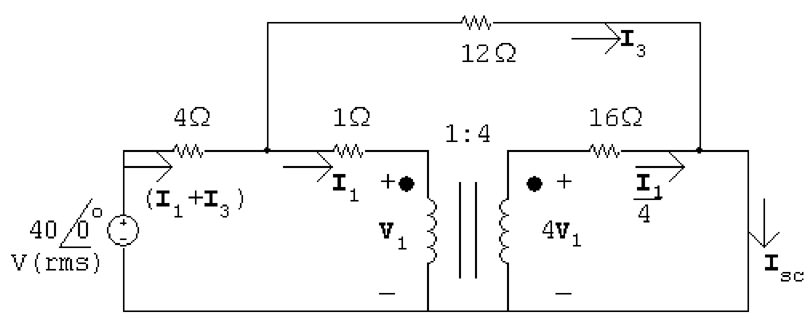
$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\rm Th}$$

$$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$$

Solving,

$$\mathbf{V}_{\rm Th} = 40\underline{/0^\circ}\,\text{V}$$

Short circuit current:



$$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore \quad \mathbf{V}_1 = \mathbf{I}_1$$

$$\therefore \quad 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$$

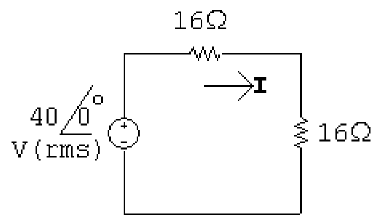
Also,

$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\rm sc} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$$

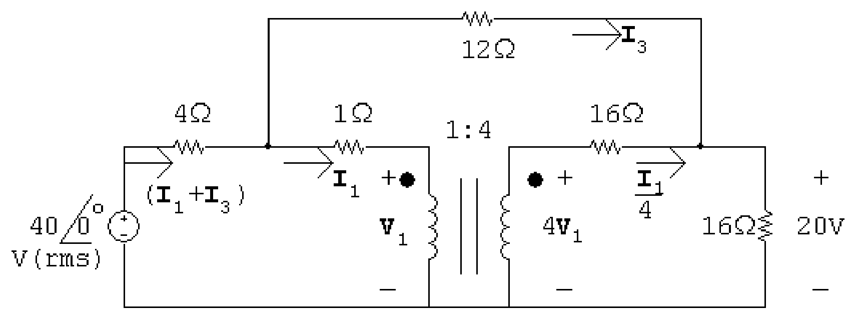
$$R_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{40}{2.5} = 16\,\Omega$$



$$\mathbf{I} = \frac{40\angle 0^\circ}{32} = 1.25\angle 0^\circ \text{ A(rms)}$$

$$P = (1.25)^2(16) = 25\text{ W}$$

[b]



$$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$$

$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore \quad \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore \quad \mathbf{I}_1 = 6\text{ A}; \qquad \mathbf{I}_3 = -0.25\text{ A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75\angle 0^\circ\text{ A}; \qquad \mathbf{V}_1 = 11\angle 0^\circ\text{ V}$$

$$P_{40V}(\text{developed}) = 40(5.75) = 230\text{ W}$$

$$\therefore \quad \%\text{ delivered } = \frac{25}{230}(100) = 10.87\%$$

[c] $P_{R_L} = 25\text{ W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\text{ W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25\text{ W}; \qquad P_{1\Omega} = (6)^2(1) = 36\text{ W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75\text{ W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\text{ W} = \sum P_{\text{dev}}$$

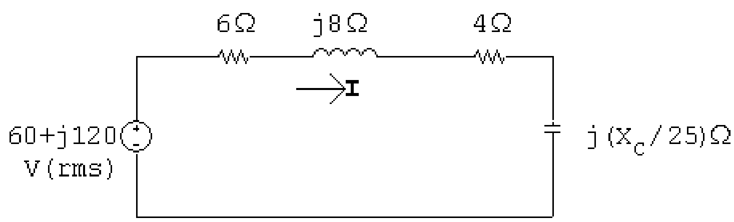
P 10.64 **[a]** Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = (15)(20\|j10) = 60 + j120\text{ V}$$

$$Z_{\text{Th}} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 + jX_C) = 4 + j\frac{X_C}{25}\,\Omega$$



Now maximize **I** by setting $(X_C/25) = -8\,\Omega$:

$$\therefore \quad C = \frac{1}{200(20\times 10^3)} = 0.25\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

$P = |\mathbf{I}|^2(4) = 720\,\text{W}$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore \quad R_o = 150\,\Omega$

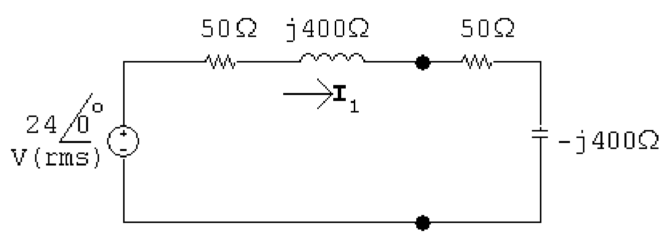
[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$P = |\mathbf{I}|^2(6) = 750\,\text{W}$

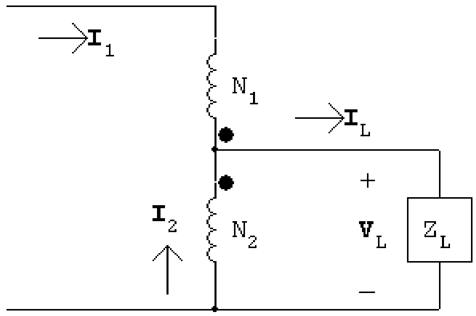
P 10.65 **[a]** $Z_{\text{ab}} = 50 - j400 = \left(1 - \dfrac{N_1}{N_2}\right)^2 Z_L = \left(1 - \dfrac{2800}{700}\right)^2 Z_L = 9Z_L$

$$\therefore \quad Z_L = \frac{1}{9}(50 - j400) = 5.556 - j44.444\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{24}{100} = 240\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -4\mathbf{I}_1 = 960\underline{/180^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = 720\underline{/180^\circ}\,\text{mA(rms)}$$

$$\mathbf{V}_\text{L} = (5.556 - j44.444)\mathbf{I}_\text{L} = -4 + j32 = 32.25\underline{/97.13^\circ}\,\text{V(rms)}$$

P 10.66 **[a]** Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_\text{med} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_\text{low} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_\text{high} = \frac{V^2}{R_1 \| R_2} = \frac{120^2}{28.8\|28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem sould not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 **[a]** $P_{\rm L} = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_{\rm L}}$

$$P_{\rm M} = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_{\rm L}}; \qquad R_1 = \frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2 V^2/P_{\rm L}}{\left(\frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}\right)\left(\frac{V^2}{P_{\rm M}}\right)} = \frac{P_{\rm M} P_{\rm L} P_{\rm M}}{P_{\rm L}(P_{\rm M} - P_{\rm L})}$$

$$P_{\rm H} = \frac{P_{\rm M}^2}{P_{\rm M} - P_{\rm L}}$$

[b] $P_{\rm H} = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for $P_{\rm M}$ as a function of $P_{\rm L}$ and $P_{\rm H}$. Thus

$$P_{\rm M} - P_{\rm L} = \frac{P_{\rm M}^2}{P_{\rm H}} \quad \text{or} \quad \frac{P_{\rm M}^2}{P_{\rm H}} - P_{\rm M} + P_{\rm L} = 0$$

$$P_{\rm M}^2 - P_{\rm M} P_{\rm H} + P_{\rm L} P_{\rm H} = 0$$

$$\therefore \quad P_{\rm M} = \frac{P_{\rm H}}{2} \pm \sqrt{\left(\frac{P_{\rm H}}{2}\right)^2 - P_{\rm L} P_{\rm H}}$$

$$= \frac{P_{\rm H}}{2} \pm P_{\rm H}\sqrt{\frac{1}{4} - \left(\frac{P_{\rm L}}{P_{\rm H}}\right)}$$

For the specified values of $P_{\rm L}$ and $P_{\rm H}$

$$P_{\rm M} = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$$\therefore \quad P_{M1} = 600\,\text{W}; \qquad P_{M2} = 400\,\text{W}$$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\text{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

CHECK: $P_{\rm H} = \frac{(120)^2(60)}{(36)(24)} = 1000\,\text{W}$

If $P_{M2} = 400\,\text{W}$

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$R_1 + R_2 = 60\,\Omega$ (as before)

$$R_1 = 24\,\Omega$$

CHECK: $P_{\rm H} = 1000\,\text{W}$

P 10.69 $R_1 + R_2 + R_3 = \frac{(120)^2}{600} = 24\,\Omega$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore \quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1 \| R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore \quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore \quad R_2 = 8\,\Omega; \qquad \therefore \quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \dfrac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

CHECK: $R_1 \| R_2 = 48.4\,\Omega$

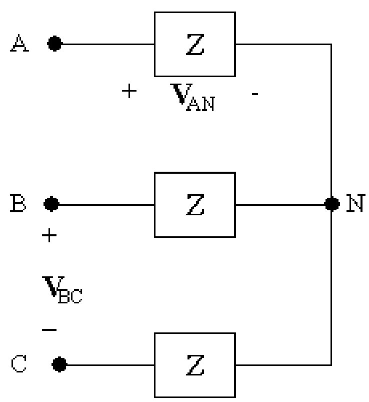
$$P_{\text{H}} = \frac{(220)^2}{48.4} = 1000\,\text{W}$$

11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{\rm AN}$ and wish to find $\mathbf{V}_{\rm BC}$. To do this, write a KVL equation to find $\mathbf{V}_{\rm AB}$, and use the known phase angle relationship between $\mathbf{V}_{\rm AB}$ and $\mathbf{V}_{\rm BC}$ to find $\mathbf{V}_{\rm BC}$.

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} + \mathbf{V}_{\rm NB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN}$$

Since $\mathbf{V}_{\rm AN}$, $\mathbf{V}_{\rm BN}$, and $\mathbf{V}_{\rm CN}$ form a balanced set, and $\mathbf{V}_{\rm AN} = 240\angle{-30^\circ}$V, and the phase sequence is positive,

$$\mathbf{V}_{\rm BN} = |\mathbf{V}_{\rm AN}|\angle{\angle\mathbf{V}_{\rm AN} - 120^\circ} = 240\angle{-30^\circ - 120^\circ} = 240\angle{-150^\circ}\text{ V}$$

Then,

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = (240\angle{-30^\circ}) - (240\angle{-150^\circ}) = 415.46\angle{0^\circ}\text{ V}$$

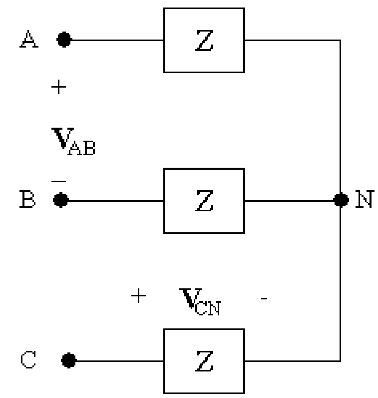
Since $\mathbf{V}_{\rm AB}$, $\mathbf{V}_{\rm BC}$, and $\mathbf{V}_{\rm CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{\rm BC}$ from $\mathbf{V}_{\rm AB}$:

$$\mathbf{V}_{\rm BC} = |\mathbf{V}_{\rm AB}|\angle{(\angle\mathbf{V}_{\rm AB} - 120^\circ)} = 415.69\angle{0^\circ - 120^\circ} = 415.69\angle{-120^\circ}\text{ V}$$

Thus,

$$\mathbf{V}_{\rm BC} = 415.69\angle{-120^\circ}\text{ V}$$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{\rm CN}$ and wish to find $\mathbf{V}_{\rm AB}$. To do this, write a KVL equation to find $\mathbf{V}_{\rm BC}$, and use the known phase angle relationship between $\mathbf{V}_{\rm AB}$ and $\mathbf{V}_{\rm BC}$ to find $\mathbf{V}_{\rm AB}$.

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} + \mathbf{V}_{\rm NC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN}$$

Since $\mathbf{V}_{\rm AN}$, $\mathbf{V}_{\rm BN}$, and $\mathbf{V}_{\rm CN}$ form a balanced set, and $\mathbf{V}_{\rm CN} = 450\angle{-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{\rm BN} = |\mathbf{V}_{\rm CN}|\angle{/\mathbf{V}_{\rm CN} - 120^\circ} = 450\angle{-23^\circ - 120^\circ} = 450\angle{-145^\circ}\ \text{V}$$

Then,

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = (450\angle{-145^\circ}) - (450\angle{-25^\circ}) = 779.42\angle{-175^\circ}\ \text{V}$$

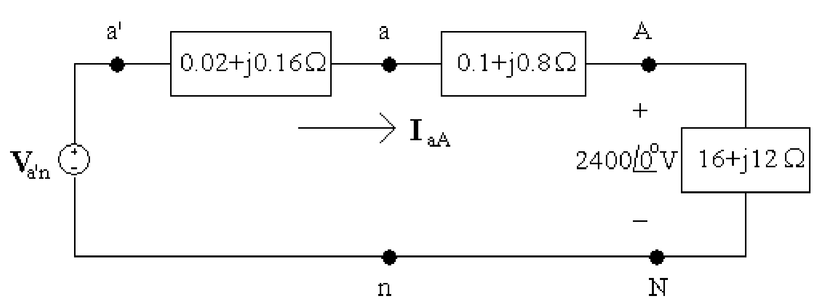
Since $\mathbf{V}_{\rm AB}$, $\mathbf{V}_{\rm BC}$, and $\mathbf{V}_{\rm CA}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\rm AB}$ from $\mathbf{V}_{\rm BC}$:

$$\mathbf{V}_{\rm AB} = |\mathbf{V}_{\rm BC}|\angle{/\mathbf{V}_{\rm BC} - 120^\circ} = 779.42\angle{-295^\circ}\ \text{V}$$

But we normally want phase angle values between $+180^\circ$ and -180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{\rm AB} = 779.42\angle{65^\circ}\ \text{V}$$

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{\rm aA}$, $\mathbf{I}_{\rm bB}$, and $\mathbf{I}_{\rm cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{\rm AN}$, has a phase angle of $0°$.

$$2400\underline{/0^\circ} = \mathbf{I}_{\rm aA}(16 + j12)$$

so

$$\mathbf{I}_{\rm aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{\rm bB}} = \underline{/\mathbf{I}_{\rm aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{\rm cC}} = \underline{/\mathbf{I}_{\rm aA}} - 120^\circ$$

so

$$\mathbf{I}_{\rm aA} = 120\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm bB} = 120\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm cC} = 120\underline{/-156.87^\circ}\,\text{A}$$

[b] The line voltages at the source are $\mathbf{V}_{\rm ab}$ $\mathbf{V}_{\rm bc}$, and $\mathbf{V}_{\rm ca}$. They form a balanced set. To find $\mathbf{V}_{\rm ab}$, use the a-phase circuit to find $\mathbf{V}_{\rm AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{\rm an} = \mathbf{V}_{\rm aA} + \mathbf{V}_{\rm AN} = (0.1 + j0.8)\mathbf{I}_{\rm aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\,\text{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\rm bc}} = \underline{/\mathbf{V}_{\rm ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\rm ca}} = \underline{/\mathbf{V}_{\rm ab}} - 120^\circ$$

so

$$\mathbf{V}_{\rm ab} = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm bc} = 4275.02\underline{/91.62^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm ca} = 4275.02\underline{/-148.38^\circ}\,\text{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{\text{a}'\text{n}} = \mathbf{V}_{\text{a}'\text{a}} + \mathbf{V}_{\text{an}} = (0.2 + j0.16)\mathbf{I}_{\text{aA}} + \mathbf{V}_{\text{an}}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\text{ V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\text{b}'\text{n}}} = \underline{/\mathbf{V}_{\text{a}'\text{n}}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\text{c}'\text{n}}} = \underline{/\mathbf{V}_{\text{a}'\text{n}}} - 120^\circ$$

so

$$\mathbf{V}_{\text{a}'\text{n}} = 2482.05\underline{/1.93^\circ}\text{ V}$$

$$\mathbf{V}_{\text{b}'\text{n}} = 2482.05\underline{/121.93^\circ}\text{ V}$$

$$\mathbf{V}_{\text{c}'\text{n}} = 2482.05\underline{/-118.07^\circ}\text{ V}$$

AP 11.4 $\mathbf{I}_{\text{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\text{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\text{ A}$

AP 11.5 $\mathbf{I}_{\text{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$

$$\mathbf{I}_{\text{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\text{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\text{ A}$$

AP 11.6 **[a]** $\mathbf{I}_{\text{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\text{ A}$

Therefore $\quad Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{\text{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\text{ A}$

Therefore $\quad Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7 $\mathbf{I}_\phi = \dfrac{110}{3.667} + \dfrac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\text{ A}$

Therefore $\quad |\mathbf{I}_{\text{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\text{ A}$

AP 11.8 **[a]** $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\text{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\,\text{VAR}$$

[b] $\text{pf} = \dfrac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$

AP 11.9 **[a]** $\mathbf{V}_{\text{AN}} = \left(\dfrac{2450}{\sqrt{3}}\right)\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{\text{AN}}\mathbf{I}^*_{\text{aA}} = S_\phi = 144 + j192\,\text{kVA}$

Therefore

$$\mathbf{I}^*_{\text{aA}} = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 101.8 - j135.7 = 169.67\underline{/-53.13^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{aA}}| = 169.67\,\text{A}$$

[b] $P = \dfrac{(2450)^2}{R}; \qquad \text{therefore} \quad R = \dfrac{(2450)^2}{144{,}000} = 41.68\,\Omega$

$$Q = \frac{(2450)^2}{X}; \qquad \text{therefore} \quad X = \frac{(2450)^2}{192{,}000} = 31.26\,\Omega$$

[c] $Z_\phi = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \dfrac{2450/\sqrt{3}}{169.67\underline{/-53.13^\circ}} = 8.34\underline{/53.13^\circ} = (5 + j6.67)\,\Omega$

$$\therefore \quad R = 5\,\Omega, \qquad X = 6.67\,\Omega$$

Problems

P 11.1 **[a]** First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 208\underline{/27^\circ}$; $\mathbf{V}_b = 208\underline{/147^\circ}$; $\mathbf{V}_c = 208\underline{/-93^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = 27^\circ - 27^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = 147^\circ - 27^\circ = 120^\circ$

$\underline{/\mathbf{V}'_c} = -93^\circ - 27^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

[b] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 4160\underline{/-18^\circ}$; $\mathbf{V}_b = 4160\underline{/-138^\circ}$; $\mathbf{V}_c = 4160\underline{/+102^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = -18^\circ + 18^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = -138^\circ + 18^\circ = -120^\circ$

$\underline{/\mathbf{V}'_c} = 102^\circ + 18^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

P 11.2 **[a]** $\mathbf{V}_a = 180\underline{/0^\circ}$ V

$\mathbf{V}_b = 180\underline{/-120^\circ}$ V

$\mathbf{V}_c = 180\underline{/-240^\circ} = 180\underline{/120^\circ}$ V

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 180\underline{/-90^\circ}$ V

$\mathbf{V}_b = 180\underline{/30^\circ}$ V

$\mathbf{V}_c = 180\underline{/-210^\circ}$ V $= 180\underline{/150^\circ}$ V

Balanced, negative phase sequence

[c] $\mathbf{V}_a = 400\underline{/-270^\circ}$ V $= 400\underline{/90^\circ}$ V

$\mathbf{V}_b = 400\underline{/120^\circ}$ V

$\mathbf{V}_c = 400\underline{/-30^\circ}$ V

Unbalanced, phase angle in b-phase

[d] $\mathbf{V}_\text{a} = 200\underline{/30^\circ}$ V

$\mathbf{V}_\text{b} = 201\underline{/150^\circ}$ V

$\mathbf{V}_\text{c} = 200\underline{/270^\circ}$ V $= 200\underline{/-90^\circ}$ V

Unbalanced, unequal amplitude in the b-phase

[e] $\mathbf{V}_\text{a} = 208\underline{/42^\circ}$ V

$\mathbf{V}_\text{b} = 208\underline{/-78^\circ}$ V

$\mathbf{V}_\text{c} = 208\underline{/-201^\circ}$ V $= 208\underline{/159^\circ}$ V

Unbalanced, phase angle in the c-phase

[f] Unbalanced; the frequencies of the waveforms are not the same for the positive sequence of Eq. 11.1

P 11.3 $\mathbf{V}_\text{a} = V_m\underline{/0^\circ} = V_m + j0$

$\mathbf{V}_\text{b} = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$

$\mathbf{V}_\text{c} = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$

$$\begin{aligned}\mathbf{V}_\text{a} + \mathbf{V}_\text{b} + \mathbf{V}_\text{c} &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

For the negative sequences of Eq. 11.2, $\mathbf{V}_\text{b}$ and $\mathbf{V}_\text{c}$ are interchanged, but the sum is still zero.

P 11.4 $\mathbf{I} = \dfrac{\mathbf{V}_\text{a} + \mathbf{V}_\text{b} + \mathbf{V}_\text{c}}{3(R_\text{W} + jX_\text{W})} = 0$

P 11.5 **[a]** $\mathbf{I}_\text{aA} = \dfrac{200}{25} = 8\underline{/0^\circ}$ A

$\mathbf{I}_\text{bB} = \dfrac{200\underline{/-120^\circ}}{30 - j40} = 4\underline{/-66.87^\circ}$ A

$\mathbf{I}_\text{cC} = \dfrac{200\underline{/120^\circ}}{80 + j60} = 2\underline{/83.13^\circ}$ A

The magnitudes are unequal and the phase angles are not 120° apart.

b] $\mathbf{I}_o = \mathbf{I}_\text{aA} + \mathbf{I}_\text{bB} + \mathbf{I}_\text{cC} = 9.96\underline{/-9.79^\circ}$ A

P 11.6 **[a]** $\mathbf{I}_{\text{aA}} = \dfrac{277\angle 0^\circ}{80 + j60} = 2.77\angle -36.87^\circ \text{ A}$

$$\mathbf{I}_{\text{bB}} = \frac{277\angle -120^\circ}{80 + j60} = 2.77\angle -156.87^\circ \text{ A}$$

$$\mathbf{I}_{\text{cC}} = \frac{277\angle 120^\circ}{80 + j60} = 2.77\angle 83.13^\circ \text{ A}$$

$$\mathbf{I}_o = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{bB}} + \mathbf{I}_{\text{cC}} = 0$$

[b] $\mathbf{V}_{\text{AN}} = (78 + j54)\mathbf{I}_{\text{aA}} = 262.79\angle -2.17^\circ \text{ V}$

[c] $\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}}$

$$\mathbf{V}_{\text{BN}} = (77 + j56)\mathbf{I}_{\text{bB}} = 263.73\angle -120.84^\circ \text{ V}$$

$$\mathbf{V}_{\text{AB}} = 262.79\angle -2.17^\circ - 263.73\angle -120.84^\circ = 452.89\angle 28.55^\circ \text{ V}$$

[d] Unbalanced — see conditions for a balanced circuit on p. 504 of the text!

P 11.7 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 40 + j30\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 20 + j15\Omega$$

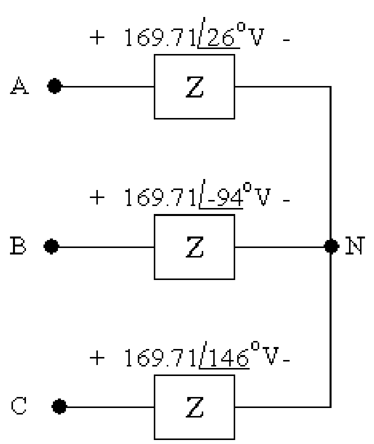
$$\frac{\mathbf{V}_N - 240}{60 + j80} + \frac{\mathbf{V}_N - 240\angle 120^\circ}{40 + j30} + \frac{\mathbf{V}_N - 240\angle -120^\circ}{20 + j15} + \frac{\mathbf{V}_N}{10} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 42.94\angle -156.32^\circ \text{ V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{10} = 4.29\angle -156.32^\circ \text{ A}$$

P 11.8 Make a sketch of the load in the frequency domain. Note that we convert the time domain line-to-neutral voltages to phasors:



Note that these three voltages form a balanced set with an abc phase sequence. First, use KVL to find $\mathbf{V}_{\rm AB}$:

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} + \mathbf{V}_{\rm NB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN}$$

$$= (169.71\underline{/26^\circ}) - (169.71\underline{/-94^\circ}) = 293.95\underline{/56^\circ}\,\text{V}$$

With an abc phase sequence,

$$\underline{/\mathbf{V}_{\rm BC}} = \underline{/\mathbf{V}_{\rm AB}} - 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\rm CA}} = \underline{/\mathbf{V}_{\rm AB}} + 120^\circ$$

so

$$\mathbf{V}_{\rm AB} = 293.95\underline{/56^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = 293.95\underline{/-64^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm CA} = 293.95\underline{/176^\circ}\,\text{V}$$

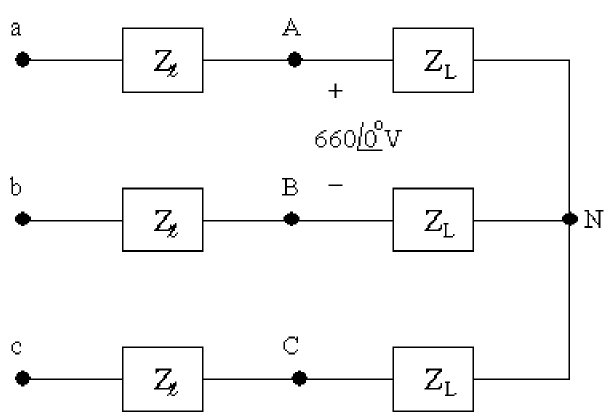
To get back to the time domain, perform an inverse phasor transform of the three line voltages, using a frequency of ω:

$$v_{\rm AB}(t) = 293.95\cos(\omega t + 56^\circ)\,\text{V}$$

$$v_{\rm BC}(t) = 293.95\cos(\omega t - 64^\circ)\,\text{V}$$

$$v_{\rm CA}(t) = 293.95\cos(\omega t + 176^\circ)\,\text{V}$$

P 11.9 Make a sketch of the three-phase line and load:



$$Z_\ell = 0.25 + j2\,\Omega/\phi$$

$$Z_{\rm L} = 30.48 + j22.86\,\Omega/\phi$$

[a] The line currents are $\mathbf{I}_{\mathrm{aA}}$, $\mathbf{I}_{\mathrm{bB}}$, and $\mathbf{I}_{\mathrm{cC}}$. To find $\mathbf{I}_{\mathrm{aA}}$, first find $\mathbf{V}_{\mathrm{AN}}$ and use Ohm's law for the a-phase load impedance. Since we are only concerned with finding voltage and current magnitudes, the phase sequence doesn't matter and we arbitrarily assume a positive phase sequence. Since we are not given any phase angles in the problem statement, we can assume the angle of $\mathbf{V}_{\mathrm{AB}}$ is $0°$. Use Fig. 11.9(a) to find $\mathbf{V}_{\mathrm{AN}}$ from $\mathbf{V}_{\mathrm{AB}}$.

$$\mathbf{V}_{\mathrm{AN}} = \frac{660}{\sqrt{3}}\underline{/(0-30^\circ)} = 381.05\underline{/-30^\circ}\,\mathrm{V}$$

Now find $\mathbf{I}_{\mathrm{aA}}$ using Ohm's law:

$$\mathbf{I}_{\mathrm{aA}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{L}}} = \frac{381.05\underline{/-30^\circ}}{30.48+j22.86} = 3.993 - j9.20 = 10\underline{/-66.87^\circ}\,\mathrm{V}$$

Thus, the magnitude of the line current is

$$|\mathbf{I}_{\mathrm{aA}}| = 10\,\mathrm{A}$$

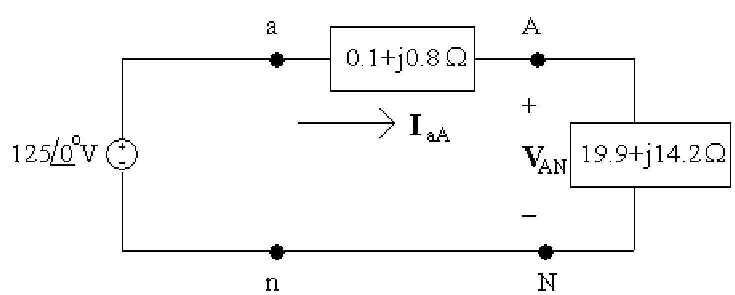
[b] The line voltage at the source is $\mathbf{V}_{\mathrm{ab}}$. From KVL on the top loop of the three-phase circuit,

$$\begin{aligned}\mathbf{V}_{\mathrm{ab}} &= \mathbf{V}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{AB}} + \mathbf{V}_{\mathrm{Bb}}\\ &= Z_\ell\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{AB}} + Z_\ell\mathbf{I}_{\mathrm{Bb}}\\ &= Z_\ell\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{AB}} - Z_\ell\mathbf{I}_{\mathrm{bB}}\\ &= (0.25+j2)(10\underline{/-66.87^\circ}) + 660\underline{/0^\circ} - (0.25+j2)(10\underline{/-173.13^\circ})\\ &= 684.71\underline{/2.10^\circ}\,\mathrm{V}\end{aligned}$$

Thus, the magnitude of the line voltage at the source is

$$|\mathbf{V}_{\mathrm{ab}}| = 684.71\,\mathrm{V}$$

P 11.10 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\mathrm{aA}} = \frac{125\underline{/0^\circ}}{0.1+j0.8+19.9+j14.2} = \frac{125\underline{/0^\circ}}{20+j15}$$

$$= 4 - j3 = 4\underline{/-36.87^\circ}\,\mathrm{A}$$

Find the other line currents using the acb phase sequence:

$$\mathbf{I}_{\rm bB} = 5\underline{/-36.87^\circ + 120^\circ} = 5\underline{/83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm cC} = 5\underline{/-36.87^\circ - 120^\circ} = 5\underline{/-156.87^\circ}\ \text{A}$$

[b] The phase voltage at the source is $\mathbf{V}_{\rm an} = 125\underline{/0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\rm an}$, from the phase voltage:

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 216.51\underline{/-30^\circ}\ \text{V}$$

Find the other line voltages using the acb phase sequence:

$$\mathbf{V}_{\rm bc} = 216.51\underline{/-30^\circ + 120^\circ} = 216.51\underline{/90^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm ca} = 216.51\underline{/-30^\circ - 120^\circ} = 216.51\underline{/-150^\circ}\ \text{V}$$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\rm AN}$. Calculate its value using $\mathbf{I}_{\rm aA}$ and the load impedance:

$$\mathbf{V}_{\rm AN} = \mathbf{I}_{\rm aA}Z_{\rm L} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\underline{/-1.36^\circ}\ \text{V}$$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\rm BN} = 122.23\underline{/-1.36^\circ + 120^\circ} = 122.23\underline{/118.64^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm CN} = 122.23\underline{/-1.36^\circ - 120^\circ} = 122.23\underline{/-121.36^\circ}\ \text{V}$$

[d] The line voltage at the load in the a-phase is $\mathbf{V}_{\rm AB}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{\rm AN}$, using Fig, 11.9(b):

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN}(\sqrt{3}\underline{/-30^\circ}) = 211.71\underline{/-31.36^\circ}\ \text{V}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\rm BC} = 211.71\underline{/-31.36^\circ + 120^\circ} = 211.71\underline{/88.69^\circ}\ \text{V}$$

$$\mathbf{V}_{\rm CA} = 211.71\underline{/-31.36^\circ - 120^\circ} = 211.71\underline{/-151.36^\circ}\ \text{V}$$

P 11.11 **[a]** $\mathbf{I}_{\rm AB} = \dfrac{480}{60 + j45} = 6.4\underline{/-36.87^\circ}\ \text{A}$

$$\mathbf{I}_{\rm BC} = 6.4\underline{/-156.87^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm CA} = 6.4\underline{/83.13^\circ}\ \text{A}$$

[b] $\mathbf{I}_{\rm aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\rm AB} = 11.09\underline{/-66.87^\circ}\ \text{A}$

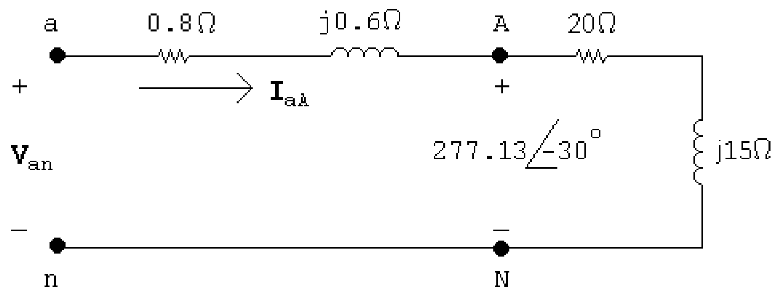
$$\mathbf{I}_{\rm bB} = 11.09\underline{/173.13^\circ}\ \text{A}$$

$$\mathbf{I}_{\rm cC} = 11.09\underline{/53.13^\circ}\ \text{A}$$

[c] Transform the Δ-connected load to a Y-connected load:

$$Z_Y = \frac{Z_\Delta}{3} = \frac{60 + j45}{3} = 20 + j15\,\Omega$$

The single-phase equivalent circuit is:



$$\mathbf{V}_{\rm an} = 277.13\underline{/-30^\circ} + (0.8 + j0.6)(11.09\underline{/-66.87^\circ})$$

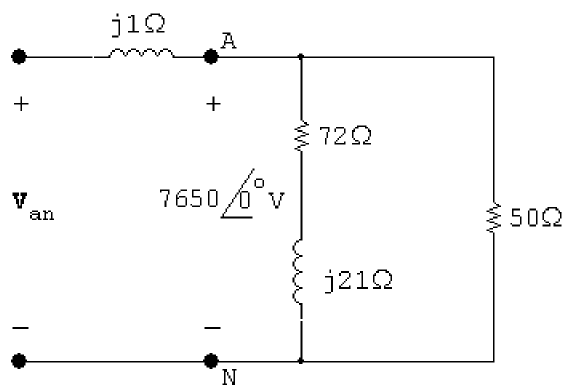
$$= 288.21\underline{/-30^\circ}\ \mathrm{V}$$

$$\mathbf{V}_{\rm ab} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\rm an} = 499.20\underline{/0^\circ}\ \mathrm{V}$$

$$\mathbf{V}_{\rm bc} = 499.20\underline{/-120^\circ}\ \mathrm{V}$$

$$\mathbf{V}_{\rm ca} = 499.20\underline{/120^\circ}\ \mathrm{V}$$

P 11.12 **[a]**



$$\mathbf{I}_{\rm aA} = \frac{7650}{72 + j21} + \frac{7650}{50} = 252.54\underline{/-6.49^\circ}\ \mathrm{A}$$

$$|\mathbf{I}_{\rm aA}| = 252.54\ \mathrm{A}$$

[b] $$\mathbf{I}_{\rm AB} = \frac{7650\sqrt{3}\underline{/30^\circ}}{150} = 88.33\underline{/30^\circ}\ \mathrm{A}$$

$$|\mathbf{I}_{\rm AB}| = 88.33\ \mathrm{A}$$

[c] $\mathbf{I}_{\rm AN} = \dfrac{7650\underline{/0^\circ}}{72 + j21} = 102\underline{/-16.26^\circ}\,\text{A}$

$|\mathbf{I}_{\rm AN}| = 102\,\text{A}$

[d] $\mathbf{V}_{\rm an} = (252.54\underline{/-6.49^\circ})(j1) + 7650\underline{/0^\circ} = 7682.66\underline{/1.87^\circ}\,\text{V}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}(7682.66) = 13{,}306.76\,\text{V}$

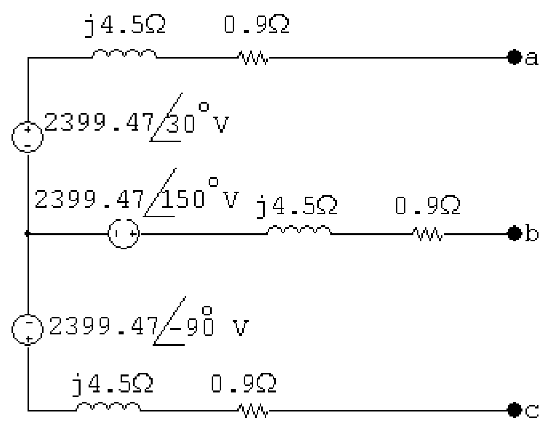
P 11.13 **[a]** Since the phase sequence is acb (negative) we have:

$\mathbf{V}_{\rm an} = 2399.47\underline{/30^\circ}\,\text{V}$

$\mathbf{V}_{\rm bn} = 2399.47\underline{/150^\circ}\,\text{V}$

$\mathbf{V}_{\rm cn} = 2399.47\underline{/-90^\circ}\,\text{V}$

$Z_Y = \dfrac{1}{3}Z_\Delta = 0.9 + j4.5\,\Omega/\phi$



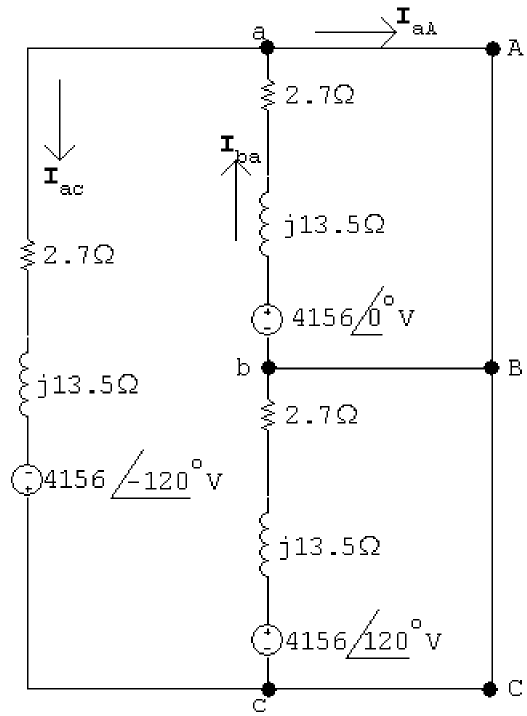
[b] $\mathbf{V}_{\rm ab} = 2399.47\underline{/30^\circ} - 2399.47\underline{/150^\circ} = 2399.47\sqrt{3}\underline{/0^\circ} = 4156\underline{/0^\circ}\,\text{V}$

Since the phase sequence is negative, it follows that

$\mathbf{V}_{\rm bc} = 4156\underline{/120^\circ}\,\text{V}$

$\mathbf{V}_{\rm ca} = 4156\underline{/-120^\circ}\,\text{V}$

[c]



$$\mathbf{I}_{\rm ba} = \frac{4156}{2.7 + j13.5} = 301.87\underline{/-78.69^\circ}\,\rm A$$

$$\mathbf{I}_{\rm ac} = 301.87\underline{/-198.69^\circ}\,\rm A$$

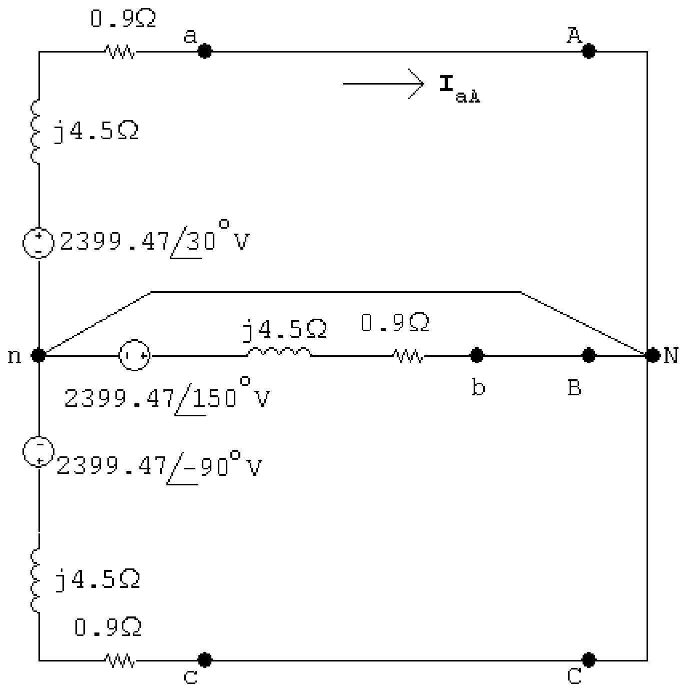
$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm ba} - \mathbf{I}_{\rm ac} = 522.86\underline{/-48.69^\circ}\,\rm A$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\rm bB} = 522.86\underline{/71.31^\circ}\,\rm A$$

$$\mathbf{I}_{\rm cC} = 522.86\underline{/-168.69^\circ}\,\rm A$$

[d]



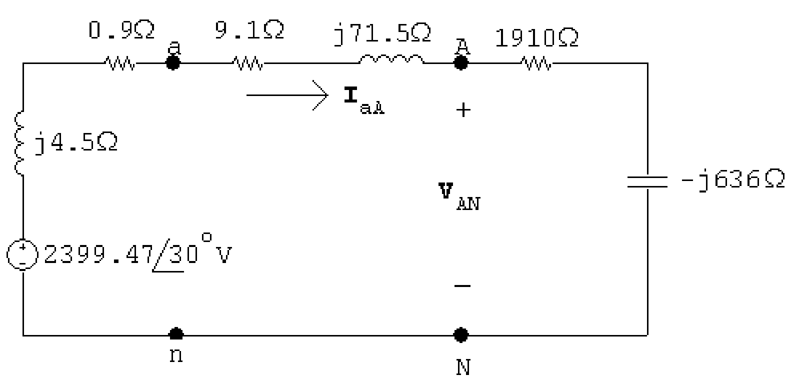
$$\mathbf{I}_{\text{aA}} = \frac{2399.47\underline{/30^\circ}}{0.9 + j4.5} = 522.86\underline{/-48.69^\circ}\text{ A}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$\mathbf{I}_{\text{bB}} = 522.86\underline{/71.31^\circ}\text{ A}$

$\mathbf{I}_{\text{cC}} = 522.86\underline{/-168.69^\circ}\text{ A}$

P 11.14 **[a]**



[b] $\mathbf{I}_{\text{aA}} = \dfrac{2399.47\underline{/30^\circ}}{1920 - j560} = 1.2\underline{/46.26^\circ}\text{ A}$

$\mathbf{V}_{\text{AN}} = (1910 - j636)(1.2\underline{/46.26^\circ}) = 2415.19\underline{/27.84^\circ}\text{ V}$

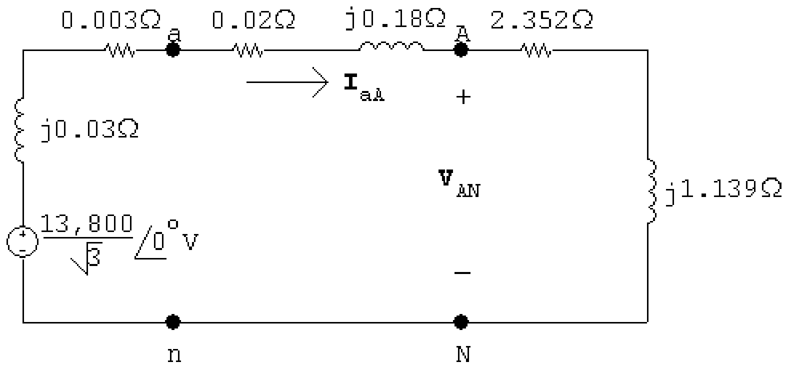
$|\mathbf{V}_{\text{AB}}| = \sqrt{3}(2415.19) = 4183.23\text{ V}$

[c] $|\mathbf{I}_{\rm ab}| = \dfrac{1.2}{\sqrt{3}} = 0.69\,\text{A}$

[d] $\mathbf{V}_{\rm an} = (1919.1 - j564.5)(1.2\underline{/46.26^\circ}) = 2400\underline{/29.87^\circ}\,\text{V}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}(2400) = 4156.92\,\text{V}$

P 11.15 **[a]**



[b] $\mathbf{I}_{\rm aA} = \dfrac{13{,}800}{\sqrt{3}(2.375 + j1.349)} = 2917\underline{/-29.6^\circ}\,\text{A}$

$|\mathbf{I}_{\rm aA}| = 2917\,\text{A}$

[c] $\mathbf{V}_{\rm AN} = (2.352 + j1.139)(2917\underline{/-29.6^\circ}) = 7622.94\underline{/-3.76^\circ}\,\text{V}$

$|\mathbf{V}_{\rm AB}| = \sqrt{3}|\mathbf{V}_{\rm AN}| = 13{,}203.31\,\text{V}$

[d] $\mathbf{V}_{\rm an} = (2.372 + j1.319)(2917\underline{/-29.6^\circ}) = 7616.93\underline{/-0.52^\circ}\,\text{V}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}|\mathbf{V}_{\rm an}| = 13{,}712.52\,\text{V}$

[e] $|\mathbf{I}_{\rm AB}| = \dfrac{|\mathbf{I}_{\rm aA}|}{\sqrt{3}} = 1684.13\,\text{A}$

[f] $|\mathbf{I}_{\rm ab}| = |\mathbf{I}_{\rm AB}| = 1684.13\,\text{A}$

P 11.16 **[a]** $\mathbf{I}_{\rm AB} = \dfrac{4160\underline{/0^\circ}}{160 + j120} = 20.8\underline{/-36.87^\circ}\,\text{A}$

$\mathbf{I}_{\rm BC} = 20.8\underline{/83.13^\circ}\,\text{A}$

$\mathbf{I}_{\rm CA} = 20.8\underline{/-156.87^\circ}\,\text{A}$

[b] $\mathbf{I}_{\rm aA} = \sqrt{3}\underline{/30^\circ}\mathbf{I}_{\rm AB} = 36.03\underline{/-6.87^\circ}\,\text{A}$

$\mathbf{I}_{\rm bB} = 36.03\underline{/113.13^\circ}\,\text{A}$

$\mathbf{I}_{\rm cC} = 36.03\underline{/-126.87^\circ}\,\text{A}$

[c] $\mathbf{I}_{\rm ba} = \mathbf{I}_{\rm AB} = 20.8\underline{/-36.87^\circ}\,\text{A};$

$\mathbf{I}_{\rm cb} = \mathbf{I}_{\rm BC} = 20.8\underline{/83.13^\circ}\,\text{A};$

$\mathbf{I}_{\rm ac} = \mathbf{I}_{\rm CA} = 20.8\underline{/-156.87^\circ}\,\text{A};$

P 11.17 **[a]** $\mathbf{I}_{\rm AB} = \dfrac{480\underline{/0^\circ}}{2.4 - j0.7} = 192\underline{/16.26^\circ}\,\text{A}$

$$\mathbf{I}_{\rm BC} = \frac{480\underline{/120^\circ}}{8 + j6} = 48\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm CA} = \frac{480\underline{/-120^\circ}}{20} = 24\underline{/-120^\circ}\,\text{A}$$

[b] $\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm AB} - \mathbf{I}_{\rm CA}$

$= 210\underline{/20.79^\circ}\,\text{A}$

$\mathbf{I}_{\rm bB} = \mathbf{I}_{\rm BC} - \mathbf{I}_{\rm AB}$

$= 178.68\underline{/-178.04^\circ}\,\text{A}$

$\mathbf{I}_{\rm cC} = \mathbf{I}_{\rm CA} - \mathbf{I}_{\rm BC}$

$= 70.7\underline{/-104.53^\circ}\,\text{A}$

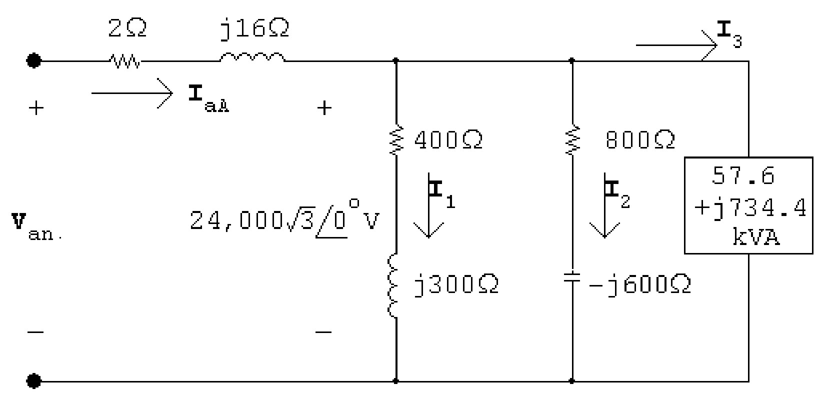
P 11.18 From the solution to Problem 11.17 we have:

$$S_{\rm AB} = (480\underline{/0^\circ})(192\underline{/-16.26^\circ}) = 88{,}473.6 - j25{,}804.8\,\text{VA}$$

$$S_{\rm BC} = (480\underline{/120^\circ})(48\underline{/-83.13^\circ}) = 18{,}432.0 + j13{,}824.0\,\text{VA}$$

$$S_{\rm CA} = (480\underline{/-120^\circ})(24\underline{/120^\circ}) = 11{,}520 + j0\,\text{VA}$$

P 11.19 **[a]**



$$\mathbf{I}_1 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{400 + j300} = 66.5 - j49.9\,\text{A}$$

$$\mathbf{I}_2 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{800 - j600} = 33.3 + j24.9\,\text{A}$$

$$\mathbf{I}_3^* = \frac{57{,}600 + j734{,}400}{24{,}000\sqrt{3}} = 1.4 + j17.7$$

$$\mathbf{I}_3 = 1.4 - j17.7\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 101.2 - j42.7\,\text{A} = 109.8\underline{/-22.8^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{an}} = (2 + j16)(101.2 - j42.7) + 24{,}000\sqrt{3} = 42{,}456.2 + j1533.2\,\text{V}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}^*_{\text{aA}} = (42{,}456.2 + j1533.8)(101.2 + j42.7)$$

$$= 4{,}229.2 + j1964.0\,\text{kVA}$$

$$S_T = 3S_\phi = 12{,}687.7 + j9892.1\,\text{kVA}$$

[b] $S_{1/\phi} = 24{,}000\sqrt{3}(66.5 + j49.9) = 2765.0 + j2073.8\,\text{kVA}$

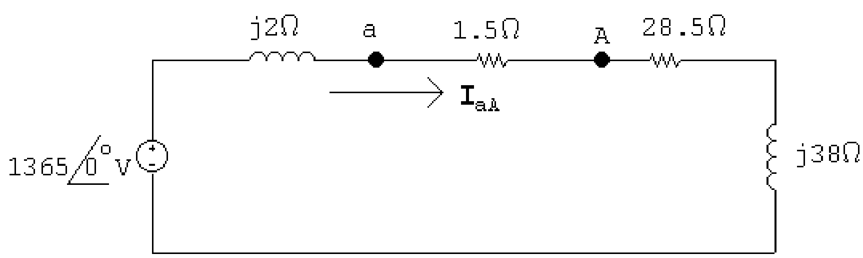
$$S_{2/\phi} = 24{,}000\sqrt{3}(33.3 - j24.9) = 1382.5 - j1036.9\,\text{kVA}$$

$$S_{3/\phi} = 57.6 + j734.4\,\text{kVA}$$

$$S_\phi(\text{load}) = 4205.1 + j1771.3\,\text{kVA}$$

$$\%\text{ delivered } = \left(\frac{4205.1}{4229.2}\right)(100) = 99.4\%$$

P 11.20 **[a]**



$$\mathbf{I}_{\text{aA}} = \frac{1365\underline{/0^\circ}}{30 + j40} = 27.3\underline{/-53.13^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 15.76\underline{/96.87^\circ}\,\text{A}$$

[b] $S_{g/\phi} = -1365\mathbf{I}^*_{\text{aA}} = -22{,}358.7 - j29{,}811.6\,\text{VA}$

$$\therefore\quad P_{\text{developed/phase}} = 22.359\,\text{kW}$$

$$P_{\text{absorbed/phase}} = |\mathbf{I}_{\text{aA}}|^2 28.5 = 21.241\,\text{kW}$$

$$\%\text{ delivered } = \frac{21.241}{22.359}(100) = 95\%$$

P 11.21 Let $p_{\rm a}$, $p_{\rm b}$, and $p_{\rm c}$ represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_{\rm a} = v_{\rm an}i_{\rm aA} = [V_m \cos\omega t][I_m \cos(\omega t - \theta_\phi)]$$

$$p_{\rm b} = v_{\rm bn}i_{\rm bB} = [V_m \cos(\omega t - 120^\circ)][I_m \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_{\rm c} = v_{\rm cn}i_{\rm cC} = [V_m \cos(\omega t + 120^\circ)][I_m \cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_{\rm a} + p_{\rm b} + p_{\rm c}$, so

$$p_T = V_m I_m[\cos\omega t \cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$$

$$+ \cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

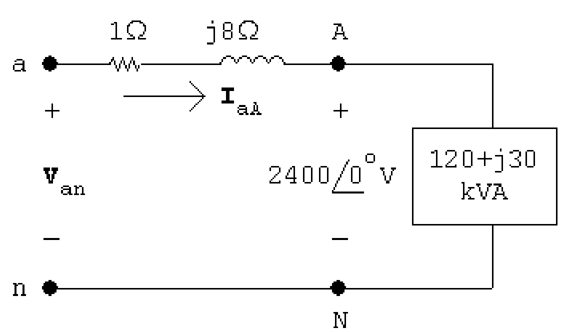
$$\begin{aligned} p_T &= V_m I_m[\cos\omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi) \\ &\quad +2\sin\omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5V_m I_m[\cos\omega t \cos(\omega t - \theta_\phi) + \sin\omega t \sin(\omega t - \theta_\phi)] \\ &= 1.5V_m I_m \cos\theta_\phi \end{aligned}$$

P 11.22 **[a]** $S_{1/\phi} = 40{,}000(0.96) - j40{,}000(0.28) = 38{,}400 - j11{,}200$ VA

$S_{2/\phi} = 60{,}000(0.8) + j60{,}000(0.6) = 48{,}000 + j36{,}000$ VA

$S_{3/\phi} = 33{,}600 + j5200$ VA

$S_{T/\phi} = S_1 + S_2 + S_3 = 120{,}000 + j30{,}000$ VA



$$\therefore \quad \mathbf{I}_{\text{aA}}^* = \frac{120{,}000 + j30{,}000}{2400} = 50 + j12.5$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 50 - j12.5\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (50 - j12.5)(1 + j8) = 2550 + j387.5 = 2579.27\angle 8.64^\circ\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2579.27) = 4467.43\,\text{V}$$

[b] $S_{g/\phi} = (2550 + j387.5)(50 + j12.5) = 122{,}656.25 + j51{,}250\,\text{VA}$

$$\%\ \text{efficiency} = \frac{120{,}000}{122{,}656.25}(100) = 97.83\%$$

P 11.23 **[a]** $S_1 = (4.864 + j3.775)\,\text{kVA}$

$$S_2 = 17.636(0.96) + j17.636(0.28) = (16.931 + j4.938)\,\text{kVA}$$

$$\sqrt{3}V_{\text{L}}I_{\text{L}}\sin\theta_3 = 13{,}853; \qquad \sin\theta_3 = \frac{13{,}853}{\sqrt{3}(208)(73.8)} = 0.521$$

Therefore $\cos\theta_3 = 0.854$

Therefore

$$P_3 = \frac{13{,}853}{0.521} \times 0.854 = 22{,}693.58\,\text{W}$$

$$S_3 = 22.694 + j13.853\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 44.49 + j22.57\,\text{kVA}$$

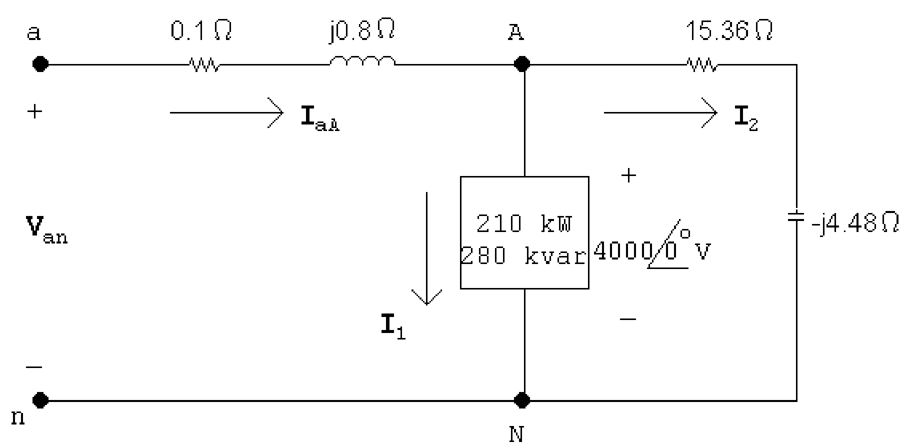
$$S_{T/\phi} = \frac{1}{3}S_T = 14.83 + j7.52\,\text{kVA}$$

$$\frac{208}{\sqrt{3}}\mathbf{I}_{\text{aA}}^* = (14.83 + j7.52)10^3; \qquad \mathbf{I}_{\text{aA}}^* = 123.49 + j62.64\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 123.49 - j62.64 = 138.46\angle -26.90^\circ\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(-26.90^\circ) = 0.892$ lagging

P 11.24



$$4000\mathbf{I}_1^* = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70\,\text{A}$$

$$\mathbf{I}_1 = 52.5 - j70\,\text{A}$$

$$\mathbf{I}_2 = \frac{4000\underline{/0^\circ}}{15.36 - j4.48} = 240 + j70\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\underline{/3.32^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 6990.62\,\text{V}$$

P 11.25 **[a]** $P_{\text{OUT}} = 746 \times 100 = 74{,}600\,\text{W}$

$$P_{\text{IN}} = 74{,}600/(0.97) = 76{,}907.22\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 76{,}907.22$$

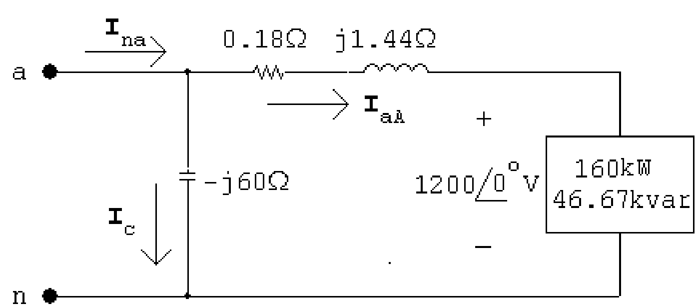
$$I_L = \frac{76{,}907.22}{\sqrt{3}(208)(0.88)} = 242.58\,\text{A}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(242.58)(0.475) = 41{,}510.12\,\text{VAR}$

P 11.26 **[a]** $\mathbf{I}_{\text{aA}}^* = \dfrac{(160 + j46.67)10^3}{1200} = 133.3 + j38.9$

$$\mathbf{I}_{\text{aA}} = 133.3 - j38.9\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 1200 + (133.3 - j38.9)(0.18 + j1.44) = 1280 + j185\,\text{V}$$



$$\mathbf{I}_{\text{C}} = \frac{1280 + j185}{-j60} = -3.1 + j21.3\,\text{A}$$

$$\mathbf{I}_{\text{na}} = (\mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{C}}) = -130.3 - j17.6 = 131.4\underline{/7.7^\circ}\,\text{A}$$

[b] $S_{g/\phi} = (1280 + j185)(-130.3 - j17.6) = -163{,}472 - j46{,}567.4\,\text{VA}$

$S_{gT} = 3S_{g/\phi} = -490.4 - j139.7\,\text{kVA}$

Therefore, the source is delivering 490.4 kW and 139.7 kvars.

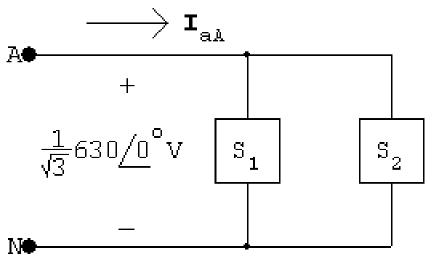
[c] $P_{\text{del}} = 490.4\,\text{kW}$

$$P_{\text{abs}} = 3(160{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.18)$$
$$= 490.4\,\text{kW} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(60) + 139.7 \times 10^3 = 223.3\,\text{kVAR}$

$$Q_{\text{abs}} = 3(46{,}666) + 3|\mathbf{I}_{\text{aA}}|^2(1.44)$$
$$= 223.3\,\text{kVAR} = Q_{\text{del}}$$

P 11.27 **[a]**



$$S_{s/\phi} = \frac{1}{3}(60)(0.96 - j0.28) \times 10^3 = 19.2 - j5.6\,\text{kVA}$$

$$S_{1/\phi} = 15\,\text{kVA}$$

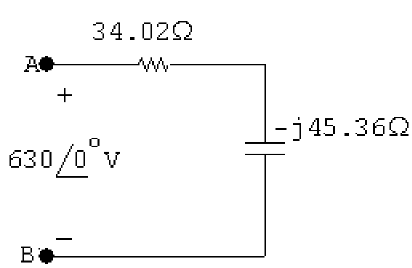
$$S_{2/\phi} = S_{s/\phi} - S_{1/\phi} = 4.2 - j5.6\,\text{kVA}$$

$$\therefore \quad \mathbf{I}_2^* = \frac{4200 - j5600}{630/\sqrt{3}} = 11.547 - j15.396\,\text{A}$$

$$\mathbf{I}_2 = 11.547 + j15.396\,\text{A}$$

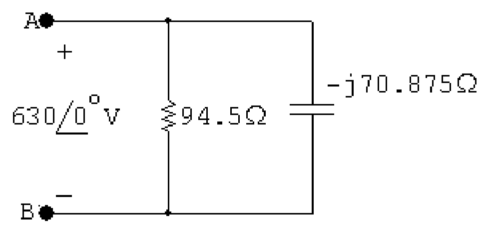
$$Z_y = \frac{630\angle 0^\circ/\sqrt{3}}{\mathbf{I}_2} = 11.34 - j15.12\,\Omega$$

$$Z_\Delta = 3Z_y = 34.02 - j45.36\,\Omega$$



[b] $R = \dfrac{(630/\sqrt{3})^2}{4200} = 31.5\,\Omega; \qquad R_\Delta = 3R = 94.5\,\Omega$

$$X_\text{L} = \frac{(630/\sqrt{3})^2}{-5600} = -23.625\,\Omega; \qquad X_\Delta = 3X_\text{L} = -70.875\,\Omega$$

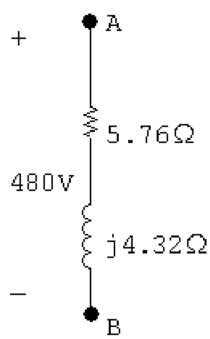


P 11.28 Assume a Δ-connect load (series):

$$S_\phi = \frac{1}{3}(96 \times 10^3)(0.8 + j0.6) = 25{,}600 + j19{,}200\,\text{VA}$$

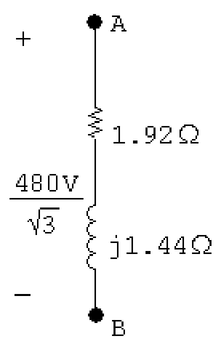
$$Z^*_{\Delta\phi} = \frac{|480|^2}{25{,}600 + j19{,}200} = 5.76 - j4.32\,\Omega$$

$$Z_{\Delta\phi} = 5.76 + 4.32\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 1.92 + j1.44\,\Omega$$



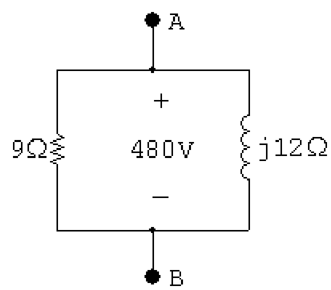
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|480|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|480|^2}{25{,}600} = 9\,\Omega$$

$$Q_\phi = \frac{|480|^2}{X_\Delta}$$

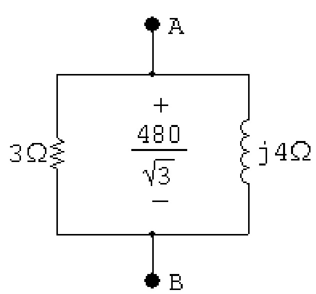
$$X_\Delta\phi = \frac{|480|^2}{19{,}200} = 12\,\Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 3\,\Omega$$

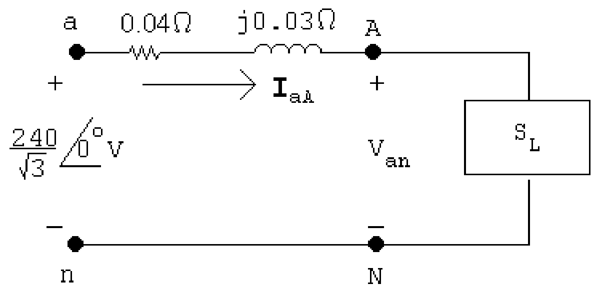
$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = 4\,\Omega$$



P 11.29 $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9806.69\,\text{VA}$

$$\mathbf{I}_{\text{aA}}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.75 + j70.77\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 70.75 - j70.77\,\text{A}$$



$$\mathbf{V}_{\text{AN}} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.75 - j70.77)$$
$$= 133.61 + j0.71 = 133.61\angle 0.30^\circ\,\text{V}$$

$$|\mathbf{V}_{\text{AB}}| = \sqrt{3}(133.61) = 231.42\,\text{V}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9403.1 + j9506.3\,\text{VA}$

$$S_L = 3S_{L/\phi} = 28{,}209 + j28{,}519\,\text{VA}$$

Check:

$$S_g = 41{,}600(0.707 + j0.707) = 29{,}411 + j29{,}420\,\text{VA}$$

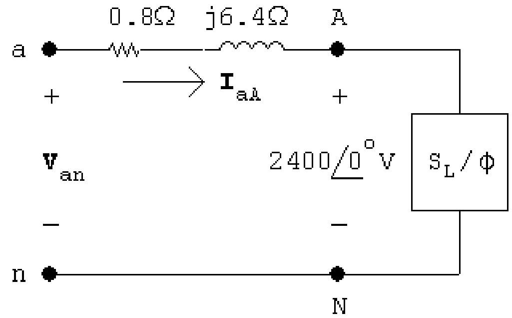
$$P_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.04) = 1202\,\text{W}$$

$$P_g = P_L + P_\ell = 28{,}209 + 1202 = 29{,}411\,\text{W} \quad \text{(checks)}$$

$$Q_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.03) = 901\,\text{VAR}$$

$$Q_g = Q_L + Q_\ell = 28{,}519 + 901 = 29{,}420\,\text{VAR} \quad \text{(checks)}$$

P 11.30 **[a]**



$$S_{L/\phi} = \frac{1}{3}\left[720 + j\frac{720}{0.8}(0.6)\right]10^3 = 240{,}000 + j180{,}000\,\text{VA}$$

$$\mathbf{I}_{\text{aA}}^* = \frac{240{,}000 + j180{,}000}{2400} = 100 + j75\,\text{A}$$

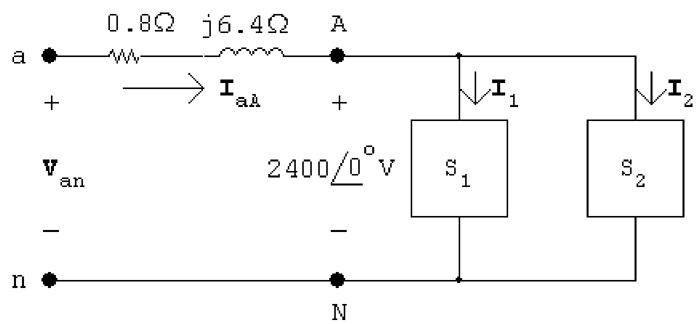
$$\mathbf{I}_{\text{aA}} = 100 - j75\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (0.8 + j6.4)(100 - j75)$$

$$= 2960 + j580 = 3016.29\underline{/11.09^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3016.29) = 5224.37\,\text{V}$$

[b]



$$\mathbf{I}_1 = 100 - j75\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(576) \times 10^3 = -j192{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j192{,}000}{2400} = -j80\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = j80\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 100 - j75 + j80 = 100 + j5\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (100 + j5)(0.8 + j6.4)$$

$$= 2448 + j644 = 2531.29\underline{/14.74^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2531.29) = 4384.33\,\text{V}$$

[c] $|\mathbf{I}_{\rm aA}| = 125\,\mathrm{A}$

$$P_{\text{loss}/\phi} = (125)^2(0.8) = 12{,}500\,\mathrm{W}$$

$$P_{g/\phi} = 240{,}000 + 12{,}500 = 252.5\,\mathrm{kW}$$

$$\%\,\eta = \frac{240}{252.5}(100) = 95.05\%$$

[d] $|\mathbf{I}_{\rm aA}| = 100.125\,\mathrm{A}$

$$P_{\ell/\phi} = (100.125)^2(0.8) = 8020\,\mathrm{W}$$

$$\%\,\eta = \frac{240{,}000}{248{,}200}(100) = 96.77\%$$

[e] $Z_{\text{cap/Y}} = -j\dfrac{2400^2}{-192{,}000} = -j30\,\Omega$

$$Z_{\text{cap}/\Delta} = 3Z_{\text{cap/Y}} = -j90\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 90; \qquad C = \frac{1}{(90)(120\pi)} = 29.47\,\mu\mathrm{F}$$

P 11.31 **[a]** From Assessment Problem 11.9, $\mathbf{I}_{\rm aA} = (101.8 - j135.7)\,\mathrm{A}$

Therefore $\mathbf{I}_{\rm cap} = j135.7\,\mathrm{A}$

Therefore $Z_{CY} = \dfrac{2450/\sqrt{3}}{j135.7} = -j10.42\,\Omega$

Therefore $C_Y = \dfrac{1}{(10.42)(2\pi)(60)} = 254.5\,\mu\mathrm{F}$

$$Z_{C\Delta} = (-j10.42)(3) = -j31.26\,\Omega$$

Therefore $C_\Delta = \dfrac{254.5}{3} = 84.84\,\mu\mathrm{F}$

[b] $C_Y = 254.5\,\mu\mathrm{F}$

[c] $|\mathbf{I}_{\rm aA}| = 101.8\,\mathrm{A}$

P 11.32 $Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}}$

$\theta = \underline{/\mathbf{V}_{\text{AN}}} - \underline{/\mathbf{I}_{\text{aA}}}$

$\theta_1 = \underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{AB}}} = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ$

Thus,

$\theta_1 = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ - \underline{/\mathbf{I}_{\text{aA}}} = \theta + 30^\circ$

Similarly,

$Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\text{CN}}}{\mathbf{I}_{\text{cC}}}$

$\theta = \underline{/\mathbf{V}_{\text{CN}}} - \underline{/\mathbf{I}_{\text{cC}}}$

$\theta_2 = \underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{CB}}} = \underline{/\mathbf{V}_{\text{BA}}} - 120^\circ = \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ$

$\underline{/\mathbf{I}_{\text{cC}}} = \underline{/\mathbf{I}_{\text{aA}}} + 120^\circ$

Thus,

$$\begin{aligned}\theta_2 &= \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ - \underline{/\mathbf{I}_{\text{aA}}} - 120^\circ = \theta_1 - 60^\circ \\ &= \theta + 30^\circ - 60^\circ = \theta - 30^\circ\end{aligned}$$

P 11.33 Use values from the negative sequence part of Example 11.1 — part (g):

$\mathbf{V}_{\text{AB}} = 199.58\underline{/-31.19^\circ}$ V

$\mathbf{I}_{\text{aA}} = 2.5\underline{/-36.87^\circ}$ A

$W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}|\cos(\underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.63\,W$

$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}|\cos(\underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.29\,W$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,W$

P 11.34 **[a]**
$$W_2 - W_1 = V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)]$$
$$= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ$$
$$- \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ]$$
$$= 2V_L I_L \sin\theta \sin 30^\circ = V_L I_L \sin\theta,$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L \sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\text{VAR},$$

$$Q_T = 3(12)^2(6) = 2592\,\text{VAR};$$

$$Z_\phi = (8 - j6)\,\Omega$$

$$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\text{VAR},$$

$$Q_T = 3(12)^2(-6) = -2592\,\text{VAR};$$

$$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$$

$$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\text{VAR},$$

$$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\text{VAR};$$

$$Z_\phi = 10\underline{/-75^\circ}\,\Omega$$

$$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80\,\text{VAR},$$

$$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.80\,\text{VAR}$$

P 11.35 $\mathbf{I}_{\text{aA}} = (\mathbf{V}_{\text{AN}}/Z_\phi) = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$$

$$W_m = |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)]$$
$$= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ)$$
$$= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi,$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.36 **[a]** $Z = 16 - j12 = 20\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{V}_{\rm AN} = 680\underline{/0^\circ}\,\text{V}; \qquad \therefore \quad \mathbf{I}_{\rm aA} = 34\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 680\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_m = (680\sqrt{3})(34)\cos(-90 - 36.87^\circ) = -24{,}027.0\,\text{W}$$

$$\sqrt{3}W_m = -41{,}616.0\,\text{VAR}$$

[b] $Q_\phi = (34^2)(-12) = -13{,}872\,\text{VAR}$

$$Q_T = 3Q_\phi = -41{,}616\,\text{VAR} = \sqrt{3}W_m$$

P 11.37 **[a]** $Z_\phi = 160 + j120 = 200\underline{/36.87^\circ}\,\Omega$

$$S_\phi = \frac{4160^2}{160 - j120} = 69{,}222.4 + j51{,}916.8\,\text{VA}$$

$$S_T = 3S_\phi = 207{,}667.2 + j155{,}750.4\,\text{VA}$$

[b] $W_{m1} = (4160)(36.03)\cos(0 + 6.87^\circ) = 148{,}808.64\,\text{W}$

$$W_{m2} = (4160)(36.03)\cos(-60^\circ + 126.87^\circ) = 58{,}877.55\,\text{W}$$

Check: $P_T = 207.7\,\text{kW} = W_{m1} + W_{m2}$.

P 11.38 **[a]** $\mathbf{I}^*_{\rm aA} = \dfrac{144(0.96 - j0.28)10^3}{7200} = 20\underline{/-16.26^\circ}\,\text{A}$

$$\mathbf{V}_{\rm BN} = 7200\underline{/-120^\circ}\,\text{V}; \qquad \mathbf{V}_{\rm CN} = 7200\underline{/120^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 7200\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$\mathbf{I}_{\rm bB} = 20\underline{/-103.74^\circ}\,\text{A}$$

$$W_{\rm m1} = (7200\sqrt{3})(20)\cos(-90^\circ + 103.74^\circ) = 242{,}278.14\,\text{W}$$

[b] Current coil in line aA, measure $\mathbf{I}_{\rm aA}$.
Voltage coil across AC, measure $\mathbf{V}_{\rm AC}$.

[c] $I_{\rm aA} = 20\underline{/16.76^\circ}\,\text{A}$

$$\mathbf{V}_{\rm AC} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm CN} = 7200\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$W_{\rm m2} = (7200\sqrt{3})(20)\cos(-30^\circ - 16.26^\circ) = 172{,}441.86\,\text{W}$$

[d] $W_{\rm m1} + W_{\rm m2} = 414.72\text{kW}$

$$P_T = 432{,}000(0.96) = 414.72\,\text{kW} = W_{\rm m1} + W_{\rm m2}$$

P 11.39 **[a]** $W_1 = |\mathbf{V}_{\rm BA}||\mathbf{I}_{\rm bB}|\cos\theta_1$

Negative phase sequence:

$$\mathbf{V}_{\rm BA} = 240\sqrt{3}\underline{/150^\circ}\,\text{V}$$

$$\mathbf{I}_{\rm aA} = \frac{240\underline{/0^\circ}}{13.33\underline{/-30^\circ}} = 18\underline{/30^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm bB} = 18\underline{/150^\circ}\,\text{A}$$

$$W_1 = (18)(240)\sqrt{3}\cos 0^\circ = 7482.46\,\text{W}$$

$$W_2 = |\mathbf{V}_{\rm CA}||\mathbf{I}_{\rm cC}|\cos\theta_2$$

$$\mathbf{V}_{\rm CA} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{\rm cC} = 18\underline{/-90^\circ}\,\text{A}$$

$$W_2 = (18)(240)\sqrt{3}\cos(-60^\circ) = 3741.23\,\text{W}$$

[b] $P_\phi = (18)^2(40/3)\cos(-30^\circ) = 3741.23\,\text{W}$

$$P_T = 3P_\phi = 11{,}223.69\,\text{W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11{,}223.69\,\text{W}$$

$$\therefore\quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.40 **[a]** Negative phase sequence:

$$\mathbf{V}_{\rm AB} = 240\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = 240\sqrt{3}\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm CA} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{\rm AB} = \frac{240\sqrt{3}\underline{/-30^\circ}}{20\underline{/30^\circ}} = 20.78\underline{/-60^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm BC} = \frac{240\sqrt{3}\underline{/90^\circ}}{60\underline{/0^\circ}} = 6.93\underline{/90^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm CA} = \frac{240\sqrt{3}\underline{/-150^\circ}}{40\underline{/-30^\circ}} = 10.39\underline{/-120^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm AB} + \mathbf{I}_{\rm AC} = 18\underline{/-30^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm cC} = \mathbf{I}_{\rm CB} + \mathbf{I}_{\rm CA} = \mathbf{I}_{\rm CA} - \mathbf{I}_{\rm BC} = 16.75\underline{/-108.06^\circ}\,\text{A}$$

$$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46\,\text{W}$$

$$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23\,\text{W}$$

[b] $W_{m1} + W_{m2} = 14{,}103.69\,\text{W}$

$$P_A = (12\sqrt{3})^2(20\cos 30^\circ) = 7482.46\,\text{W}$$

$$P_B = (4\sqrt{3})^2(60) = 2880\,\text{W}$$

$$P_C = (6\sqrt{3})^2[40\cos(-30^\circ)] = 3741.23\,\text{W}$$

$$P_A + P_B + P_C = 14{,}103.69 = W_{m1} + W_{m2}$$

P 11.41 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.7498$

$\therefore \quad \phi = 36.86^\circ$

$\therefore \quad 2400|\mathbf{I}_L|\cos 66.87^\circ = 40{,}823.09$

$|\mathbf{I}_L| = 43.3\,\text{A}$

$|Z_\phi| = \dfrac{2400/\sqrt{3}}{43.3} = 32\,\Omega \qquad \therefore \quad Z_\phi = 32\underline{/36.87^\circ}\,\Omega$

P 11.42 **[a]** $Z = \dfrac{1}{3}Z_\Delta = 4.48 + j15.36 = 16\underline{/73.74^\circ}\,\Omega$

$$\mathbf{I}_{aA} = \frac{600\underline{/0^\circ}}{16\underline{/73.74^\circ}} = 37.5\underline{/-73.74^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 37.5\underline{/-193.74^\circ}\,\text{A}$$

$$\mathbf{V}_{AC} = 600\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{BC} = 600\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_1 = (600\sqrt{3})(37.5)\cos(-30 + 73.74^\circ) = 28{,}156.15\,\text{W}$$

$$W_2 = (600\sqrt{3})(37.5)\cos(-90 + 193.74^\circ) = -9256.15\,\text{W}$$

[b] $W_1 + W_2 = 18{,}900\,\text{W}$

$$P_T = 3(37.5)^2(13.44/3) = 18{,}900\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = 64{,}800\,\text{VAR}$

$$Q_T = 3(37.5)^2(46.08/3) = 64{,}800\,\text{VAR}$$

P 11.43 From the solution to Prob. 11.17 we have

$$\mathbf{I}_{\rm aA} = 210\underline{/20.79^\circ}\,\text{A} \qquad \text{and} \qquad \mathbf{I}_{\rm bB} = 178.68\underline{/-178.04^\circ}\,\text{A}$$

[a] $W_1 = |\mathbf{V}_{\rm ac}|\,|\mathbf{I}_{\rm aA}|\cos(\theta_{\rm ac} - \theta_{\rm aA})$

$= 480(210)\cos(60^\circ - 20.79^\circ) = 78{,}103.2\,\text{W}$

[b] $W_2 = |\mathbf{V}_{\rm bc}|\,|\mathbf{I}_{\rm bB}|\cos(\theta_{\rm bc} - \theta_{\rm bB})$

$= 480(178.68)\cos(120^\circ + 178.04^\circ) = 40{,}317.7\,\text{W}$

[c] $W_1 + W_2 = 118{,}421\,\text{W}$

$P_{\rm AB} = (192)^2(2.4) = 88{,}473.6\,\text{W}$

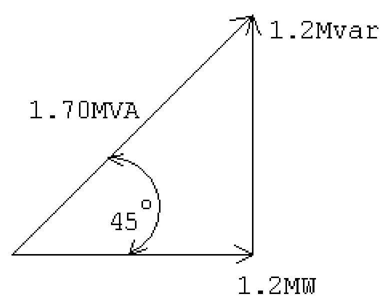
$P_{\rm BC} = (48)^2(8) = 18{,}432\,\text{W}$

$P_{\rm CA} = (24)^2(20) = 11{,}520\,\text{W}$

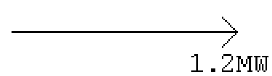
$P_{\rm AB} + P_{\rm BC} + P_{\rm CA} = 118{,}425.7$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

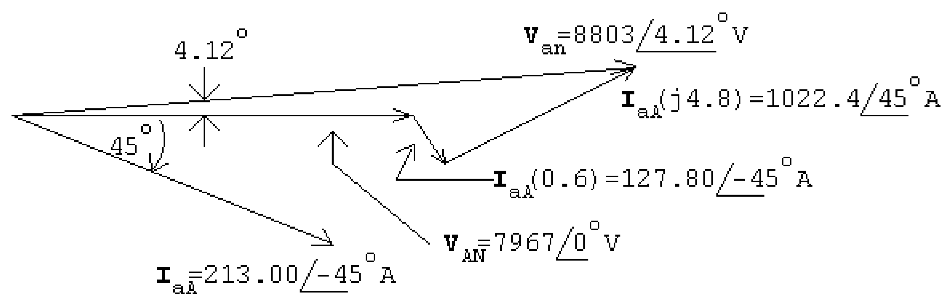
P 11.44 **[a]** For one phase,



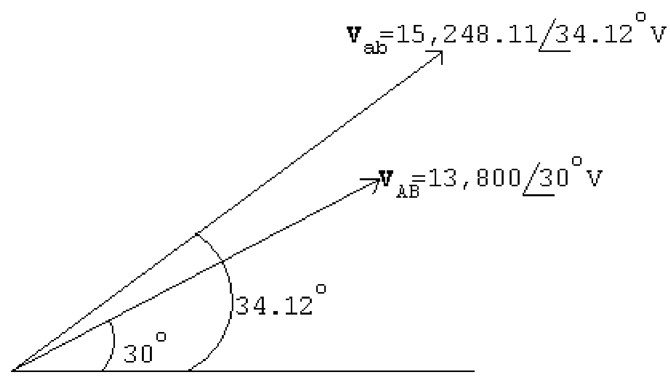
[b]



[c]



[d]



P 11.45 **[a]** $Q = \dfrac{|\mathbf{V}|^2}{X_{\rm C}}$

$$\therefore \quad |X_{\rm C}| = \frac{(13{,}800)^2}{1.2\times10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[b] $|X_{\rm C}| = \dfrac{(13{,}800/\sqrt{3})^2}{1.2\times10^6} = \dfrac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.46 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{\rm aA} = -j1.2\times10^6$$

or $\quad \mathbf{I}^*_{\rm aA} = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{\rm aA} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6+j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.47 Before the capacitors are added the total line loss is

$$P_{\rm L} = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_{\rm L} = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.48 **[a]** $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}_{\rm aA}^* = 80 \times 10^3 + j200 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{\rm aA}^* = \frac{80\sqrt{3} - j1000\sqrt{3}}{13.8} = 10.04 - j125.51\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\rm aA} = 10.04 + j125.51\,\text{A}$$

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\angle 0^\circ + (0.6 + j4.8)(10.04 + j125.51)$$
$$= 7371.01 + j123.50 = 7372.04\angle 0.96^\circ\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\rm ab}| = \sqrt{3}(7372.04) = 12{,}768.75\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.49 **[a]** $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}_{\rm aA}^* = (80 + j200) \times 10^3$

$$\mathbf{I}_{\rm aA}^* = \frac{80\sqrt{3} + j200\sqrt{3}}{13.8} = 10.04 + j25.1\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\rm aA} = 10.04 - j25.1\,\text{A}$$

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\angle 0^\circ + (0.6 + j4.8)(10.04 - j25.1)$$
$$= 8093.95 + j33.13 = 8094.02\angle 0.23^\circ\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\rm ab}| = \sqrt{3}(8094.02) = 14{,}019.25\,\text{V}$$

[b] Yes: $13\,\text{kV} < 14{,}019.25 < 14.6\,\text{kV}$

[c] $P_{\rm loss} = 3|10.04 + j125.51|^2(0.6) = 28.54\,\text{kW}$

[d] $P_{\rm loss} = 3|10.04 + j25.1|^2(0.6) = 1.32\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.

12
Introduction to the Laplace Transform

Assessment Problems

AP 12.1 **[a]** $\cosh \beta t = \dfrac{e^{\beta t} + e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\cosh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} [e^{-(s-\beta)t} + e^{-(s+\beta)t}]dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\Bigg|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)}\Bigg|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} + \frac{1}{s+\beta}\right) = \frac{s}{s^2-\beta^2}$$

[b] $\sinh \beta t = \dfrac{e^{\beta t} - e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\sinh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} \left[e^{-(s-\beta)t} - e^{-(s+\beta)t}\right] dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\right]_{0^-}^{\infty} - \frac{1}{2}\left[\frac{e^{-(s+\beta)t}}{-(s+\beta)}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} - \frac{1}{s+\beta}\right) = \frac{\beta}{(s^2-\beta^2)}$$

AP 12.2 **[a]** Let $f(t) = te^{-at}$:

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

Now, $\quad \mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

So, $\mathcal{L}\{t \cdot te^{-at}\} = -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^2}$

[b] Let $f(t) = e^{-at}\sinh\beta t,$ then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2 - \beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2-\beta^2} - 0 = \frac{\beta s}{(s+a)^2-\beta^2}$$

[c] Let $f(t) = \cos\omega t$. Then

$$F(s) = \frac{s}{(s^2+\omega^2)} \quad \text{and} \quad \frac{dF(s)}{ds} = \frac{-(s^2-\omega^2)}{(s^2+\omega^2)^2}$$

Therefore $\mathcal{L}\{t\cos\omega t\} = -\frac{dF(s)}{ds} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$

AP 12.3 $F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$

$$K_1 = \frac{6-26+26}{(1)(2)} = 3; \qquad K_2 = \frac{24-52+26}{(-1)(1)} = 2$$

$$K_3 = \frac{54-78+26}{(-2)(-1)} = 1$$

Therefore $f(t) = [3e^{-t} + 2e^{-2t} + e^{-3t}]\,u(t)$

AP 12.4 $F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{K_1}{s+3} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$

$$K_1 = \frac{63-189+134}{1(2)} = 4; \qquad K_2 = \frac{112-252+134}{(-1)(1)} = 6$$

$$K_3 = \frac{175-315+134}{(-2)(-1)} = -3$$

$$f(t) = [4e^{-3t} + 6e^{-4t} - 3e^{-5t}]u(t)$$

AP 12.5 $F(s) = \frac{10(s^2+119)}{(s+5)(s^2+10s+169)}$

$$s_{1,2} = -5 + \sqrt{25-169} = -5 + j12$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+5-j12} + \frac{K_2^*}{s+5+j12}$$

$$K_1 = \frac{10(25+119)}{25-50+169} = 10$$

$$K_2 = \frac{10[(-5+j12)^2+119]}{(j12)(j24)} = j4.167 = 4.167\underline{/90^\circ}$$

Therefore

$$\begin{aligned} f(t) &= [10e^{-5t} + 8.33e^{-5t}\cos(12t+90^\circ)]\,u(t) \\ &= [10e^{-5t} - 8.33e^{-5t}\sin 12t]\,u(t) \end{aligned}$$

AP 12.6 $F(s) = \dfrac{4s^2+7s+1}{s(s+1)^2} = \dfrac{K_0}{s} + \dfrac{K_1}{(s+1)^2} + \dfrac{K_2}{s+1}$

$$K_0 = \frac{1}{(1)^2} = 1; \qquad K_1 = \frac{4-7+1}{-1} = 2$$

$$\begin{aligned} K_2 &= \frac{d}{ds}\left[\frac{4s^2+7s+1}{s}\right]_{s=-1} = \left.\frac{s(8s+7)-(4s^2+7s+1)}{s^2}\right|_{s=-1} \\ &= \frac{1+2}{1} = 3 \end{aligned}$$

Therefore $\quad f(t) = [1 + 2te^{-t} + 3e^{-t}]\,u(t)$

AP 12.7 $F(s) = \dfrac{40}{(s^2+4s+5)^2} = \dfrac{40}{(s+2-j1)^2(s+2+j1)^2}$

$$= \frac{K_1}{(s+2-j1)^2} + \frac{K_2}{(s+2-j1)} + \frac{K_1^*}{(s+2+j1)^2} + \frac{K_2^*}{(s+2+j1)}$$

$$K_1 = \frac{40}{(j2)^2} = -10 = 10\underline{/180^\circ} \quad \text{and} \quad K_1^* = -10$$

$$K_2 = \frac{d}{ds}\left[\frac{40}{(s+2+j1)^2}\right]_{s=-2+j1} = \frac{-80}{(j2)^3} = -j10 = 10\underline{/-90^\circ}$$

$$K_2^* = j10$$

Therefore

$$f(t) = [20te^{-2t}\cos(t + 180°) + 20e^{-2t}\cos(t - 90°)]\,u(t)$$
$$= 20e^{-2t}[\sin t - t\cos t]\,u(t)$$

AP 12.8 $F(s) = \dfrac{5s^2 + 29s + 32}{(s+2)(s+4)} = \dfrac{5s^2 + 29s + 32}{s^2 + 6s + 8} = 5 - \dfrac{s+8}{(s+2)(s+4)}$

$$\frac{s+8}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{-2+8}{2} = 3; \qquad K_2 = \frac{-4+8}{-2} = -2$$

Therefore,

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$f(t) = 5\delta(t) + [-3e^{-2t} + 2e^{-4t}]u(t)$$

AP 12.9 $F(s) = \dfrac{2s^3 + 8s^2 + 2s - 4}{s^2 + 5s + 4} = 2s - 2 + \dfrac{4(s+1)}{(s+1)(s+4)} = 2s - 2 + \dfrac{4}{s+4}$

$$f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}\,u(t)$$

AP 12.10

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{7s^3[1 + (9/s) + (134/7s^2)]}{s^3[1 + (3/s)][1 + (4/s)][1 + (5/s)]}\right] = 7$$

$$\therefore \quad f(0^+) = 7$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{7s^3 + 63s^2 + 134s}{(s+3)(s+4)(s+5)}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{s^3[4 + (7/s) + (1/s^2)]}{s^3[1 + (1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{4s^2+7s+1}{(s+1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{40s}{s^4[1+(4/s)+(5/s^2)]^2}\right] = 0$$

$$\therefore \quad f(0^+) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{40s}{(s^2+4s+5)^2}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

Problems

P 12.1 **[a]** $f(t) = 5t[u(t) - u(t-2)] + 10[u(t-2) - u(t-6)] +$

$$(-5t + 40)[u(t-6) - u(t-8)]$$

[b] $f(t) = (10 \sin \pi t)[u(t) - u(t-2)]$

[c] $f(t) = 4t[u(t) - u(t-5)]$

P 12.2 **[a]** $(10 + t)[u(t + 10) - u(t)] + (10 - t)[u(t) - u(t - 10)]$

$$= (t + 10)u(t + 10) - 2tu(t) + (t - 10)u(t - 10)$$

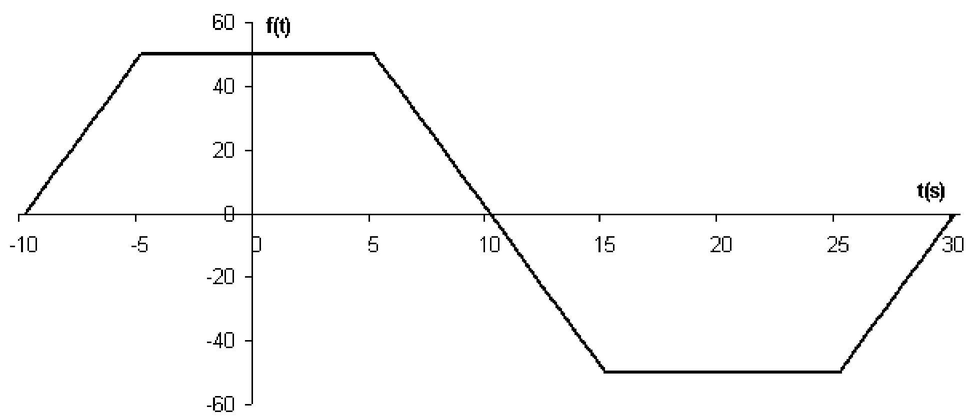
[b] $(-24 - 8t)[u(t + 3) - u(t + 2)] - 8[u(t + 2) - u(t + 1)] + 8t[u(t + 1) - u(t - 1)]$

$$+8[u(t - 1) - u(t - 2)] + (24 - 8t)[u(t - 2) - u(t - 3)]$$

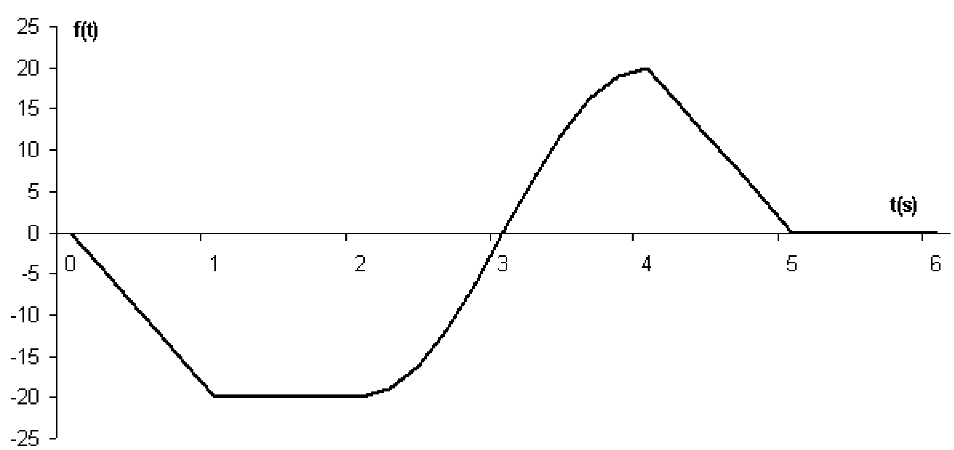
$$= -8(t + 3)u(t + 3) + 8(t + 2)u(t + 2) + 8(t + 1)u(t + 1) - 8(t - 1)u(t - 1)$$

$$-8(t - 2)u(t - 2) + 8(t - 3)u(t - 3)$$

P 12.3



P 12.4 **[a]**



[b] $f(t) = -20t[u(t) - u(t-1)] - 20[u(t-1) - u(t-2)]$
$+20\cos(\frac{\pi}{2}t)[u(t-2) - u(t-4)]$
$+(100 - 20t)[u(t-4) - u(t-5)]$

P 12.5 **[a]** $A = \left(\frac{1}{2}\right) bh = \left(\frac{1}{2}\right)(2\varepsilon)\left(\frac{1}{\varepsilon}\right) = 1$

[b] 0; **[c]** ∞

P 12.6 **[a]** $I = \int_{-1}^{3}(t^3 + 2)\delta(t)\,dt + \int_{-1}^{3} 8(t^3 + 2)\delta(t-1)\,dt$

$$= (0^3 + 2) + 8(1^3 + 2) = 2 + 8(3) = 26$$

[b] $I = \int_{-2}^{2} t^2\delta(t)\,dt + \int_{-2}^{2} t^2\delta(t+1.5)\,dt + \int_{-2}^{2} t^2\delta(t-3)\,dt$

$$= 0^2 + (-1.5)^2 + 0 = 2.25$$

P 12.7 $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(4 + j\omega)}{(9 + j\omega)} \cdot \pi\delta(\omega) \cdot e^{jt\omega}\,d\omega = \left(\frac{1}{2\pi}\right)\left(\frac{4 + j0}{9 + j0}\pi e^{jt0}\right) = \frac{2}{9}$

P 12.8 As $\varepsilon \to 0$ the amplitude $\to \infty$; the duration $\to 0$; and the area is independent of ε, i.e.,

$$A = \int_{-\infty}^{\infty}\frac{\varepsilon}{\pi}\frac{1}{\varepsilon^2 + t^2}\,dt = 1$$

P 12.9 $F(s) = \int_{-\varepsilon}^{\varepsilon}\frac{1}{2\varepsilon}e^{-st}\,dt = \frac{e^{s\varepsilon} - e^{-s\varepsilon}}{2\varepsilon s}$

$$F(s) = \frac{1}{2s}\lim_{\varepsilon\to 0}\left[\frac{se^{s\varepsilon} + se^{-s\varepsilon}}{1}\right] = \frac{1}{2s} \cdot \frac{2s}{1} = 1$$

P 12.10 **[a]** Let $dv = \delta'(t-a)\,dt, \qquad v = \delta(t-a)$

$u = f(t), \qquad du = f'(t)\,dt$

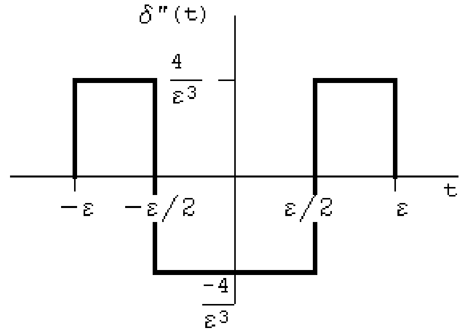
Therefore

$$\int_{-\infty}^{\infty} f(t)\delta'(t-a)\,dt = f(t)\delta(t-a)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\delta(t-a)f'(t)\,dt$$

$$= 0 - f'(a)$$

[b] $\mathcal{L}\{\delta'(t)\} = \int_{0^-}^{\infty}\delta'(t)e^{-st}\,dt = -\left[\frac{d(e^{-st})}{dt}\right]_{t=0} = -\left[-se^{-st}\right]_{t=0} = s$

P 12.11



$$F(s) = \int_{-\varepsilon}^{-\varepsilon/2} \frac{4}{\varepsilon^3} e^{-st}\,dt + \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{-4}{\varepsilon^3}\right) e^{-st}\,dt + \int_{\varepsilon/2}^{\varepsilon} \frac{4}{\varepsilon^3} e^{-st}\,dt$$

Therefore $\quad F(s) = \dfrac{4}{s\varepsilon^3}[e^{s\varepsilon} - 2e^{s\varepsilon/2} + 2e^{-s\varepsilon/2} - e^{-s\varepsilon}]$

$$\mathcal{L}\{\delta''(t)\} = \lim_{\varepsilon \to 0} F(s)$$

After applying L'Hopital's rule three times, we have

$$\lim_{\varepsilon \to 0} \frac{2s}{3}\left[se^{s\varepsilon} - \frac{s}{4}e^{s\varepsilon/2} - \frac{s}{4}e^{-s\varepsilon/2} + se^{-s\varepsilon}\right] = \frac{2s}{3}\left(\frac{3s}{2}\right)$$

Therefore $\quad \mathcal{L}\{\delta''(t)\} = s^2$

P 12.12 $\mathcal{L}\left\{\dfrac{d^n f(t)}{dt^n}\right\} = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots,$

Therefore

$$\mathcal{L}\{\delta^{(n)}(t)\} = s^n(1) - s^{n-1}\delta(0^-) - s^{n-2}\delta'(0^-) - \cdots = s^n$$

P 12.13 **[a]** $\mathcal{L}\{t\} = \dfrac{1}{s^2};\qquad$ therefore $\quad \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

[b] $\sin \omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{j2}$

Therefore

$$\mathcal{L}\{\sin \omega t\} = \left(\frac{1}{j2}\right)\left(\frac{1}{s - j\omega} - \frac{1}{s + j\omega}\right) = \left(\frac{1}{j2}\right)\left(\frac{2j\omega}{s^2 + \omega^2}\right)$$

$$= \frac{\omega}{s^2 + \omega^2}$$

[c] $\sin(\omega t + \theta) = (\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

Therefore

$$\mathcal{L}\{\sin(\omega t+\theta)\} = \cos\theta\mathcal{L}\{\sin\omega t\} + \sin\theta\mathcal{L}\{\cos\omega t\}$$
$$= \frac{\omega\cos\theta + s\sin\theta}{s^2+\omega^2}$$

[d] $\mathcal{L}\{t\} = \int_0^\infty te^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty = 0 - \frac{1}{s^2}(0-1) = \frac{1}{s^2}$

[e] $f(t) = \cosh t\cosh\theta + \sinh t\sinh\theta$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2-1}$$

$$\therefore \quad \mathcal{L}\{\cosh(t+\theta)\} = \cosh\theta\left[\frac{s}{s^2-1}\right] + \sinh\theta\left[\frac{1}{s^2-1}\right]$$
$$= \frac{\sinh\theta + s[\cosh\theta]}{s^2-1}$$

P 12.14 **[a]** $\mathcal{L}\{te^{-at}\} = \int_{0^-}^\infty te^{-(s+a)t}\,dt$

$$= \frac{e^{-(s+a)t}}{(s+a)^2}\Big[-(s+a)t-1\Big]_{0^-}^\infty$$
$$= 0 + \frac{1}{(s+a)^2}$$
$$\therefore \quad \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

[b]

$$\mathcal{L}\left\{\frac{d}{dt}(te^{-at})\right\} = \frac{s}{(s+a)^2} - 0$$
$$= \frac{s}{(s+a)^2}$$

[c] $\frac{d}{dt}(te^{-at}) = -ate^{-at} + e^{-at}$

$$\mathcal{L}\{-ate^{-at}+e^{-at}\} = \frac{-a}{(s+a)^2} + \frac{1}{(s+a)} = \frac{-a}{(s+a)^2} + \frac{s+a}{(s+a)^2}$$

$$\therefore \quad \mathcal{L}\left\{\frac{d}{dt}(te^{-at})\right\} = \frac{s}{(s+a)^2} \qquad \text{CHECKS}$$

P 12.15 **[a]** $\mathcal{L}\{f'(t)\} = \int_{-\varepsilon}^{\varepsilon} \frac{e^{-st}}{2\varepsilon}\,dt + \int_{\varepsilon}^{\infty} -\,ae^{-a(t-\varepsilon)}e^{-st}\,dt$

$$= \frac{1}{2s\varepsilon}(e^{s\varepsilon} - e^{-s\varepsilon}) - \left(\frac{a}{s+a}\right)e^{-s\varepsilon} = F(s)$$

$$\lim_{\varepsilon\to 0} F(s) = 1 - \frac{a}{s+a} = \frac{s}{s+a}$$

[b] $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$

Therefore $\mathcal{L}\{f'(t)\} = sF(s) - f(0^-) = \frac{s}{s+a} - 0 = \frac{s}{s+a}$

P 12.16 $\mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^{\infty} [e^{-at}f(t)]e^{-st}\,dt = \int_{0^-}^{\infty} f(t)e^{-(s+a)t}\,dt = F(s+a)$

P 12.17 **[a]** $\mathcal{L}\left\{\int_{0^-}^{t} e^{-ax}\,dx\right\} = \frac{F(s)}{s} = \frac{1}{s(s+a)}$

[b] $\mathcal{L}\left\{\int_{0^-}^{t} y\,dy\right\} = \frac{1}{s}\left(\frac{1}{s^2}\right) = \frac{1}{s^3}$

[c] $\int_{0^-}^{t} e^{-ax}\,dx = \frac{1}{a} - \frac{e^{-at}}{a}$

$$\mathcal{L}\left\{\frac{1}{a} - \frac{e^{-at}}{a}\right\} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right] = \frac{1}{s(s+a)}$$

$$\int_{0^-}^{t} y\,dy = \frac{t^2}{2}; \qquad \mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\cdot\frac{2}{s^3} = \frac{1}{s^3}$$

P 12.18 **[a]** $\mathcal{L}\left\{\frac{d\sin\omega t}{dt}\right\} = \frac{s\omega}{s^2+\omega^2} - 0$

[b] $\mathcal{L}\left\{\frac{d\cos\omega t}{dt}\right\} = \frac{s^2}{s^2+\omega^2} - 0$

[c] $\mathcal{L}\left\{\frac{d^3(t^2)}{dt^3}\right\} = s^3\left(\frac{2}{s^3}\right) - s^2(0) - s(0) - 2(0) = 2$

[d] $\frac{d\sin\omega t}{dt} = (\cos\omega t)\cdot\omega, \qquad \mathcal{L}\{\omega\cos\omega t\} = \frac{\omega s}{s^2+\omega^2}$

$$\frac{d\cos\omega t}{dt} = -\omega\sin\omega t + \delta(t)$$

$$\mathcal{L}\{-\omega\sin\omega t + \delta(t)\} = -\frac{\omega^2}{s^2+\omega^2} + 1 = \frac{s^2}{s^2+\omega^2}$$

$$\frac{d^2(t^2)}{dt^2} = 2u(t); \qquad \frac{d^3(t^2)}{dt^3} = 2\delta(t); \qquad \mathcal{L}\{2\delta(t)\} = 2$$

P 12.19 **[a]** $f(t) = 5t[u(t) - u(t-2)]$

$$+(20-5t)[u(t-2) - u(t-6)]$$

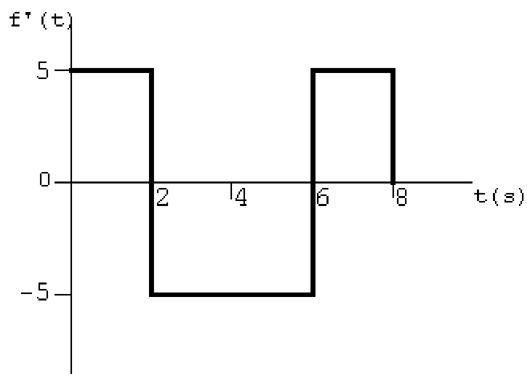
$$+(5t-40)[u(t-6) - u(t-8)]$$

$$= 5tu(t) - 10(t-2)u(t-2)$$

$$+10(t-6)u(t-6) - 5(t-8)u(t-8)$$

$$\therefore \quad F(s) = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s^2}$$

[b]



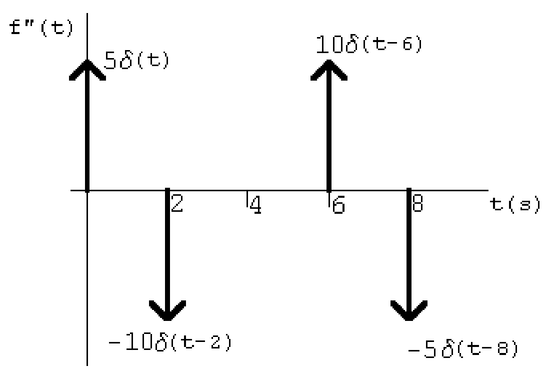
$$f'(t) = 5[u(t) - u(t-2)] - 5[u(t-2) - u(t-6)]$$

$$+5[u(t-6) - u(t-8)]$$

$$= 5u(t) - 10u(t-2) + 10u(t-6) - 5u(t-8)$$

$$\mathcal{L}\{f'(t)\} = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s}$$

[c]



$$f''(t) = 5\delta(t) - 10\delta(t-2) + 10\delta(t-6) - 5\delta(t-8)$$

$$\mathcal{L}\{f''(t)\} = 5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]$$

P 12.20 **[a]** $\int_{0^-}^{t} x\,dx = \frac{t^2}{2}$

$$\mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\int_{0^-}^{\infty} t^2 e^{-st}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-st}}{-s^3}(s^2t^2 + 2st + 2)\Big|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2s^3}(2) = \frac{1}{s^3}$$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3}$$

[b] $\mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{\mathcal{L}\{t\}}{s} = \frac{1/s^2}{s} = \frac{1}{s^3}$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3} \qquad \text{CHECKS}$$

P 12.21 **[a]** $\mathcal{L}\{40e^{-8(t-3)}u(t-3)\} = \frac{40e^{-3s}}{(s+8)}$

[b] First rewrite $f(t)$ as

$$\begin{aligned} f(t) &= (5t-10)u(t-2) + (40-10t)u(t-4) \\ &\quad +(10t-80)u(t-8) + (50-5t)u(t-10) \\ &= 5(t-2)u(t-2) - 10(t-4)u(t-4) \\ &\quad +10(t-8)u(t-8) - 5(t-10)u(t-10) \end{aligned}$$

$$\therefore \quad F(s) = \frac{5[e^{-2s} - 2e^{-4s} + 2e^{-8s} - e^{-10s}]}{s^2}$$

P 12.22 $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st}\,dt$

Let $\quad u = at, \qquad du = a\,dt, \qquad u = 0^- \quad$ when $\quad t = 0^-$

and $\quad u = \infty \quad$ when $\quad t = \infty$

Therefore $\quad \mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s}\frac{du}{a} = \frac{1}{a}F(s/a)$

P 12.23 **[a]** $f_1(t) = e^{-at}\sin\omega t; \qquad F_1(s) = \frac{\omega}{(s+a)^2 + \omega^2}$

$$F(s) = sF_1(s) - f_1(0^-) = \frac{s\omega}{(s+a)^2 + \omega^2} - 0$$

[b] $f_1(t) = e^{-at}\cos\omega t; \qquad F_1(s) = \frac{s+a}{(s+a)^2+\omega^2}$

$$F(s) = \frac{F_1(s)}{s} = \frac{s+a}{s[(s+a)^2+\omega^2]}$$

[c] $\frac{d}{dt}[e^{-at}\sin\omega t] = \omega e^{-at}\cos\omega t - ae^{-at}\sin\omega t$

Therefore $\quad F(s) = \frac{\omega(s+a)-\omega a}{(s+a)^2+\omega^2} = \frac{\omega s}{(s+a)^2+\omega^2}$

$$\int_{0^-}^{t} e^{-ax}\cos\omega x\,dx = \frac{-ae^{-at}\cos\omega t + \omega e^{-at}\sin\omega t + a}{a^2+\omega^2}$$

Therefore

$$F(s) = \frac{1}{a^2+\omega^2}\left[\frac{-a(s+a)}{(s+a)^2+\omega^2} + \frac{\omega^2}{(s+a)^2+\omega^2} + \frac{a}{s}\right]$$

$$= \frac{s+a}{s[(s+a)^2+\omega^2]}$$

P 12.24 **[a]** $\frac{dF(s)}{ds} = \frac{d}{ds}\left[\int_{0^-}^{\infty} f(t)e^{-st}\,dt\right] = -\int_{0^-}^{\infty} tf(t)e^{-st}\,dt$

Therefore $\quad \mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds}$

[b] $\frac{d^2F(s)}{ds^2} = \int_{0^-}^{\infty} t^2 f(t)e^{-st}\,dt; \qquad \frac{d^3F(s)}{ds^3} = \int_{0^-}^{\infty} -t^3 f(t)e^{-st}\,dt$

Therefore $\quad \frac{d^nF(s)}{ds^n} = (-1)^n\int_{0^-}^{\infty} t^n f(t)e^{-st}\,dt = (-1)^n\mathcal{L}\{t^n f(t)\}$

[c] $\mathcal{L}\{t^5\} = \mathcal{L}\{t^4 t\} = (-1)^4\frac{d^4}{ds^4}\left(\frac{1}{s^2}\right) = \frac{120}{s^6}$

$$\mathcal{L}\{t\sin\beta t\} = (-1)^1\frac{d}{ds}\left(\frac{\beta}{s^2+\beta^2}\right) = \frac{2\beta s}{(s^2+\beta^2)^2}$$

$\mathcal{L}\{te^{-t}\cosh t\}$:

From Assessment Problem 12.1(a),

$$F(s) = \mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

$$\frac{dF}{ds} = \frac{(s^2-1)1 - s(2s)}{(s^2-1)^2} = -\frac{s^2+1}{(s^2-1)^2}$$

Therefore $\quad -\frac{dF}{ds} = \frac{s^2+1}{(s^2-1)^2}$

Thus

$$\mathcal{L}\{t\cosh t\} = \frac{s^2+1}{(s^2-1)^2}$$

$$\mathcal{L}\{e^{-t}t\cosh t\} = \frac{(s+1)^2+1}{[(s+1)^2-1]^2} = \frac{s^2+2s+2}{s^2(s+2)^2}$$

P 12.25 **[a]** $\int_s^\infty F(u)du = \int_s^\infty \left[\int_{0^-}^\infty f(t)e^{-ut}\,dt\right]du = \int_{0^-}^\infty \left[\int_s^\infty f(t)e^{-ut}\,du\right]dt$

$$= \int_{0^-}^\infty f(t)\int_s^\infty e^{-ut}\,du\,dt = \int_{0^-}^\infty f(t)\left[\frac{e^{-tu}}{-t}\Big|_s^\infty\right]dt$$

$$= \int_{0^-}^\infty f(t)\left[\frac{-e^{-st}}{-t}\right]dt = \mathcal{L}\left\{\frac{f(t)}{t}\right\}$$

[b] $\mathcal{L}\{t\sin\beta t\} = \dfrac{2\beta s}{(s^2+\beta^2)^2}$

therefore $\quad \mathcal{L}\left\{\dfrac{t\sin\beta t}{t}\right\} = \displaystyle\int_s^\infty \left[\frac{2\beta u}{(u^2+\beta^2)^2}\right]du$

Let $\omega = u^2+\beta^2$, then $\omega = s^2+\beta^2$ when $u = s$, and $\omega = \infty$ when $u = \infty$; also $d\omega = 2u\,du$, thus

$$\mathcal{L}\left\{\frac{t\sin\beta t}{t}\right\} = \beta\int_{s^2+\beta^2}^\infty \left[\frac{d\omega}{\omega^2}\right] = \beta\left(\frac{-1}{\omega}\right)\Big|_{s^2+\beta^2}^\infty = \frac{\beta}{s^2+\beta^2}$$

P 12.26 $I_g(s) = \dfrac{1.2s}{s^2+1}; \qquad \dfrac{1}{RC} = 1.6; \qquad \dfrac{1}{LC} = 1; \qquad \dfrac{1}{C} = 1.6$

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_g(s)$$

$$V(s)\left[\frac{1}{R} + \frac{1}{Ls} + sC\right] = I_g(s)$$

$$V(s) = \frac{I_g(s)}{\frac{1}{R}+\frac{1}{Ls}+sC} = \frac{LsI_g(s)}{\frac{L}{R}s+1+s^2LC} = \frac{\frac{1}{C}sI_g(s)}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$

$$= \frac{(1.6)(1.2)s^2}{(s^2+1.6s+1)(s^2+1)} = \frac{1.92s^2}{(s^2+1.6s+1)(s^2+1)}$$

P 12.27 **[a]** $\dfrac{v_o - V_{\rm dc}}{R} + \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + C\frac{dv_o}{dt} = 0$

$$\therefore \qquad v_o + \frac{R}{L}\int_0^t v_o\,dx + RC\frac{dv_o}{dt} = V_{\rm dc}$$

[b] $V_o + \frac{R}{L}\frac{V_o}{s} + RCsV_o = \frac{V_{\rm dc}}{s}$

$\therefore \quad sLV_o + RV_o + RCLs^2V_o = LV_{\rm dc}$

$\therefore \quad V_o(s) = \frac{(1/RC)V_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$

[c] $i_o = \frac{1}{L}\int_0^t v_o\,dx$

$I_o(s) = \frac{V_o}{sL} = \frac{(1/RCL)V_{\rm dc}}{s[s^2 + (1/RC)s + (1/LC)]}$

P 12.28 **[a]** $\frac{1}{LC} = \frac{1}{(200 \times 10^{-3})(100 \times 10^{-9})} = 50 \times 10^6$

$\frac{1}{RC} = \frac{1}{(5000)(100 \times 10^{-9})} = 2000$

$V_o(s) = \frac{70{,}000}{s^2 + 2000s + 50 \times 10^6}$

$s_{1,2} = -1000 \pm j7000$ rad/s

$$V_o(s) = \frac{70{,}000}{(s + 1000 - j7000)(s + 1000 + j7000)}$$

$$= \frac{K_1}{s + 1000 - j7000} + \frac{K_1^*}{s + 1000 + j7000}$$

$K_1 = \frac{70{,}000}{j14{,}000} = 5\underline{/-90^\circ}$

$v_o(t) = 10e^{-1000t}\cos(7000t - 90^\circ)u(t)$ V

$= 10e^{-1000t}\sin(7000t)u(t)$ V

[b] $I_o(s) = \frac{35(10{,}000)}{s(s + 1000 - j7000)(s + 1000 + j7000)}$

$$= \frac{K_1}{s} + \frac{K_2}{s + 1000 - j7000} + \frac{K_2^*}{s + 1000 + j7000}$$

$K_1 = \frac{35(10{,}000)}{50 \times 10^6} = 7\,$mA

$K_2 = \frac{35(10{,}000)}{(-1000 + j7000)(j14{,}000)} = 3.54\underline{/171.87^\circ}$ mA

$i_o(t) = [7 + 7.07e^{-1000t}\cos(7000t + 171.87^\circ)]u(t)$ mA

P 12.29 **[a]** $I_{\rm dc} = \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + \dfrac{v_o}{R} + C\dfrac{dv_o}{dt}$

[b] $\dfrac{I_{\rm dc}}{s} = \dfrac{V_o(s)}{sL} + \dfrac{V_o(s)}{R} + sCV_o(s)$

$$\therefore\quad V_o(s) = \frac{I_{\rm dc}/C}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = C\dfrac{dv_o}{dt}$

$$\therefore\quad I_o(s) = sCV_o(s) = \frac{sI_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

P 12.30 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(1\times 10^3)(2\times 10^{-6})} = 500$

$$\frac{1}{LC} = \frac{1}{(12.5)(2\times 10^{-6})} = 40{,}000$$

$$V_o(s) = \frac{500{,}000 I_{\rm dc}}{s + 500s + 40{,}000}$$

$$= \frac{500{,}000 I_{\rm dc}}{(s+100)(s+400)}$$

$$= \frac{15{,}000}{(s+100)(s+400)}$$

$$= \frac{K_1}{s+100} + \frac{K_2}{s+400}$$

$$K_1 = \frac{15{,}000}{300} = 50; \qquad K_2 = \frac{15{,}000}{-300} = -50$$

$$V_o(s) = \frac{50}{s+100} - \frac{50}{s+400}$$

$$v_o(t) = [50e^{-100t} - 50e^{-400t}]u(t)\,\mathrm{V}$$

[b] $I_o(s) = \dfrac{0.03s}{(s+100)(s+400)}$

$$= \frac{K_1}{s+100} + \frac{K_2}{s+400}$$

$$K_1 = \frac{0.03(-100)}{300} = -0.01$$

$$K_2 = \frac{0.03(-400)}{-300} = 0.04$$

$$I_o(s) = \frac{-0.01}{s+100} + \frac{0.04}{s+400}$$

$$i_o(t) = (40e^{-400t} - 10e^{-100t})u(t)\,\text{mA}$$

[c] $i_o(0) = 40 - 10 = 30\,\text{mA}$

Yes. The initial inductor current is zero by hypothesis, the initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at $t = 0$ the source current appears in the capacitor.

P 12.31 **[a]** $C\frac{dv_1}{dt} + \frac{v_1 - v_2}{R} = i_g$

$$\frac{1}{L}\int_0^t v_2\,d\tau + \frac{v_2 - v_1}{R} = 0$$

or

$$C\frac{dv_1}{dt} + \frac{v_1}{R} - \frac{v_2}{R} = i_g$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + \frac{1}{L}\int_0^t v_2\,d\tau = 0$$

[b] $CsV_1(s) + \frac{V_1(s)}{R} - \frac{V_2(s)}{R} = I_g(s)$

$$-\frac{V_1(s)}{R} + \frac{V_2(s)}{R} + \frac{V_2(s)}{sL} = 0$$

or

$$(RCs + 1)V_1(s) - V_2(s) = RI_g(s)$$

$$-sLV_1(s) + (R + sL)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

P 12.32 $\frac{1}{C} = 5 \times 10^6; \qquad \frac{1}{LC} = 25 \times 10^6; \qquad \frac{R}{L} = 8000$

$$V_2(s) = \frac{(6 \times 10^{-3})(5 \times 10^6)}{s^2 + 8000s + 25 \times 10^6}$$

$$s_{1,2} = -4000 \pm j3000$$

$$V_2(s) = \frac{30{,}000}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{30{,}000}{j6000} = -j5 = 5\underline{/-90^\circ}$$

$$v_2(t) = 10e^{-4000t}\cos(3000t - 90^\circ)$$
$$= [10e^{-4000t}\sin 3000t]u(t)\,\text{V}$$

P 12.33 **[a]** For $t \geq 0^+$:

$$\frac{v_o}{R} + C\frac{dv_o}{dt} + i_o = 0$$

$$v_o = L\frac{di_o}{dt}; \qquad \frac{dv_o}{dt} = L\frac{d^2i_o}{dt^2}$$

$$\therefore \qquad \frac{L}{R}\frac{di_o}{dt} + LC\frac{d^2i_o}{dt^2}$$

or $$\frac{d^2i_o}{dt^2} + \frac{1}{RC}\frac{di_o}{dt} + \frac{1}{LC}i_o = 0$$

[b] $$s^2I_o(s) - sI_{\text{dc}} - 0 + \frac{1}{RC}[sI_o(s) - I_{\text{dc}}] + \frac{1}{LC}I_o(s) = 0$$

$$I_o(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = I_{\text{dc}}(s + 1/RC)$$

$$I_o(s) = \frac{I_{\text{dc}}[s + (1/RC)]}{[s^2 + (1/RC)s + (1/LC)]}$$

P 12.34 $$\frac{1}{RC} = 8000; \qquad \frac{1}{LC} = 16 \times 10^6$$

$$I_o(s) = \frac{0.005(s + 8000)}{s^2 + 8000s + 16 \times 10^6}$$

$$s_{1,2} = -4000$$

$$I_o(s) = \frac{0.005(s + 8000)}{(s + 4000)^2} = \frac{K_1}{(s + 4000)^2} + \frac{K_2}{s + 4000}$$

$$K_1 = 0.005(s + 8000)\Big|_{s=-4000} = 20$$

$$K_2 = \frac{d}{ds}\left[0.005(s + 8000)\right]_{s=-4000} = 0.005$$

$$I_o(s) = \frac{20}{(s + 4000)^2} + \frac{0.005}{s + 4000}$$

$$i_o(t) = [20te^{-4000t} + 0.005e^{-4000t}]u(t)\,\text{V}$$

P 12.35 **[a]** $300 = 60i_1 + 25\frac{di_1}{dt} + 10\frac{d}{dt}(i_2 - i_1) + 5\frac{d}{dt}(i_1 - i_2) - 10\frac{di_1}{dt}$

$0 = 5\frac{d}{dt}(i_2 - i_1) + 10\frac{di_1}{dt} + 40i_2$

Simplifying the above equations gives:

$300 = 60i_1 + 10\frac{di_1}{dt} + 5\frac{di_2}{dt}$

$0 = 40i_2 + 5\frac{di_1}{dt} + 5\frac{di_2}{dt}$

[b] $\frac{300}{s} = (10s + 60)I_1(s) + 5sI_2(s)$

$0 = 5sI_1(s) + (5s + 40)I_2(s)$

[c] Solving the equations in (b),

$I_1(s) = \frac{60(s + 8)}{s(s + 4)(s + 24)}$

$I_2(s) = \frac{-60}{(s + 4)(s + 24)}$

[d] $I_1(s) = \frac{K_1}{s} + \frac{K_2}{s + 4} + \frac{K_3}{s + 24}$

$K_1 = \frac{(60)(8)}{(4)(24)} = 5; \qquad K_2 = \frac{(60)(4)}{(-4)(20)} = -3$

$K_3 = \frac{(60)(-16)}{(-24)(-20)} = -2$

$I_1(s) = \left(\frac{5}{s} - \frac{3}{s + 4} - \frac{2}{s + 24}\right)$

$i_1(t) = (5 - 3e^{-4t} - 2e^{-24t})u(t)\,\text{A}$

$I_2(s) = \frac{K_1}{s + 4} + \frac{K_2}{s + 24}$

$K_1 = \frac{-60}{20} = -3; \qquad K_2 = \frac{-60}{-20} = 3$

$I_2(s) = \left(\frac{-3}{s + 4} + \frac{3}{s + 24}\right)$

$i_2(t) = (3e^{-24t} - 3e^{-4t})u(t)\,\text{A}$

[e] $i_1(\infty) = 5\,\text{A}; \qquad i_2(\infty) = 0\,\text{A}$

[f] Yes, at $t = \infty$

$$i_1 = \frac{300}{60} = 5\,\text{A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 10 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

P 12.36 From Problem 12.26:

$$V(s) = \frac{1.92s^2}{(s^2 + 1.6s + 1)(s^2 + 1)}$$

$$s^2 + 1.6s + 1 = (s + 0.8 + j0.6)(s + 0.8 - j0.6); \qquad s^2 + 1 = (s - j1)(s + j1)$$

Therefore

$$V(s) = \frac{1.92s^2}{(s + 0.8 + j0.6)(s + 0.8 - j0.6)(s - j1)(s + j1)}$$

$$= \frac{K_1}{s + 0.8 - j0.6} + \frac{K_1^*}{s + 0.8 + j0.6} + \frac{K_2}{s - j1} + \frac{K_2^*}{s + j1}$$

$$K_1 = \frac{1.92s^2}{(s + 0.8 + j0.6)(s^2 + 1)}\bigg|_{s=-0.8+j0.6} = 1\underline{/-126.87^\circ}$$

$$K_2 = \frac{1.92s^2}{(s + j1)(s^2 + 1.6s + 1)}\bigg|_{s=j1} = 0.6\underline{/0^\circ}$$

Therefore

$$v(t) = [2e^{-0.8t}\cos(0.6t - 126.87^\circ) + 1.2\cos(t)]u(t)\,\text{V}$$

P 12.37 **[a]** $F(s) = \dfrac{K_1}{s + 1} + \dfrac{K_2}{s + 2} + \dfrac{K_3}{s + 4}$

$$K_1 = \frac{8s^2 + 37s + 32}{(s + 2)(s + 4)}\bigg|_{s=-1} = 1$$

$$K_2 = \frac{8s^2 + 37s + 32}{(s + 1)(s + 4)}\bigg|_{s=-2} = 5$$

$$K_3 = \frac{8s^2 + 37s + 32}{(s + 1)(s + 2)}\bigg|_{s=-4} = 2$$

$$f(t) = [e^{-t} + 5e^{-2t} + 2e^{-4t}]u(t)$$

[b] $F(s) = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3} + \frac{K_4}{s+5}$

$$K_1 = \frac{8s^3 + 89s^2 + 311s + 300}{(s+2)(s+3)(s+5)}\bigg|_{s=0} = 10$$

$$K_2 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+3)(s+5)}\bigg|_{s=-2} = 5$$

$$K_3 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+2)(s+5)}\bigg|_{s=-3} = -8$$

$$K_4 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+2)(s+3)}\bigg|_{s=-5} = 1$$

$$f(t) = [10 + 5e^{-2t} - 8e^{-3t} + e^{-5t}]u(t)$$

[c] $F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2-j} + \frac{K_2^*}{s+2+j}$

$$K_1 = \frac{22s^2 + 60s + 58}{s^2 + 4s + 5}\bigg|_{s=-1} = 10$$

$$K_2 = \frac{22s^2 + 60s + 58}{(s+1)(s+2+j)}\bigg|_{s=-2+j} = 6 + j8 = 10\underline{/53.13^\circ}$$

$$f(t) = [10e^{-t} + 20e^{-2t}\cos(t + 53.13^\circ)]u(t)$$

[d] $F(s) = \frac{K_1}{s} + \frac{K_2}{s+7-j} + \frac{K_2^*}{s+7+j}$

$$K_1 = \frac{250(s+7)(s+14)}{s^2 + 14s + 50}\bigg|_{s=0} = 490$$

$$K_2 = \frac{250(s+7)(s+14)}{s(s+7+j)}\bigg|_{s=-7+j} = 125\underline{/-163.74^\circ}$$

$$f(t) = [490 + 250e^{-7t}\cos(t - 163.74^\circ)]u(t)$$

P 12.38 **[a]** $F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+5}$

$$K_1 = \frac{100}{s+5}\bigg|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100}{s+5}\right] = \frac{-100}{(s+5)^2}\bigg|_{s=0} = -4$$

$$K_3 = \frac{100}{s^2}\bigg|_{s=-5} = 4$$

$$f(t) = [20t - 4 + 4e^{-5t}]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^2} + \dfrac{K_3}{s+1}$

$$K_1 = \frac{50(s+5)}{(s+1)^2}\bigg|_{s=0} = 250$$

$$K_2 = \frac{50(s+5)}{s}\bigg|_{s=-1} = -200$$

$$K_3 = \frac{d}{ds}\left[\frac{50(s+5)}{s}\right] = \left[\frac{50}{s} - \frac{50(s+5)}{s^2}\right]_{s=-1} = -250$$

$$f(t) = [250 - 200te^{-t} - 250e^{-t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+3-j} + \dfrac{K_3^*}{s+3+j}$

$$K_1 = \frac{100(s+3)}{s^2+6s+10}\bigg|_{s=0} = 30$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+3)}{s^2+6s+10}\right]$$

$$= \left[\frac{100}{s^2+6s+10} - \frac{100(s+3)(2s+6)}{(s^2+6s+10)^2}\right]_{s=0} = 10 - 18 = -8$$

$$K_3 = \frac{100(s+3)}{s^2(s+3+j)}\bigg|_{s=-3+j} = 4 + j3 = 5\underline{/36.87^\circ}$$

$$f(t) = [30t - 8 + 10e^{-3t}\cos(t + 36.87^\circ)]u(t)$$

[d] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^3} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$

$$K_1 = \frac{5(s+2)^2}{(s+1)^3}\bigg|_{s=0} = 20$$

$$K_2 = \frac{5(s+2)^2}{s}\bigg|_{s=-1} = -5$$

$$K_3 = \frac{d}{ds}\left[\frac{5(s+2)^2}{s}\right] = \left[\frac{10(s+2)}{s} - \frac{5(s+2)^2}{s^2}\right]_{s=-1}$$

$$= -10 - 5 = -15$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{10(s+2)}{s} - \frac{5(s+2)^2}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{10}{s} - \frac{10(s+2)}{s^2} - \frac{10(s+2)}{s^2} + \frac{10(s+2)^2}{s^3}\right]_{s=-1}$$

$$= \frac{1}{2}(-10 - 10 - 10 - 10) = -20$$

$$f(t) = [20 - 2.5t^2e^{-t} - 15te^{-t} - 20e^{-t}]u(t)$$

[e] $$F(s) = \frac{K_1}{s} + \frac{K_2}{(s+2-j)^2} + \frac{K_2^*}{(s+2+j)^2} + \frac{K_3}{s+2-j} + \frac{K_3^*}{s+2-j}$$

$$K_1 = \left.\frac{400}{(s^2+4s+5)^2}\right|_{s=0} = 16$$

$$K_2 = \left.\frac{400}{s(s+2+j)^2}\right|_{s=-2+j} = 44.72\underline{/26.57^\circ}$$

$$K_3 = \frac{d}{ds}\left[\frac{400}{s(s+2+j)^2}\right] = \left[\frac{-400}{s^2(s+2+j)^2} + \frac{-800}{s(s+2+j)^3}\right]_{s=-2+j}$$

$$= 12 + j16 - 20 + j40 = -8 + j56 = 56.57\underline{/98.13^\circ}$$

$$f(t) = [16 + 89.44te^{-2t}\cos(t + 26.57^\circ) + 113.14e^{-2t}\cos(t + 98.13^\circ)]u(t)$$

P 12.39 **[a]**

$$F(s) = \frac{5s^2 + 38s + 80}{s^2 + 6s + 8}$$

Long division: $s^2 + 6s + 8 \overline{)\,5s^2 + 38s + 80}$ gives quotient 5; subtracting $5s^2 + 30s + 40$ leaves remainder $8s + 40$.

$$F(s) = 5 + \frac{8s+40}{s^2+6s+8} = 5 + \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \left.\frac{8s+40}{s+4}\right|_{s=-2} = 12$$

$$K_2 = \left.\frac{8s+40}{s+2}\right|_{s=-4} = -4$$

$$f(t) = 5\delta(t) + [12e^{-2t} - 4e^{-4t}]u(t)$$

[b]

$$F(s) = \frac{10s^2 + 512s + 7186}{s^2 + 48s + 625}$$

Long division: $s^2 + 48s + 625 \overline{)\,10s^2 + 512s + 7186}$ gives quotient 10; subtracting $10s^2 + 480s + 6250$ leaves remainder $32s + 936$.

$$F(s) = 10 + \frac{32s+936}{s^2+48s+625} = 10 + \frac{K_1}{s+24-j7} + \frac{K_2^*}{s+24+j7}$$

$$K_1 = \left.\frac{32s+936}{s+24+j7}\right|_{s=-24+j7} = 16 - j12 = 20\underline{/-36.87^\circ}$$

$$f(t) = 10\delta(t) + [40e^{-24t}\cos(7t - 36.87^\circ)]u(t)$$

[c]

$$F(s) = \frac{s^3 + 5s^2 - 50s - 100}{s^2 + 15s + 50}$$

Long division of $s^3 + 5s^2 - 50s - 100$ by $s^2 + 15s + 50$:

$$\begin{array}{r|l} & s - 10 \\ \hline s^2 + 15s + 50 & s^3 + 5s^2 - 50s - 100 \\ & s^3 + 15s^2 + 50s \\ \hline & -10s^2 - 100s - 100 \\ & -10s^2 - 150s - 500 \\ \hline & 50s + 400 \end{array}$$

$$F(s) = s - 10 + \frac{K_1}{s+5} + \frac{K_2}{s+10}$$

$$K_1 = \left.\frac{50s + 400}{s + 10}\right|_{s=-5} = 30$$

$$K_2 = \left.\frac{50s + 400}{s + 5}\right|_{s=-10} = 20$$

$$f(t) = \delta'(t) - 10\delta(t) + [30e^{-5t} + 20e^{-10t}]u(t)$$

P 12.40 **[a]** $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s + 1 - j2} + \dfrac{K_3^*}{s + 1 + j2}$

$$K_1 = \left.\frac{100(s+1)}{s^2 + 2s + 5}\right|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+1)}{s^2 + 2s + 5}\right] = \left[\frac{100}{s^2 + 2s + 5} - \frac{100(s+1)(2s+2)}{(s^2 + 2s + 5)^2}\right]_{s=0}$$

$$= 20 - 8 = 12$$

$$K_3 = \left.\frac{100(s+1)}{s^2(s + 1 + j2)}\right|_{s=-1+j2} = -6 + j8 = 10\underline{/126.87^\circ}$$

$$f(t) = [20t + 12 + 20e^{-t}\cos(2t + 126.87^\circ)]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+5)^3} + \dfrac{K_3}{(s+5)^2} + \dfrac{K_4}{s+5}$

$$K_1 = \left.\frac{500}{(s+5)^3}\right|_{s=0} = 4$$

$$K_2 = \left.\frac{500}{s}\right|_{s=-5} = -100$$

$$K_3 = \frac{d}{ds}\left[\frac{500}{s}\right] = \left.\frac{-500}{s^2}\right|_{s=-5} = -20$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{-500}{s^2}\right] = \left.\frac{1}{2}\frac{1000}{(s^3)}\right|_{s=-5} = -4$$

$$f(t) = [4 - 50t^2e^{-5t} - 20te^{-5t} - 4e^{-5t}]u(t)$$

[c] $F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^3} + \frac{K_3}{(s+1)^2} + \frac{K_4}{s+1}$

$$K_1 = \frac{40(s+2)}{(s+1)^3}\Big|_{s=0} = 80$$

$$K_2 = \frac{40(s+2)}{s}\Big|_{s=-1} = -40$$

$$K_3 = \frac{d}{ds}\left[\frac{40(s+2)}{s}\right] = \left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]_{s=-1} = -40 - 40 = -80$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{-40}{s^2} - \frac{40}{s^2} + \frac{80(s+2)}{s^3}\right]_{s=-1} = \frac{1}{2}(-40 - 40 - 80) = -80$$

$$f(t) = [80 - 20t^2e^{-t} - 80te^{-t} - 80e^{-t}]u(t)$$

[d] $F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^4} + \frac{K_3}{(s+1)^3} + \frac{K_4}{(s+1)^2} + \frac{K_5}{s+1}$

$$K_1 = \frac{(s+5)^2}{(s+1)^4}\Big|_{s=0} = 25$$

$$K_2 = \frac{(s+5)^2}{s}\Big|_{s=-1} = -16$$

$$K_3 = \frac{d}{ds}\left[\frac{(s+5)^2}{s}\right] = \left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]_{s=-1}$$

$$= -8 - 16 = -24$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{2(s+5)^2}{s^3}\right]_{s=-1}$$

$$= \frac{1}{2}(-2 - 8 - 8 - 32) = -25$$

$$K_5 = \frac{1}{6}\frac{d}{ds}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{2(s+5)^2}{s^3}\right]$$

$$= \frac{1}{6}\left[\frac{-2}{s^2} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} + \frac{4(s+5)}{s^3} - \frac{6(s+5)^2}{s^4}\right]_{s=-1}$$

$$= \frac{1}{6}(-2 - 2 - 16 - 2 - 16 - 16 - 96) = -25$$

$$f(t) = [25 - (8/3)t^3e^{-t} - 12t^2e^{-t} - 25te^{-t} - 25e^{-t}]u(t)$$

P 12.41 $f(t) = \mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}\right\}$

$$= Ke^{-\alpha t}e^{j\beta t} + K^* e^{-\alpha t}e^{-j\beta t}$$

$$= |K|e^{-\alpha t}[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}]$$

$$= |K|e^{-\alpha t}[e^{j(\beta t+\theta)} + e^{-j(\beta t+\theta)}]$$

$$= 2|K|e^{-\alpha t}\cos(\beta t+\theta)$$

P 12.42 **[a]** $\mathcal{L}\{t^n f(t)\} = (-1)^n\left[\frac{d^nF(s)}{ds^n}\right]$

Let $f(t) = 1$, then $F(s) = \frac{1}{s}$, thus $\frac{d^nF(s)}{ds^n} = \frac{(-1)^n n!}{s^{(n+1)}}$

Therefore $\mathcal{L}\{t^n\} = (-1)^n\left[\frac{(-1)^n n!}{s^{(n+1)}}\right] = \frac{n!}{s^{(n+1)}}$

It follows that $\mathcal{L}\{t^{(r-1)}\} = \frac{(r-1)!}{s^r}$

and $\mathcal{L}\{t^{(r-1)}e^{-at}\} = \frac{(r-1)!}{(s+a)^r}$

Therefore $\frac{K}{(r-1)!}\mathcal{L}\{t^{r-1}e^{-at}\} = \frac{K}{(s+a)^r} = \mathcal{L}\left\{\frac{Kt^{r-1}e^{-at}}{(r-1)!}\right\}$

[b] $f(t) = \mathcal{L}^{-1}\left\{\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}\right\}$

Therefore

$$f(t) = \frac{Kt^{r-1}}{(r-1)!}e^{-(\alpha-j\beta)t} + \frac{K^*t^{r-1}}{(r-1)!}e^{-(\alpha+j\beta)t}$$

$$= \frac{|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\left[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}\right]$$

$$= \left[\frac{2|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\right]\cos(\beta t+\theta)$$

P 12.43 **[a]** $\lim_{s\to\infty} sV(s) = \lim_{s\to\infty}\left[\frac{1.92s^3}{s^4[1+(1.6/s)+(1/s^2)][1+(1/s^2)]}\right] = 0$

Therefore $v(0^+) = 0$

[b] No, V has a pair of poles on the imaginary axis.

P 12.44 **[a]** $sF(s) = \dfrac{8s^3 + 37s^2 + 32s}{(s+1)(s+2)(s+4)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 8, \qquad \therefore \quad f(0^+) = 8$$

[b] $sF(s) = \dfrac{8s^3 + 89s^2 + 311s + 300}{(s+2)(s^2+8s+15)}$

$$\lim_{s\to 0} sF(s) = 10; \qquad \therefore \quad f(\infty) = 10$$

$$\lim_{s\to \infty} sF(s) = 8, \qquad \therefore \quad f(0^+) = 8$$

[c] $sF(s) = \dfrac{22s^3 + 60s^2 + 58s}{(s+1)(s^2+4s+5)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 22, \qquad \therefore \quad f(0^+) = 22$$

[d] $sF(s) = \dfrac{250(s+7)(s+14)}{(s^2+14s+50)}$

$$\lim_{s\to 0} sF(s) = \frac{250(7)(14)}{50} = 490, \qquad \therefore \quad f(\infty) = 490$$

$$\lim_{s\to \infty} sF(s) = 250, \qquad \therefore \quad f(0^+) = 250$$

P 12.45 **[a]** $sF(s) = \dfrac{100}{s(s+5)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[b] $sF(s) = \dfrac{50(s+5)}{(s+1)^2}$

$$\lim_{s\to 0} sF(s) = 250, \qquad \therefore \quad f(\infty) = 250$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[c] $sF(s) = \dfrac{100(s+3)}{s(s^2+6s+10)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[d] $sF(s) = \dfrac{5(s+2)^2}{(s+1)^3}$

$$\lim_{s\to 0} sF(s) = 20, \qquad \therefore \quad f(\infty) = 20$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[e] $sF(s) = \dfrac{400}{(s^2+4s+5)^2}$

$$\lim_{s\to 0} sF(s) = 16, \qquad \therefore \quad f(\infty) = 16$$

$$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

P 12.46 All of the $F(s)$ functions referenced in this problem are improper rational functions, and thus the corresponding $f(t)$ functions contain impulses ($\delta(t)$). Thus, neither the initial value theorem nor the final value theorem may be applied to these $F(s)$ functions!

P 12.47 $sV_o(s) = \dfrac{sV_{\text{dc}}/RC}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{V_{\text{dc}}/RCL)}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = \frac{V_{\text{dc}}/RLC}{1/LC} = \frac{V_{\text{dc}}}{R}, \qquad \therefore \quad i_o(\infty) = \frac{V_{\text{dc}}}{R}$$

$$\lim_{s\to \infty} sI_o(s) = 0, \qquad \therefore \quad i_o(0^+) = 0$$

P 12.48 $sV_o(s) = \dfrac{(I_{\text{dc}}/C)s}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{s^2 I_{\text{dc}}}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$$

$$\lim_{s\to \infty} sI_o(s) = I_{\text{dc}}, \qquad \therefore \quad v_o(0^+) = I_{\text{dc}}$$

P 12.49 **[a]** $sF(s) = \frac{100(s+1)}{s(s^2+2s+5)}$

$F(s)$ has a second-order pole at the origin, so we cannot use the final value theorem here.

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[b] $sF(s) = \frac{500}{(s+5)^3}$

$\lim_{s\to 0} sF(s) = 4, \qquad \therefore \quad f(\infty) = 4$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[c] $sF(s) = \frac{40(s+2)}{(s+1)^3}$

$\lim_{s\to 0} sF(s) = 80, \qquad \therefore \quad f(\infty) = 80$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[d] $sF(s) = \frac{(s+5)^2}{(s+1)^4}$

$\lim_{s\to 0} sF(s) = 25, \qquad \therefore \quad f(\infty) = 25$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

P 12.50 $sI_o(s) = \frac{I_{\text{dc}}s[s+(1/RC)]}{s^2+(1/RC)s+(1/LC)}$

$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$

$\lim_{s\to\infty} sI_o(s) = I_{\text{dc}}, \qquad \therefore \quad i_o(0^+) = I_{\text{dc}}$

13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

Therefore $\quad Y = \frac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$

[b] $-z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000$ rad/s

$-z_1 = -40{,}000 - j30{,}000$ rad/s

$-z_2 = -40{,}000 + j30{,}000$ rad/s

$-p_1 = 0$ rad/s

AP 13.2 **[a]** $Z = 2000 + \frac{1}{Y} = 2000 + \frac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

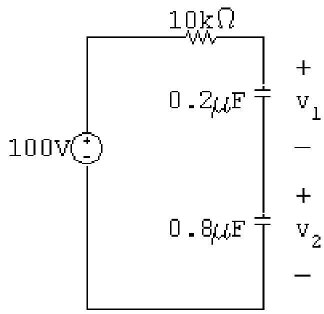
[b] $-z_1 = -z_2 = -50{,}000$ rad/s

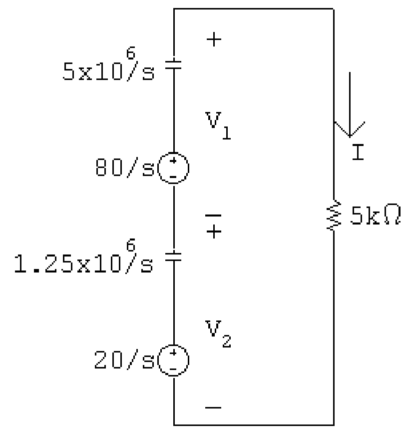
$-p_1 = -40{,}000 - j30{,}000$ rad/s

$-p_2 = -40{,}000 + j30{,}000$ rad/s

AP 13.3 **[a]** At $t = 0^-$, $\quad 0.2v_1 = 0.8v_2; \quad v_1 = 4v_2; \quad v_1 + v_2 = 100\,\text{V}$

Therefore $\quad v_1(0^-) = 80\,\text{V} = v_1(0^+); \quad v_2(0^-) = 20\,\text{V} = v_2(0^+)$





$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

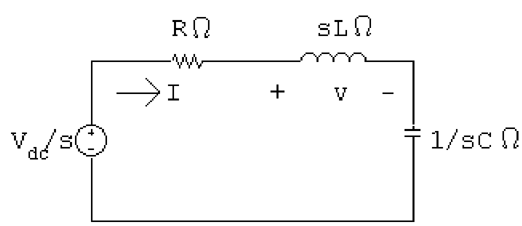
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\text{mA}; \qquad v_1 = 80e^{-1250t}u(t)\,\text{V}$

$v_2 = 20e^{-1250t}u(t)\,\text{V}$

AP 13.4 **[a]**



$$I = \frac{V_{\text{dc}}/s}{R + sL + (1/sC)} = \frac{V_{\text{dc}}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{\text{dc}}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{s^2 + 1.2s + 1}$$

[b] $I = \dfrac{40}{(s+0.6-j0.8)(s+0.6+j0.8)} = \dfrac{K_1}{s+0.6-j0.8} + \dfrac{K_1^*}{s+0.6+j0.8}$

$$K_1 = \frac{40}{j1.6} = -j25 = 25\angle -90^\circ; \qquad K_1^* = 25\angle 90^\circ$$

$$i = 50e^{-0.6t}\cos(0.8t - 90^\circ) = [50e^{-0.6t}\sin 0.8t]u(t)\,\text{A}$$

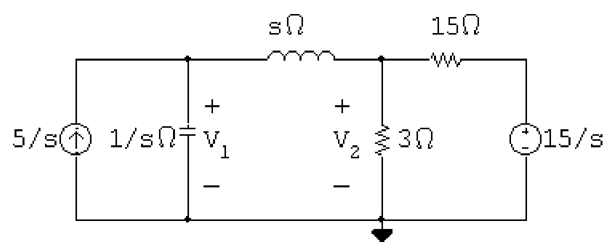
[c] $V = sLI = \dfrac{160s}{s^2+1.2s+1} = \dfrac{160s}{(s+0.6-j0.8)(s+0.6+j0.8)}$

$$= \frac{K_1}{s+0.6-j0.8} + \frac{K_1^*}{s+0.6+j0.8}$$

$$K_1 = \frac{160(-0.6+j0.8)}{j1.6} = 100\angle 36.87^\circ$$

[d] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87^\circ)]u(t)\,\text{V}$

AP 13.5 **[a]**



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2+2.5s+1)}, \qquad V_2 = \frac{2.5(s^2+6)}{s(s^2+2.5s+1)}$$

[b] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s+0.5} + \frac{5/3}{s+2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s+0.5} + \frac{25/3}{s+2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\,\text{V} \quad \text{and}$$

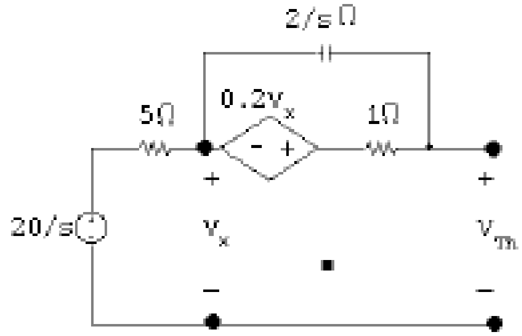
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\,\text{V}$$

[c] $v_1(0^+) = 15 - \dfrac{50}{3} + \dfrac{5}{3} = 0$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\,\text{V}$$

[d] $v_1(\infty) = 15\,\text{V}; \qquad v_2(\infty) = 15\,\text{V}$

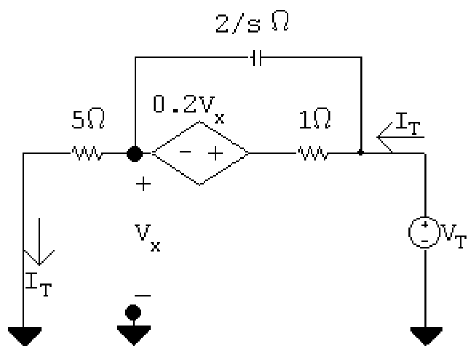
AP 13.6 **[a]**



With no load across terminals a–b, $\quad V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\text{Th}}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\text{Th}}\right] = 0$$

therefore $\quad V_{\text{Th}} = \dfrac{20(s+2.4)}{s(s+2)}$



$V_x = 5I_T \quad$ and $\quad Z_{\text{Th}} = \dfrac{V_T}{I_T}$

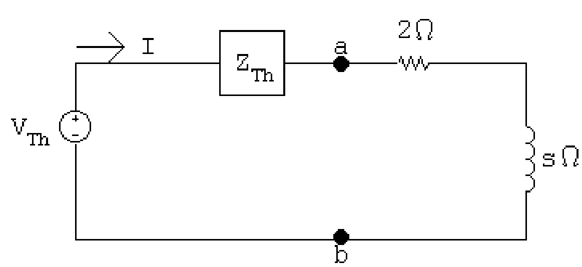
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$14I_T = V_T s - 5sI_T + 2V_T; \qquad$ therefore $\quad Z_{\text{Th}} = \dfrac{5(s+2.8)}{s+2}$

[b]



$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + 2 + s} = \frac{20(s+2.4)}{s(s+3)(s+6)}$$

AP 13.7 **[a]** $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3$, $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s+1)(s+3)$ and $N_1 = 60(s+2)$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s+2)}{(s+1)(s+3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s+1} + \frac{2.5}{s+3}$$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\,\text{A}$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\frac{di_1(0.54931)}{dt} = -2.89\,\text{A/s}$

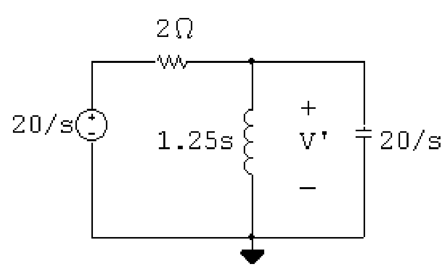
[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2\left(\frac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\frac{M}{12}\right)\left(\frac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \frac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (Checks)

AP 13.8 **[a]** The s-domain circuit with the voltage source acting alone is

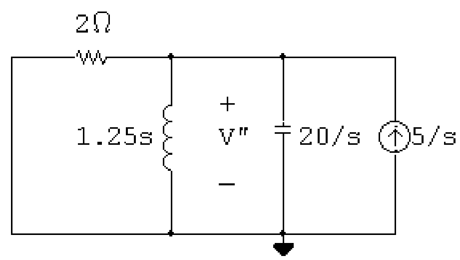


$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[b] With the current source acting alone,



$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\,\text{V}$

AP 13.9 **[a]** $\dfrac{V_o}{s+2} + \dfrac{V_o s}{10} = I_g;$ therefore $\dfrac{V_o}{I_g} = H(s) = \dfrac{10(s+2)}{s^2+2s+10}$

[b] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 **[a]** $V_o = \dfrac{10(s+2)}{s^2+2s+10} \cdot \dfrac{1}{s} = \dfrac{K_0}{s} + \dfrac{K_1}{s+1-j3} + \dfrac{K_1^*}{s+1+j3}$

$K_0 = 2;\quad K_1 = (5/3)\underline{/-126.87^\circ};\quad K_1^* = (5/3)\underline{/126.87^\circ}$

$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\,\text{V}$

[b] $V_o = \dfrac{10(s+2)}{s^2+2s+10} \cdot 1 = \dfrac{K_2}{s+1-j3} + \dfrac{K_2^*}{s+1+j3}$

$K_2 = 5.27\underline{/-18.43^\circ};\quad K_2^* = 5.27\underline{/18.43^\circ}$

$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\,\text{V}$

AP 13.11 **[a]** $H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$

$$\begin{aligned} v_o(t) &= 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t \\ &= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t \end{aligned}$$

Therefore $H(s) = \dfrac{9600(s+70)}{(s+70)^2 + (240)^2} - \dfrac{2800(240)}{(s+70)^2 + (240)^2}$

$$= \frac{9600s}{s^2 + 140s + 62{,}500}$$

[b] $V_o(s) = H(s)\cdot\dfrac{1}{s} = \dfrac{9600}{s^2 + 140s + 62{,}500}$

$$= \frac{K_1}{s + 70 - j240} + \frac{K_1^*}{s + 70 + j240}$$

$$K_1 = \frac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$$

Therefore

$$v_o(t) = [40e^{-70t}\cos(240t - 90^\circ)]u(t)\,\text{V} = [40e^{-70t}\sin 240t]u(t)\,\text{V}$$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2 + 2s + 10}$$

Therefore $H(j4) = \dfrac{10(2+j4)}{10 - 16 + j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$$v_o = (10)(4.47)\cos(4t - 63.43^\circ) = 44.7\cos(4t - 63.43^\circ)\,\text{V}$$

AP 13.13 **[a]** Let $R_1 = 10\,\text{k}\Omega$, $R_2 = 50\,\text{k}\Omega$, $C = 400$ pF, $R_2C = 2\times 10^{-5}$

then $V_1 = V_2 = \dfrac{V_gR_2}{R_2 + (1/sC)}$

Also $\dfrac{V_1 - V_g}{R_1} + \dfrac{V_1 - V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \dfrac{R_2Cs - 1}{R_2Cs + 1}$

It follows that $H(j50{,}000) = \dfrac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t + 90^\circ)\,\text{V}$

[b] Replacing R_2 by R_x gives us $\quad H(s) = \dfrac{R_xCs - 1}{R_xCs + 1}$

Therefore

$$H(j50{,}000) = \frac{j20 \times 10^{-6}R_x - 1}{j20 \times 10^{-6}R_x + 1} = \frac{R_x + j50{,}000}{R_x - j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

Problems

P 13.1 $I_{sc_{ab}} = I_N = \frac{-LI_0}{sL} = \frac{-I_0}{s}; \qquad Z_N = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.2 $i = \frac{1}{L}\int_{0^-}^{t} v\,d\tau + I_0; \qquad$ therefore $\quad I = \left(\frac{1}{L}\right)\left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.3 $V_{\text{Th}} = V_{\text{ab}} = CV_o\left(\frac{1}{sC}\right) = \frac{V_o}{s}; \qquad Z_{\text{Th}} = \frac{1}{sC}$

P 13.4 **[a]** $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{0.0025[s^2 + 16 \times 10^7 s + 10^{10}]}{s}$$

[b] Zeros at -62.5 rad/s and -1.6×10^8 rad/s

Pole at 0.

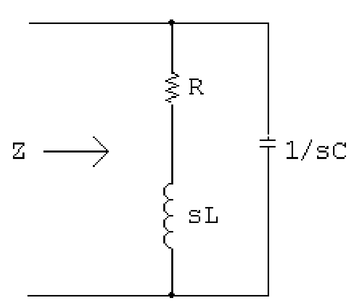
P 13.5 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{4 \times 10^6 s}{s^2 + 2000s + 64 \times 10^4}$$

[b] zero at $-z_1 = 0$

poles at $-p_1 = -400$ rad/s and $-p_2 = -1600$ rad/s

P 13.6 **[a]**



$$Z = \frac{(R + sL)(1/sC)}{R + sL + (1/sC)} = \frac{(1/C)(s + R/L)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = \frac{250}{0.08} = 3125; \qquad \frac{1}{LC} = \frac{1}{(0.08)(0.5 \times 10^{-6})} = 25 \times 10^6$$

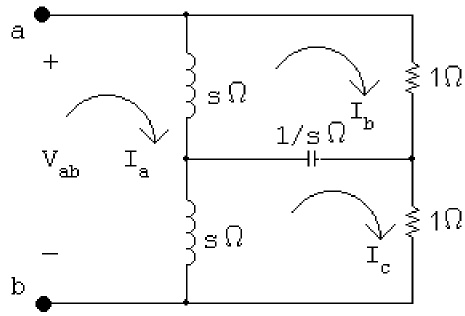
$$Z = \frac{2 \times 10^6(s + 3125)}{s^2 + 3125s + 25 \times 10^6}$$

[b] $Z = \dfrac{2 \times 10^6(s + 3125)}{(s + 1562.5 - j4749.6)(s + 1562.5 + j4749.6)}$

$-z_1 = -3125$ rad/s; $\quad -p_1 = -1562.5 + j4749.6$ rad/s

$-p_2 = -1562.5 - j4749.6$ rad/s

P 13.7 Transform the Y-connection of the two resistors and the capacitor into the equivalent delta-connection:



where

$$Z_{\rm a} = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{1/s} = s + 2$$

$$Z_{\rm b} = Z_{\rm c} = \frac{(1/s)(1) + (1)(1/s) + (1)(1)}{1} = \frac{s+2}{s}$$

Then

$$Z_{\rm ab} = Z_{\rm a}\|[(s\|Z_{\rm c}) + (s\|Z_{\rm b})] = Z_{\rm a}\|2(s\|Z_{\rm b})$$

$$s\|Z_{\rm b} = \frac{s+2}{s + (s+2)/s} = \frac{s(s+2)}{s^2 + s + 2}$$

$$Z_{\rm ab} = (s+2)\|\frac{2s(s+2)}{s^2+s+2} = \frac{2s(s+2)^2}{(s+2)(s^2+s+2) + 2s(s+2)}$$

$$= \frac{2s(s+2)}{s^2+3s+2} = \frac{2s}{s+1}$$

One zero at the origin (0 rad/s); one pole at -1 rad/s.

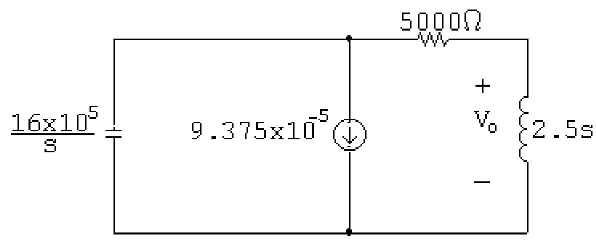
P 13.8 $Z_1 = \dfrac{16}{s} + s\|4 = \dfrac{16}{s} + \dfrac{4s}{s+4} = \dfrac{4(s^2 + 4s + 16)}{s(s+4)}$

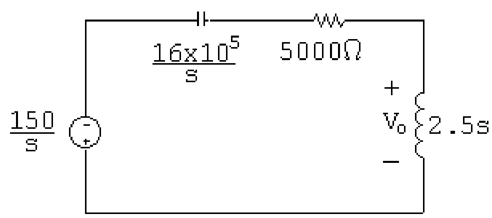
$$Z_{\rm ab} = 4\|\frac{4(s^2+4s+16)}{s(s+4)} = \frac{16(s^2+4s+16)}{8s^2+32s+64}$$

$$= \frac{2(s^2 + 4s + 16)}{s^2 + 4s + 8} = \frac{2(s + 2 + j3.46)(s + 2 - j3.46)}{(s + 2 + j2)(s + 2 - j2)}$$

Zeros at $-2 + j3.46$ rad/s and $-2 - j3.46$ rad/s; poles at $-2 + j2$ rad/s and $-2 - j2$ rad/s.

P 13.9 **[a]** For $t > 0$:





[b] $V_o = \frac{2.5s}{(16 \times 10^5)/s + 5000 + 2.5s}\left(\frac{-150}{s}\right)$

$$= \frac{-150s}{s^2 + 2000s + 64 \times 10^4}$$

$$= \frac{-150s}{(s + 400)(s + 1600)}$$

[c] $V_o = \frac{K_1}{s + 400} + \frac{K_2}{s + 1600}$

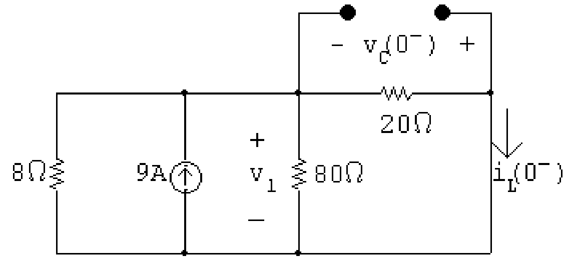
$$K_1 = \frac{-150s}{s + 1600}\bigg|_{s=-400} = 50$$

$$K_2 = \frac{-150s}{s + 400}\bigg|_{s=-1600} = -200$$

$$V_o = \frac{50}{s + 400} - \frac{200}{s + 1600}$$

$$v_o(t) = (50e^{-400t} - 200e^{-1600t})u(t)\,\text{V}$$

P 13.10 **[a]** For $t < 0$:



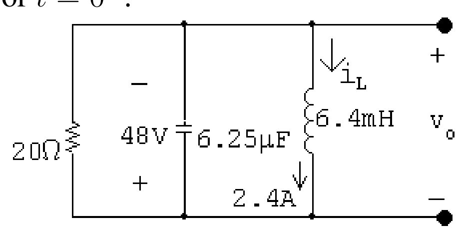
$$\frac{1}{R_e} = \frac{1}{8} + \frac{1}{80} + \frac{1}{20} = 0.1875; \qquad R_e = 5.33\,\Omega$$

$$v_1 = (9)(5.33) = 48\,\text{V}$$

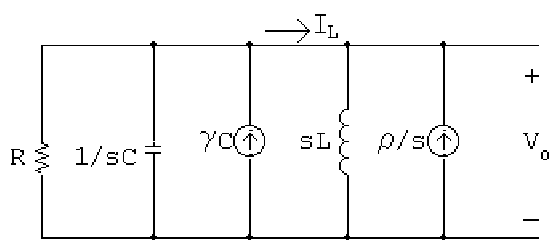
$$i_\text{L}(0^-) = \frac{48}{20} = 2.4\,\text{A}$$

$$v_\text{C}(0^-) = -v_1 = -48\,\text{V}$$

For $t = 0^+$:



s-domain circuit:



where

$R = 20\,\Omega; \qquad C = 6.25\,\mu\text{F}; \qquad \gamma = -48\,\text{V};$

$L = 6.4\,\text{mH}; \qquad$ and $\qquad \rho = -2.4\,\text{A}$

[b] $$\frac{V_o}{R} + V_o sC - \gamma C + \frac{V_o}{sL} - \frac{\rho}{s} = 0$$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-2.4}{(-48)(6.25 \times 10^{-6})} = 8000$$

$$\frac{1}{RC} = \frac{1}{(20)(6.25 \times 10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(6.4 \times 10^{-3})(6.25 \times 10^{-6})} = 25 \times 10^6$$

$$V_o = \frac{-48(s + 8000)}{s^2 + 8000s + 25 \times 10^6}$$

[c] $$I_L = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{0.0064s} + \frac{2.4}{s}$$

$$= \frac{-7500(s + 8000)}{s(s^2 + 8000s + 25 \times 10^6)} + \frac{2.4}{s} = \frac{2.4(s + 4875)}{(s^2 + 8000s + 25 \times 10^6)}$$

[d] $$V_o = \frac{-48(s + 8000)}{s^2 + 8000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{-48(s + 8000)}{s + 4000 + j3000}\bigg|_{s=-4000+j3000} = 40\underline{/126.87^\circ}$$

$$v_o(t) = [80e^{-4000t}\cos(3000t + 126.87^\circ)]u(t)\ \text{V}$$

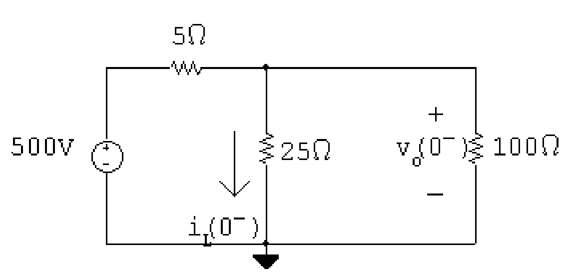
[e] $$I_L = \frac{2.4(s + 4875)}{s^2 + 8000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 4000 - j3000} + \frac{K_1^*}{s + 4000 + j3000}$$

$$K_1 = \frac{2.4(s + 4875)}{s + 4000 + j3000}\bigg|_{s=-4000+j3000} = 1.25\underline{/-16.26^\circ}$$

$$i_L(t) = [2.5e^{-4000t}\cos(3000t - 16.26^\circ)]u(t)\ \text{A}$$

P 13.11 For $t < 0$:

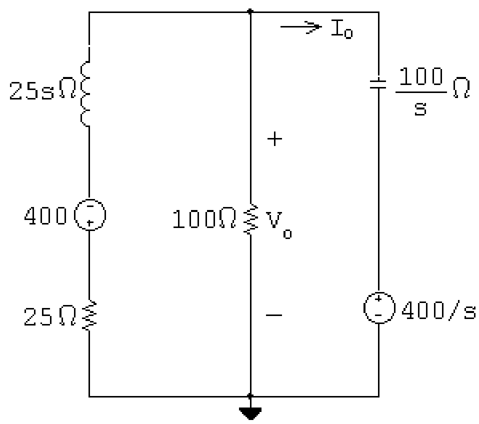


$$\frac{v_o(0^-) - 500}{5} + \frac{v_o(0^-)}{25} + \frac{v_o(0^-)}{100} = 0$$

$$25v_o(0^-) = 10{,}000 \qquad \therefore \qquad v_o(0^-) = 400\,\text{V}$$

$$i_L(0^-) = \frac{v_o(0^-)}{25} = \frac{400}{25} = 16\,\text{A}$$

For $t > 0$:



$$\frac{V_o + 400}{25 + 25s} + \frac{V_o}{100} + \frac{V_o - (400/s)}{100/s} = 0$$

$$V_o\left(\frac{1}{25 + 25s} + \frac{1}{100} + \frac{s}{100}\right) = 4 - \frac{400}{25 + 25s}$$

$$\therefore \qquad V_o = \frac{400(s-3)}{s^2 + 2s + 5}$$

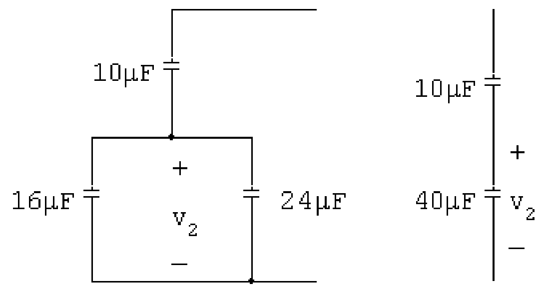
$$I_o = \frac{V_o - (400/s)}{100/s} = \frac{-20s - 20}{s^2 + 2s + 5}$$

$$= \frac{K_1}{s + 1 - j2} + \frac{K_1^*}{s + 1 + j2}$$

$$K_1 = \frac{-20(s+1)}{s + 1 + j2}\bigg|_{s=-1+j2} = -10$$

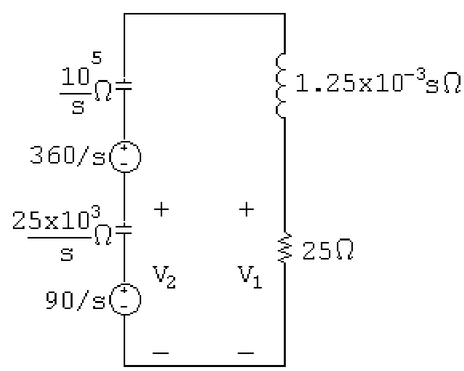
$$i_o(t) = [-20e^{-t}\cos 2t]u(t)\,\text{A}$$

P 13.12 **[a]** For $t < 0$:



$$V_2 = \frac{10}{10+40}(450) = 90\,\text{V}$$

For $t > 0$:



[b] $V_1 = \dfrac{25(450/s)}{(125{,}000/s) + 25 + 1.25 \times 10^{-3}s}$

$$= \frac{9 \times 10^6}{s^2 + 20,000s + 10^8} = \frac{9 \times 10^6}{(s + 10{,}000)^2}$$

$$v_1(t) = (9 \times 10^6 te^{-10{,}000t})u(t)\,\text{V}$$

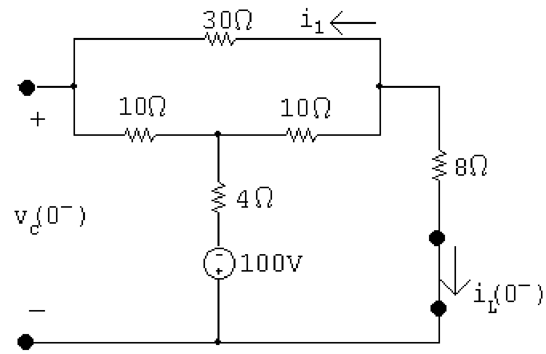
[c] $V_2 = \dfrac{90}{s} - \dfrac{(25{,}000/s)(450/s)}{(125{,}000/s) + 1.25 \times 10^{-3}s + 25}$

$$= \frac{90(s + 20{,}000)}{s^2 + 20{,}000s + 10^8}$$

$$= \frac{900{,}000}{(s + 10{,}000)^2} + \frac{90}{s + 10{,}000}$$

$$v_2(t) = [9 \times 10^5 te^{-10{,}000t} + 90e^{-10{,}000t}]u(t)\,\text{V}$$

P 13.13 **[a]** For $t < 0$:

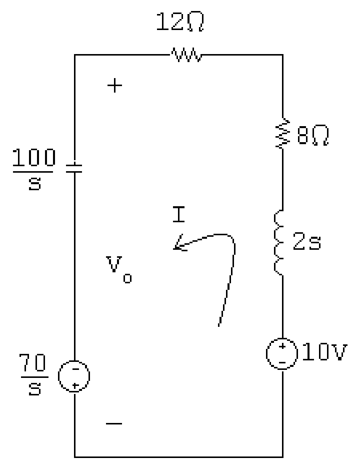


$$i_L(0^-) = \frac{-100}{4 + 10\|40 + 8} = \frac{-100}{20} = -5\,\text{A}$$

$$i_1 = \frac{10}{50}(5) = 1\,\text{A}$$

$$v_C(0^-) = 10(1) + 4(5) - 100 = -70\,\text{V}$$

For $t > 0$:



[b] $(20 + 2s + 100/s)I = 10 + \dfrac{70}{s}$

$$\therefore \qquad I = \frac{5(s+7)}{s^2 + 10s + 50}$$

$$V_o = \frac{100}{s}I - \frac{70}{s}$$

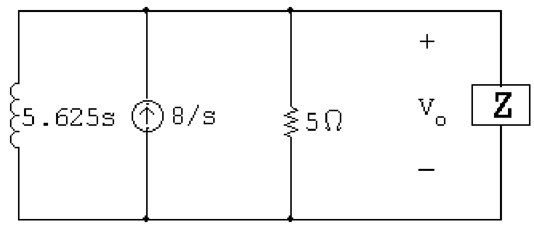
$$= \frac{-70s^2 - 200s}{s(s^2 + 10s + 50)} = \frac{-70(s + 20/7)}{s^2 + 10s + 50}$$

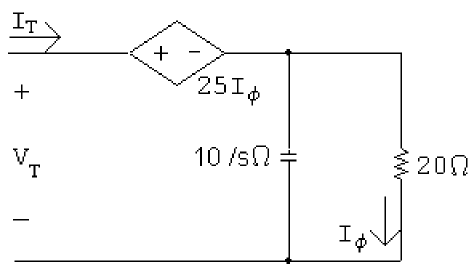
$$= \frac{K_1}{s + 5 - j5} + \frac{K_1^*}{s + 5 + j5}$$

$$K_1 = \left.\frac{-70(s + 20/7)}{s + 5 + j5}\right|_{s=-5+j5} = 38.1\underline{/-156.8^\circ}$$

[c] $v_o(t) = 76.2e^{-5t}\cos(5t - 156.8^\circ)u(t)\,\text{V}$

P 13.14 **[a]** $i_L(0^-) = i_L(0^+) = \frac{24}{3} = 8\,\text{A}$ directed upward





$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20 + (10/s)}\right] I_T = \frac{25I_T(10/s)}{20 + (10/s)} + \left(\frac{200}{10 + 20s}\right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250 + 200}{20s + 10} = \frac{45}{2s + 1}$$

$$\frac{V_o}{5} + \frac{V_o(2s + 1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

$$\frac{[9s + (2s + 1)s + 8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2 + 10s + 8] = 360$$

$$V_o = \frac{360}{2s^2 + 10s + 8} = \frac{180}{s^2 + 5s + 4}$$

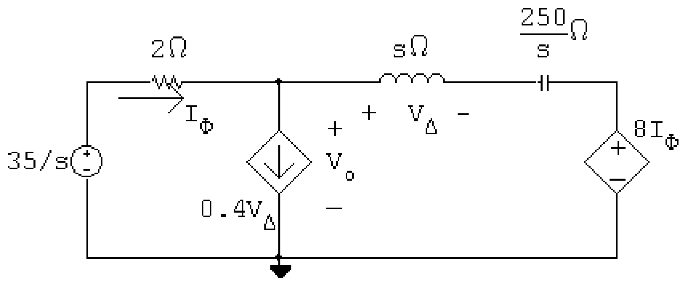
[b] $V_o = \frac{180}{(s + 1)(s + 4)} = \frac{K_1}{s + 1} + \frac{K_2}{s + 4}$

$$K_1 = \frac{180}{3} = 60; \qquad K_2 = \frac{180}{-3} = -60$$

$$V_o = \frac{60}{s + 1} - \frac{60}{s + 4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t)\,\text{V}$$

P 13.15 **[a]**



$$\frac{V_o - 35/s}{2} + 0.4V_\Delta + \frac{V_o - 8I_\phi}{s + (250/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 8I_\phi}{s + (250/s)}\right] s; \qquad I_\phi = \frac{(35/s) - V_o}{2}$$

Solving for V_o yields:

$$V_o = \frac{29.4s^2 + 56s + 1750}{s(s^2 + 2s + 50)} = \frac{29.4s^2 + 56s + 1750}{s(s + 1 - j7)(s + 1 + j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s + 1 - j7} + \frac{K_2^*}{s + 1 + j7}$$

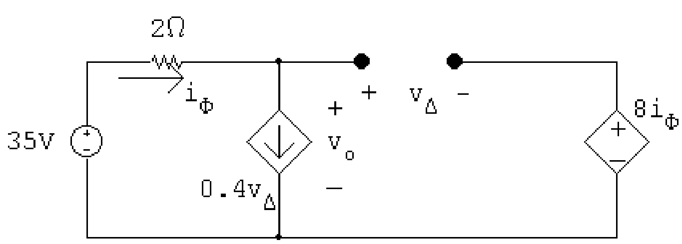
$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50}\bigg|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s + 1 + j7)}\bigg|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86\underline{/167.91^\circ}$$

$$\therefore \quad v_o(t) = [35 + 5.73e^{-t}\cos(7t + 167.91^\circ)]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 35 + 5.73\cos(167.91^\circ) = 29.4\,\text{V}$

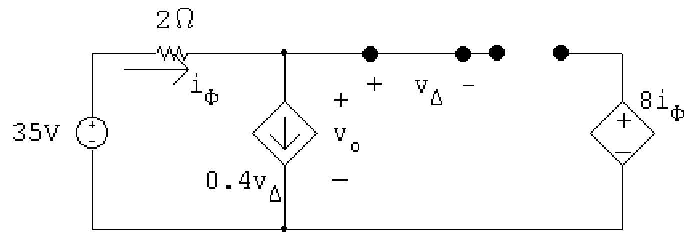


$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \qquad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2v_\Delta$$

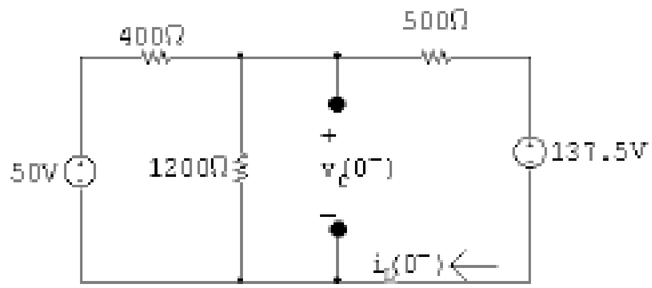
$$v_o + (0.8)\frac{v_o}{4.2} = 35; \qquad \therefore \quad v_o(0^+) = 29.4\,\text{V(Checks)}$$

At $t = \infty$, the circuit is



$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 35\,\text{V(Checks)}$

P 13.16 **[a]** For $t < 0$:



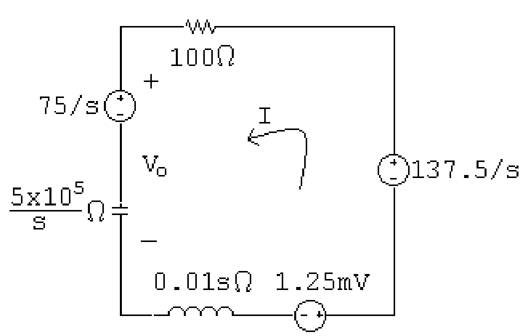
$$\frac{V_c - 50}{400} + \frac{V_c}{1200} + \frac{V_c - 137.5}{500} = 0$$

$$V_c\left(\frac{1}{400} + \frac{1}{1200} + \frac{1}{500}\right) = \frac{50}{400} + \frac{137.5}{500}$$

$$V_c = 75\,\text{V}$$

$$i_L(0^-) = \frac{75 - 137.5}{500} = -0.125\,\text{A}$$

For $t > 0$:



[b] $V_o = \dfrac{5 \times 10^5}{s} I + \dfrac{75}{s}$

$$0 = -\frac{137.5}{s} + 100I + \frac{5 \times 10^5}{s} I + \frac{75}{s} - 1.25 \times 10^{-3} + 0.01sI$$

$$I\left(100 + \frac{5\times 10^5}{s} + 0.01s\right) = \frac{62.5}{s} + 1.25\times 10^{-3}$$

$$\therefore \quad I = \frac{6250 + 0.125s}{s^2 + 10^4 s + 5\times 10^7}$$

$$V_o = \frac{5\times 10^5}{s}\left(\frac{6250 + 0.125s}{s^2 + 10^4 s + 5\times 10^7}\right) + \frac{75}{s}$$

$$= \frac{75s^2 + 812{,}500s + 6875\times 10^6}{s(s^2 + 10^4 s + 5\times 10^7)}$$

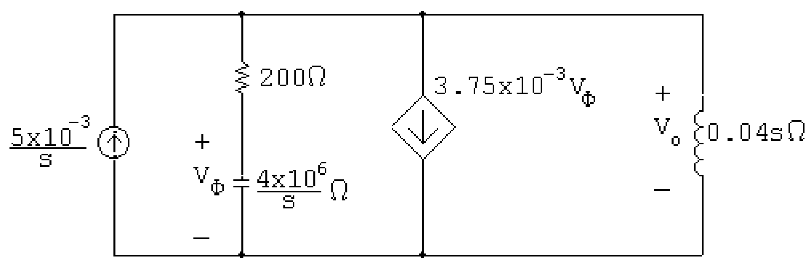
[c] $V_o = \frac{K_1}{s} + \frac{K_2}{s + 5000 - j5000} + \frac{K_2^*}{s + 5000 + j5000}$

$$K_1 = \frac{75s^2 + 812{,}500s + 6875\times 10^6}{s^2 + 10^4 s + 5\times 10^7}\bigg|_{s=0} = 137.5$$

$$K_2 = \frac{75s^2 + 812{,}500s + 6875\times 10^6}{s(s + 5000 + j5000)}\bigg|_{s=-5000+j5000} = 40.02\underline{/141.34^\circ}$$

$$v_o(t) = [137.5 + 80.04e^{-5000t}\cos(5000t + 141.34^\circ)]u(t)\ \text{V}$$

P 13.17



$$\frac{5\times 10^{-3}}{s} = \frac{V_o}{200 + 4\times 10^6/s} + 3.75\times 10^{-3}V_\phi + \frac{V_o}{0.04s}$$

$$V_\phi = \frac{4\times 10^6/s}{200 + 4\times 10^6/s}V_o = \frac{4\times 10^6 V_o}{200s + 4\times 10^6}$$

$$\therefore \qquad \frac{5\times 10^{-3}}{s} = \frac{V_o s}{200s + 4\times 10^6} + \frac{15{,}000V_o}{200s + 4\times 10^6} + \frac{25V_o}{s}$$

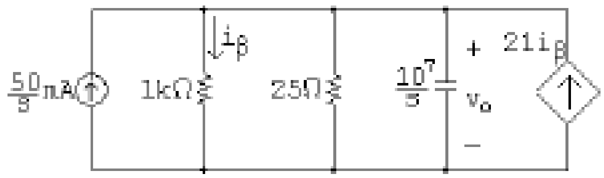
$$\therefore \quad V_o = \frac{s + 20{,}000}{s^2 + 20{,}000s + 10^8} = \frac{K_1}{(s + 10{,}000)^2} + \frac{K_2}{s + 10{,}000}$$

$K_1 = 10{,}000; \qquad K_2 = 1$

$$V_o = \frac{10{,}000}{(s + 10{,}000)^2} + \frac{1}{s + 10{,}000}$$

$$v_o(t) = [10{,}000te^{-10{,}000t} + e^{-10{,}000t}]u(t)\ \text{V}$$

P 13.18 $v_o(0^-) = v_o(0^+) = 0$



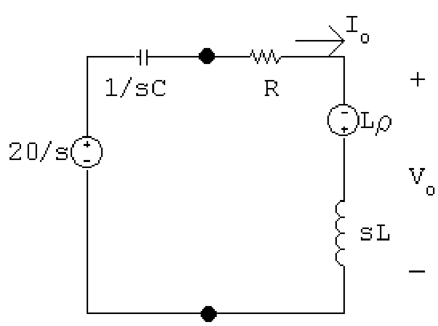
$$-\frac{0.05}{s} + \frac{V_o}{1000} + \frac{V_o}{25} - 21\frac{V_o}{1000} + \frac{V_o}{10^7/s} = 0$$

$$V_o\left(\frac{20}{1000} + \frac{s}{10^7}\right) = \frac{0.05}{s}$$

$$\therefore \qquad V_o = \frac{500{,}000}{s(s + 200{,}000)} = \frac{2.5}{s} - \frac{2.5}{s + 200{,}000}$$

$$v_o(t) = [2.5 - 2.5e^{-200{,}000t}]u(t)\,\text{V}$$

P 13.19 **[a]** $i_o(0^-) = \dfrac{20}{4000} = 5\,\text{mA}$



$$I_o = \frac{20/s + L\rho}{R + sL + 1/sC}$$

$$= \frac{20/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{40 + s(0.005)}{s^2 + 8000s + 16 \times 10^6}$$

$$V_o = -L\rho + sLI_o = -0.0025 + \frac{0.0025s(s + 8000)}{s^2 + 8000s + 16 \times 10^6}$$

$$= \frac{-40{,}000}{(s + 4000)^2}$$

$$v_o(t) = -40{,}000te^{-4000t}u(t)\,\text{V}$$

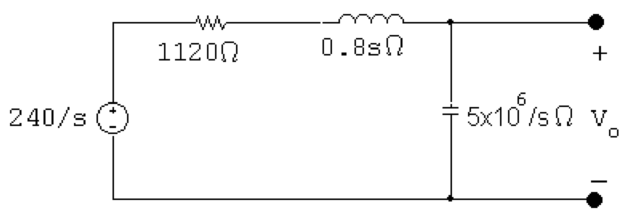
[b] $I_o = \dfrac{0.005(s+8000)}{s^2+8000s+16\times 10^6}$

$$= \frac{K_1}{(s+4000)^2} + \frac{K_2}{s+4000}$$

$K_1 = 20 \qquad K_2 = 0.005$

$i_o(t) = [20te^{-4000t} + 0.005e^{-4000t}]u(t)\text{ A}$

P 13.20



$$V_o = \frac{5\times 10^6/s}{1120+0.8s+5\times 10^6/s}\left(\frac{240}{s}\right)$$

$$= \frac{12\times 10^8}{s(0.8s^2+1120s+5\times 10^6)}$$

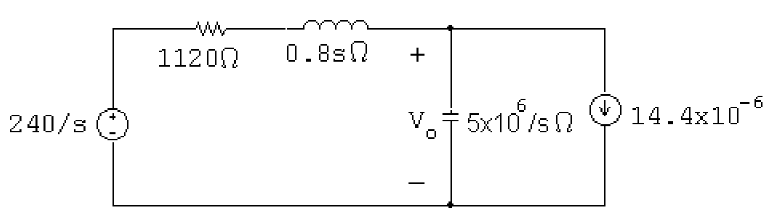
$$= \frac{15\times 10^8}{s(s^2+1400s+625\times 10^4)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+700-j2400} + \frac{K_2^*}{s+700+j2400}$$

$K_1 = 240; \qquad K_2 = 125\underline{/163.74^\circ}$

$v_o(t) = [240 + 250e^{-700t}\cos(2400t + 163.74^\circ)]u(t)\text{ V}$

P 13.21



$$\frac{V_o - 240/s}{1120+0.8s} + \frac{V_o s}{5\times 10^6} + 14.4\times 10^{-6} = 0$$

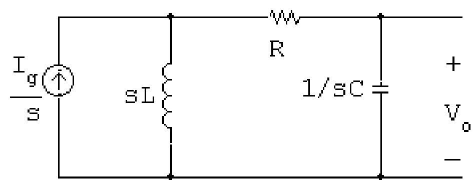
$$V_o\left(\frac{1}{1120+0.8s}+\frac{s}{5\times10^6}\right)=\frac{240/s}{0.8s+1120}-14.4\times10^{-6}$$

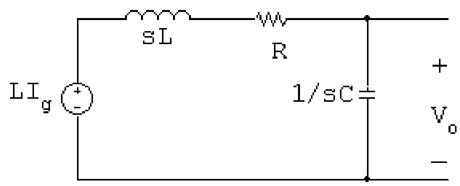
$$V_o=\frac{-72s^2-100{,}800s+15\times10^8}{s(s^2+1400s+625\times10^4)}$$

$$=\frac{240}{s}+\frac{162.5/\!\underline{163.74^\circ}}{s+700-j2400}+\frac{162.5/\!\underline{-163.74^\circ}}{s+700+j2400}$$

$$\therefore\qquad v_o(t)=[240+325e^{-700t}\cos(2400t+163.74^\circ)]u(t)\text{ V}$$

P 13.22 **[a]**





$$V_o=\frac{(1/sC)(LI_g)}{R+sL+(1/sC)}=\frac{I_g/C}{s^2+(R/L)s+(1/LC)}$$

$$\frac{I_g}{C}=\frac{15}{0.1}=150$$

$$\frac{R}{L}=7;\qquad \frac{1}{LC}=10$$

$$V_o=\frac{150}{s^2+7s+10}$$

[b] $sV_o=\dfrac{150s}{s^2+7s+10}$

$$\lim_{s\to0}sV_o=0;\qquad\therefore\quad v_o(\infty)=0$$

$$\lim_{s\to\infty}sV_o=0;\qquad\therefore\quad v_o(0^+)=0$$

[c] $V_o=\dfrac{150}{(s+2)(s+5)}=\dfrac{50}{s+2}+\dfrac{-50}{s+5}$

$$v_o=[50e^{-2t}-50e^{-5t}]u(t)\text{ V}$$

P 13.23 $I_L = \frac{I_g}{s} - \frac{V_o}{1/sC} = \frac{I_g}{s} - sCV_o$

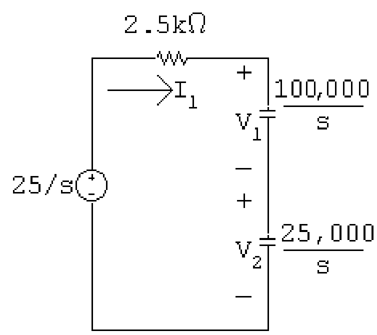
$$I_L = \frac{15}{s} - \frac{15s}{(s+2)(s+5)} = \frac{15}{s} - \left[\frac{-10}{s+2} + \frac{25}{s+5}\right]$$

$$i_L(t) = [15 + 10e^{-2t} - 25e^{-5t}]u(t)\,\text{A}$$

Check:

$$i_L(0^+) = 0 \quad \text{(ok)}; \qquad i_L(\infty) = 15 \quad \text{(ok)}$$

P 13.24 **[a]**



[b] $I_1 = \frac{25/s}{2500 + (125{,}000/s)} = \frac{0.01}{s+50}$

$$V_1 = \frac{(100{,}000/s)(25/s)}{2500 + (125{,}000/s)} = \frac{1000}{s(s+50)}$$

$$V_2 = \frac{(25{,}000/s)(25/s)}{2500 + (125{,}000/s)} = \frac{250}{s(s+50)}$$

[c] $i_1(t) = 10e^{-50t}u(t)\,\text{mA}$

$$V_1 = \frac{20}{s} - \frac{20}{s+50} \qquad \therefore \qquad v_1(t) = (20 - 20e^{-50t})u(t)\,\text{V}$$

$$V_2 = \frac{5}{s} - \frac{5}{s+50} \qquad \therefore \qquad v_2(t) = (5 - 5e^{-50t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 10\,\text{mA}$

$$i_1(0^+) = \frac{25}{2.5 \times 10^{-3}} = 10\,\text{mA(Checks)}$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0\text{(Checks)}$$

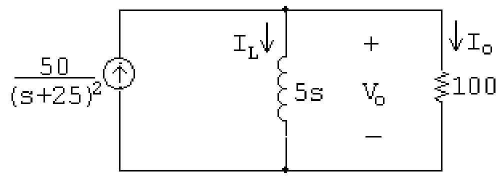
$$v_1(\infty) = 20\,\text{V}; \qquad v_2(\infty) = 5\,\text{V(Checks)}$$

$v_1(\infty) + v_2(\infty) = 25\,\text{V(Checks)}$

$(10 \times 10^{-6})v_1(\infty) = 200\,\mu\text{C}$

$(40 \times 10^{-6})v_2(\infty) = 200\,\mu\text{C(Checks)}$

P 13.25 **[a]**



$$100 \| 5s = \frac{500s}{5s+100} = \frac{100s}{s+20}$$

$$V_o = \frac{100s}{s+20}\left[\frac{50}{(s+25)^2}\right] = \frac{5000s}{(s+20)(s+25)^2}$$

$$I_o = \frac{V_o}{100} = \frac{50s}{(s+20)(s+25)^2}$$

$$I_L = \frac{V_o}{5s} = \frac{1000}{(s+20)(s+25)^2}$$

[b] $$V_o = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$$

$$K_1 = \frac{5000s}{(s+25)^2}\Big|_{s=-20} = -4000$$

$$K_2 = \frac{5000s}{(s+20)}\Big|_{s=-25} = 25{,}000$$

$$K_3 = \frac{d}{ds}\left[\frac{5000s}{s+20}\right]_{s=-25} = \left[\frac{5000}{s+20} - \frac{5000s}{(s+20)^2}\right]_{s=-25} = 4000$$

$$v_o(t) = [-4000e^{-20t} + 25{,}000te^{-25t} + 4000e^{-25t}]u(t)\,\text{V}$$

$$I_o = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$$

$$K_1 = \frac{50s}{(s+25)^2}\Big|_{s=-20} = -40$$

$$K_2 = \frac{50s}{(s+20)}\Big|_{s=-25} = 250$$

$$K_3 = \frac{d}{ds}\left[\frac{50s}{s+20}\right]_{s=-25} = \left[\frac{50}{s+20} - \frac{50s}{(s+20)^2}\right]_{s=-25} = 40$$

$$i_o(t) = [-40e^{-20t} + 250te^{-25t} + 40e^{-25t}]u(t)\,\mathrm{V}$$

$$I_L = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$$

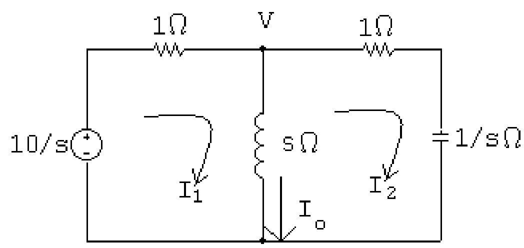
$$K_1 = \frac{1000}{(s+25)^2}\bigg|_{s=-20} = 40$$

$$K_2 = \frac{1000}{(s+20)}\bigg|_{s=-25} = -200$$

$$K_3 = \frac{d}{ds}\left[\frac{1000}{s+20}\right]_{s=-25} = \left[-\frac{1000}{(s+20)^2}\right]_{s=-25} = -40$$

$$i_L(t) = [40e^{-20t} - 200te^{-25t} - 40e^{-25t}]u(t)\,\mathrm{V}$$

P 13.26



$$\frac{10}{s} = (s+1)I_1 - sI_2$$

$$0 = -sI_1 + \left(s+1+\frac{1}{s}\right)I_2$$

In standard form,

$$s(s+1)I_1 - s^2I_2 = 10$$

$$-s^2I_1 + (s^2+s+1)I_2 = 0$$

$$\Delta = \begin{vmatrix} s(s+1) & -s^2 \\ -s^2 & (s^2+s+1) \end{vmatrix} = 2s(s^2+s+0.5)$$

$$N_1 = \begin{vmatrix} 10 & -s^2 \\ 0 & (s^2+s+1) \end{vmatrix} = 10(s^2+s+1)$$

$$N_2 = \begin{vmatrix} s(s+1) & 10 \\ -s^2 & 0 \end{vmatrix} = 10s^2$$

$$I_1 = \frac{N_1}{\Delta}; \qquad I_2 = \frac{N_2}{\Delta}; \qquad I_0 = I_1 - I_2$$

$$\therefore \qquad I_o = \frac{N_1 - N_2}{\Delta} = \frac{5(s+1)}{s(s^2+s+0.5)}$$

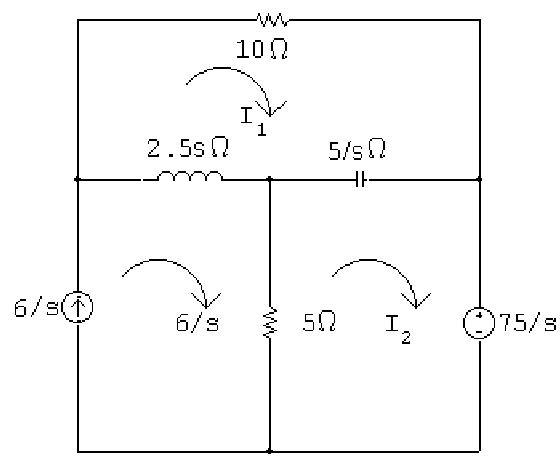
$$= \frac{K_1}{s} + \frac{K_2}{s+0.5-j0.5} + \frac{K_2^*}{s+0.5+j0.5}$$

$$K_1 = \frac{5}{0.5} = 10$$

$$K_2 = \frac{5(-0.5+j0.5+1)}{(-0.5+j0.5)(j1)} = 5\underline{/-180^\circ}$$

$$i_o(t) = [10 - 10e^{-t/2}\cos 0.5t]u(t)\,\text{A}$$

P 13.27 **[a]**



$$0 = 2.5s(I_1 - 6/s) + \frac{5}{s}(I_1 - I_2) + 10I_1$$

$$\frac{-75}{s} = \frac{5}{s}(I_2 - I_1) + 5(I_2 - 6/s)$$

or

$$(s^2+4s+2)I_1 - 2I_2 = 6s$$

$$-I_1 + (s+1)I_2 = -9$$

$$\Delta = \begin{vmatrix} (s^2+4s+2) & -2 \\ -1 & (s+1) \end{vmatrix} = s(s+2)(s+3)$$

$$N_1 = \begin{vmatrix} 6s & -2 \\ -9 & (s+1) \end{vmatrix} = 6(s^2 + s - 3)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{6(s^2 + s - 3)}{s(s+2)(s+3)}$$

$$N_2 = \begin{vmatrix} (s^2 + 4s + 2) & 6s \\ -1 & -9 \end{vmatrix} = -9s^2 - 30s - 18$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-9s^2 - 30s - 18}{s(s+2)(s+3)}$$

[b] $sI_1 = \dfrac{6(s^2 + s - 3)}{(s+2)(s+3)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 6\,\text{A}; \qquad \lim_{s\to 0} sI_1 = i_1(\infty) = -3\,\text{A}$$

$$sI_2 = \frac{-9s^2 - 30s - 18}{(s+2)(s+3)}$$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = -9\,\text{A}; \qquad \lim_{s\to 0} sI_2 = i_2(\infty) = -3\,\text{A}$$

[c] $I_1 = \dfrac{6(s^2 + s - 3)}{s(s+2)(s+3)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+2} + \dfrac{K_3}{s+3}$

$$K_1 = \frac{6(-3)}{6} = -3; \qquad K_2 = \frac{6(4 - 2 - 3)}{(-2)(1)} = 3$$

$$K_3 = \frac{6(9 - 3 - 3)}{(-3)(-1)} = 6$$

$$i_1(t) = [-3 + 3e^{-2t} + 6e^{-3t}]u(t)\,\text{A}$$

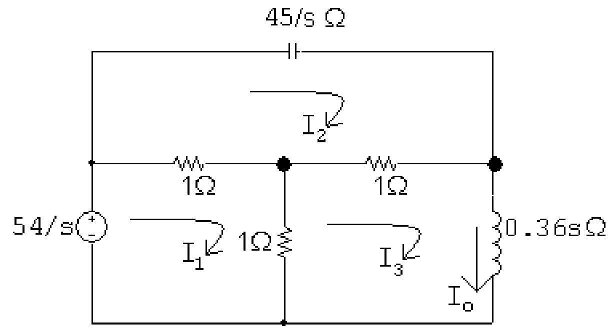
$$I_2 = \frac{-9s^2 - 30s - 18}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{-18}{6} = -3; \qquad K_2 = \frac{-36 + 60 - 18}{(-2)(1)} = -3$$

$$K_3 = \frac{-81 + 90 - 18}{(-3)(-1)} = -3$$

$$i_2(t) = [-3 - 3e^{-2t} - 3e^{-3t}]u(t)\,\text{A}$$

P 13.28 **[a]**



$$\frac{54}{s} = 2I_1 - I_2 - I_3$$

$$0 = -I_1 + \left(2 + \frac{45}{s}\right) I_2 - I_3$$

$$0 = -I_1 - I_2 + (2 + 0.36s)I_3$$

$$\Delta = \begin{vmatrix} 2 & -1 & -1 \\ -1 & (2s+45)/s & -1 \\ -1 & -1 & (0.36s+2) \end{vmatrix} = \frac{1.08(s+5)(s+25)}{s}$$

$$N_2 = \begin{vmatrix} 2 & (54/s) & -1 \\ -1 & 0 & -1 \\ -1 & 0 & (0.36s+2) \end{vmatrix} = \frac{162}{s}(0.12s+1)$$

$$N_3 = \begin{vmatrix} 2 & -1 & (54/s) \\ -1 & (2s+45)/s & 0 \\ -1 & -1 & 0 \end{vmatrix} = \frac{162}{s^2}(s+15)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{150(0.12s+1)}{(s+5)(s+25)}$$

$$V_o = \frac{45}{s} I_2 = \frac{6750(0.12s+1)}{s(s+5)(s+25)}$$

$$I_3 = \frac{N_3}{\Delta} = \frac{150(s+15)}{s(s+5)(s+25)} = I_o$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s+5} + \dfrac{K_3}{s+25}$

$$K_1 = \frac{6750}{125} = 54; \quad K_2 = \frac{6750(-0.6+1)}{(-5)(20)} = -27$$

$$K_3 = \frac{6750(-3+1)}{(-25)(-20)} = -27$$

$$\therefore \quad v_o(t) = [54 - 27e^{-5t} - 27e^{-25t}]u(t)\,\text{V}$$

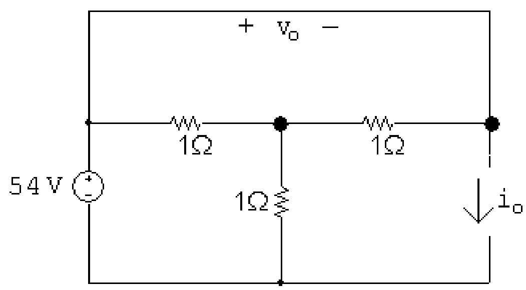
$$I_o = \frac{K_1}{s} + \frac{K_2}{s+5} + \frac{K_3}{s+25}$$

$$K_1 = \frac{150(15)}{(5)(25)} = 18; \quad K_2 = \frac{150(10)}{(-5)(20)} = -15$$

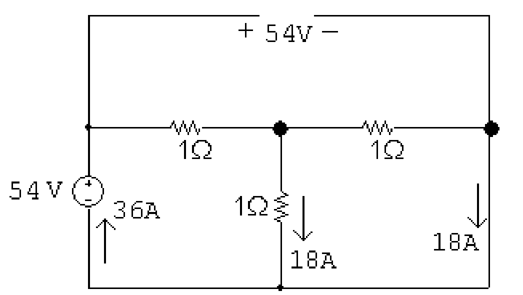
$$K_3 = \frac{150(-10)}{(-25)(-20)} = -3$$

$$\therefore \quad i_o(t) = [18 - 15e^{-5t} - 3e^{-25t}]u(t)\,\text{A}$$

[c] At $t = 0^+$ the circuit is



Both v_o and i_o are zero, which agrees with our solutions in part (a).
At $t = \infty$ the circuit is



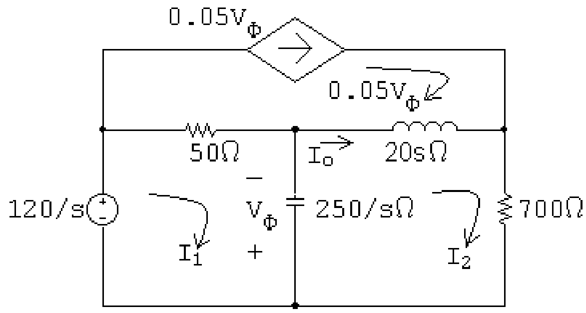
Our solutions predict $v_o(\infty) = 54$ V and $i_o(\infty) = 18$ A.
Also observe from the circuit at $t = 0^+$ that the voltage across the inductor is 54 V. Our solution predicts

$$v_L(0^+) = 0.36\frac{di_o(0^+)}{dt} = 0.36(75+75) = 54\,\text{V}$$

At $t = 0^+$ the current in the capacitive branch is $(1/2)(54/1.5) = 18$ A. From our solution we have

$$sI_2 = \frac{150(0.12 + 1/s)}{(1+5/s)(1+25/s)} \quad \text{and} \quad \lim_{s\to\infty} sI_2 = i_2(0^+) = 150(0.12) = 18\,\text{A}$$

P 13.29 **[a]**



$$\frac{120}{s} = 50(I_1 - 0.05V_\phi) + \frac{250}{s}(I_1 - I_2)$$

$$\frac{120}{s} = 50I_1 - 2.5\left(\frac{250}{s}\right)(I_2 - I_1) + \frac{250}{s}I_1 - \frac{250}{s}I_2;$$

$$0 = \frac{250}{s}(I_2 - I_1) + 20s(I_2 - 0.05V_\phi) + 700I_2$$

$$0 = \frac{250}{s}(I_2 - I_1) + 20s\left[I_2 - 0.05\left(\frac{250}{s}\right)(I_2 - I_1)\right]V_\phi) + 700I_2$$

Simplifying,

$$(50s + 875)I_1 - 875I_2 = 120$$

$$250(s - 1)I_1 + (20s^2 + 450s + 250)I_2 = 0$$

$$\Delta = \begin{vmatrix} (50s + 875) & -875 \\ 250(s - 1) & (20s^2 + 450s + 250) \end{vmatrix} = 1000s(s^2 + 40s + 625)$$

$$N_1 = \begin{vmatrix} 120 & -875 \\ 0 & (20s^2 + 450s + 250) \end{vmatrix} = 1200(2s^2 + 45s + 25)$$

$$N_2 = \begin{vmatrix} (50s + 875) & 120 \\ 250(s - 1) & 0 \end{vmatrix} = -30{,}000(s - 1)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1.2(2s^2 + 45s + 25)}{s(s^2 + 40s + 625)}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-30(s - 1)}{s(s^2 + 40s + 625)}$$

$$I_o = I_2 - 0.05V_\phi = I_2 - 0.05\left[\frac{250}{s}(I_2 - I_1)\right]$$

$$I_2 - I_1 = \frac{-2.4s(s + 35)}{s(s^2 + 40s + 625)}$$

$$\frac{250}{s}(I_2 - I_1) = \frac{-600(s+35)}{s(s^2+40s+625)}$$

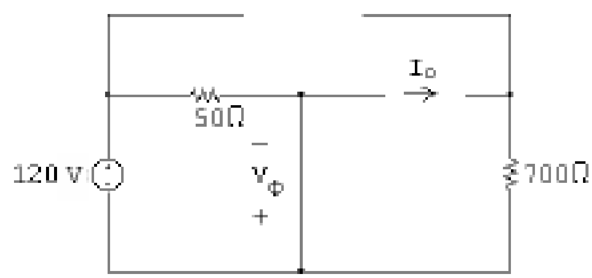
$$\therefore \qquad I_o = \frac{-30(s-1)}{s(s^2+40s+625)} + \frac{30(s+35)}{s(s^2+40s+625)} = \frac{1080}{s(s^2+40s+625)}$$

[b] $sI_o = \dfrac{1080}{(s^2+40s+625)}$

$$i_o(0^+) = \lim_{s\to\infty} sI_o = 0$$

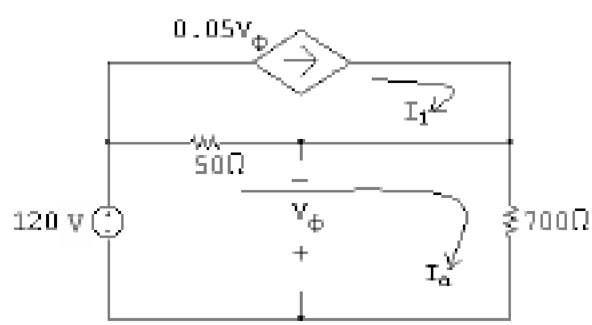
$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{1080}{625} = 1728\,\text{mA}$$

[c] At $t = 0^+$ the circuit is



$i_0(0^+) = 0$ (Checks)

At $t = \infty$ the circuit is



$$120 = 50(i_\text{a} - i_1) + 700i_\text{a}$$

$$= 50(i_\text{a} - 0.05v_\phi) + 700i_\text{a} = 750i_\text{a} - 2.5v_\phi$$

$$v_\phi = -700i_\text{a} \qquad \therefore \qquad 120 = 750i_\text{a} + 1750i_\text{a} = 2500i_\text{a}$$

$$i_\text{a} = \frac{120}{2500} = 48\,\text{mA}$$

$$v_\phi = -700i_\text{a} = -33.60\,\text{V}$$

$$i_o(\infty) = 48\times 10^{-3} - 0.05(-33.60) = 48\times 10^{-3} + 1.68 = 1728\,\text{mA (Checks)}$$

[d] $I_o = \dfrac{1080}{s(s^2+40s+625)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+20-j15} + \dfrac{K_2^*}{s+20+j15}$

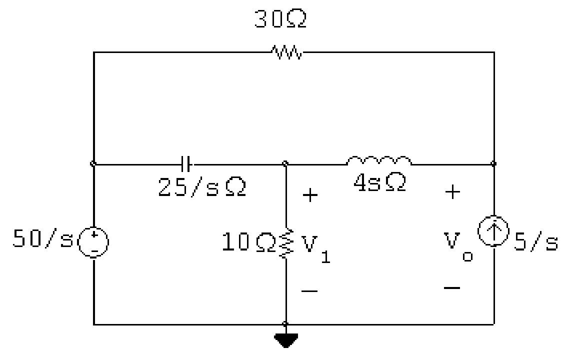
$$K_1 = \frac{1080}{625} = 1.728$$

$$K_2 = \frac{1080}{(-20+j15)(j30)} = 1.44\underline{/126.87^\circ}$$

$$i_o(t) = [1728 + 2880e^{-20t}\cos(15t+126.87^\circ)]u(t)\,\text{mA}$$

Check: $i_o(0^+) = 0\,\text{mA}$; $i_o(\infty) = 1728\,\text{mA}$

P 13.30 **[a]**



$$\frac{V_1}{10} + \frac{V_1 - 50/s}{25/s} + \frac{V_1 - V_o}{4s} = 0$$

$$\frac{-5}{s} + \frac{V_o - V_1}{4s} + \frac{V_o - 50/s}{30} = 0$$

Simplifying,

$$(4s^2+10s+25)V_1 - 25V_o = 200s$$

$$-15V_1 + (2s+15)V_o = 400$$

$$\Delta = \begin{vmatrix} (4s^2+10s+25) & -25 \\ -15 & (2s+15) \end{vmatrix} = 8s(s+5)^2$$

$$N_o = \begin{vmatrix} (4s^2+10s+25) & 200s \\ -15 & 400 \end{vmatrix} = 200(8s^2+35s+50)$$

$$V_o = \frac{N_o}{\Delta} = \frac{200(8s^2+35s+50)}{8s(s+5)^2} = \frac{K_1}{s} + \frac{K_2}{(s+5)^2} + \frac{K_3}{s+5}$$

$$K_1 = \frac{(25)(50)}{25} = 50; \quad K_2 = \frac{25(200-175+50)}{-5} = -375$$

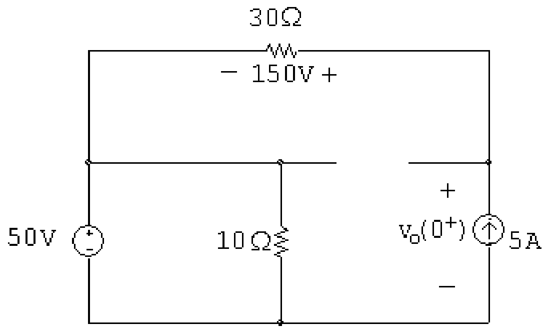
$$K_3 = 25\frac{d}{ds}\left[\frac{8s^2+35s+50}{s}\right]_{s=-5} = 25\left[\frac{s(16s+35)-(8s^2+35s+50)}{s^2}\right]_{s=-5}$$

$$= -5(-45) - 75 = 150$$

$$\therefore \quad V_o = \frac{50}{s} - \frac{375}{(s+5)^2} + \frac{150}{s+5}$$

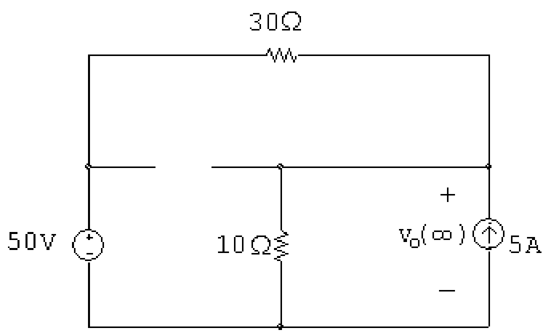
[b] $v_o(t) = [50 - 375te^{-5t} + 150e^{-5t}]u(t)\,\text{V}$

[c] At $t = 0^+$:



$$v_o(0^+) = 50 + 150 = 200\,\text{V}(\text{Checks})$$

At $t = \infty$:

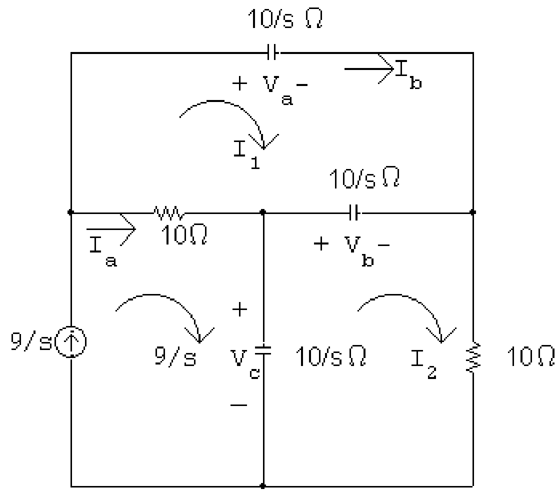


$$\frac{v_o(\infty)}{10} - 5 + \frac{v_o(\infty) - 50}{30} = 0$$

$$\therefore \quad 3v_o(\infty) - 150 + v_o(\infty) - 50 = 0; \qquad \therefore \quad 4v_o(\infty) = 200$$

$$\therefore \qquad v_o(\infty) = 50\,\text{V}(\text{Checks})$$

P 13.31 **[a]**



$$\frac{10}{s}I_1 + \frac{10}{s}(I_1 - I_2) + 10(I_1 - 9/s) = 0$$

$$\frac{10}{s}(I_2 - 9/s) + \frac{10}{s}(I_2 - I_1) + 10I_2 = 0$$

Simplifying,

$$(s+2)I_1 - I_2 = 9$$

$$-I_1 + (s+2)I_2 = \frac{9}{s}$$

$$\Delta = \begin{vmatrix} (s+2) & -1 \\ -1 & (s+2) \end{vmatrix} = s^2 + 4s + 3 = (s+1)(s+3)$$

$$N_1 = \begin{vmatrix} 9 & -1 \\ 9/s & (s+2) \end{vmatrix} = \frac{9s^2 + 18s + 9}{s} = \frac{9}{s}(s+1)^2$$

$$I_1 = \frac{N_1}{\Delta} = \frac{9}{s}\left[\frac{(s+1)^2}{(s+1)(s+3)}\right] = \frac{9(s+1)}{s(s+3)}$$

$$N_2 = \begin{vmatrix} (s+2) & 9 \\ -1 & 9/s \end{vmatrix} = \frac{18}{s}(s+1)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{18(s+1)}{s(s+1)(s+3)} = \frac{18}{s(s+3)}$$

$$I_{\rm a} = \frac{9}{s} - I_1 = \frac{9}{s} - \frac{9(s+1)}{s(s+3)} = \frac{6}{s} - \frac{6}{s+3}$$

$$I_{\rm b} = I_1 = \frac{9(s+1)}{s(s+3)} = \frac{3}{s} + \frac{6}{s+3}$$

[b] $i_a(t) = 6(1 - e^{-3t})u(t)\,\text{A}$

$i_b(t) = 3(1 + 2e^{-3t})u(t)\,\text{A}$

[c] $V_a = \frac{10}{s}I_b = \frac{10}{s}\left(\frac{3}{s} + \frac{6}{s+3}\right)$

$= \frac{30}{s^2} + \frac{60}{s(s+3)} = \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$

$V_b = \frac{10}{s}(I_2 - I_1) = \frac{10}{s}\left[\left(\frac{6}{s} - \frac{6}{s+3}\right) - \left(\frac{3}{s} + \frac{6}{s+3}\right)\right]$

$= \frac{10}{s}\left[\frac{3}{s} - \frac{12}{s+3}\right] = \frac{30}{s^2} - \frac{40}{s} + \frac{40}{s+3}$

$V_c = \frac{10}{s}(9/s - I_2) = \frac{10}{s}\left(\frac{9}{s} - \frac{6}{s} + \frac{6}{s+3}\right)$

$= \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$

[d] $v_a(t) = [30t + 20 - 20e^{-3t}]u(t)\,\text{V}$

$v_b(t) = [30t - 40 + 40e^{-3t}]u(t)\,\text{V}$

$v_c(t) = [30t + 20 - 20e^{-3t}]u(t)\,\text{V}$

[e] Calculating the time when the capacitor voltage drop first reaches 1000 V:

$30t + 20 - 20e^{-3t} = 1000$ or $30t - 40 + 40e^{-3t} = 1000$

Note that in either of these expressions the exponential term over time becomes is negligible when compared to the other terms. Thus,

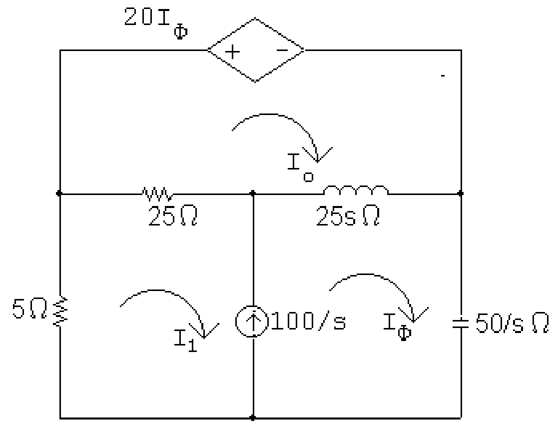
$30t + 20 = 1000$ or $30t - 40 = 1000$

Thus,

$t = \frac{980}{30} = 32.67\,\text{s}$ or $t = \frac{1040}{30} = 34.67\,\text{s}$

Therefore, the breakdown will occur at $t = 32.67$ s.

P 13.32 **[a]**



$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

$$\frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) + 25s(I_\phi - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s} \quad \therefore \quad I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-25s - 5)I_\phi + (25s + 25)I_o = -2500/s$$

$$(50/s + 25s + 30)I_\phi + (-25s - 25)I_o = 3000/s$$

$$\Delta = \begin{vmatrix} -5(5s+1) & 25(s+1) \\ \frac{5}{s}(5s^2+6s+10) & -25(s+1) \end{vmatrix} = -625(s+1)(1+2/s)$$

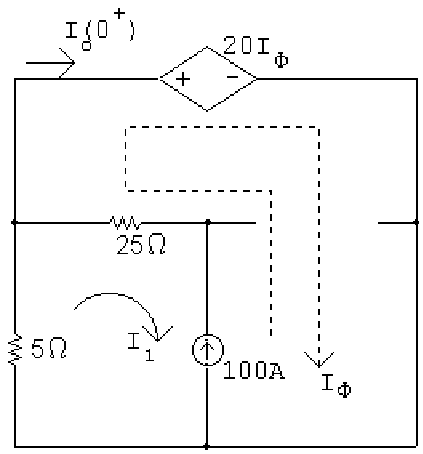
$$N_2 = \begin{vmatrix} -5(5s+1) & -2500/s \\ \frac{5}{s}(5s^2+6s+10) & 3000/s \end{vmatrix} = -12{,}500\frac{s^2 - 4.8s - 10}{s^2}$$

$$I_o = \frac{N_2}{\Delta} = \frac{20(s^2 - 4.8s - 10)}{s(s+1)(s+2)}$$

[b] $i_o(0^+) = \lim_{s\to\infty} sI_o = 20\,\text{A}$

$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{-200}{2} = -100\,\text{A}$$

[c] At $t = 0^+$ the circuit is

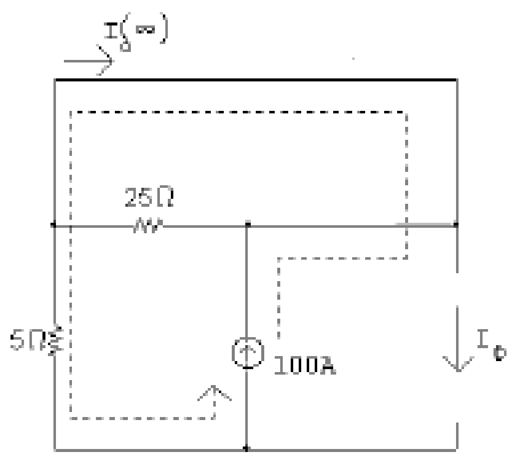


$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore \quad 20I_\phi + 5(I_\phi - 100) = 0; \qquad 25I_\phi = 500$$

$$\therefore \quad I_\phi = I_o(0^+) = 20\,\text{A}(\text{Checks})$$

At $t = \infty$ the circuit is



$$I_o(\infty) = -100\,\text{A}(\text{Checks})$$

[d] $$I_o = \frac{20(s^2 - 4.8s - 10)}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = \frac{-200}{(1)(2)} = -100; \qquad K_2 = \frac{20(1 + 4.8 - 10)}{(-1)(1)} = 84$$

$$K_3 = \frac{20(4 + 9.6 - 10)}{(-2)(-1)} = 36$$

$$I_o = \frac{-100}{s} + \frac{84}{s+1} + \frac{36}{s+2}$$

$$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t)\,\text{A}$$

$$i_o(\infty) = -100\,\text{A(Checks)}$$

$$i_o(0^+) = -100 + 84 + 36 = 20\,\text{A(Checks)}$$

P 13.33 $v_C = 12 \times 10^5 te^{-5000t}\,\text{V}, \quad C = 5\,\mu\text{F};$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\text{A}$$

$i_C > 0$ when $1 > 5000t$ or $i_C < 0$ when $0 < t < 200\,\mu$s

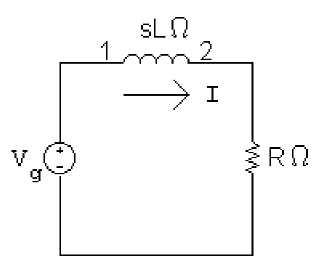
and $i_C < 0$ when $t > 200\,\mu$s

$i_C = 0$ when $1 - 5000t = 0,$ or $t = 200\,\mu$s

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34 **[a]** The s-domain equivalent circuit is



$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2 + \omega^2}$$

$$I = \frac{K_0}{s + R/L} + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_0 = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}, \qquad K_1 = \frac{V_m\underline{/\phi - 90^\circ - \theta(\omega)}}{2\sqrt{R^2 + \omega^2 L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}e^{-(R/L)t} + \frac{V_m\sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2 + \omega^2 L^2}}$$

[b] $i_{ss}(t) = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin[\omega t + \phi - \theta(\omega)]$

[c] $i_{\text{tr}} = \dfrac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}e^{-(R/L)t}$

[d] $\mathbf{I} = \dfrac{\mathbf{V}_g}{R+j\omega L}, \qquad \mathbf{V}_g = V_m\angle\phi - 90^\circ$

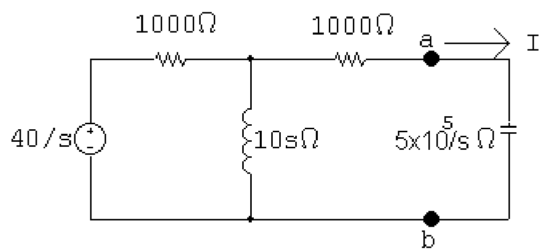
Therefore $\quad \mathbf{I} = \dfrac{V_m\angle\phi-90^\circ}{\sqrt{R^2+\omega^2L^2}\angle\theta(\omega)} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\angle\phi-90^\circ-\theta(\omega)$

Therefore $\quad i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\sin[\omega t+\phi-\theta(\omega)]$

[e] The transient component vanishes when

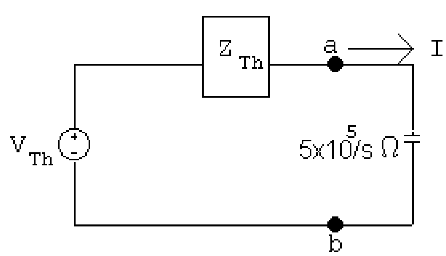
$$\omega L\cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.35



$$V_{\text{Th}} = \frac{10s}{10s+1000}\cdot\frac{40}{s} = \frac{400}{10s+1000} = \frac{40}{s+100}$$

$$Z_{\text{Th}} = 1000 + 1000\|10s = 1000 + \frac{10{,}000s}{10s+1000} = \frac{2000(s+50)}{s+100}$$



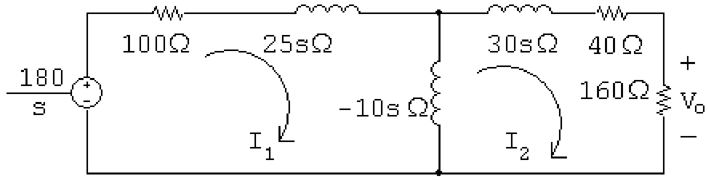
$$I = \frac{40/(s+100)}{(5\times10^5)/s + 2000(s+50)/(s+100)} = \frac{40s}{2000s^2+600{,}000s+5\times10^7}$$

$$= \frac{0.02s}{s^2+300s+25{,}000} = \frac{K_1}{s+150-j50} + \frac{K_1^*}{s+150+j50}$$

$$K_1 = \left.\frac{0.02s}{s+150+j50}\right|_{s=-150+j50} = 31.62\times10^{-3}\angle 71.57^\circ$$

$$i(t) = 63.25e^{-150t}\cos(50t+71.57^\circ)u(t)\,\text{mA}$$

P 13.36 **[a]**



$$\frac{180}{s} = (100 + 15s)I_1 + 10sI_2$$

$$0 = 10sI_1 + (20s + 200)I_2$$

$$\Delta = \begin{vmatrix} 15s + 100 & 10s \\ 10s & 20s + 200 \end{vmatrix} = 200(s + 5)(s + 20)$$

$$N_2 = \begin{vmatrix} 15s + 100 & 180/s \\ 10s & 0 \end{vmatrix} = -1800$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-9}{(s + 5)(s + 20)}$$

$$V_o = 160I_2 = \frac{-1440}{(s + 5)(s + 20)}$$

[b] $sV_o = \dfrac{-1440s}{(s + 5)(s + 20)}$

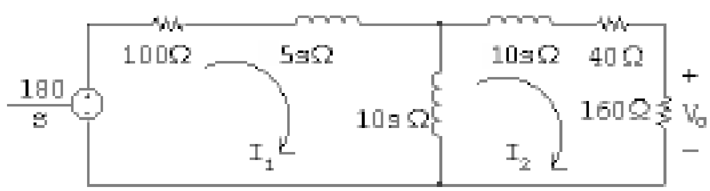
$$\lim_{s\to 0} sV_o = v_o(\infty) = 0\,\text{V}$$

$$\lim_{s\to\infty} sV_o = v_o(0^+) = 0\,\text{V}$$

[c] $V_o = \dfrac{-96}{s + 5} + \dfrac{96}{s + 20}$

$$v_o(t) = [-96e^{-5t} + 96e^{-20t}]u(t)\,\text{V}$$

P 13.37



$$\frac{180}{s} = (100 + 15s)I_1 - 10sI_2$$

$$0 = -10sI_1 + (20s + 200)I_2$$

$$\Delta = \begin{vmatrix} 15s + 100 & -10s \\ -10s & 20s + 200 \end{vmatrix} = 200(s + 5)(s + 20)$$

$$N_2 = \begin{vmatrix} 15s + 100 & 180/s \\ -10s & 0 \end{vmatrix} = 1800$$

$$I_2 = \frac{N_2}{\Delta} = \frac{9}{(s + 5)(s + 20)}$$

$$V_o = 160I_2 = \frac{1440}{(s + 5)(s + 20)} = \frac{96}{s + 5} - \frac{96}{s + 20}$$

$$v_o(t) = [96e^{-5t} - 96e^{-20t}]u(t)\,\text{V}$$

P 13.38 **[a]** $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

$$W = 4(15)^2 + 9(100) + 150(6) = 2700\,\text{J}$$

[b] $120i_1 + 8\frac{di_1}{dt} - 6\frac{di_2}{dt} = 0$

$$270i_2 + 18\frac{di_2}{dt} - 6\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$$120I_1 + 8(sI_1 - 15) - 6(sI_2 + 10) = 0$$

$$270I_2 + 18(sI_2 + 10) - 6(sI_1 - 15) = 0$$

In standard form,

$$(8s + 120)I_1 - 6sI_2 = 180$$

$$-6sI_1 + (18s + 270)I_2 = -270$$

$$\Delta = \begin{vmatrix} 8s + 120 & -6s \\ -6s & 18s + 270 \end{vmatrix} = 108(s + 10)(s + 30)$$

$$N_1 = \begin{vmatrix} 180 & -6s \\ -270 & 18s + 270 \end{vmatrix} = 1620(s + 30)$$

$$N_2 = \begin{vmatrix} 8s + 120 & 180 \\ -6s & -270 \end{vmatrix} = -1080(s + 30)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1620(s+30)}{108(s+10)(s+30)} = \frac{15}{s+10}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-1080(s+30)}{108(s+10)(s+30)} = \frac{-10}{s+10}$$

[c] $i_1(t) = 15e^{-10t}u(t)\,\text{A}; \qquad i_2(t) = -10e^{-10t}u(t)\,\text{A}$

[d] $W_{120\Omega} = \int_0^\infty (225e^{-20t})(120)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$

$$W_{270\Omega} = \int_0^\infty (100e^{-20t})(270)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$$

$W_{120\Omega} + W_{270\Omega} = 2700\,\text{J}$ (Checks)

[e] $W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = 900 + 900 - 900 = 900\,\text{J}$

With the dot reversed the s-domain equations are

$$(8s+120)I_1 + 6sI_2 = 60$$

$$6sI_1 + (18s+270)I_2 = -90$$

As before, $\Delta = 108(s+10)(s+30)$. Now,

$$N_1 = \begin{vmatrix} 60 & 6s \\ -90 & 18s+270 \end{vmatrix} = 1620(s+10)$$

$$N_2 = \begin{vmatrix} 8s+120 & 60 \\ 6s & -90 \end{vmatrix} = -1080(s+10)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15}{s+30}; \qquad I_2 = \frac{N_2}{\Delta} = \frac{-10}{s+30}$$

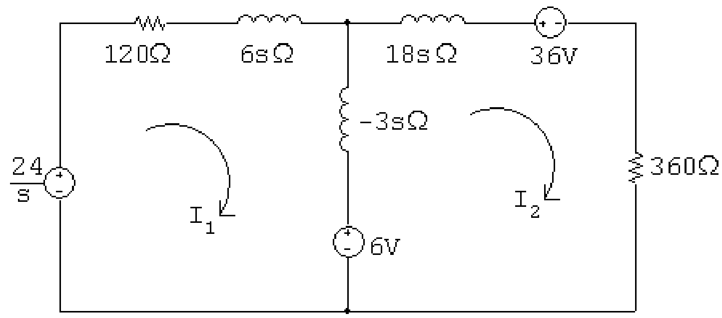
$i_1(t) = 15e^{-30t}u(t)\,\text{A}; \qquad i_2(t) = -10e^{-30t}u(t)\,\text{A}$

$$W_{270\Omega} = \int_0^\infty (100e^{-60t})(270)\,dt = 450\,\text{J}$$

$$W_{120\Omega} = \int_0^\infty (225e^{-60t})(120)\,dt = 450\,\text{J}$$

$W_{120\Omega} + W_{270\Omega} = 900\,\text{J}$ (Checks)

P 13.39 **[a]** s-domain equivalent circuit is



Note: $\quad i_2(0^+) = -\frac{20}{10} = -2\text{ A}$

[b] $\frac{24}{s} = (120+3s)I_1 + 3sI_2 + 6$

$0 = -6 + 3sI_1 + (360+15s)I_2 + 36$

In standard form,

$(s+40)I_1 + sI_2 = (8/s) - 2$

$sI_1 + (5s+120)I_2 = -10$

$$\Delta = \begin{vmatrix} s+40 & s \\ s & 5s+120 \end{vmatrix} = 4(s+20)(s+60)$$

$$N_1 = \begin{vmatrix} (8/s)-2 & s \\ -10 & 5s+120 \end{vmatrix} = \frac{-200(s-4.8)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{-50(s-4.8)}{s(s+20)(s+60)}$$

[c] $sI_1 = \frac{-50(s-4.8)}{(s+20)(s+60)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 0\text{ A}$$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = \frac{(-50)(-4.8)}{(20)(60)} = 0.2\text{ A}$$

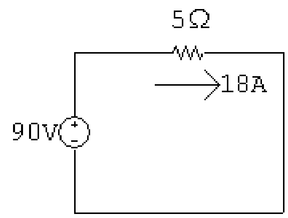
[d] $I_1 = \frac{K_1}{s} + \frac{K_2}{s+20} + \frac{K_3}{s+60}$

$$K_1 = \frac{240}{1200} = 0.2; \qquad K_2 = \frac{-50(-20)+240}{(-20)(40)} = -1.55$$

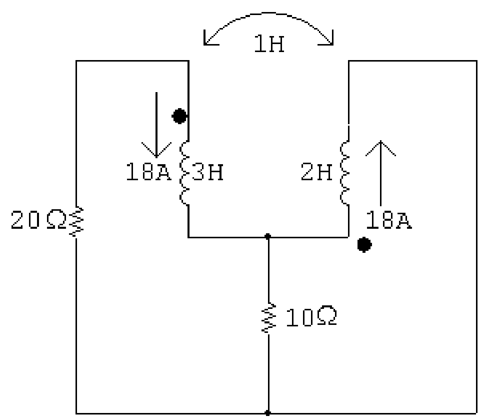
$$K_3 = \frac{-50(-60) + 240}{(-60)(-40)} = 1.35$$

$$i_1(t) = [0.2 - 1.55e^{-20t} + 1.35e^{-60t}]u(t)\,\text{A}$$

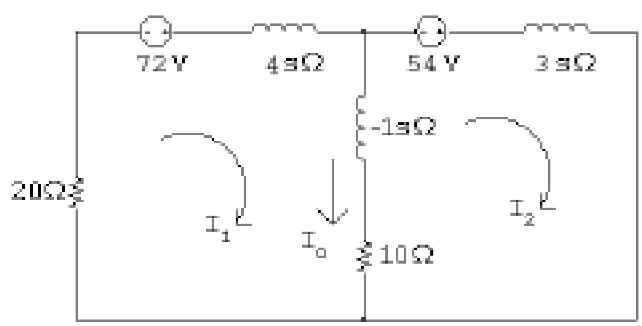
P 13.40 For $t < 0$:



For $t > 0^+$:



$18 \times 4 = 72; \qquad 18 \times 3 = 54$



$$20I_1 - 72 + 4sI_1 + s(I_2 - I_1) + 10(I_1 - I_2) = 0$$

$$-54 + 3sI_2 + 10(I_2 - I_1) + s(I_1 - I_2) = 0$$

In standard form,

$$(3s + 30)I_1 + (s - 10)I_2 = 72$$

$$(s-10)I_1 + (2s+10)I_2 = 54$$

$$\therefore \quad \Delta = \begin{vmatrix} (3s+30) & (s-10) \\ (s-10) & (2s+10) \end{vmatrix} = 5(s+2)(s+20)$$

$$N_1 = \begin{vmatrix} 72 & (s-10) \\ 54 & (2s+10) \end{vmatrix} = 90s + 1260$$

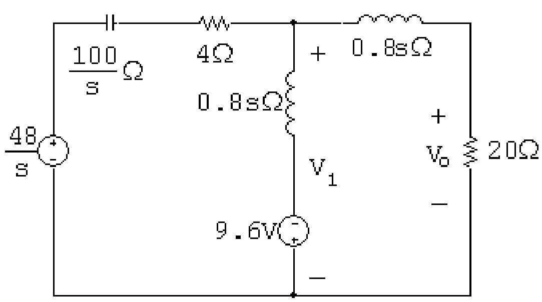
$$N_2 = \begin{vmatrix} (3s+30) & 72 \\ (s-10) & 54 \end{vmatrix} = 90s + 2340$$

$$I_o = I_1 - I_2 = \frac{N_1}{\Delta} - \frac{N_2}{\Delta} = \frac{-1080}{5(s+2)(s+20)}$$

$$= \frac{-216}{(s+2)(s+20)} - \frac{12}{s+2} - \frac{12}{s+20}$$

$$i_o(t) = [12e^{-2t} + 12e^{-20t}]u(t)\,\text{A}$$

P 13.41 The s-domain equivalent circuit is



$$\frac{V_1 - 48/s}{4 + (100/s)} + \frac{V_1 + 9.6}{0.8s} + \frac{V_1}{0.8s + 20} = 0$$

$$V_1 = \frac{-1200}{s^2 + 10s + 125}$$

$$V_o = \frac{20}{0.8s + 20}V_1 = \frac{-30{,}000}{(s+25)(s+5-j10)(s+5+j10)}$$

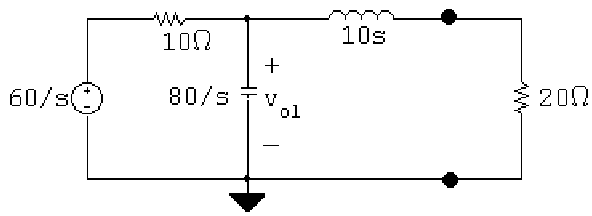
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \frac{-30{,}000}{s^2 + 10s + 125}\bigg|_{s=-25} = -60$$

$$K_2 = \frac{-30{,}000}{(s+25)(s+5+j10)}\bigg|_{s=-5+j10} = 67.08\underline{/63.43^\circ}$$

$$v_o(t) = [-60e^{-25t} + 134.16e^{-5t}\cos(10t + 63.43^\circ)]u(t)\ \text{V}$$

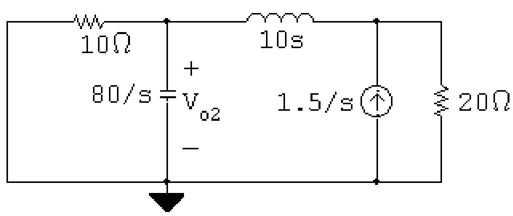
P 13.42 **[a]** Voltage source acting alone:

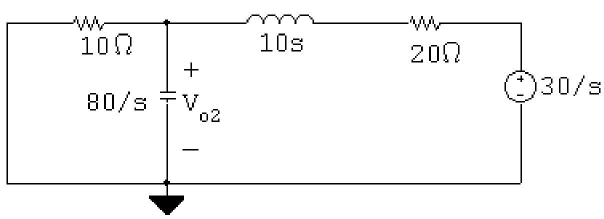


$$\frac{V_{o1} - 60/s}{10} + \frac{V_{o1}s}{80} + \frac{V_{o1}}{20 + 10s} = 0$$

$$\therefore\quad V_{o1} = \frac{480(s+2)}{s(s+4)(s+6)}$$

Current source acting alone:





$$\frac{V_{o2}}{10} + \frac{V_{o2}s}{80} + \frac{V_{o2} - 30/s}{10(s+2)} = 0$$

$$\therefore\quad V_{o2} = \frac{240}{s(s+4)(s+6)}$$

$$V_o = V_{o1} + V_{o2} = \frac{480(s+2) + 240}{s(s+4)(s+6)} = \frac{480(s+2.5)}{s(s+4)(s+6)}$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s+4} + \dfrac{K_3}{s+6}$

$$K_1 = \frac{(480)(2.5)}{(4)(6)} = 50; \qquad K_2 = \frac{480(-1.5)}{(-4)(2)} = 90; \qquad K_3 = \frac{480(-3.5)}{(-6)(-2)} = -140$$

$$v_o(t) = [50 + 90e^{-4t} - 140e^{-6t}]u(t)\ \text{V}$$

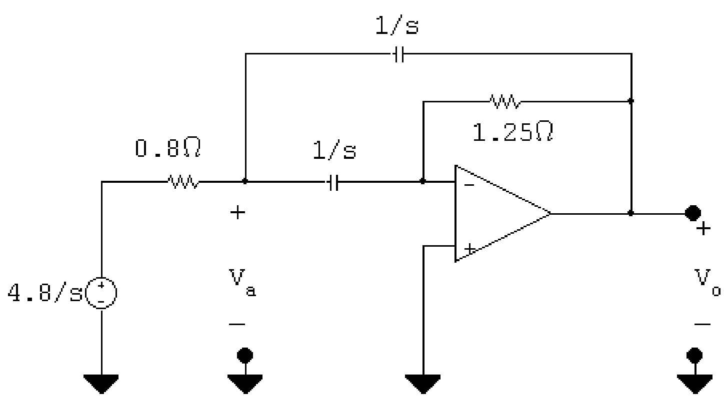
P 13.43 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

Substitution and simplification lead directly to Eq. 13.90.

P 13.44



$$\frac{V_a - 4.8/s}{0.8} + \frac{V_a}{1/s} + \frac{V_a - V_o}{1/s} = 0$$

$$\frac{0 - V_a}{1/s} + \frac{0 - V_o}{1.25} = 0$$

$$V_a = \frac{-V_o}{1.25s}$$

$$V_a(2s + 1.25) - sV_o = 6/s$$

$$-V_o\left[\frac{(2s+1.25)}{1.25s}+s\right]=6/s$$

$$-V_o\left[\frac{125s^2+2s+1.25}{1.25s}\right]=6/s$$

$$V_o=\frac{-7.5}{1.25s^2+2s+1.25}=\frac{-6}{s^2+1.6s+1}$$

$$=\frac{K_1}{s+0.8-j0.6}+\frac{K_1^*}{s+0.8+j0.6}$$

$$K_1=\left.\frac{-6}{s+0.8+j0.6}\right|_{s=-0.8+j0.6}=5\underline{/90^\circ}$$

$$v_o(t)=10e^{-0.8t}\cos(0.6t+90^\circ)u(t)\,\text{V}=-10e^{-0.8t}\sin(0.6t)u(t)\,\text{V}$$

P 13.45 **[a]** $V_o=-\dfrac{Z_f}{Z_i}V_g$

$$Z_f=\frac{10^7}{s}\|1000=\frac{10^{10}/s}{10^7/s+1000}=\frac{10^{10}}{1000s+10^7}=\frac{10^7}{s+10^4}$$

$$Z_i=\frac{2\times10^6}{s}+400=\frac{400s+2\times10^6}{s}=\frac{400}{s}(s+5000)$$

$$V_g=\frac{20{,}000}{s^2}$$

$$\therefore\quad V_o=\frac{-10^7/(s+10^4)}{(400/s)(s+5000)}\cdot\frac{20{,}000}{s^2}=\frac{-5\times10^8}{s(s+5000)(s+10{,}000)}$$

[b] $V_o=\dfrac{K_1}{s}+\dfrac{K_2}{s+5000}+\dfrac{K_3}{s+10{,}000}$

$$K_1=\left.\frac{-5\times10^8}{(s+5000)(s+10{,}000)}\right|_{s=0}=-10$$

$$K_2=\left.\frac{-5\times10^8}{s(s+10{,}000)}\right|_{s=-5000}=20$$

$$K_3=\left.\frac{-5\times10^8}{s(s+5000)}\right|_{s=-10{,}000}=-10$$

$$\therefore\quad v_o(t)=[-10+20e^{-5000t}-10e^{-10{,}000t}]u(t)\,\text{V}$$

[c] $-10 + 20e^{-5000t_s} - 10e^{-10,000t_s} = -5$

Let $x = e^{-5000t_s}$. Then

$10x^2 - 20x + 5 = 0$

Solving,

$x = 0.292893$

$e^{-5000t_s} = 0.292893 \quad \therefore \quad t_s = 245.6\,\mu\text{s}$

[d] $v_g = m\,tu(t); \qquad V_g = \dfrac{m}{s^2}$

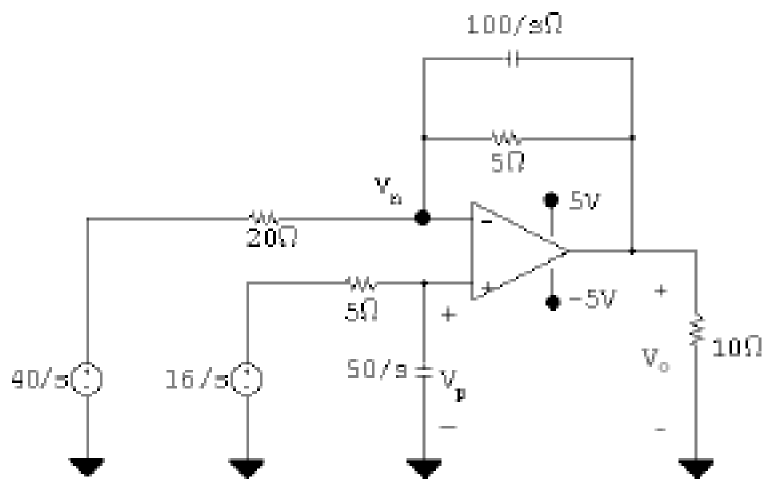
$$V_o = \frac{-10^7 s}{400(s+5000)(s+10{,}000)} \cdot \frac{m}{s^2}$$

$$= \frac{-25{,}000m}{s(s+5000)(s+10{,}000)}$$

$$K_1 = \frac{-25{,}000m}{(5000)(10{,}000)} = -5 \times 10^{-4}m$$

$\therefore \quad -5 = -5 \times 10^{-4}m \qquad \therefore \quad m = 10{,}000\,\text{V/s}$

P 13.46 **[a]**



$$V_p = \frac{50/s}{5 + 50/s} V_{g2} = \frac{50}{5s+50} V_{g2}$$

$$\frac{V_p - 40/s}{20} + \frac{V_p - V_o}{5} + \frac{V_p - V_o}{100/s} = 0$$

$$V_p\left(\frac{1}{20} + \frac{1}{5} + \frac{s}{100}\right) - V_o\left(\frac{1}{5} + \frac{s}{100}\right) = \frac{2}{s}$$

$$\frac{s+25}{100}\left(\frac{50}{5s+50}\right)\frac{16}{s} - \frac{2}{s} = V_o\left(\frac{1}{5} + \frac{s}{100}\right) = V_o\left(\frac{s+20}{100}\right)$$

$$V_o = \frac{100}{s+20}\left[\frac{16(s+25)}{10(s+10)(s)} - \frac{2}{s}\right] = \frac{-40s+2000}{s(s+10)(s+20)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+20}$$

$K_1 = 10; \qquad K_2 = -24; \qquad K_3 = 14$

$\therefore \qquad v_o(t) = [10 - 24e^{-10t} + 14e^{-20t}]u(t)\,\mathrm{V}$

[b] $10 - 24e^{-10t} + 14e^{-20t} = 5$

Let $x = e^{-10t_s}$. Then

$10 - 24x + 14x^2 = 5$

$14x^2 - 24x + 5 = 0$

$x = 0.242691$

$e^{-10t_s} = 0.242691 \qquad \therefore \qquad t_s = 141.60\,\mathrm{ms}$

P 13.47 Let v_{o1} equal the output voltage of the first op amp. Then

$$V_{o1} = \frac{-Z_{f1}}{Z_{A1}}V_g \quad \text{where} \quad Z_{f1} = 25 \times 10^3\,\Omega$$

$$Z_{A1} = 25{,}000 + \frac{25{,}000(20 \times 10^4/s)}{25{,}000 + (20 \times 10^4/s)}$$

$$= \frac{25{,}000(s+16)}{(s+8)}\,\Omega$$

$$\therefore \qquad V_{o1} = \frac{-(s+8)}{(s+16)}V_g$$

Also,

$$V_o = \frac{-Z_{f2}}{Z_{A2}}V_{o1} \quad \text{where} \quad Z_{f2} = \frac{2 \times 10^8}{s}\,\Omega \text{ and } Z_{A2} = 25{,}000\,\Omega$$

$$\therefore \qquad V_o = \frac{-8000}{s}V_{o1} = \frac{-8000}{s}\left[\frac{-(s+8)}{(s+16)}\right]V_g$$

$$= \frac{8000(s+8)}{s(s+16)}V_g$$

$$v_g(t) = 16u(t)\,\mathrm{mV}; \quad \therefore \qquad V_g = \frac{16 \times 10^{-3}}{s}$$

$$V_o = \frac{128(s+8)}{s^2(s+16)} = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+16}$$

$$K_1 = \frac{128(8)}{16} = 64$$

$$K_2 = 128\frac{d}{ds}\left[\frac{s+8}{s+16}\right]_{s=0} = 4$$

$$K_3 = \frac{128(-8)}{256} = -4$$

$$v_o(t) = [64t + 4 - 4e^{-16t}]u(t)\text{ V}$$

The op amp will saturate when $v_o = \pm 6$ V. Hence, saturation will occur when

$$64t + 4 - 4e^{-16t} = 6 \quad \text{or} \quad 16t - 0.5 = e^{-16t}$$

This equation can be solved by trial and error. First note that $t > 0.5/16$ or $t > 31.25$ ms.
Try 40 ms:

$$0.64 - 0.5 = 0.14; \qquad e^{-0.64} = 0.53$$

Try 50 ms:

$$0.80 - 0.5 = 0.30; \qquad e^{-0.80} = 0.45$$

Try 60 ms:

$$0.96 - 0.5 = 0.46; \qquad e^{-0.96} = 0.38$$

Further trial and error gives

$$t_{\text{sat}} \cong 56.5\,\text{ms}$$

P 13.48 **[a]** Let v_a be the voltage across the $0.5\,\mu$F capacitor, positive at the upper terminal. Let v_b be the voltage across the $100\,\text{k}\Omega$ resistor, positive at the upper terminal. Also note

$$\frac{10^6}{0.5s} = \frac{2\times 10^6}{s} \quad \text{and} \quad \frac{10^6}{0.25s} = \frac{4\times 10^6}{s}; \qquad V_g = \frac{0.5}{s}$$

$$\frac{sV_\text{a}}{2\times 10^6} + \frac{V_\text{a} - (0.5/s)}{200{,}000} + \frac{V_\text{a}}{200{,}000} = 0$$

$$sV_a + 10V_a - \frac{5}{s} + 10V_a = 0$$

$$V_a = \frac{5}{s(s+20)}$$

$$\frac{0 - V_a}{200{,}000} + \frac{(0 - V_b)s}{4 \times 10^6} = 0$$

$$\therefore \qquad V_b = -\frac{20}{s}V_a = \frac{-100}{s^2(s+20)}$$

$$\frac{V_b}{100{,}000} + \frac{(V_b - 0)s}{4 \times 10^6} + \frac{(V_b - V_o)s}{4 \times 10^6} = 0$$

$$40V_b + sV_b + sV_b = sV_o$$

$$\therefore \qquad V_o = \frac{2(s+20)V_b}{s}; \qquad V_o = 2\left(\frac{-100}{s^3}\right) = \frac{-200}{s^3}$$

[b] $v_o(t) = -100t^2u(t)$ V

[c] $-100t^2 = -4; \qquad t = 0.2\,\text{s} = 200\,\text{ms}$

P 13.49 **[a]** $\dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC}$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{200}{s + 200}; \qquad -p_1 = -200\,\text{rad/s}$$

[b] $\dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{RCs}{RCs + 1} = \dfrac{s}{s + (1/RC)}$

$$= \frac{s}{s + 200}; \qquad z_1 = 0, \quad -p_1 = -200\,\text{rad/s}$$

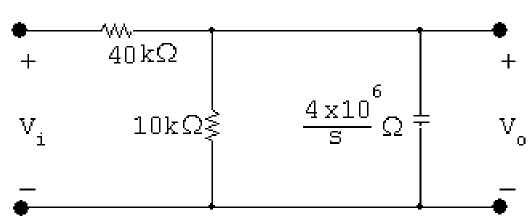
[c] $\dfrac{V_o}{V_i} = \dfrac{sL}{R + sL} = \dfrac{s}{s + R/L} = \dfrac{s}{s + 8000}$

$$z_1 = 0; \qquad -p_1 = -8000\,\text{rad/s}$$

[d] $\dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + (R/L)} = \dfrac{8000}{s + 8000}$

$$-p_1 = -8000\,\text{rad/s}$$

[e]



$$\frac{V_o s}{4 \times 10^6} + \frac{V_o}{10{,}000} + \frac{V_o - V_i}{40{,}000} = 0$$

$$sV_o + 400V_o + 100V_o = 100V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{100}{s + 500}$$

$$-p_1 = -500\,\text{rad/s}$$

P 13.50 **[a]** Let $R_1 = 250\,\text{k}\Omega$; $R_2 = 125\,\text{k}\Omega$; $C_2 = 1.6\,\text{nF}$; and $C_f = 0.4\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2+C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 2.5 \times 10^9$$

$$\frac{1}{R_2C_2} = \frac{62.5 \times 10^7}{125 \times 10^3} = 5000\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{2 \times 10^{-9}}{(0.64 \times 10^{-18})(125 \times 10^3)} = 25{,}000\,\text{rad/s}$$

$$\therefore \quad Z_f = \frac{2.5 \times 10^9(s + 5000)}{s(s + 25{,}000)}\Omega$$

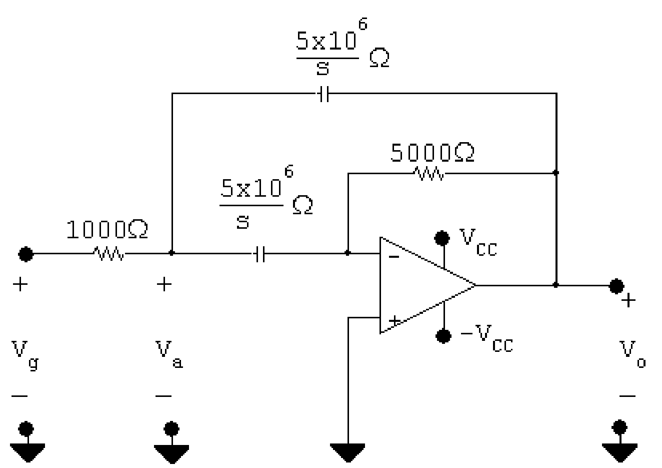
$$Z_i = R_1 = 250 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-10^4(s + 5000)}{s(s + 25{,}000)}$$

[b] $-z_1 = -5000\,\text{rad/s}$

$$-p_1 = 0; \qquad -p_2 = -25{,}000\,\text{rad/s}$$

P 13.51 **[a]**



$$\frac{V_a - V_g}{1000} + \frac{sV_a}{5 \times 10^6} + \frac{(V_a - V_o)s}{5 \times 10^6} = 0$$

$$5000V_a - 5000V_g + 2sV_a - sV_o = 0$$

$$(5000 + 2s)V_a - sV_o = 5000V_g$$

$$\frac{(0 - V_a)s}{5 \times 10^6} + \frac{0 - V_o}{5000} = 0$$

$$-sV_a - 1000V_o = 0; \quad \therefore \qquad V_a = \frac{-1000}{s}V_o$$

$$(2s + 5000)\left(\frac{-1000}{s}\right)V_o - sV_o = 5000V_g$$

$$1000V_o(2s + 5000) + s^2V_o = -5000sV_g$$

$$V_o(s^2 + 2000s + 5 \times 10^6) = -5000sV_g$$

$$\frac{V_o}{V_g} = \frac{-5000s}{s^2 + 2000s + 5 \times 10^6}$$

$$s_{1,2} = -1000 \pm \sqrt{10^6 - 5 \times 10^6} = -1000 \pm j2000$$

$$\frac{V_o}{V_g} = \frac{-5000s}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

[b] $z_1 = 0; \qquad -p_1 = -1000 + j2000; \qquad -p_2 = -1000 - j2000$

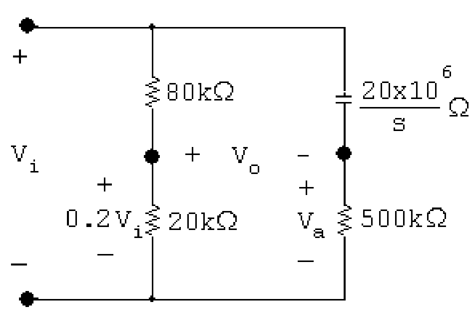
P 13.52 **[a]** $Z_i = 1000 + \dfrac{5 \times 10^6}{s} = \dfrac{1000(s + 5000)}{s}$

$$Z_f = \frac{40 \times 10^6}{s} \| 40{,}000 = \frac{40 \times 10^6}{s + 1000}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-40 \times 10^6/(s + 1000)}{1000(s + 5000)/s} = \frac{-40{,}000s}{(s + 1000)(s + 5000)}$$

[b] Zero at $s = 0$; Poles at $-p_1 = -1000$ rad/s and $-p_2 = -5000$ rad/s

P 13.53 **[a]**



$$V_a = \frac{V_i}{500{,}000 + [(20 \times 10^6)/s]}(500{,}000) = \frac{s}{s + 40}V_i$$

$$0.2V_i = V_o + V_a$$

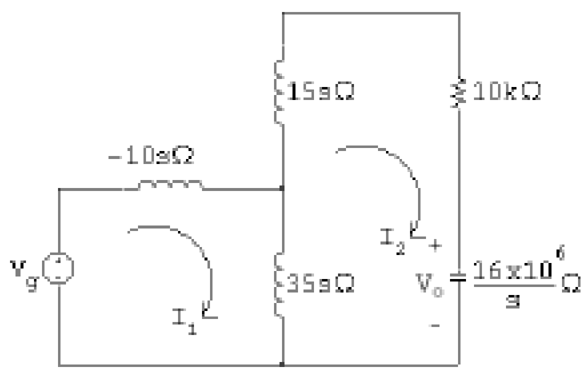
$$\therefore \qquad V_o = 0.2V_i - \frac{s}{s+40}V_i$$

$$\frac{V_o}{V_i} = \frac{0.2(s+40)-s}{s+40} = \frac{-0.8s+8}{s+40} = \frac{-0.8(s-10)}{s+40}$$

[b] $-z_1 = 10\,\text{rad/s}$

$-p_1 = -40\,\text{rad/s}$

P 13.54



$$V_g = 25sI_1 - 35sI_2$$

$$0 = -35sI_1 + \left(50s + 10{,}000 + \frac{16\times 10^6}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} 25s & -35s \\ -35s & 50s+10{,}000+16\times 10^6/s \end{vmatrix} = 25(s+2000)(s+8000)$$

$$N_2 = \begin{vmatrix} 25s & V_g \\ -35s & 0 \end{vmatrix} = 35sV_g$$

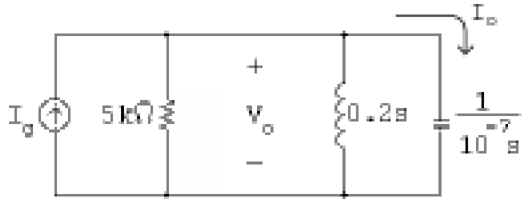
$$I_2 = \frac{N_2}{\Delta} = \frac{35sV_g}{25(s+2000)(s+8000)}$$

$$V_o = \frac{16\times 10^6}{s}I_2 = \frac{22.4\times 10^6 V_g}{(s+2000)(s+8000)}$$

$$H(s) = \frac{V_o}{V_g} = \frac{22.4\times 10^6}{(s+2000)(s+8000)}$$

$$\therefore \qquad -p_1 = -2000 \text{ rad/s}; \qquad -p_2 = -8000 \text{ rad/s}$$

P 13.55 **[a]**



$$\frac{V_o}{5000} + \frac{V_o}{0.2s} + V_o(10^{-7})s = I_g$$

$$\therefore \quad V_o = \frac{10 \times 10^6 s}{s^2 + 2000s + 50 \times 10^6} \cdot I_g$$

$$I_g = \frac{0.1s}{s^2 + 10^8}; \qquad I_o = \frac{V_o s}{10 \times 10^6}$$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + 2000s + 50 \times 10^6}$$

[b] $I_o = \dfrac{(s^2)(0.1s)}{(s + 1000 - j7000)(s + 1000 + j7000)(s^2 + 10^8)}$

$$I_o = \frac{0.1s^3}{(s + 1000 - j7000)(s + 1000 + j7000)(s + j10^4)(s - j10^4)}$$

[c] Damped sinusoid of the form

$$Me^{-1000t}\cos(7000t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N\cos(10^4 t + \theta_2)$$

[e] $I_o = \dfrac{K_1}{s + 1000 - j7000} + \dfrac{K_1^*}{s + 1000 + j7000} + \dfrac{K_2}{s - j10^4} + \dfrac{K_2^*}{s + j10^4}$

$$K_1 = \frac{0.1(-1000 + j7000)^3}{(j14{,}000)(-1000 - j3000)(-1000 + j17{,}000)} = 46.90 \times 10^{-3}\underline{/-140.54^\circ}$$

$$K_2 = \frac{0.1(j10^4)^3}{(j20{,}000)(1000 + j3000)(1000 + j17{,}000)} = 92.85 \times 10^{-3}\underline{/21.80^\circ}$$

$$i_o(t) = [93.8e^{-1000t}\cos(7000t - 140.54^\circ) + 185.7\cos(10^4 t + 21.80^\circ)]\ \text{mA}$$

Test:

$$i_o(0) = 93.8\cos(-140.54^\circ) + 185.7\cos(21.80^\circ)\ \text{mA} = 100\ \text{mA}$$

$$Z = \frac{1}{Y}; \qquad Y = \frac{1}{5000} + \frac{1}{j2000} + \frac{1}{-j1000} = \frac{2 + j5}{10{,}000}$$

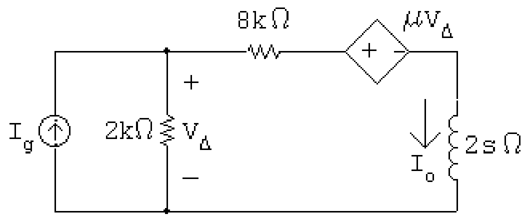
$$\therefore \quad Z = \frac{10{,}000}{2 + j5} = 1856.95\underline{/-68.2^\circ}\ \Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z = (0.1\underline{/0^\circ})(1856.95\underline{/-68.2^\circ}) = 185.695\underline{/-68.2^\circ}\text{ V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{-j1000} = 185.7\underline{/21.80^\circ}\text{ mA}$$

$$i_{oss} = 185.7\cos(10^4 t + 21.80^\circ)\text{ mA(Checks)}$$

P 13.56 **[a]**



$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore \quad I_o = \frac{1000(1-\mu)}{s + 1000(5-\mu)} I_g$$

$$\therefore \quad H(s) = \frac{1000(1-\mu)}{s + 1000(5-\mu)}$$

[b] $\mu < 5$

[c]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20{,}000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15{,}000/s(s+1000)$
5	$-4000/s$	$-20{,}000/s^2$
6	$-5000/(s-1000)$	$-25{,}000/s(s-1000)$

$\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \qquad i_o = [2.5 - 2.5e^{-8000t}]u(t)\text{ A}$$

$\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \qquad i_o = [1 - e^{-5000t}]u(t)\text{ A}$$

$\mu = 4$:

$$I_o = \frac{-15}{s} + \frac{15}{s+1000}; \qquad i_o = [-15 + 15e^{-1000t}]u(t)\text{ A}$$

$\mu = 5$:

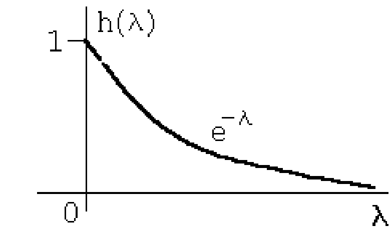
$$I_o = \frac{-20{,}000}{s^2}; \qquad i_o = -20{,}000t\,u(t)\text{ A}$$

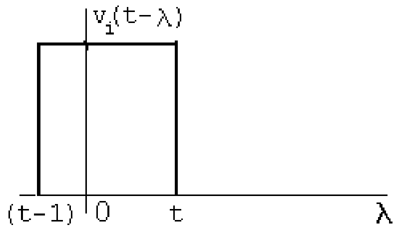
$\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s - 1000}; \qquad i_o = 25[1 - e^{1000t}]u(t)\,\text{A}$$

P 13.57 $H(s) = \frac{V_o}{V_i} = \frac{1}{s+1}; \qquad h(t) = e^{-t}$

For $\quad 0 \le t \le 1$:





$$v_o = \int_0^t e^{-\lambda}\,d\lambda = (1 - e^{-t})\,\text{V}$$

For $\quad 1 \le t \le \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda}\,d\lambda = (e-1)e^{-t}\,\text{V}$$

P 13.58 $H(s) = \frac{V_o}{V_i} = \frac{s}{s+1} = 1 - \frac{1}{s+1}; \qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

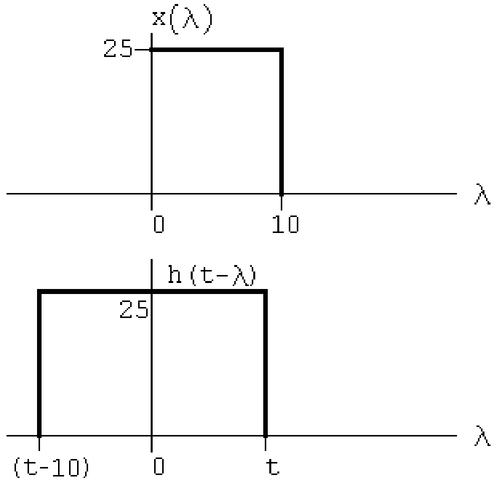
For $\quad 0 \le t \le 1$:

$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\,d\lambda = 1 + [e^{-\lambda}]\,\Big|_0^t = e^{-t}\,\text{V}$$

For $\quad 1 \le t \le \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda})\,d\lambda = e^{-\lambda}\,\Big|_{t-1}^t = (1-e)e^{-t}\,\text{V}$$

P 13.59 **[a]**

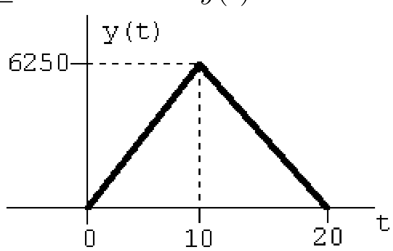


$t < 0: \qquad y(t) = 0$

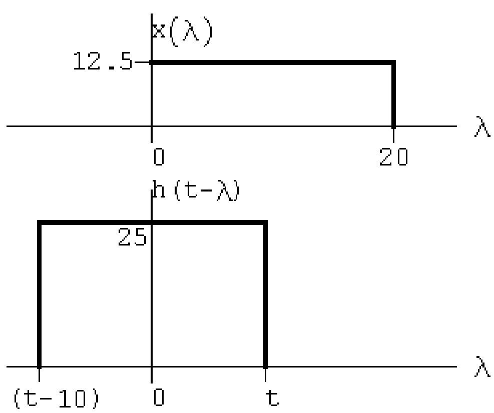
$0 \leq t \leq 10: \qquad y(t) = \int_0^t 625\, d\lambda = 625t$

$10 \leq t \leq 20: \qquad y(t) = \int_{t-10}^{10} 625\, d\lambda = 625(10 - t + 10) = 625(20 - t)$

$20 \leq t < \infty: \qquad y(t) = 0$



[b]



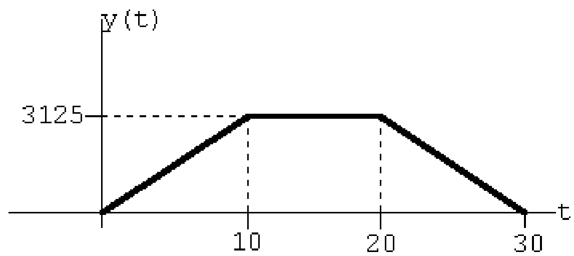
$t < 0: \qquad y(t) = 0$

$0 \leq t \leq 10: \qquad y(t) = \int_0^t 312.5\, d\lambda = 312.5t$

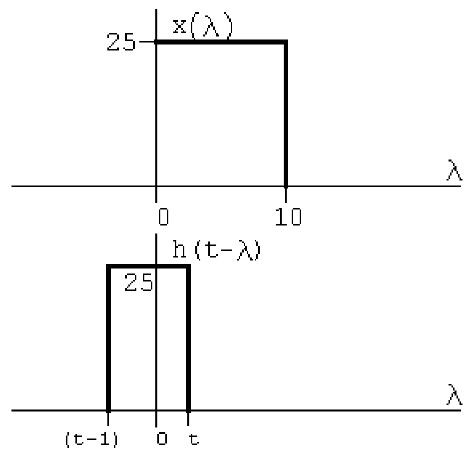
$$10 \le t \le 20: \qquad y(t) = \int_{t-10}^{t} 312.5\, d\lambda = 3125$$

$$20 \le t \le 30: \qquad y(t) = \int_{t-10}^{20} 312.5\, d\lambda = 312.5(30 - t)$$

$$30 \le t < \infty: \qquad y(t) = 0$$



[c]



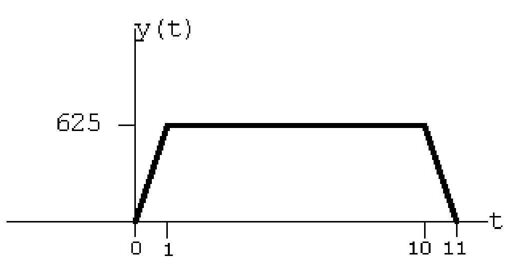
$$t < 0: \qquad y(t) = 0$$

$$0 \le t \le 1: \qquad y(t) = \int_{0}^{t} 625\, d\lambda = 625t$$

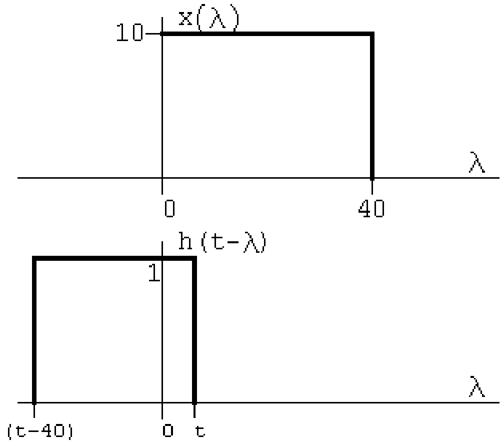
$$1 \le t \le 10: \qquad y(t) = \int_{t-1}^{t} 625\, d\lambda = 625$$

$$10 \le t \le 11: \qquad y(t) = \int_{t-1}^{10} 625\, d\lambda = 625(11 - t)$$

$$11 \le t < \infty: \qquad y(t) = 0$$

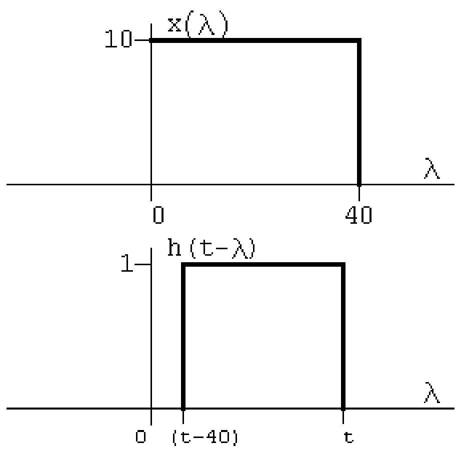


P 13.60 **[a]** $0 \leq t \leq 40$:



$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

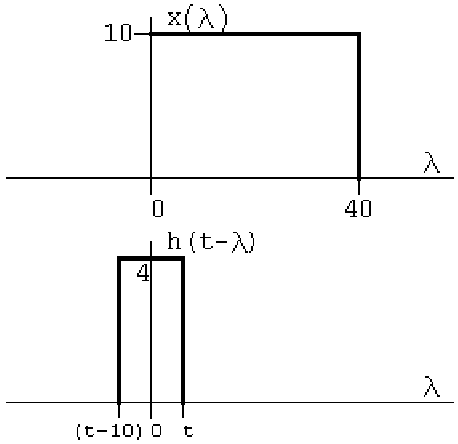
$40 \leq t \leq 80$:



$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

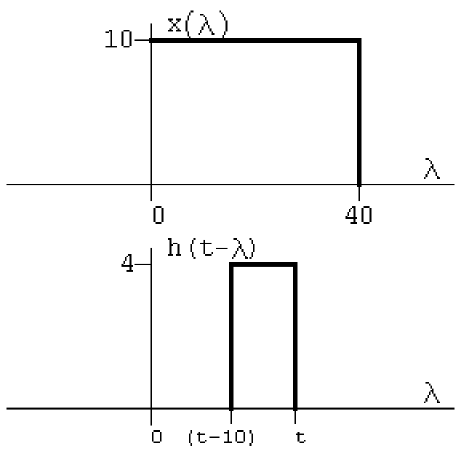
$t \geq 80: \qquad y(t) = 0$

[b] $0 \leq t \leq 10$:



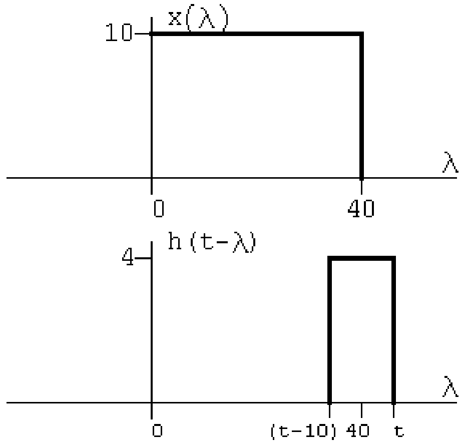
$$y(t) = \int_0^t 40\, d\lambda = 40\lambda \Big|_0^t = 40t$$

$10 \leq t \leq 40$:



$$y(t) = \int_{t-10}^t 40\, d\lambda = 40\lambda \Big|_{t-10}^t = 400$$

$40 \le t \le 50$:



$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50 - t)$$

$$t \ge 50: \qquad y(t) = 0$$

[c] The expressions are

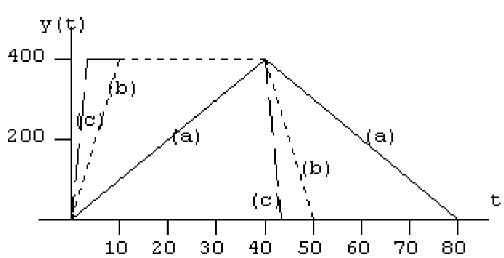
$$0 \le t \le 1: \qquad y(t) = \int_0^t 400\,d\lambda = 400\lambda \Big|_0^t = 400t$$

$$1 \le t \le 40: \qquad y(t) = \int_{t-1}^t 400\,d\lambda = 400\lambda \Big|_{t-1}^t = 400$$

$$40 \le t \le 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41 - t)$$

$$41 \le t < \infty: \qquad y(t) = 0$$

[d]



[e] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \to \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\to 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.61 **[a]** $-1 \leq t \leq 4$:

$$v_o = \int_0^{t+1} 10\lambda\, d\lambda = 5\lambda^2 \Big|_0^{t+1} = 5t^2 + 10t + 5\,\mathrm{V}$$

$4 \leq t \leq 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda\, d\lambda = 5\lambda^2 \Big|_{t-4}^{t+1} = 50t - 75\,\mathrm{V}$$

$9 \leq t \leq 14$:

$$v_o = 10\int_{t-4}^{10} \lambda\, d\lambda + 10\int_{10}^{t+1} 10\, d\lambda$$

$$= 5\lambda^2 \Big|_{t-4}^{10} + 100\lambda \Big|_{10}^{t+1} = -5t^2 + 140t - 480\,\mathrm{V}$$

$14 \leq t \leq 19$:

$$v_o = 100\int_{t-4}^{t+1} d\lambda = 500\,\mathrm{V}$$

$19 \leq t \leq 24$:

$$v_o = \int_{t-4}^{20} 100\, d\lambda + \int_{20}^{t+1} 10(30-\lambda)\, d\lambda$$

$$= 100\lambda \Big|_{t-4}^{20} + 300\lambda \Big|_{20}^{t+1} - 5\lambda^2 \Big|_{20}^{t+1}$$

$$= -5t^2 + 190t - 1305\,\mathrm{V}$$

$24 \leq t \leq 29$:

$$v_o = 10\int_{t-4}^{t+1} (30-\lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{t+1} - 5\lambda^2 \Big|_{t-4}^{t+1}$$

$$= 1575 - 50t\,\mathrm{V}$$

$29 \leq t \leq 34$:

$$v_o = 10\int_{t-4}^{30} (30-\lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{30} - 5\lambda^2 \Big|_{t-4}^{30}$$

$$= 5t^2 - 340t + 5780\,\mathrm{V}$$

Summary:

$v_o = 0 \qquad -\infty \leq t \leq -1$

$v_o = 5t^2 + 10t + 5\,\mathrm{V} \qquad -1 \leq t \leq 4$

$v_o = 50t - 75\,\mathrm{V} \qquad 4 \leq t \leq 9$

$v_o = -5t^2 + 140t - 480\,\mathrm{V} \qquad 9 \leq t \leq 14$

$v_o = 500\,\text{V} \qquad 14 \leq t \leq 19$

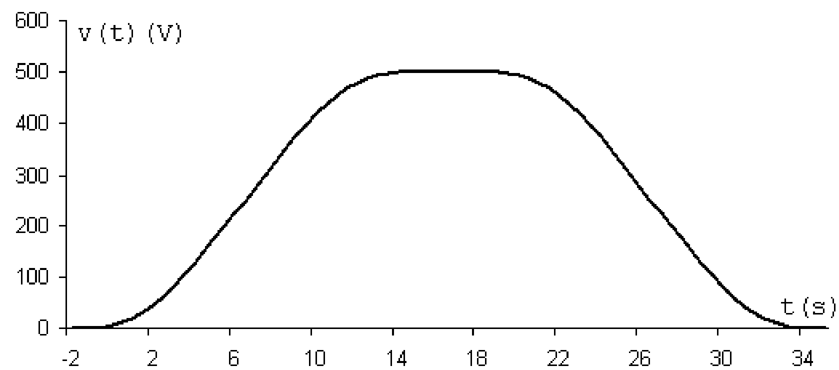
$v_o = -5t^2 + 190t - 1305\,\text{V} \qquad 19 \leq t \leq 24$

$v_o = 1575 - 50t\,\text{V} \qquad 24 \leq t \leq 29$

$v_o = 5t^2 - 340t + 5780\,\text{V} \qquad 29 \leq t \leq 34$

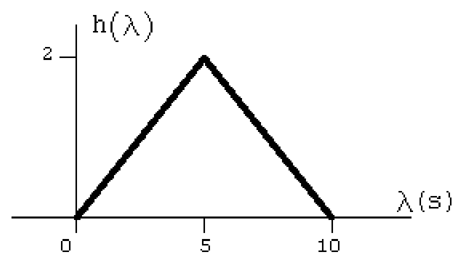
$v_o = 0\,\text{V} \qquad 34 \leq t \leq \infty$

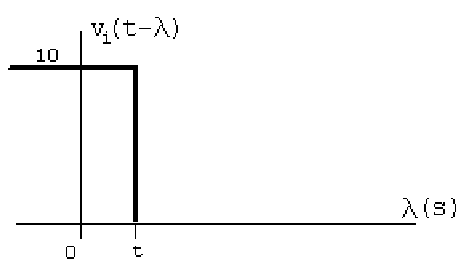
[b]



P 13.62 **[a]** $h(\lambda) = \frac{2}{5}\lambda \qquad 0 \leq \lambda \leq 5$

$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \qquad 5 \leq \lambda \leq 10$





$0 \leq t \leq 5$:

$$v_o = 10\int_0^t \frac{2}{5}\lambda\,d\lambda = 2t^2$$

$5 \le t \le 10$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\,d\lambda + 10\int_5^t \left(4 - \frac{2}{5}\lambda\right)\,d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^t - \frac{4\lambda^2}{2}\Big|_5^t$$

$$= -100 + 40t - 2t^2$$

$10 \le t \le \infty$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\,d\lambda + 10\int_5^{10} \left(4 - \frac{2}{5}\lambda\right)\,d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^{10} - \frac{4\lambda^2}{2}\Big|_5^{10}$$

$$= 50 + 200 - 150 = 100$$

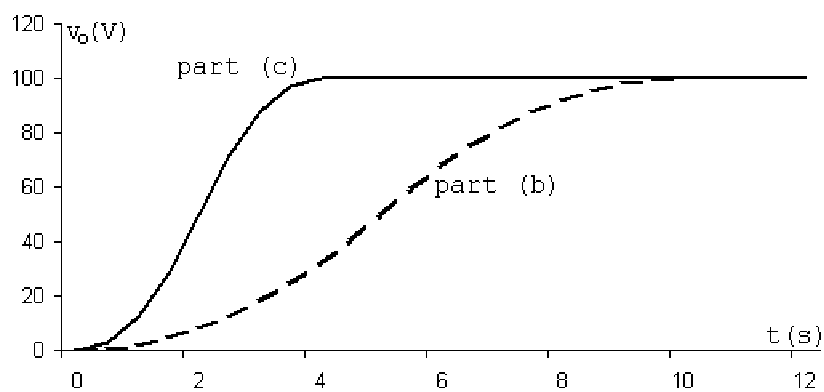
Summary:

$v_o = 2t^2$ V $\qquad 0 \le t \le 5$

$v_o = 40t - 100 - 2t^2$ V $\qquad 5 \le t \le 10$

$v_o = 100$ V $\qquad 10 \le t \le \infty$

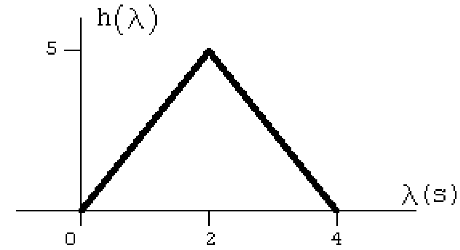
[b]

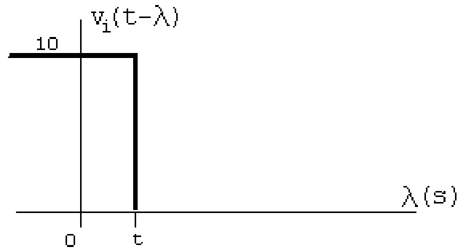


[c] Area $= \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10$ so $h = 5$

$h(\lambda) = \frac{5}{2}\lambda \qquad 0 \le \lambda \le 2$

$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \qquad 2 \le \lambda \le 4$





$0 \le t \le 2$:

$$v_o = 10\int_0^t \frac{5}{2}\lambda\,d\lambda = 12.5t^2$$

$2 \le t \le 4$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^t \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^t - \frac{25\lambda^2}{2}\Big|_2^t$$

$$= -100 + 100t - 12.5t^2$$

$4 \le t \le \infty$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^4 \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^4 - \frac{25\lambda^2}{2}\Big|_2^4$$

$$= 50 + 200 - 150 = 100$$

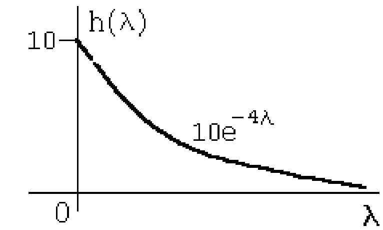
$v_o = 12.5t^2\,\mathrm{V} \qquad 0 \le t \le 2$

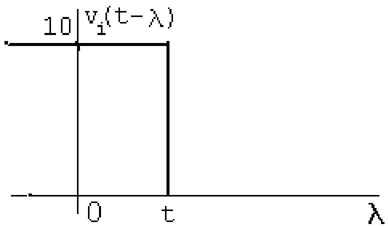
$v_o = 100t - 100 - 12.5t^2\,\mathrm{V} \qquad 2 \le t \le 4$

$v_o = 100\,\mathrm{V} \qquad 4 \le t \le \infty$

[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.63 **[a]**



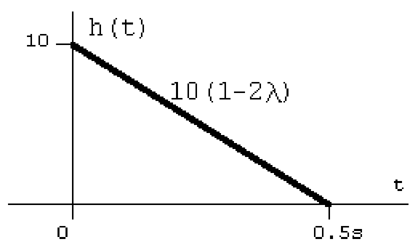


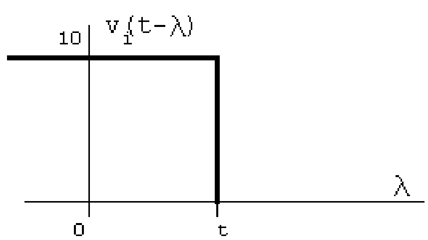
$$v_o = \int_0^t 10(10e^{-4\lambda})\,d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\Big|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t})\,\text{V}, \qquad 0 \le t \le \infty$$

[b]





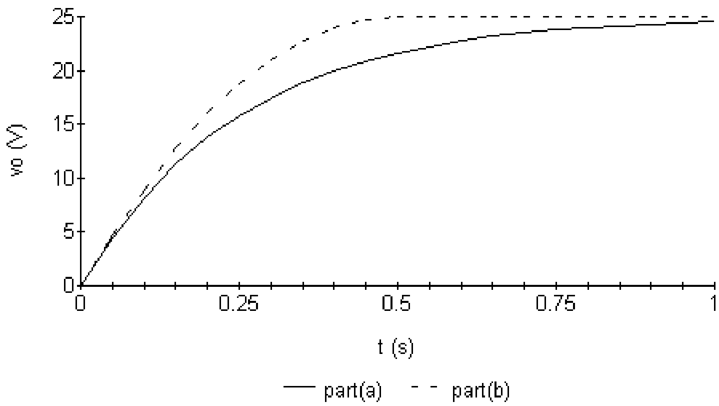
$0 \le t \le 0.5$:

$$v_o = \int_0^t 100(1 - 2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1 - t)$$

$0.5 \le t \le \infty$:

$$v_o = \int_0^{0.5} 100(1 - 2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$

[c]



P 13.64 **[a]** From Problem 13.49(a)

$$H(s) = \frac{200}{s+200}$$

$$h(\lambda) = 200e^{-200\lambda}$$

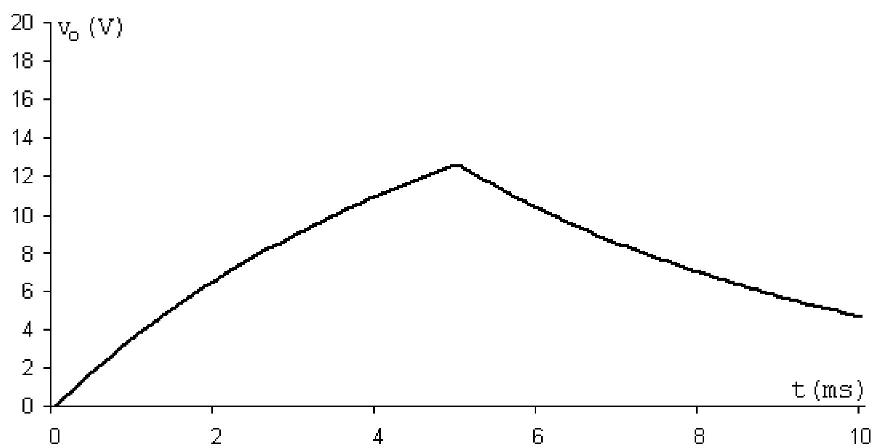
$0 \leq t \leq 5\,\text{ms}$:

$$v_o = \int_0^t 20(200)e^{-200\lambda}\,d\lambda = 20(1-e^{-200t})\,\text{V}$$

$5\,\text{ms} \leq t \leq \infty$:

$$v_o = \int_{t-5\times10^{-3}}^{t} 20(200)e^{-200\lambda}\,d\lambda = 20(e^1-1)e^{-200t}\,\text{V}$$

[b]



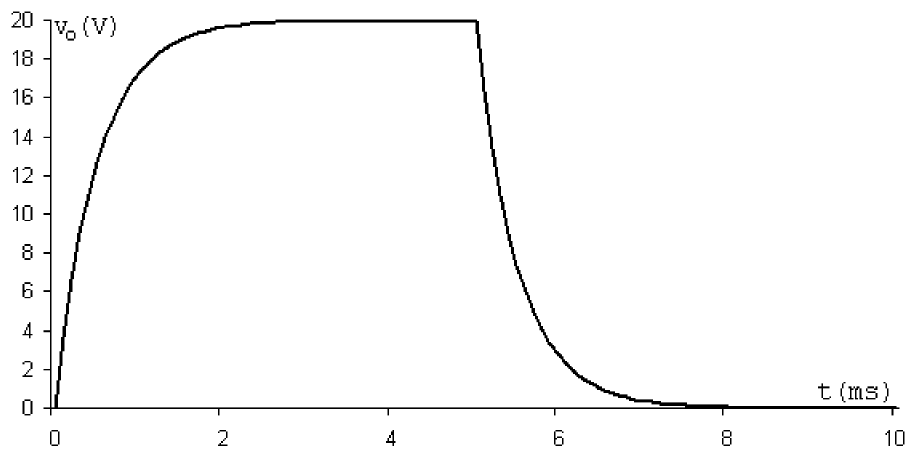
P 13.65 **[a]** $H(s) = \dfrac{2000}{s+2000} \qquad \therefore \quad h(\lambda) = 2000e^{-2000\lambda}$

$0 \le t \le 5\,\text{ms}$:

$$v_o = \int_0^t 20(2000)e^{-2000\lambda}\,d\lambda = 20(1 - e^{-2000t})\text{ V}$$

$5\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-5\times10^{-3}}^t 20(2000)e^{-2000\lambda}\,d\lambda = 20(e^{10} - 1)e^{-2000t}\text{ V}$$



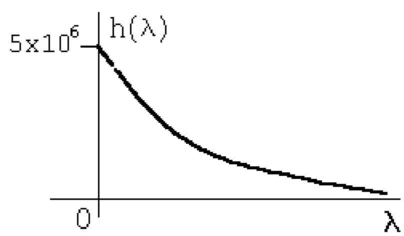
[b] decrease

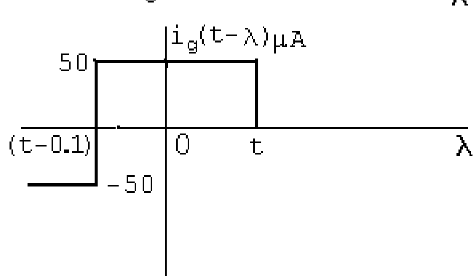
[c] The circuit with $R = 5\,\text{k}\Omega$.

P 13.66 **[a]** $I_g = \dfrac{V_o}{10^5} + \dfrac{V_o s}{5\times10^6} = \dfrac{V_o(s+50)}{5\times10^6}$

$$\frac{V_o}{I_g} = H(s) = \frac{5\times10^6}{s+50}$$

$$h(\lambda) = 5\times10^6 e^{-50\lambda}u(\lambda)$$



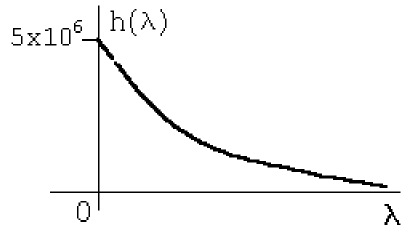


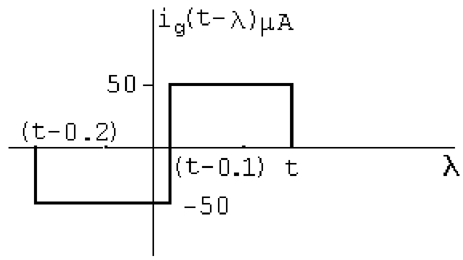
$0 \le t \le 0.1\,\text{s}$:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6)e^{-50\lambda}\,d\lambda = 250\frac{e^{-50\lambda}}{-50}\Big|_0^t$$

$$= 5(1 - e^{-50t})\,\text{V}$$

$0.1\,\text{s} \le t \le 0.2\,\text{s}$:





$$v_o = \int_0^{t-0.1} (-50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\,d\lambda)$$

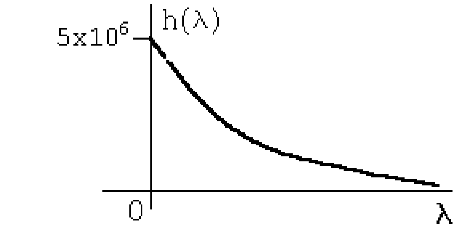
$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\,d\lambda)$$

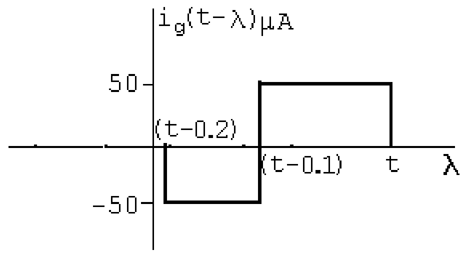
$$= -250\frac{e^{-50\lambda}}{-50}\Big|_0^{t-0.1} + 250\frac{e^{-50\lambda}}{-50}\Big|_{t-0.1}^{t}$$

$$= 5\left[e^{-50(t-0.1)} - 1\right] - 5\left[e^{-50t} - e^{-50(t-0.1)}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V}$$

$0.2\,\text{s} \le t \le \infty$:





$$v_o = \int_{t-0.2}^{t-0.1} -250e^{-50\lambda}\,d\lambda + \int_{t-0.1}^{t} 250e^{-50\lambda}\,d\lambda$$

$$= 5e^{-50\lambda}\Big|_{t-0.2}^{t-0.1} - 5e^{-50\lambda}\Big|_{t-0.1}^{t}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V}$$

Summary:

$$v_o = 5(1 - e^{-50t})\,\text{V} \qquad 0 \le t \le 0.1\,\text{s}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V} \qquad 0.1\,\text{s} \le t \le 0.2\,\text{s}$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V} \qquad 0.2\,\text{s} \le t \le \infty$$

[b] $I_o = \dfrac{V_o s}{5 \times 10^6} = \dfrac{s}{5 \times 10^6} \cdot \dfrac{5 \times 10^6 I_g}{s + 50}$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s+50} = 1 - \frac{50}{s+50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

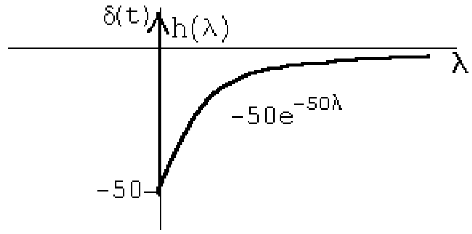
$0 < t < 0.1\,\text{s}$:

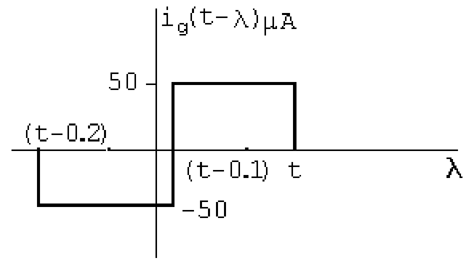
$$i_o = \int_0^t (50 \times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$= 50 \times 10^{-6} - \left[50 \times 50 \times 10^{-6}\frac{e^{-50\lambda}}{-50}\right]\Big|_0^t$$

$$= 50 \times 10^{-6} + 50 \times 10^{-6}[e^{-50t} - 1] = 50e^{-50t}\,\mu\text{A}$$

$0.1\,\mathrm{s} < t < 0.2\,\mathrm{s}$:





$$i_o = \int_0^{t-0.1} (-50 \times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

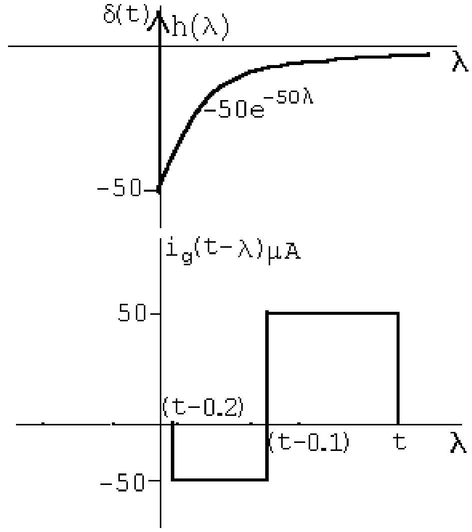
$$+ \int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= -50 \times 10^{-6} + 2500 \times 10^{-6} \frac{e^{-50\lambda}}{-50}\Big|_0^{t-0.1} - 2500 \times 10^{-6} \frac{e^{-50\lambda}}{-50}\Big|_{t-0.1}^{t}$$

$$= -50 \times 10^{-6} - 50 \times 10^{-6}[e^{-50(t-0.1)} - 1] + 50 \times 10^{-6}[e^{-50t} - e^{-50(t-0.1)}]$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)}\,\mu\mathrm{A}$$

$0.2\,\text{s} < t < \infty$:



$$i_o = \int_{t-0.2}^{t-0.1} (-50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\,\mu\text{A}$$

Summary:

$i_0 = 50e^{-50t}\,\mu\text{A} \qquad 0 \le t \le 0.1\,\text{s}$

$i_0 = 50e^{-50t} - 100e^{-50(t-0.1)}\,\mu\text{A} \qquad 0.1\,\text{s} \le t \le 0.2\,\text{s}$

$i_0 = 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\,\mu\text{A} \qquad 0.2\,\text{s} \le t \le \infty$

[c] At $\quad t = 0.1^-$:

$$v_o = 5(1 - e^{-5}) = 4.97\,\text{V}; \qquad i_{100\text{k}\Omega} = \frac{4.97}{0.1} = 49.66\,\mu\text{A}; \qquad i_g = 50\,\mu\text{A}$$

$\therefore \quad i_o = 50 - 49.66 = 0.34\,\mu\text{A}$

From the solution for i_o we have $i_o(0.1^-) = 50e^{-5} = 0.34\,\mu\text{A}$ (Checks)
At $\quad t = 0.1^+$:

$$v_o(0.1^+) = v_o(0.1^-) = 4.97\,\mu\text{V}; \qquad i_{100\text{k}\Omega} = 49.66\,\mu\text{A}; \qquad i_g = -50\,\mu\text{A}$$

$\therefore \quad i_o(0.1^+) = -(50 + 49.66) = -99.66\,\mu\text{A}$

From the solution for i_o we have

$i_o(0.1^+) = 50e^{-5} - 100 = -99.66\,\mu\text{A} \quad$ (Checks)

At $t = 0.2^-$:

$$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93\,\mu\text{V}$$

$$i_{100\text{k}\Omega} = -49.33\,\mu\text{A} \qquad i_g = -50\,\mu\text{A}$$

$$i_o = i_g - i_{100\text{k}\Omega} = -50 + 49.33 = -0.67\,\mu\text{A}$$

From the solution for i_o, $i_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67\,\mu$A (Checks)
At $t = 0.2^+$:

$$v_o(0.2^+) = i_o(0.2^-) = -4.93\,\text{V}; \qquad i_{100\text{k}\Omega} = -49.33\,\mu\text{A}; \qquad i_g = 0$$

$$i_o = i_g - i_{100\text{k}\Omega} = 49.33\,\mu\text{A}$$

From the solution for i_o,
$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33\,\mu\text{A}(\text{Checks})$

P 13.67 $H(s) = \dfrac{V_o}{V_i} = \dfrac{5}{5 + 2.5s} = \dfrac{2}{s+2}$

$$h(\lambda) = 2e^{-2\lambda}; \qquad h(t-\lambda) = 2e^{-2(t-\lambda)} = 2e^{-2t}e^{2\lambda}$$

$$\frac{T}{2} = \frac{\pi}{2}; \qquad T = \pi\,\text{s}; \qquad f = \frac{1}{\pi}\,\text{Hz}$$

$$v_i(\lambda) = (20\sin 2\lambda)[u(\lambda) - u(\lambda - \pi/2)]$$

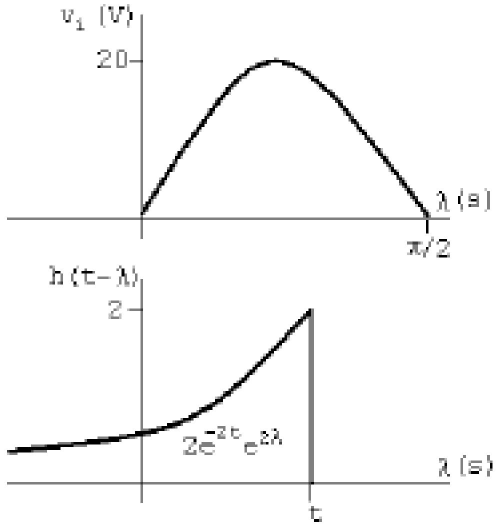
$(\pi/2)\,\text{s} \leq t \leq \infty$:

$$v_o = \int_0^{\pi/2} (2e^{-2t}e^{2\lambda})(20\sin 2\lambda)\,d\lambda = 40e^{-2t}\int_0^{\pi/2} e^{2\lambda}\sin 2\lambda\,d\lambda$$

$$= 40e^{-2t}\left[\frac{e^{2\lambda}}{8}(2\sin 2\lambda - 2\cos 2\lambda)\right]_0^{\pi/2} = 10e^{-2t}[e^{\pi}(\sin\pi - \cos\pi) - 1(0-1)]$$

$$= 10e^{-2t}(e^{\pi} + 1) = 10(e^{\pi} + 1)e^{-2t}\,\text{V}$$

$$v_o(2.2) = 241.41e^{-4.4} = 2.96\,\text{V}$$

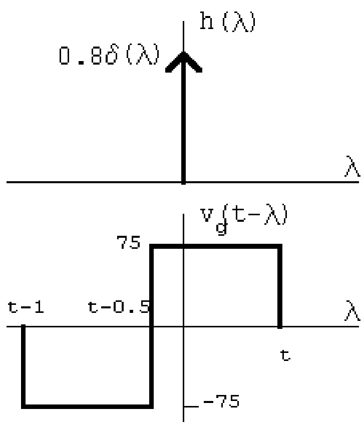


P 13.68 **[a]** $V_o = \dfrac{16}{20} V_g$

$$\therefore \qquad H(s) = \frac{V_o}{V_g} = \frac{4}{5}$$

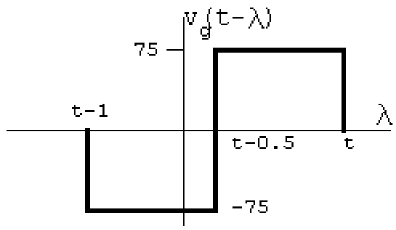
$$h(\lambda) = 0.8\delta(\lambda)$$

[b]



$$0 < t < 0.5\,\text{s} : \qquad v_o = \int_0^t 75[0.8\delta(\lambda)]\, d\lambda = 60\,\text{A}$$

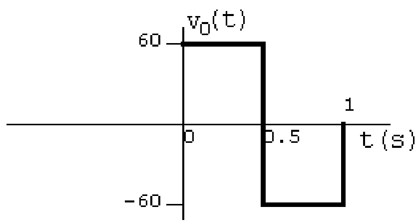
$0.5\,\text{s} \le t \le 1.0\,\text{s}$:



$$v_o = \int_0^{t-0.5} -75[0.8\delta(\lambda)]\,d\lambda = -60\,\text{A}$$

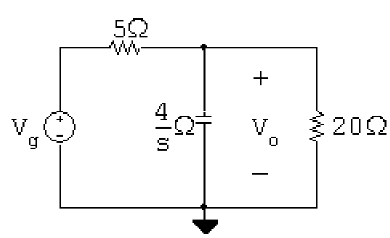
$1\,\text{s} < t < \infty: \qquad v_o = 0$

[c]



Yes, because the circuit has no memory.

P 13.69 **[a]**

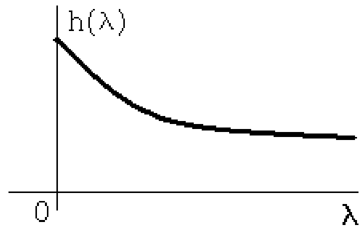


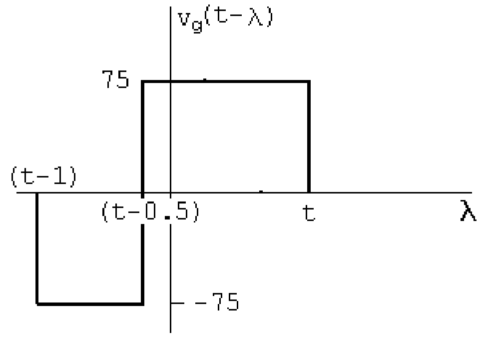
$$\frac{V_o - V_g}{5} + \frac{V_o s}{4} + \frac{V_o}{20} = 0$$

$$(5s + 5)V_o = 4V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.8}{s+1}; \qquad h(\lambda) = 0.8e^{-\lambda}u(\lambda)$$

[b]



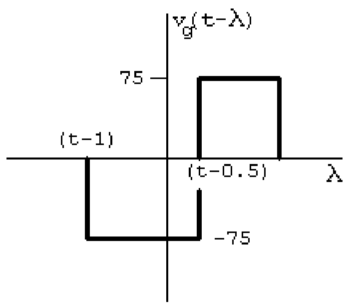


$0 \leq t \leq 0.5\,\text{s}$;

$$v_o = \int_0^t 75(0.8e^{-\lambda})\,d\lambda = 60\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 60 - 60e^{-t}\,\text{V}, \qquad 0 \leq t \leq 0.5\,\text{s}$$

$0.5\,\text{s} \leq t \leq 1\,\text{s}$:



$$v_o = \int_0^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-t} - 60\,\text{V}, \qquad 0.5\,\text{s} \leq t \leq 1\,\text{s}$$

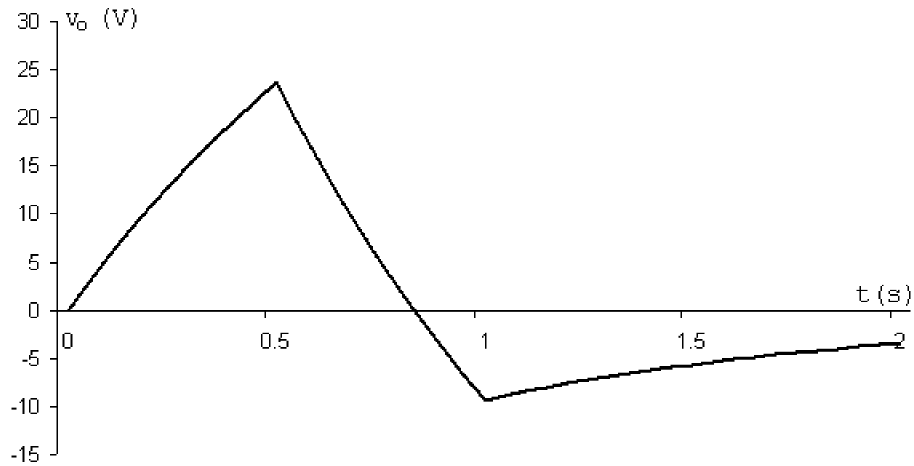
$1\,\text{s} \le t \le \infty$;

$$v_o = \int_{t-1}^{t-0.5} (-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

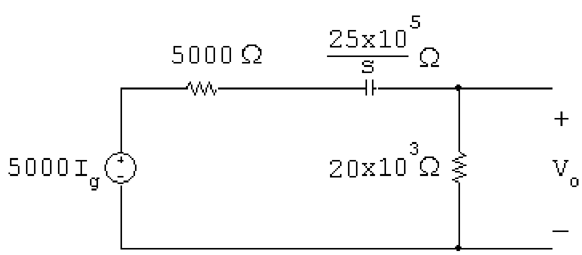
$$= 120e^{-(t-0.5)} - 60e^{-(t-1)} - 60e^{-t}\,\text{V}, \qquad 1\,\text{s} \le t \le \infty$$

[c]



[d] No, the circuit has memory because of the capacitive storage element.

P 13.70



$$V_o = \frac{20 \times 10^3}{5000 + 25 \times 10^5/s + 20 \times 10^3}(5000I_g)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s + 100}$$

$$H(s) = 4000\left[1 - \frac{100}{s+100}\right] = 4000 - \frac{4 \times 10^5}{s + 100}$$

$$h(\lambda) = 4000\delta(\lambda) - 400{,}000e^{-100\lambda}u(\lambda)$$

$$v_o = \int_0^{10^{-3}} (-20\times 10^{-3})[4000\delta(\lambda) - 400{,}000e^{-100\lambda}]\,d\lambda$$

$$+\int_{10^{-3}}^{5\times 10^{-3}} (10\times 10^{-3})[-400{,}000e^{-100\lambda}]\,d\lambda$$

$$= -80 + 8000\int_0^{10^{-3}} e^{-100\lambda}\,d\lambda - \int_{10^{-3}}^{5\times 10^{-3}} 4000e^{-100\lambda}\,d\lambda$$

$$= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1})$$

$$v_o(5\times 10^{-3}) = 40e^{-0.5} - 120e^{-0.1} = 24.26 - 108.58 = -84.32\,\text{V}$$

Alternate solution (not using the convolution integral):

$$I_g = \int_0^{4\times 10^{-3}} (10\times 10^{-3})e^{-st}\,dt + \int_{4\times 10^{-3}}^{6\times 10^{-3}} (-20\times 10^{-3})e^{-st}\,dt$$

$$= 10^{-3}\frac{e^{-st}}{-s}\Bigg|_0^{4\times 10^{-3}} - 20\times 10^{-3}\frac{e^{-st}}{-s}\Bigg|_{4\times 10^{-3}}^{6\times 10^{-3}}$$

$$= 10\times 10^{-3}\left[\frac{1}{s} - \frac{e^{-4\times 10^{-3}s}}{s}\right] + 20\times 10^{-3}\left[\frac{e^{-6\times 10^{-3}s} - e^{-4\times 10^{-3}s}}{s}\right]$$

$$= \frac{10\times 10^{-3}}{s} - \frac{30\times 10^{-3}}{s}e^{-4\times 10^{-3}s} + \frac{20\times 10^{-3}}{s}e^{-6\times 10^{-3}s}$$

$$V_o = I_gH(s) = \frac{40}{s+100} - \frac{120e^{-4\times 10^{-3}s}}{s+100} + \frac{80e^{-6\times 10^{-3}s}}{s+100}$$

Now use the operational transform $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$:

$$v_o = 40e^{-100t} - 120e^{-100(t-4\times 10^{-3})}u(t - 4\times 10^{-3})$$

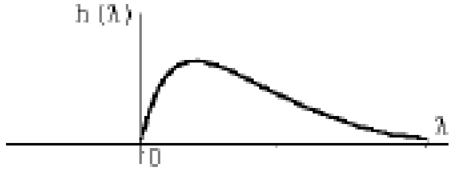
$$+\,80e^{-100(t-6\times 10^{-3})}u(t - 6\times 10^{-3})\,\text{V}$$

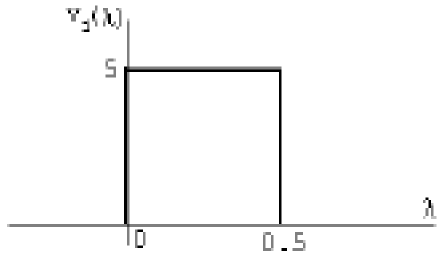
$$v_o(5\times 10^{-3}) = 40e^{-0.5} - 120e^{-0.1} + 80(0) = -84.32\,\text{V (Checks)}$$

P 13.71 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{100}{s^2 + 20s + 100} = \frac{100}{(s+10)^2}$$

$$h(\lambda) = 100\lambda e^{-10\lambda}u(\lambda)$$





$0 \leq t \leq 0.5$:

$$v_o = 500\int_0^t \lambda e^{-10\lambda}\,d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\left.\right|_0^t\right\}$$

$$= 5[1 - e^{-10t}(10t + 1)]$$

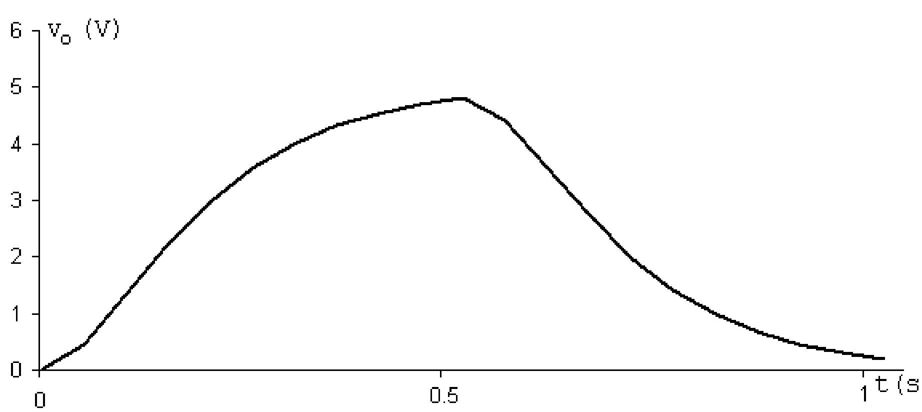
$0.5 \leq t \leq \infty$:

$$v_o = 500\int_{t-0.5}^t \lambda e^{-10\lambda}\,d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\left.\right|_{t-0.5}^t\right\}$$

$$= 5e^{-10t}[e^5(10t - 4) - 10t - 1]$$

[b]



P 13.72 $H(s) = \dfrac{16s}{40 + 4s + 16s} = \dfrac{0.8s}{s+2} = 0.8\left(1 - \dfrac{2}{s+2}\right) = 0.8 - \dfrac{1.6}{s+2}$

$$h(\lambda) = 0.8\delta(\lambda) - 1.6e^{-2\lambda}u(\lambda)$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 1.6e^{-2\lambda}]\,d\lambda = \int_0^t 60\delta(\lambda)\,d\lambda - 120\int_0^t e^{-2\lambda}\,d\lambda$$

$$= 60 - 120\frac{e^{-2\lambda}}{-2}\Big|_0^t = 60 + 60(e^{-2t} - 1)$$

$$= 60e^{-2t}u(t)\,\text{V}$$

P 13.73 **[a]** $Y(s) = \int_0^\infty y(t)e^{-st}\,dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\,d\lambda\right]dt$$

$$= \int_0^\infty\int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\,d\lambda\,dt$$

$$= \int_0^\infty h(\lambda)\int_0^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$$

But $\quad x(t-\lambda) = 0 \quad$ when $\quad t < \lambda$

Therefore $\quad Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$

Let $\quad u = t - \lambda; \quad du = dt; \quad u = 0, \quad t = \lambda; \quad u = \infty, \quad t = \infty$

$$Y(s) = \int_0^\infty h(\lambda)\int_0^\infty e^{-s(u+\lambda)}x(u)\,du\,d\lambda$$

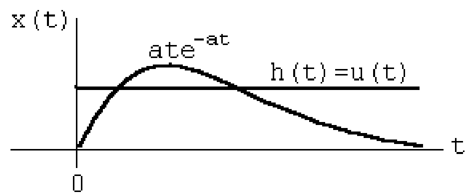
$$= \int_0^\infty h(\lambda)e^{-s\lambda}\int_0^\infty e^{-su}x(u)\,du\,d\lambda$$

$$= \int_0^\infty h(\lambda)e^{-s\lambda}X(s)\,d\lambda = H(s)\,X(s)$$

We are using one-sided Laplace transforms; therefore $h(t)$ and $X(t)$ are assumed zero for $t < 0$.

[b] $F(s) = \frac{a}{s(s+a)^2} = \frac{1}{s} \cdot \frac{a}{(s+a)^2} = H(s)X(s)$

$\therefore \quad h(t) = u(t), \qquad x(t) = at\, e^{-at}u(t)$



$$\therefore \quad f(t) = \int_0^t (1)a\lambda e^{-a\lambda}\, d\lambda = a\left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1)\right]\Bigg|_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a}e^{-at} - te^{-at}\right]u(t)$$

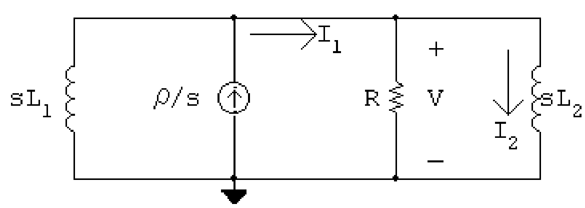
Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right]u(t)$$

P 13.74 **[a]** The s-domain circuit is



The node-voltage equation is $\quad \frac{V}{sL_1} + \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho}{s}$

Therefore $\quad V = \frac{\rho R}{s + (R/L_e)} \quad$ where $\quad L_e = \frac{L_1 L_2}{L_1 + L_2}$

Therefore $\quad v = \rho R e^{-(R/L_e)t}u(t)$ V

[b] $I_1 = \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \frac{K_0}{s} + \frac{K_1}{s + (R/L_e)}$

$$K_0 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_1 = \frac{\rho L_2}{L_1 + L_2}$$

Thus we have $i_1 = \frac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)$ A

[c] $I_2 = \frac{V}{sL_2} = \frac{(\rho R/L_2)}{s[s + (R/L_e)]} = \frac{K_2}{s} + \frac{K_3}{s + (R/L_e)}$

$$K_2 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_3 = \frac{-\rho L_1}{L_1 + L_2}$$

Therefore $i_2 = \frac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

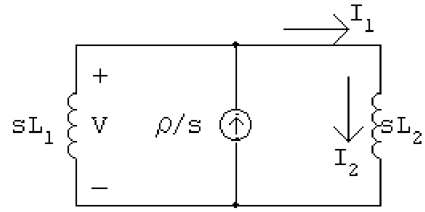
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.75 **[a]** As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$$i_1(t) \to \frac{\rho L_1}{L_1 + L_2} \quad \text{as} \quad R \to \infty$$

$$i_2(t) \to \frac{\rho L_1}{L_1 + L_2} \quad \text{as} \quad R \to \infty$$

[b] The s-domain circuit is



$$\frac{V}{sL_1} + \frac{V}{sL_2} = \frac{\rho}{s}; \qquad \text{therefore} \quad V = \frac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$$

Therefore $\quad v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $\quad i_1 = i_2 = \dfrac{\rho L_1}{L_1 + L_2} u(t)$ A

P 13.76 $\quad H(j3) = \dfrac{4(3 + j3)}{-9 + j24 + 41} = 0.42\underline{/8.13^\circ}$

$\therefore \quad v_o(t) = 16.97 \cos(3t + 8.13^\circ)$ V

P 13.77 **[a]** $H(s) = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_f C_f)} = \frac{10^8}{s + 1000}$$

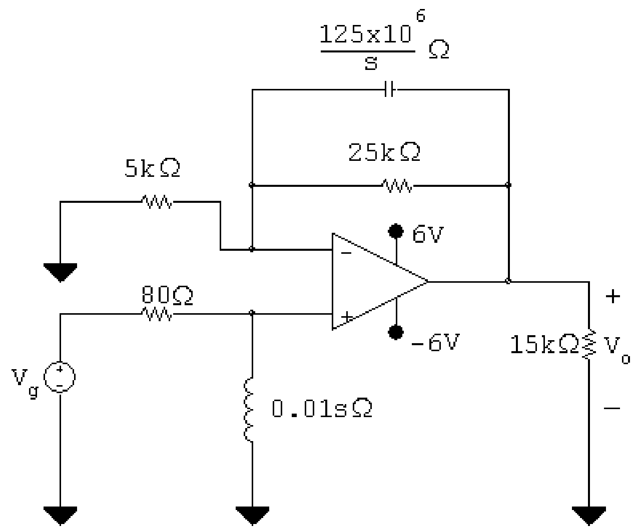
$$Z_i = \frac{R_i[s + (1/R_i C_i)]}{s} = \frac{10{,}000(s + 400)}{s}$$

$$H(s) = \frac{-10^4 s}{(s + 400)(s + 1000)}$$

[b] $H(j400) = \dfrac{-10^4(j400)}{(400 + j400)(1000 + j400)} = 6.565\underline{/-156.8^\circ}$

$v_o(t) = 13.13 \cos(400t - 156.8^\circ)$ V

P 13.78 **[a]**



$$V_p = \frac{0.01s}{80 + 0.01s} V_g = \frac{s}{s + 8000} V_g$$

$$\frac{V_n}{5000} + \frac{V_n - V_o}{25{,}000} + (V_n - V_o)8 \times 10^{-9}s = 0$$

$$5V_n + V_n - V_o + (V_n - V_o)2 \times 10^{-4}s = 0$$

$$6V_n + 2 \times 10^{-4}sV_n = V_o + 2 \times 10^{-4}sV_o$$

$$2 \times 10^{-4}V_n(s + 30{,}000) = 2 \times 10^{-4}V_o(s + 5000)$$

$$V_n = V_p$$

$$V_o = \frac{s + 30{,}000}{s + 5000} V_f = \left(\frac{s + 30{,}000}{s + 5000}\right)\left(\frac{sV_g}{s + 8000}\right)$$

$$H(s) = \frac{V_o}{V_g} = \frac{s(s + 30{,}000)}{(s + 5000)(s + 8000)}$$

[b] $v_g = 0.6u(t); \qquad V_g = \dfrac{0.6}{s}$

$$V_o = \frac{0.6(s + 30{,}000)}{(s + 5000)(s + 8000)} = \frac{K_1}{s + 5000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{0.6(25{,}000)}{3000} = 5; \qquad K_2 = \frac{0.6(22{,}000)}{-3000} = -4.4$$

$$\therefore \quad v_o(t) = (5e^{-5000t} - 4.4e^{-8000t})u(t)\,\text{V}$$

[c] $V_g = 2\cos 10{,}000t\,\text{V}$

$$H(j\omega) = \frac{j10{,}000(30{,}000 + j10{,}000)}{(5000 + j10{,}000)(8000 + j10{,}000)} = 2.21\underline{/-6.34^\circ}$$

$$\therefore \qquad v_o = 4.42\cos(10{,}000t - 6.34^\circ)\,\text{V}$$

P 13.79 $V_o = \dfrac{50}{s+8000} - \dfrac{20}{s+5000} = \dfrac{30(s+3000)}{(s+5000)(s+8000)}$

$$V_o = H(s)V_g = H(s)\left(\frac{30}{s}\right)$$

$$\therefore \quad H(s) = \frac{s(s+3000)}{(s+5000)(s+8000)}$$

$$H(j6000) = \frac{(j6000)(3000+j6000)}{(5000+j6000)(8000+j6000)} = 0.52\underline{/66.37^\circ}$$

$$\therefore \quad v_o(t) = 61.84\cos(6000t + 66.37^\circ)\,\text{V}$$

P 13.80 Original charge on C_1; $\quad q_1 = V_0C_1$

The charge transferred to C_2; $\quad q_2 = V_0C_e = \dfrac{V_0C_1C_2}{C_1+C_2}$

The charge remaining on C_1; $\quad q_1' = q_1 - q_2 = \dfrac{V_0C_1^2}{C_1+C_2}$

Therefore $\quad V_2 = \dfrac{q_2}{C_2} = \dfrac{V_0C_1}{C_1+C_2} \quad$ and $\quad V_1 = \dfrac{q_1'}{C_1} = \dfrac{V_0C_1}{C_1+C_2}$

P 13.81 **[a]** $Z_1 = \dfrac{1/C_1}{s+1/R_1C_1} = \dfrac{25\times 10^{10}}{s+20\times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s+1/R_2C_2} = \frac{6.25\times 10^{10}}{s+12{,}500}\Omega$$

$$\frac{V_o}{Z_2} + \frac{V_o - 10/s}{Z_1} = 0$$

$$\frac{V_o(s+12{,}500)}{6.25\times 10^{10}} + \frac{V_o(s+20\times 10^4)}{25\times 10^{10}} = \frac{10}{s}\,\frac{(s+20\times 10^4)}{25\times 10^{10}}$$

$$V_o = \frac{2(s+200{,}000)}{s(s+50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore \quad v_o = [8 - 6e^{-50{,}000t}]u(t)\,\text{V}$$

[b] $I_0 = \frac{V_0}{Z_2} = \frac{2(s+200{,}000)(s+12{,}500)}{s(s+50{,}000)6.25\times10^{10}}$

$$= 32\times10^{-12}\left[1+\frac{162{,}500s+25\times10^{8}}{s(s+50{,}000)}\right]$$

$$= 32\times10^{-12}\left[1+\frac{K_1}{s}+\frac{K_2}{s+50{,}000}\right]$$

$K_1 = 50{,}000; \qquad K_2 = 112{,}500$

$i_o = 32\delta(t) + [1.6\times10^{6} + 3.6\times10^{6}e^{-50{,}000t}]u(t)$ pA

[c] When $C_1 = 64\,\text{pF}$

$$Z_1 = \frac{156.25\times10^{8}}{s+12{,}500}\Omega$$

$$\frac{V_0(s+12{,}500)}{625\times10^{8}} + \frac{V_0(s+12{,}500)}{156.25\times10^{8}} = \frac{10}{s}\,\frac{(s+12{,}500)}{156.25\times10^{8}}$$

$$\therefore\quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$v_o = 8u(t)$ V

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\,\frac{(s+12{,}500)}{6.25\times10^{10}} = 128\times10^{-12}\left[1+\frac{12{,}500}{s}\right]$$

$i_o(t) = 128\delta(t) + 1.6\times10^{-6}u(t)$ pA

P 13.82 Let $a = \frac{1}{R_1C_1} = \frac{1}{R_2C_2}$

Then $Z_1 = \frac{1}{C_1(s+a)}$ and $Z_2 = \frac{1}{C_2(s+a)}$

$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_oC_1(s+a) = (10/s)C_1(s+a)$$

$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1+C_2}\right)$$

Thus, v_o is the input scaled by the factor $\frac{C_1}{C_1+C_2}$.

P 13.83 **[a]** For $t < 0, \quad 0.5v_1 = 2v_2;$ therefore $v_1 = 4v_2$

$v_1 + v_2 = 100;$ therefore $v_1(0^-) = 80\,\text{V}$

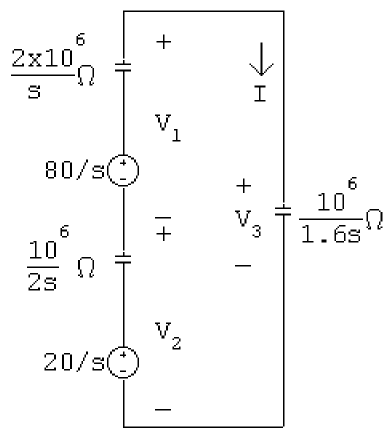
[b] $v_2(0^-) = 20\,\text{V}$

[c] $v_3(0^-) = 0\,\text{V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$$i(t) = 32\delta(t)\,\mu\text{A}$$



[e] $v_1(0^+) = -\dfrac{10^6}{0.5}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16\,\text{V}$

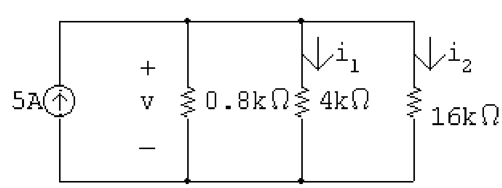
[f] $v_2(0^+) = -\dfrac{10^6}{2}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4\,\text{V}$

[g] $V_3 = \dfrac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \dfrac{20}{s}$

$v_3(t) = 20u(t)\,\text{V}; \qquad v_3(0^+) = 20\,\text{V}$

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

P 13.84 **[a]** For $t < 0$:



$R_{\text{eq}} = 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$

$i_1(0^-) = \dfrac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \dfrac{3200}{16{,}000} = 0.2\,\text{A}$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

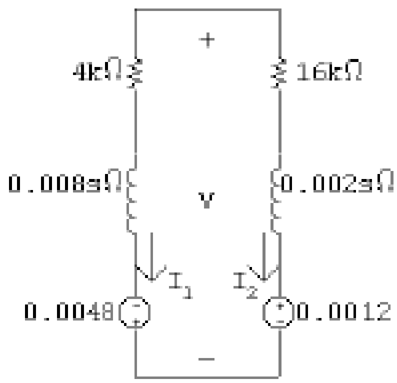
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2\times10^6}$$

$$i_1(t) = 0.6e^{-2\times10^6 t}u(t)\,\text{A}$$

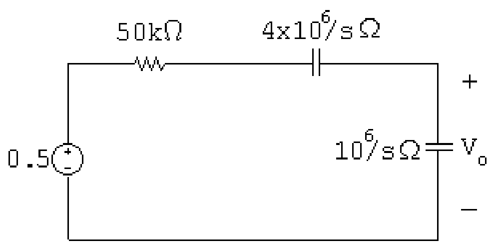
[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times10^6 t}u(t)\,\text{A}$

[g] $V = -0.0064 + (0.008s + 4000)I_1 = \dfrac{-0.0016(s + 6.5\times10^6)}{s + 2\times10^6}$

$$= -1.6\times10^{-3} - \frac{7200}{s + 2\times10^6}$$

$$v(t) = [-1.6\times10^{-3}\delta(t)] - [7200e^{-2\times10^6 t}u(t)]\,\text{V}$$

P 13.85 **[a]**



$$V_o = \frac{0.5}{50{,}000 + 5\times10^6/s}\cdot\frac{10^6}{s}$$

$$\frac{500{,}000}{50{,}000s + 5 \times 10^6} = \frac{10}{s + 100}$$

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

[b] At $t = 0$ the current in the $1\,\mu$F capacitor is $10\delta(t)\,\mu$A

$$\therefore \qquad v_o(0^+) = 10^6 \int_{0^-}^{0^+} 10 \times 10^{-6}\delta(t)\,dt = 10\,\text{V}$$

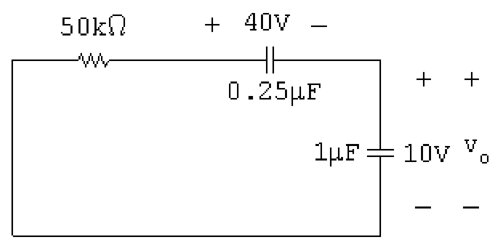
After the impulsive current has charged the $1\,\mu$F capacitor to 10 V it discharges through the 50 kΩ resistor.

$$C_e = \frac{C_1C_2}{C_1 + C_2} = \frac{0.25}{1.25} = 0.2\,\mu\text{F}$$

$$\tau = (50{,}000)(0.2 \times 10^{-6}) = 10^{-2}$$

$$\frac{1}{\tau} = 100 \text{ (Checks)}$$

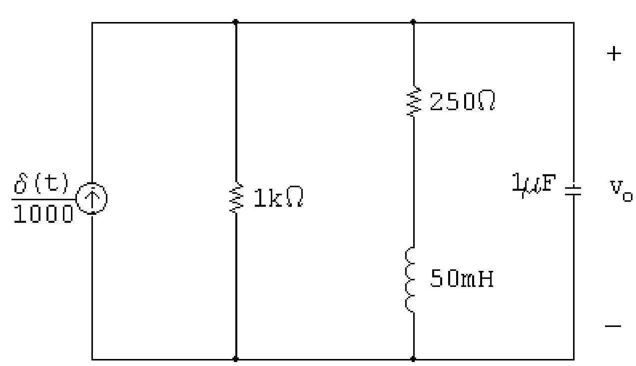
Note – after the impulsive current passes the circuit becomes



The solution for v_o in this circuit is also

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

P 13.86 **[a]** After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



Therefore $\quad v_o(0^+) = 10^6 \int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000}\right] dt = 1000\,\text{V}$

Therefore $w_C = (0.5)Cv^2 = 0.5\,\text{J}$

[b] $i_{\text{L}}(0^+) = 0;$ therefore $w_{\text{L}} = 0\,\text{J}$

[c] $V_o(10^{-6})s + \dfrac{V_o}{250 + 0.05s} + \dfrac{V_o}{1000} = 10^{-3}$

Therefore

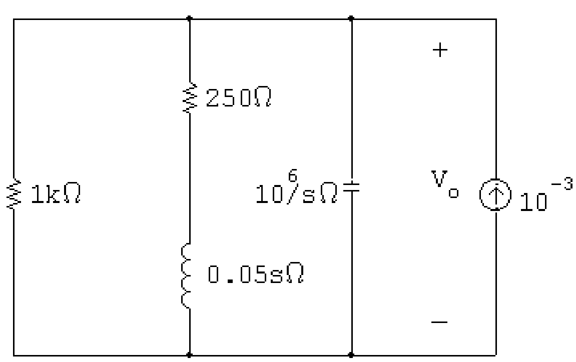
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$$K_1 = 559.02\underline{/-26.57^\circ}; \qquad K_1^* = 559.02\underline{/26.57^\circ}$$

$$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\text{V}$$

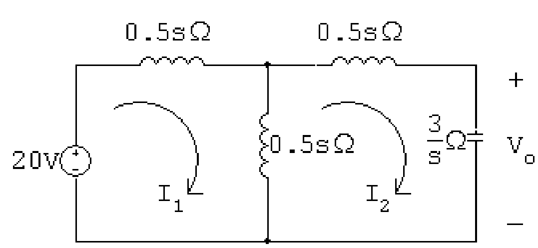
[d] The s-domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.87 **[a]**



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s+3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2+4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s+3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2+60}{s} = \frac{20(s^2+3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2+3)}{s(0.75)(s^2+4)} = \frac{80}{3} \cdot \frac{s^2+3}{s(s^2+4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4+3}{(j2)(j4)}\right] = \frac{10}{3}\underline{/0^\circ}$$

$$\therefore \quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right] u(t)\,\text{A}$$

[b] $$N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2+4)} = \frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \frac{40}{3}(\cos 2t)u(t)\,\text{A}$$

[c] $$V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} = \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\,\text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to $(80/3)$ A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to $(40/3)$ A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\,\text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\,\mathrm{A/s}$$

From the circuit diagram we have

$$20\delta(t) = 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt}$$

$$= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t$$

$$= 20\delta(t)$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior. Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate. Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior. Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.

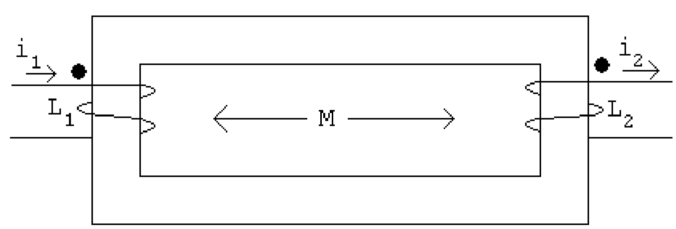
Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20.

For the given polarity dots and reference directions of i_1 and i_2 we have

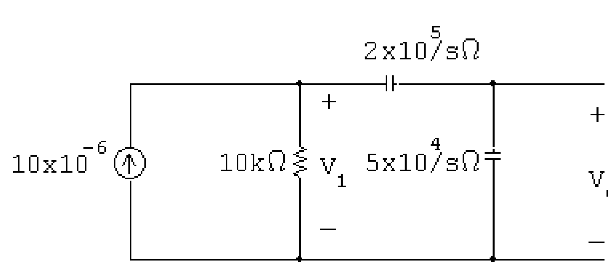
$$\lambda(0^+) = L_1 i_1(0^+) + Mi_1(0^+) - L_2 i_2(0^+) - Mi_2(0^+)$$

$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(Checks)}$$



P 13.88 **[a]**



$$\frac{V_1}{10^4} + \frac{V_1}{[(2\times 10^5)/s)] + [(5\times 10^4)/s]} = 10^{-5}$$

$$\frac{V_1}{10^4} + \frac{sV_1}{25\times 10^4} = 10^{-5}$$

$$25V_1 + sV_1 = 2.5$$

$$V_1 = \frac{2.5}{s+25}$$

$$V_o = \left(\frac{sV_1}{25\times 10^4}\right)\left(\frac{5\times 10^4}{s}\right) = \frac{1}{5}V_1$$

$$\therefore \qquad V_o = \frac{0.5}{s+25}; \qquad v_o = 0.5e^{-25t}u(t)\,\text{V}$$

[b] $v_o(0^+) = 0.5\,\text{V}$

$$v_o(0^+) = \frac{10^6}{20}\int_{0^-}^{0^+} 10\times 10^{-6}\delta(x)\,dx = 0.5\,\text{V (Checks)}$$

$$C_e = \frac{(5)(20)}{25} = 4\,\mu\text{F}$$

$$\tau = RC_e = (10\times 10^3)(4\times 10^{-6}) = 4\times 10^{-2}\,\text{s}; \qquad \frac{1}{\tau} = \frac{100}{4} = 25 \text{ (Checks)}$$

Yes, the impulsive current establishes an instantaneous charge on each capacitor. After the impulsive current vanishes the capacitors discharge exponentially to zero volts.

P 13.89 **[a]** The circuit parameters are

$$R_\text{a} = \frac{120^2}{1200} = 12\,\Omega \qquad R_\text{b} = \frac{120^2}{1800} = 8\,\Omega \qquad X_\text{a} = \frac{120^2}{350} = \frac{1440}{35}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\text{ A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\text{ A(rms)}$$

$$\therefore \quad \mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\text{ A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\,\text{A} \qquad \text{and} \qquad i_L = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\,\text{A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\,\text{A} \qquad \text{and} \qquad i_L(0^-) = i_L(0^+) = 25\sqrt{2}\,\text{A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically. Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_0)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore \quad V_0 = \frac{(R_a/L_\ell)V_g + I_0 R_a}{s + [R_a(L_a + L_\ell)]/L_a L_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0 R_a = 300\sqrt{2}$ V. Also,

$$V_g = V_0 + I_L(j) = 120 + \left(25 - j\frac{35}{12}\right) j = 122.92 + 25j \text{ V(rms)}$$

$$v_g(t) = 122.92\sqrt{2}\cos\omega t - 25\sqrt{2}\sin\omega t \text{ V}, \text{ with } \omega = 120\pi \text{ rad/s}.$$

Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\,\text{V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)\text{ V}$$

Initially,

$$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\,\text{V}$$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

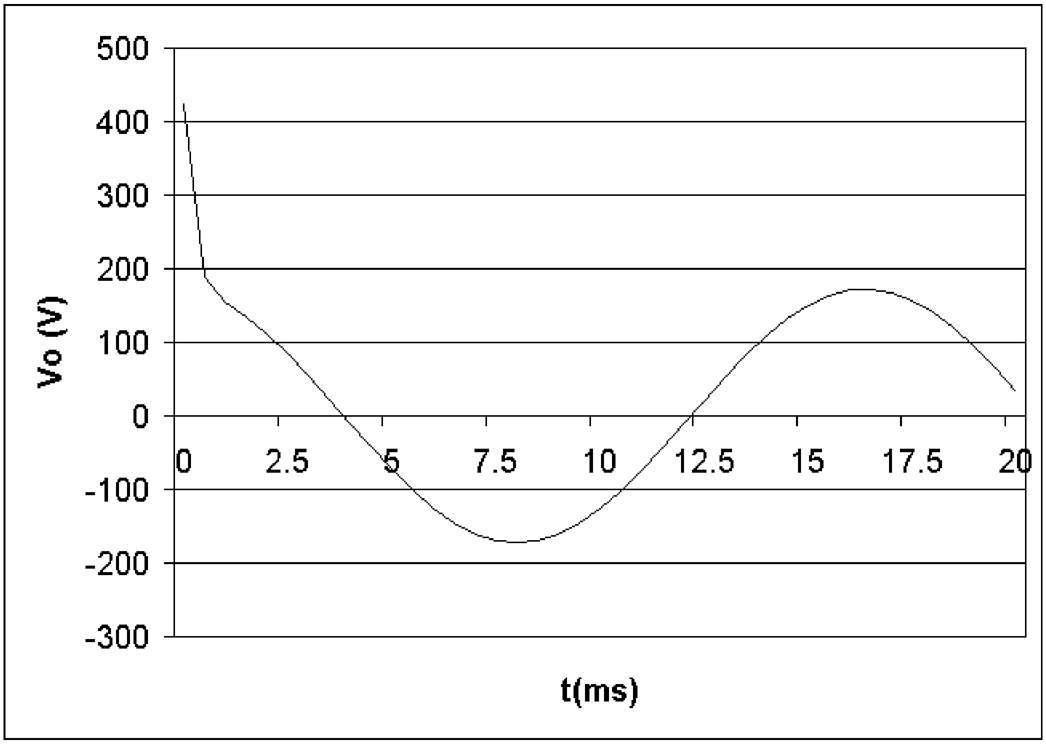
[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{j1440} = 0$$

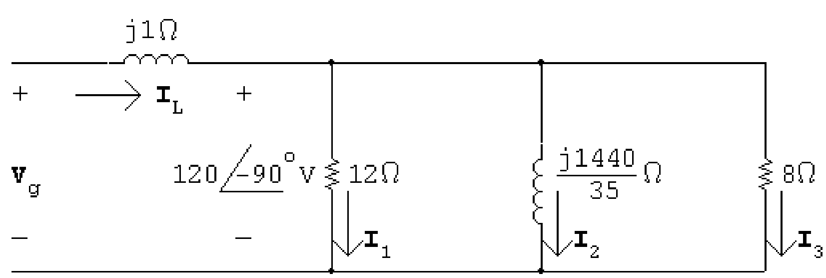
$$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}\text{ V(rms)}$$

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.



P 13.90 **[a]** At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\mathbf{V}_g = -j120 + 25 - j\frac{35}{12}$$

$$= 25 - j122.92$$

$$v_g = 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t$$

$$\therefore \quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

Use a variation of the s-domain circuit in Fig.13.60, where

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\text{A}; \qquad i_2(0) = -2.92\sqrt{2}\text{A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l}{[s + R_a(L_l + L_a)/L_aL_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_lL_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_lL_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\therefore \quad V_o = \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

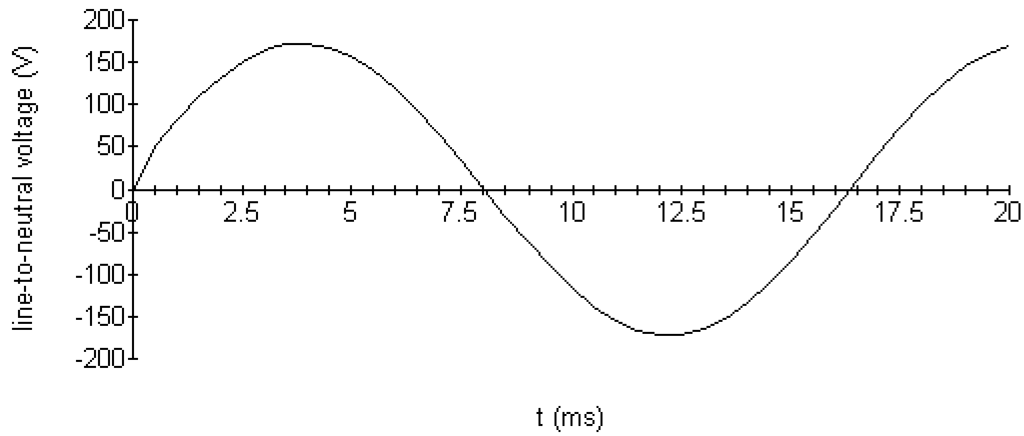
$$K_1 = -14.55\sqrt{2} \qquad K_2 = 61.03\sqrt{2}\underline{/-83.15^\circ}$$

$$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$$

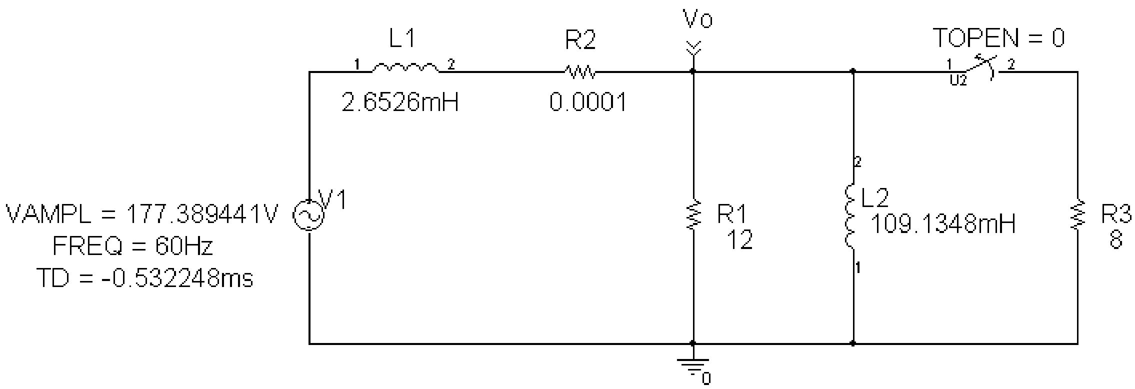
Check:

$$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$$

[b]

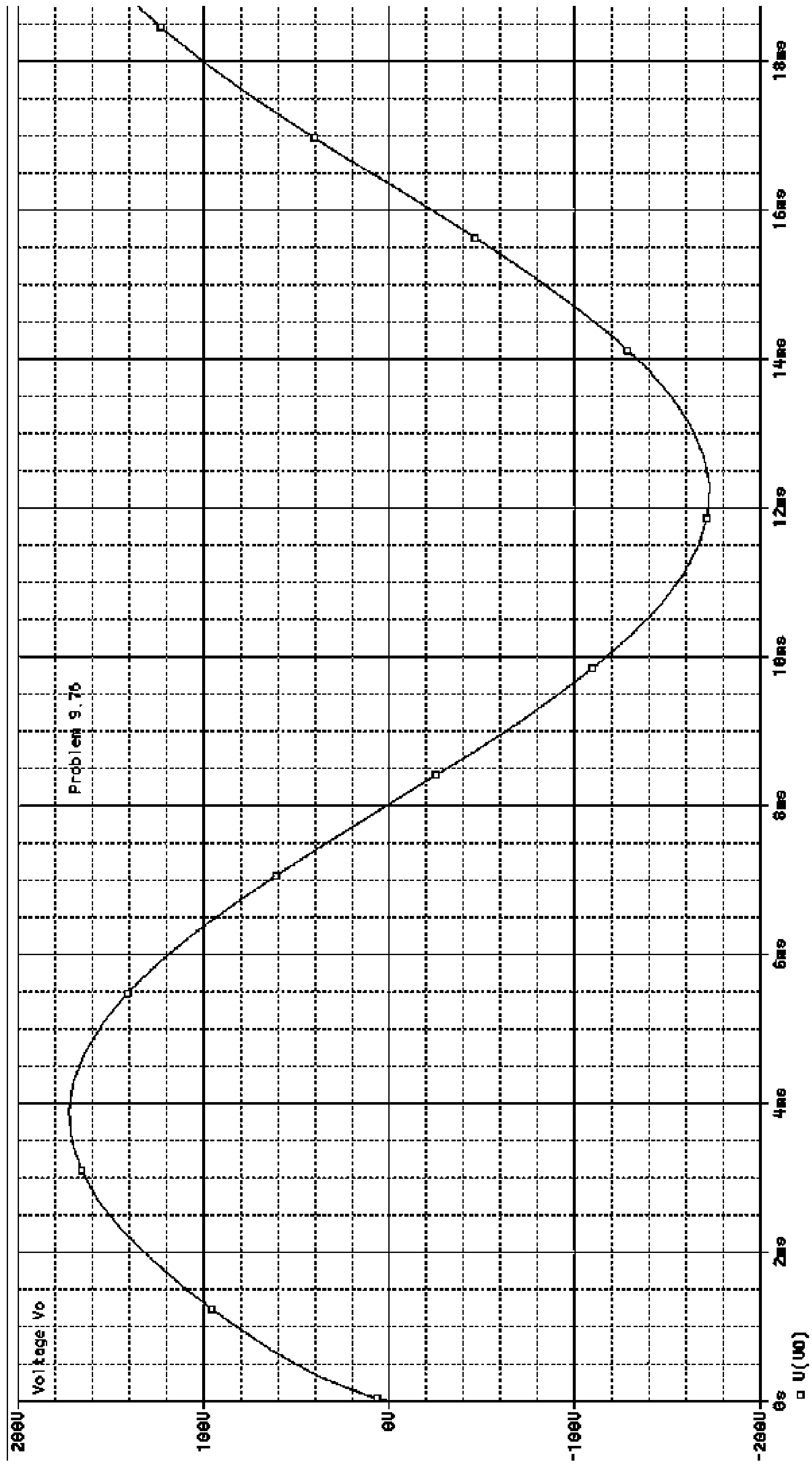

200
150
100
50
0
-50
-100
-150
-200
line-to-neutral voltage (V)
0
2.5
5
7.5
10
12.5
15
17.5
20
t (ms)

PSpice schematic



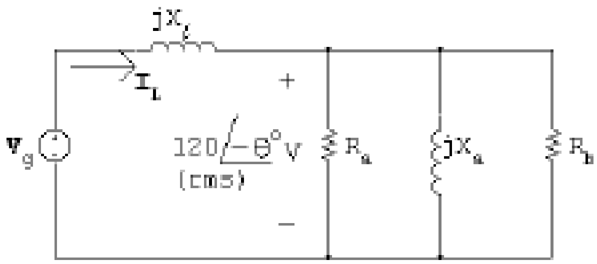
PSpice output file

```
**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]


 ****     CIRCUIT DESCRIPTION


******************************************************************************



** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



**** INCLUDING p9_76-SCHEMATIC1.net ****
* source P9_76
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  12
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
.END
```


Voltage Vo
Problem 9.75
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
□ V(VO)

[c] In Prob. 13.89 the line-to-neutral voltage spikes at $300\sqrt{2}$ V. In part (a) the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.91 **[a]** First find V$_g$ before R_b is disconnected. The phasor domain circuit is



$$\mathbf{I}_L = \frac{120\angle -\theta^\circ}{R_a} + \frac{120\angle -\theta^\circ}{R_b} + \frac{120\angle -\theta^\circ}{jX_a}$$

$$= \frac{120\angle -\theta^\circ}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120\angle -\theta^\circ + \frac{120\angle -\theta^\circ}{R_a R_b X_a}[R_a R_b + j(R_a + R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120\angle -\theta^\circ}{1440}(1475 + j300)$$

$$= \frac{25}{12}\angle -\theta^\circ(59 + j12) = 125.43\angle(-\theta + 11.50)^\circ$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t - \theta + 11.50^\circ)\text{V}$$

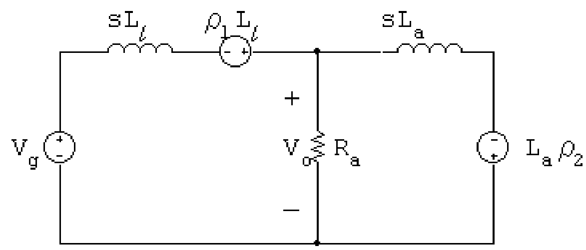
Let $\beta = -\theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t \cos\beta - \sin 120\pi t \sin\beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\,\text{H}; \qquad L_a = \frac{12}{35\pi}\,\text{H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120\underline{/-\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

$$= \frac{120\underline{/-\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17\underline{/(-\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t - \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(-\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta - 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120\underline{/-\theta^\circ}}{j(1440/35)} = \frac{35}{12}\underline{/(-\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t - \theta - 90^\circ)\mathrm{A}$$

$$\rho_2 = i_2(0^+) = -\frac{35}{12}\sqrt{2}\sin\theta = -2.92\sqrt{2}\sin\theta\mathrm{A}$$

$$\therefore \quad \rho_1 - \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad V_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}400}$$

Since $\beta = -\theta + 11.50^\circ$, K_1 reduces to

$$K_1 = -121.18\sqrt{2}\cos\theta - 14.55\sqrt{2}\sin\theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$$-14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$$

or $\quad \tan\theta = 12.29$

Therefore, $\quad \theta = 85.35^\circ$

[b] When $\theta = 85.35^\circ$, $\beta = -73.85^\circ$

$$\therefore \quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85^\circ) + j120\pi\cos(-73.85^\circ)]}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

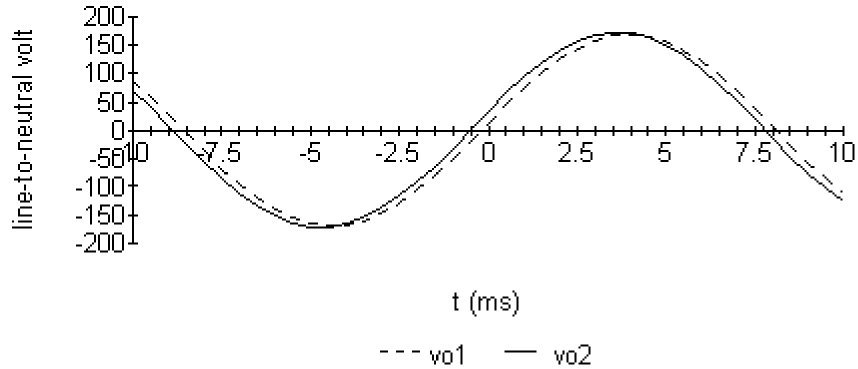
$$= 61.03\sqrt{2}\underline{/-78.50^\circ}$$

$$\therefore \quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50^\circ)\mathrm{V} \quad t > 0$$

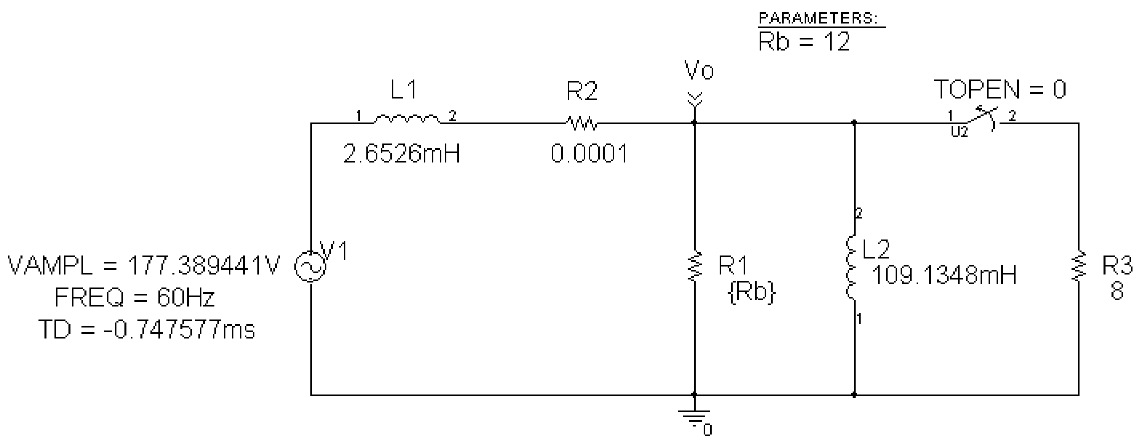
$$= 172.61\cos(120\pi t - 78.50^\circ)\mathrm{V} \quad t > 0$$

[c] $v_{o1} = 169.71\cos(120\pi t - 85.35^\circ)\text{V} \qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50^\circ)\text{V} \qquad t > 0$

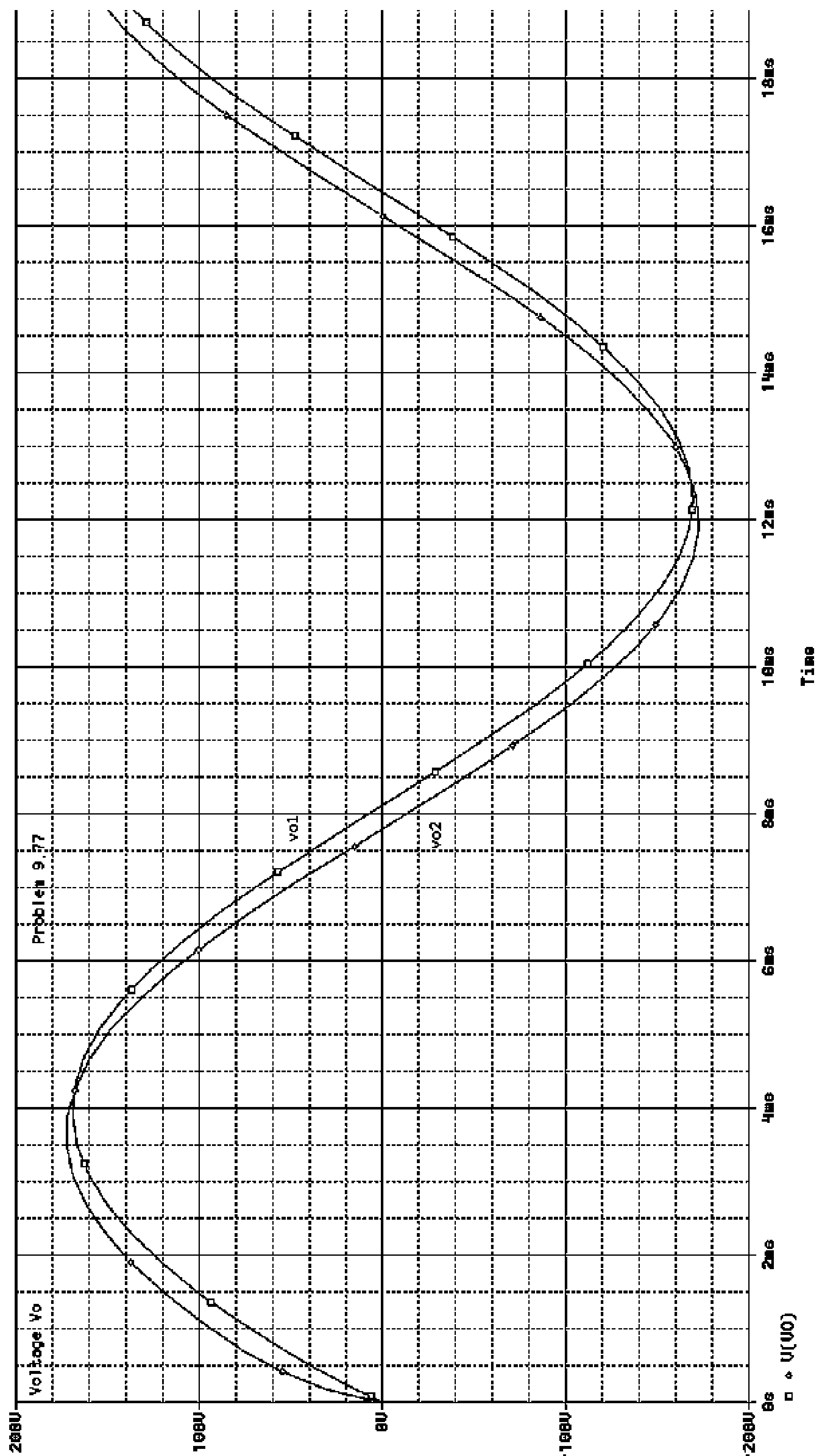


PSpice schematic



PSpice output file

```
** Creating circuit file "p9_77-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.STEP PARAM Rb LIST 4.8  12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"



**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1          N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1          N00637 N01311  2.6526mH IC=0
L_L2          0 VO  109.1348mH IC=0
R_R1          0 VO  {Rb}
R_R2          VO N01311  0.0001
R_R3          0 N01959  8
X_U2          VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
```



Voltage Vo
Problem 9.77
vo1
vo2
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
Time
V(VO)

14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1 $f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi$ krad/s

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 **[a]** $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi$ krad/s

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi$ rad/s

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/-87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3 $\omega_c = \dfrac{R}{L} = \dfrac{5000}{3.5 \times 10^{-3}} = 1.43$ Mrad/s

AP 14.4 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3\times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/sC)$ and R_L. Then

$$Z = \frac{R_L}{(R_LCs+1)}$$

Thus $$H(s) = \frac{Z}{R+Z} = \frac{R_L}{R(R_LCs+1)+R_L}$$

$$= \frac{(1/RC)}{s+\frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s+\frac{1}{K}\left(\frac{1}{RC}\right)}$$

where $$K = \frac{R_L}{R+R_L}$$

AP 14.6 $\omega_o^2 = \dfrac{1}{LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(24\pi\times 10^3)^2(0.1\times 10^{-6})} = 1.76\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi\times 10^3)(1.76\times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7 $\omega_o = 2\pi(2000) = 4000\pi$ rad/s;

$\beta = 2\pi(500) = 1000\pi$ rad/s; $\qquad R = 250\,\Omega$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2(1.27)} = 4.97\,\text{mH}$$

AP 14.8 $\omega_o^2 = \dfrac{1}{LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(10^4\pi)^2(0.2\times 10^{-6})} = 5.07\,\text{mH}$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2\times 10^{-6})} = 3.98\,\text{k}\Omega$$

AP 14.9 $\omega_o^2 = \frac{1}{LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{1}{(4000\pi)^2(0.2 \times 10^{-6})} = 31.66\,\text{mH}$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(4000\pi)(0.2 \times 10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi \text{ rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500 \times 10^{-9})(5)} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi \text{ krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{RC} \quad \text{so} \quad L = \frac{RQ}{\omega_o} = \frac{100}{40\pi \times 10^3} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92\,\text{nF}$$

Problems

P 14.1 **[a]** $\omega_c = \dfrac{R}{L} = \dfrac{127}{10\times 10^{-3}} = 12.7\,\text{krad/s}$

$$\therefore \quad f_c = \frac{\omega_c}{2\pi} = \frac{12{,}700}{2\pi} = 2021.27\,\text{Hz}$$

[b] $H(s) = \dfrac{\omega_c}{s+\omega_c} = \dfrac{12{,}700}{s+12{,}700}$

$$H(j\omega) = \frac{12{,}700}{12{,}700 + j\omega}$$

$$H(j\omega_c) = \frac{12{,}700}{12{,}700 + j12{,}700} = 0.7071\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}700}{12{,}700 + j2540} = 0.981\underline{/-11.31^\circ}$$

$$H(j5\omega_c) = \frac{12{,}700}{12{,}700 + j63{,}500} = 0.196\underline{/-78.69^\circ}$$

[c] $v_o(t)|_{\omega_c} = 7.07\cos(12{,}700t - 45^\circ)\,\text{V}$

$$v_o(t)|_{0.2\omega_c} = 9.81\cos(2540t - 11.31^\circ)\,\text{V}$$

$$v_o(t)|_{5\omega_c} = 1.96\cos(63{,}500t - 78.69^\circ)\,\text{V}$$

P 14.2 **[a]** $\omega_o = \dfrac{R}{L} = 2000\pi\ \text{rad/s}$

$$R = L\omega_o = (0.005)(2000\pi) = 31.42\,\Omega$$

[b] $R_e = 31.42\|270 = 28.14\,\Omega$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 5628\ \text{rad/s}$$

$$\therefore \quad f_{\text{loaded}} = \frac{\omega_{\text{loaded}}}{2\pi} = 895.77\,\text{Hz}$$

P 14.3 Note: add the resistor to the cirucit in Fig. 14.4(a).

[a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{sL + R + R_l} = \dfrac{(R/L)}{s + (R+R_l)/L}$

[b] $H(j\omega) = \frac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\max}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \frac{R}{R + R_l}$

[d] $|H(j\omega_c)| = \frac{R}{\sqrt{2}(R + R_l)} = \frac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore \quad \omega_c^2 = \left(\frac{R + R_l}{L}\right)^2; \qquad \therefore \quad \omega_c = (R + R_l)/L$$

[e] Note – add $75\,\Omega$ resistor in series with the 10 mH inductor.

$$\omega_c = \frac{127 + 75}{0.01} = 20{,}200 \text{ rad/s}$$

$$H(j\omega) = \frac{12{,}700}{20{,}200 + j\omega}$$

$$H(j0) = 0.6287$$

$$H(j20{,}200) = \frac{0.6287}{\sqrt{2}}\underline{/-45^\circ} = 0.4446\underline{/-45^\circ}$$

$$H(j6060) = \frac{12{,}700}{20{,}200 + j6060} = 0.6022\underline{/-16.70^\circ}$$

$$H(j60{,}600) = \frac{12{,}700}{20{,}200 + j60{,}600} = 0.1988\underline{/-71.57^\circ}$$

P 14.4 **[a]** $\omega_c = \frac{1}{RC} = \frac{1}{(10^3)(100 \times 10^{-9})} = 10\,\text{krad/s}$

$$f_c = \frac{\omega_c}{2\pi} = 1591.55\,\text{Hz}$$

[b] $H(j\omega) = \frac{\omega_c}{s + \omega_c} = \frac{10{,}000}{s + 10{,}000}$

$$H(j\omega) = \frac{10{,}000}{10{,}000 + j\omega}$$

$$H(j\omega_c) = \frac{10{,}000}{10{,}000 + j10{,}000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{10{,}000}{10{,}000 + j1000} = 0.9950\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{10{,}000}{10{,}000 + j100{,}000} = 0.0995\underline{/-84.29^\circ}$$

[c] $v_o(t)|_{\omega_c} = 200(0.7071)\cos(10{,}000t - 45^\circ)$

$= 141.42\cos(10{,}000t - 45^\circ)$ mV

$v_o(t)|_{0.1\omega_c} = 200(0.9950)\cos(1000t - 5.71^\circ)$

$= 199.01\cos(1000t - 5.71^\circ)$ mV

$v_o(t)|_{10\omega_c} = 200(0.0995)\cos(100{,}000t - 84.29^\circ)$

$= 19.90\cos(100{,}000t - 84.29^\circ)$ mV

P 14.5 **[a]** Let $Z = \frac{R_L(1/sC)}{R_L + 1/sC} = \frac{R_L}{R_LCs + 1}$

Then $H(s) = \frac{Z}{Z + R}$

$$= \frac{R_L}{RR_LCs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\frac{R + R_L}{RR_LC}\right)}$$

[b] $|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + [(R + R_L)/RR_LC]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$

[c] $|H(j\omega)|_{\max} = \frac{R_L}{R + R_L}$

[d] $|H(j\omega_c)| = \frac{R_L}{\sqrt{2}(R + R_L)} = \frac{(1/RC)}{\sqrt{\omega_c^2 + [(R + R_L)/RR_LC]^2}}$

$\therefore \quad \omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC}(1 + (R/R_L))$

[e] $\omega_c = \frac{1}{(10^3)(10^{-7})}[1 + (10^3/10^4)] = 10{,}000(1 + 0.1) = 11{,}000$ rad/s

$$H(j0) = \frac{10{,}000}{11{,}000} = 0.9091\underline{/0^\circ}$$

$$H(j\omega_c) = \frac{10{,}000}{11{,}000 + j11{,}000} = 0.6428\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{10{,}000}{11{,}000 + j1100} = 0.9046\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{10{,}000}{11{,}000 + j110{,}000} = 0.0905\underline{/-84.29^\circ}$$

P 14.6 **[a]** $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{50{,}000}{2\pi} = \dfrac{50}{2\pi} \times 10^3 = 7957.75\,\text{Hz}$

[b] $\dfrac{1}{RC} = 50 \times 10^3$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40\,\Omega$$

[c] $\omega_c = \dfrac{1}{RC}\left(1 + \dfrac{R}{R_L}\right)$

$$\therefore \quad \frac{R}{R_L} = 0.05 \qquad \therefore \quad R_L = 20R = 800\,\Omega$$

[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{800}{840} = 0.9524$

P 14.7 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(50 \times 10^3)(5 \times 10^{-9})} = 4000\,\text{rad/s}$

$$f_c = \frac{4000}{2\pi} = 636.62\,\text{Hz}$$

[b] $H(s) = \dfrac{s}{s + \omega_c} \qquad \therefore \qquad H(j\omega) = \dfrac{j\omega}{4000 + j\omega}$

$$H(j\omega_c) = H(j4000) = \frac{j4000}{4000 + j4000} = 0.7071\underline{/45^\circ}$$

$$H(j0.2\omega_c) = H(j800) = \frac{j800}{4000 + j800} = 0.1961\underline{/78.69^\circ}$$

$$H(j5\omega_c) = H(j20,000) = \frac{j20{,}000}{4000 + j20{,}000} = 0.9806\underline{/11.31^\circ}$$

[c] $v_o(t)|_{\omega_c} = (0.7071)(500)\cos(4000t + 45^\circ)$

$$= 353.55\cos(4000t + 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.2\omega_c} = (0.1961)(500)\cos(800t + 78.69^\circ)$$

$$= 98.06\cos(800t + 78.69^\circ)\,\text{mV}$$

$$v_o(t)|_{5\omega_c} = (0.9806)(500)\cos(20{,}000t + 11.31^\circ)$$

$$= 490.29\cos(20{,}000t + 11.31^\circ)\,\text{mV}$$

P 14.8 **[a]** $H(s) = \frac{V_o}{V_i} = \frac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R + R_c} \cdot \frac{s}{[s + (1/(R + R_c)C)]}$$

[b] $H(j\omega) = \frac{R}{R + R_c} \cdot \frac{j\omega}{j\omega + (1/(R + R_c)C)}$

$$|H(j\omega)| = \frac{R}{R + R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$.

[c] $|H(j\omega)|_{\max} = \frac{R}{R + R_c}$

[d] $|H(j\omega_c)| = \frac{R\omega_c}{(R + R_c)\sqrt{\omega_c^2 + [1/(R + R_c)C]^2}}$

$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R + R_c)}$ when

$$\therefore \quad \omega_c^2 = \frac{1}{(R + R_c)^2C^2}$$

or $\omega_c = \frac{1}{(R + R_c)C}$

[e] $\omega_c = \frac{1}{(62.5 \times 10^3)(5 \times 10^{-9})} = 3200$ rad/s

$$\frac{R}{R + R_c} = \frac{50}{62.5} = 0.8$$

$$\therefore \qquad H(j\omega) = \frac{0.8j\omega}{3200 + j\omega}$$

$$H(j\omega_c) = \frac{(0.8)j3200}{3200 + j3200} = 0.5657\underline{/45^\circ}$$

$$H(j0.2\omega_c) = \frac{(0.8)j640}{3200 + j640} = 0.1569\underline{/78.69^\circ}$$

$$H(j5\omega_c) = \frac{(0.8)j16{,}000}{3200 + j16{,}000} = 0.7845\underline{/11.31^\circ}$$

P 14.9 **[a]** $\omega_c = \frac{1}{RC} = 2\pi(300) = 600\pi$ rad/s

$$\therefore \quad R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100 \times 10^{-9})} = 5305.16\,\Omega$$

[b] $R_e = 5305.16 \| 47{,}000 = 4767.08\,\Omega$

$$\omega_c = \frac{1}{R_e C} = \frac{1}{(4767.08)(100 \times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86\,\text{Hz}$$

P 14.10 **[a]** $\omega_c = \dfrac{R}{L}$ so $R = \omega_c L = (25 \times 10^3)(5 \times 10^{-3}) = 125\,\Omega$

[b] $\omega_c(\text{loaded}) = \dfrac{R}{L} \cdot \dfrac{R_L}{R + R_L} = 24{,}000 \text{ rad/s}$

$$\therefore \quad \frac{R_L}{R + R_L} = \frac{\omega_c(\text{loaded})}{\omega_c(\text{unloaded})} = \frac{24{,}000}{25{,}000} = 0.96$$

$$R_L = 0.96(R + R_L) \quad \therefore \quad 0.04 R_L = 0.96 R = (0.96)(125)$$

$$\therefore \quad R_L = \frac{(0.96)(125)}{0.04} = 3\,\text{k}\Omega$$

P 14.11 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

P 14.12 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110\,\text{krad/s}$

$$f_o = \frac{\omega_o}{2\pi} = 17.51\,\text{kHz}$$

$$\beta = 121 - 100 = 21\,\text{krad/s} \quad \text{or} \quad 3.34\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{110}{21} = 5.24$$

P 14.13 $\beta = \dfrac{\omega_o}{Q} = \dfrac{50{,}000}{4} = 12.5\,\text{krad/s}; \qquad \dfrac{12{,}500}{2\pi} = 1.99\,\text{kHz}$

$$\omega_{c2} = 50{,}000\left[\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 56.64\,\text{krad/s}$$

$$f_{c2} = \frac{56.64\,\text{k}}{2\pi} = 9.01\,\text{kHz}$$

$$\omega_{c1} = 50{,}000\left[-\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 44.14\,\text{krad/s}$$

$$f_{c1} = \frac{44.14\,\text{k}}{2\pi} = 7.02\,\text{kHz}$$

P 14.14 **[a]** $\omega_o^2 = \dfrac{1}{LC} \quad \text{so} \quad L = \dfrac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000\left[-\dfrac{1}{4} + \sqrt{1 + \dfrac{1}{16}}\right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\dfrac{1}{4} + \sqrt{1 + \dfrac{1}{16}}\right] = 10.25\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

P 14.15 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10 \times 10^{-3})(10 \times 10^{-9})} = 10^{10}$

$$\omega_o = 10^5\,\text{rad/s} = 100\,\text{krad/s}$$

[b] $f_o = \dfrac{\omega_o}{2\pi} = \dfrac{10^5}{2\pi} = 15.92\,\text{kHz}$

[c] $Q = \omega_o RC = (100 \times 10^3)(8000)(10 \times 10^{-9}) = 8$

[d] $\omega_{c1} = \omega_o\left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 10^5\left[-\dfrac{1}{16} + \sqrt{1 + \dfrac{1}{256}}\right] = 93.95\,\text{krad/s}$

[e] $\therefore \quad f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 14.96\,\text{kHz}$

[f] $\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 106.45\,\text{krad/s}$

[g] $\therefore\quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 16.94\,\text{kHz}$

[h] $\beta = \frac{\omega_o}{Q} = \frac{10^5}{8} = 12.5\,\text{krad/s or } 1.99\,\text{kHz}$

P 14.16 **[a]** $L = \frac{1}{\omega_o^2 C} = \frac{1}{(50\times 10^{-9})(20\times 10^3)^2} = 50\,\text{mH}$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(20\times 10^3)(50\times 10^{-9})} = 5\,\text{k}\Omega$$

[b] $\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 20{,}000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$$= 22.10\,\text{krad/s} \qquad \therefore \qquad f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52\,\text{kHz}$$

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 20{,}000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 18.10\,\text{krad/s} \qquad \therefore \qquad f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88\,\text{kHz}$$

[c] $\beta = \frac{\omega_o}{Q} = \frac{20{,}000}{5} = 4000\,\text{rad/s} \quad \text{or} \quad 636.62\,\text{Hz}$

P 14.17 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(40\times 10^{-3})(40\times 10^{-9})} = 625\times 10^6$

$$\omega_o = 25\times 10^3\,\text{rad/s} = 25\,\text{krad/s}$$

$$f_o = \frac{25{,}000}{2\pi} = 3978.87\,\text{Hz}$$

[b] $Q = \frac{\omega_o L}{R + R_i} = \frac{(25\times 10^3)(40\times 10^{-3})}{200} = 5$

[c] $\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 25{,}000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$$= 22.62\,\text{krad/s} \quad \text{or} \quad 3.60\,\text{kHz}$$

[d] $w_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 25{,}000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$$= 27.62\,\text{krad/s} \quad \text{or} \quad 4.40\,\text{kHz}$$

[e] $\beta = \omega_{c2} - \omega_{c1} = 27.62 - 22.62 = 5\,\text{krad/s}$

or

$$\beta = \frac{\omega_o}{Q} = \frac{25{,}000}{5} = 5\,\text{krad/s} \quad \text{or} \quad 795.77\,\text{Hz}$$

P 14.18 **[a]** $H(s) = \dfrac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.17 we have

$$H(s) = \frac{4500s}{s^2 + 5000s + 625 \times 10^6}$$

$$\therefore \quad H(j\omega) = \frac{4500j\omega}{(625 \times 10^6 - \omega^2) + j5000\omega}$$

$$H(j\omega_o) = \frac{j4500(25 \times 10^3)}{j5000(25 \times 10^3)} = 0.9\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 500(0.9)\cos 25{,}000t = 450\cos 25{,}000t\,\text{mV}$$

[b] From the solution to Problem 14.17,

$$\omega_{c1} = 22.62\,\text{krad/s}$$

$$H(j22.62\,\text{k}) = \frac{j4500(22.62 \times 10^3)}{(113.12 + j113.12) \times 10^6} = 0.6364\underline{/45^\circ}$$

$$\therefore \quad v_o(t) = 500(0.6364)\cos(22{,}620t + 45^\circ) = 318.2\cos(22{,}620t + 45^\circ)\,\text{mV}$$

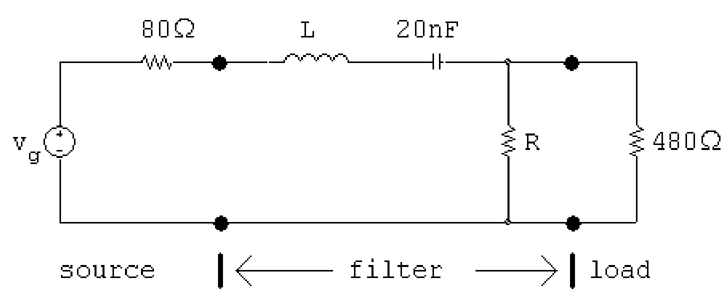
[c] From the solution to Problem 14.17,

$$\omega_{c2} = 27.62\,\text{krad/s}$$

$$H(j27.62\,\text{k}) = \frac{j4500(27.62 \times 10^3)}{(-138.12 + j138.12) \times 10^6} = 0.6364\underline{/-45^\circ}$$

$$\therefore \quad v_o(t) = 500(0.6364)\cos(27{,}620t - 45^\circ) = 318.2\cos(27{,}620t - 45^\circ)\,\text{mV}$$

P 14.19 **[a]**



[b] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50 \times 10^3)^2(20 \times 10^{-9})} = 20\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{6.25} = 160\,\Omega$$

[c] $R_e = 160\|480 = 120\,\Omega$

$$R_e + R_i = 120 + 80 = 200\,\Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(50\times10^3)(20\times10^{-3})}{200} = 5$$

[d] $\beta_{\text{system}} = \dfrac{\omega_o}{Q_{\text{system}}} = \dfrac{50\times10^3}{5} = 10\,\text{krad/s}$

$$\beta_{\text{system}}(\text{Hz}) = \frac{10{,}000}{2\pi} = 1591.55\,\text{Hz}$$

P 14.20 **[a]** $\dfrac{V_o}{V_i} = \dfrac{Z}{Z+R}$ where $Z = \dfrac{1}{Y}$

and $Y = sC + \dfrac{1}{sL} + \dfrac{1}{R_L} = \dfrac{LCR_L s^2 + sL + R_L}{R_L L s}$

$$H(s) = \frac{R_L L s}{R_L RLC s^2 + (R+R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R+R_L}$$

[b] $\beta_L = \left(\dfrac{R+R_L}{R_L}\right)\dfrac{1}{RC}$

[c] $\beta_U = \dfrac{1}{RC}$

$$\therefore\quad \beta_L = \left(\frac{R+R_L}{R_L}\right)\beta_U = \left(1 + \frac{R}{R_L}\right)\beta_U$$

[d] $Q_L = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_U = \omega_o RC$

$$\therefore\quad Q_L = \left(\frac{R_L}{R+R_L}\right)Q_U = \frac{1}{[1+(R/R_L)]}Q_U$$

[f] $H(j\omega) = \dfrac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a cutoff frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2$ or $(\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$\beta = \left(1 + \dfrac{R}{R_L}\right)\dfrac{1}{RC}$ and $\omega_o^2 = \dfrac{1}{LC}$

P 14.21 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5\times 10^{-3})(200\times 10^{-12})} = 10^{12}$

$\omega_o = 1\,\text{Mrad/s}$

[b] $\beta = \dfrac{R + R_L}{R_L}\cdot\dfrac{1}{RC} = \left(\dfrac{500\times 10^3}{400\times 10^3}\right)\left(\dfrac{1}{(100\times 10^3)(200\times 10^{-12})}\right) = 62.5\,\text{krad/s}$

[c] $Q = \dfrac{\omega_o}{\beta} = \dfrac{10^6}{62.5\times 10^3} = 16$

[d] $H(j\omega_o) = \dfrac{R_L}{R + R_L} = 0.8\underline{/0^\circ}$

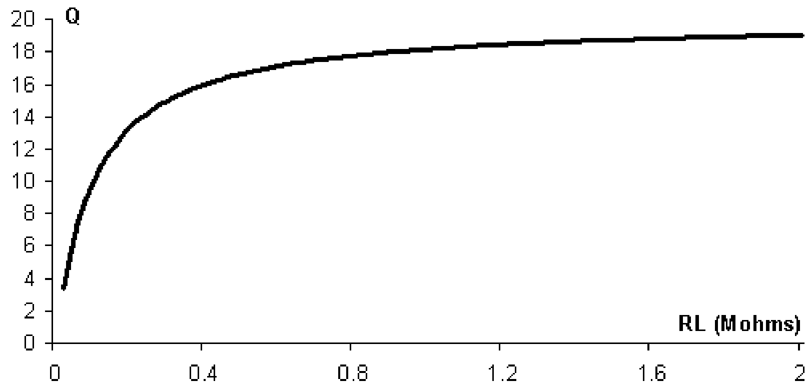
$$\therefore \quad v_o(t) = 250(0.8)\cos(10^6 t) = 200\cos 10^6 t\,\text{mV}$$

[e] $\beta = \left(1 + \dfrac{R}{R_L}\right)\dfrac{1}{RC} = \left(1 + \dfrac{100}{R_L}\right)(50\times 10^3)$ rad/s

$\omega_o = 10^6$ rad/s

$Q = \dfrac{\omega_o}{\beta} = \dfrac{20}{1 + (100/R_L)}$ where R_L is in kilohms

[f]



P 14.22 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(2 \times 10^{-6})(50 \times 10^{-12})} = 10^{16}$

$\omega_o = 100\,\text{Mrad/s}$

$Q_u = \omega_o RC = (100 \times 10^6)(2.4 \times 10^3)(50 \times 10^{-12}) = 12$

$\therefore \quad \left(\dfrac{R_L}{R + R_L}\right) 12 = 7.5; \qquad \therefore \quad R_L = \dfrac{7.5}{4.5} R = 4\,\text{k}\Omega$

P 14.23 **[a]** In analyzing the circuit qualitatively we visualize v_i as a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.

At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.

At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.

At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.

At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.

Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $|H(j\omega)| = \dfrac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$

$|H(j\omega)| = \dfrac{1}{\sqrt{2}}$ when $\omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$

or $\pm\,\omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o\left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c_2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

[e] It follows from the equations derived in part (b) that

$$\beta = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$

P 14.24 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-6})(20 \times 10^{-9})} = 10^{12}$

$\therefore \quad \omega_o = 1\,\text{Mrad/s}$

[b] $f_o = \frac{\omega_o}{2\pi} = 159.15\,\text{kHz}$

[c] $Q = \omega_o RC = (10^6)(750)(20 \times 10^{-9}) = 15$

[d] $\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 10^6 \left[-\frac{1}{30} + \sqrt{1 + \frac{1}{900}} \right]$

$= 967.22\,\text{krad/s}$

[e] $f_{c1} = \frac{\omega_{c1}}{2\pi} = 153.94\,\text{kHz}$

[f] $\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 10^6 \left[\frac{1}{30} + \sqrt{1 + \frac{1}{900}} \right]$

$= 1.03\,\text{Mrad/s}$

[g] $f_{c2} = \frac{\omega_{c2}}{2\pi} = 164.55\,\text{kHz}$

[h] $\beta = f_{c2} - f_{c1} = 10.61\,\text{kHz}$

P 14.25 **[a]** $\omega_o = 2\pi f_o = 8\pi\,\text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2(0.5 \times 10^{-6})} = 3.17\,\text{mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5 \times 10^{-6})} = 397.89\,\Omega$$

[b] $f_{c2} = f_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 4000 \left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$

$= 4.42\,\text{kHz}$

$$f_{c1} = f_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 4000 \left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}} \right]$$

$= 3.62\,\text{kHz}$

[c] $\beta = f_{c2} - f_{c1} = 800\,\text{Hz}$

or

$$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800\,\text{Hz}$$

P 14.26 **[a]** $R_e = 397.89 \| 1000 = 284.63\,\Omega$

$$Q = \omega_o R_e C = (8000\pi)(284.63)(0.5 \times 10^{-6}) = 3.58$$

[b] $\beta = \dfrac{f_o}{Q} = \dfrac{4000}{3.58} = 1.12\,\text{kHz}$

[c] $f_{c2} = 4000\left[\dfrac{1}{7.15} + \sqrt{1 + \dfrac{1}{7.15^2}}\right] = 4.60\,\text{kHz}$

[d] $f_{c1} = 4000\left[-\dfrac{1}{7.15} + \sqrt{1 + \dfrac{1}{7.15^2}}\right] = 3.48\,\text{kHz}$

P 14.27 **[a]** Let $Z = \dfrac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $H(s) = \dfrac{V_o}{V_i} = \dfrac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$
$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $\quad K = \dfrac{R_L}{R + R_L}; \quad \omega_o^2 = \dfrac{1}{LC}; \quad \beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j\omega_o) = 0$$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \frac{K\left[(\omega_o/\omega)^2 - 1\right]}{\left\{\left[(\omega_o/\omega)^2 - 1\right] + j\beta/\omega\right\}}$

$$H(j\infty) = \frac{-K}{-1} = K$$

[h] $H(j\omega) = \frac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j0) = H(j\infty) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore \quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2$ or $(\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L}$ and $\omega_o^2 = \frac{1}{LC}$

P 14.28 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(10^{-6})(4 \times 10^{-12})} = 0.25 \times 10^{18} = 25 \times 10^{16}$

$\omega_o = 5 \times 10^8 = 500$ Mrad/s

$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} = \frac{(30)(150)}{180} \cdot \frac{1}{10^{-6}} = 25$ Mrad/s $= 3.98$ MHz

$$Q = \frac{\omega_o}{\beta} = \frac{500\,\text{M}}{25\,\text{M}} = 20$$

[b] $H(j0) = \dfrac{R_L}{R+R_L} = \dfrac{150}{180} = 0.8333$

$$H(j\infty) = \frac{R_L}{R+R_L} = 0.8333$$

[c] $f_{c2} = \dfrac{250}{\pi}\left[\dfrac{1}{40} + \sqrt{1+\dfrac{1}{1600}}\right] = 81.59\,\text{MHz}$

$$f_{c2} = \frac{250}{\pi}\left[-\frac{1}{40} + \sqrt{1+\frac{1}{1600}}\right] = 77.61\,\text{MHz}$$

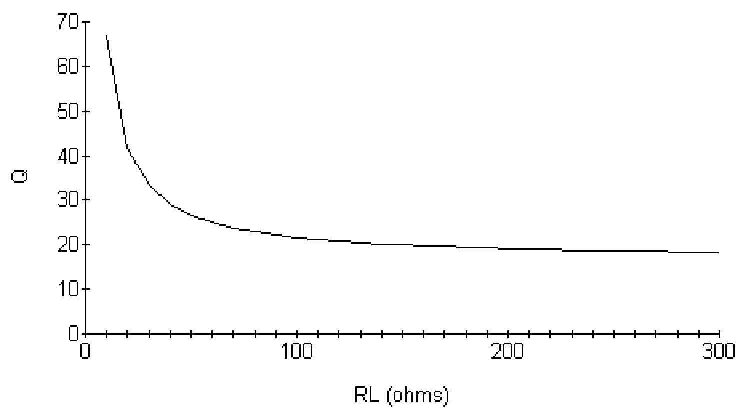
Check: $\beta = f_{c2} - f_{c1} = 3.98\,\text{MHz}$.

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{500\times 10^6}{\frac{RR_L}{R+R_L}\cdot\frac{1}{L}}$

$$= \frac{500(R+R_L)}{RR_L} = \frac{50}{3}\left(1+\frac{30}{R_L}\right)$$

where R_L is in ohms.

[e]



P 14.29 **[a]** $\omega_o^2 = \dfrac{1}{LC} = 10^{12}$

$$\therefore \quad L = \frac{1}{(10^{12})(400\times 10^{-12})} = 2.5\,\text{mH}$$

$$\frac{R_L}{R+R_L} = 0.96; \qquad \therefore \quad 0.04R_L = 0.96R$$

$$\therefore \qquad R_L = 24R \quad \therefore \quad R = \frac{36{,}000}{24} = 1.5\,\text{k}\Omega$$

[b] $\beta = \left(\dfrac{R_L}{R+R_L}\right) R \cdot \dfrac{1}{L} = 576\times 10^3$

$$Q = \frac{\omega_o}{\beta} = \frac{10^6}{576\times 10^3} = 1.74$$

P 14.30 Refer to Sections E.5 and E.7.

[a] $\omega_n = 10^5$

$$2\zeta\omega_n = 50{,}000, \quad \zeta = 0.25$$

$$\omega_o = \sqrt{2}\omega_p = \sqrt{2}\omega_n\sqrt{1 - 2\zeta^2} = 132{,}287.57\,\text{rad/s}$$

$$\therefore \quad \omega = 0$$

$$\omega = 132{,}287.57\,\text{rad/s}$$

[b] $\omega_p = \omega_n\sqrt{1 - 2\zeta^2} = 93{,}541.43\,\text{rad/s}$

P 14.31 **[a]** Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \qquad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus $\beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|\mathbf{V}_{697\,\text{Hz}}| = |V_{941\,\text{Hz}}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\,\text{Hz}}| = |V_{\text{peak}}|\frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852\,\text{Hz}}| = |V_{\text{peak}}|\frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this same property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\,\text{Hz}}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.32 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filter's inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\,|V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.33 From Problem 14.31 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\,\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore \quad \frac{0.00744|V_{\text{ring-peak}}|}{0.948|V_{\text{DTMF-peak}}|} = 0.5$$

$$\therefore \quad |V_{\text{ring-peak}}| = 63.7|V_{\text{DTMF-peak}}|$$

Thus, the 20 Hz signal can be 63.7 times as large as the DTMF tones.

15

Active Filter Circuits

Assessment Problems

AP 15.1 $H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2 $H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$$

AP 15.3 $\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi$ rad/s

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \quad \therefore \quad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order prototype Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2 C}\right)s + \left(\frac{1}{R_1 R_2 C^2}\right)}$$

Equate the transfer functions. For $C = 1$F,

$$\frac{2}{R_2 C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 1.414\,\Omega$$

$$\frac{1}{R_1 R_2 C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 0.707\,\Omega$$

AP 15.5 $Q = 8, K = 5, \omega_o = 1000$ rad/s, $C = 1\,\mu$F

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\frac{1}{R_1 C}\right)s}{s^2 + \left(\frac{2}{R_3 C}\right)s + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)} = \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3 C}, \quad \therefore \quad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2\times 10^6}{(125)(1)} = 16\,\mathrm{k}\Omega$$

$$K\beta = \frac{1}{R_1C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1\times 10^{-6})} = 1.6\,\mathrm{k}\Omega$$

$$\omega_o^2 = \frac{R_1+R_2}{R_1R_2R_3C^2}$$

$$10^6 = \frac{(1600+R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600+R_2)10^6}{256\times 10^5}, \quad 246R_2 = 16{,}000, \quad R_2 = 65.04\,\Omega$$

AP 15.6 $\omega_o = 1000\,\mathrm{rad/s}; \qquad Q = 4;$

$C = 2\,\mu\mathrm{F}$

$$H(s) = \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right]s + \left(\dfrac{1}{R^2C^2}\right)}$$

$$= \frac{s^2+\omega_o^2}{s^2+\beta s+\omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_oC} = \frac{1}{(1000)(2\times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2\times 10^{-6}) = 0.25$$

$$1-\sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \qquad \sigma = 0.9375$$

Problems

P 15.1 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.2 **[a]** $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1}{C_2}\frac{[s + (1/R_1C_1)]}{[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.

As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$. Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.3 **[a]** $Z_f = \dfrac{(1/C_2)}{s + (1/R_2C_2)}$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2}\frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $H(j\omega) = -\dfrac{1}{R_1C_2}\dfrac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$

$$H(j0) = 0$$

[c] $H(j\infty) = 0$

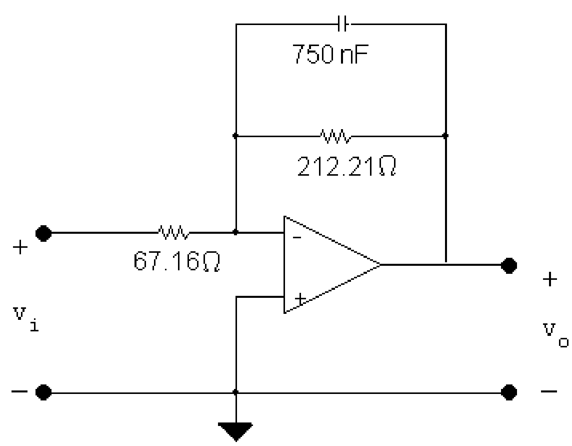
[d] As $\omega \to 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_f = 0$ and therefore $v_o = 0$.

P 15.4 **[a]** $K = 10^{(10/20)} = 3.16 = \dfrac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21\,\Omega$$

$$R_1 = \frac{R_2}{K} = \frac{212.21}{3.16} = 67.11\,\Omega$$

[b]

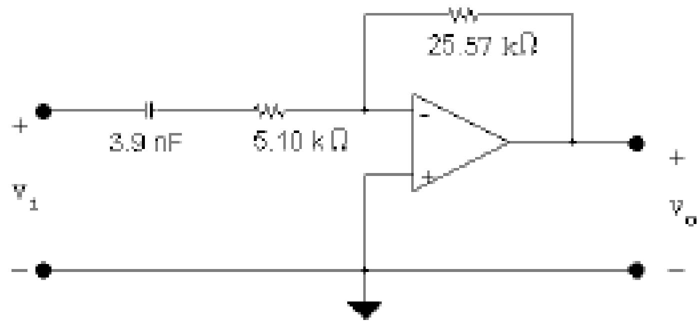


P 15.5 **[a]** $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{(2\pi)(8 \times 10^3)(3.9 \times 10^{-9})} = 5.10\,\text{k}\Omega$

$$K = 10^{(14/20)} = 5.01 = \frac{R_2}{R_1}$$

$$\therefore \quad R_2 = 5.01R_1 = 25.57\,\text{k}\Omega$$

[b]



P 15.6 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R \frac{C}{k_m k_f} = \frac{1}{k_f} RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{V_0}{V_i} = \frac{R/L}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.7 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s + (R'/L')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.8 $$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \frac{\beta s}{s^2 \beta s + \omega_o^2}$$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R;\ L' = \dfrac{k_m}{k_f} L;$ and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L}\right) = k_f \beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f} LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f\omega_o}{k_f\beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.9 **[a]** $L = 1\,\text{H}; \qquad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{20} = 0.05\,\Omega$$

[b] $k_f = \dfrac{\omega_o'}{\omega_o} = 40{,}000; \qquad k_m = \dfrac{R'}{R} = \dfrac{5000}{0.05} = 100{,}000$

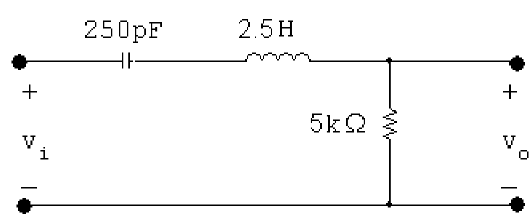
Thus,

$$R' = k_mR = (0.05)(100{,}000) = 5\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{100{,}000}{40{,}000}(1) = 2.5\,\text{H}$$

$$C' = \frac{C}{k_mk_f} = \frac{1}{(40{,}000)(100{,}000)} = 250\,\text{pF}$$

[c]



P 15.10 **[a]** Since $\omega_o^2 = 1/LC$ and $\omega_o = 1$ rad/s,

$$C = \frac{1}{L} = \frac{1}{Q}$$

[b] $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 16\,\text{H}; \qquad C = \frac{1}{L} = 0.0625\,\text{F}$$

$$\therefore \quad k_m = \frac{R'}{R} = 10{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 25{,}000$$

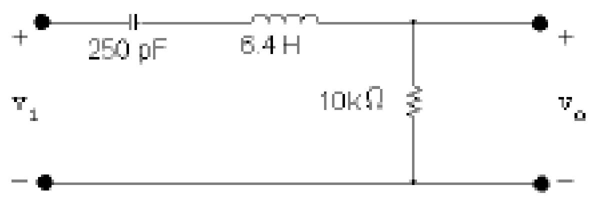
Thus

$$R' = k_m R = 10\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{10{,}000}{25{,}000}(16) = 6.4\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10{,}000)(25{,}000)} = 250\,\text{pF}$$

[d]



[e] $$H'(s) = \frac{\frac{1}{16}\left(\frac{s}{25{,}000}\right)}{\left(\frac{s}{25{,}000}\right)^2 + \frac{1}{16}\left(\frac{s}{25{,}000}\right) + 1}$$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

P 15.11 **[a]** Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 25\,\Omega$$

$$k_m = \frac{R'}{R} = \frac{40{,}000}{25} = 1600; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{1600}{50{,}000}(1) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50{,}000)} = 12.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 25\,\text{F}$$

$$L = \frac{1}{25} = 40\,\text{mH}; \qquad R = 1\,\Omega$$

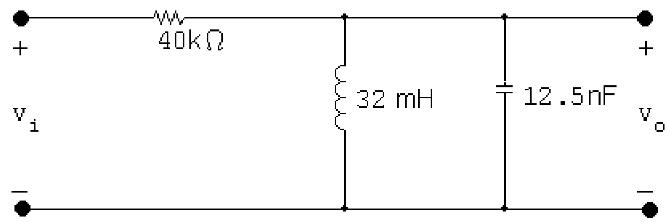
$$k_m = \frac{R'}{R} = 40{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{40{,}000}{50{,}000}(0.04) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40{,}000)(50{,}000)} = 12.5\,\text{nF}$$

[b]



P 15.12 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = k_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)|_{s=s/k_f}$$

P 15.13 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R; \qquad L' = \frac{k_m}{k_f} L;$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right)s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}} \\ &= H(s)|_{s=s/k_f} \end{aligned}$$

P 15.14 **[a]** For the circuit in Fig. P15.14(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \frac{1}{s}}{\frac{1}{Q} + s + \frac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.14(b)

$$\begin{aligned} H(s) = \frac{V_o}{V_i} &= \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}} \\ &= \frac{Q(s^2 + 1)}{Qs^2 + s + Q} \end{aligned}$$

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $k_f = \dfrac{\omega_o'}{\omega_o} = 10^4; \qquad Q = 8;$

Replace s with s/k_f.

$$H'(s) = \frac{\left(\frac{s}{10^4}\right)^2 + 1}{\left(\frac{s}{10^4}\right)^2 + \frac{1}{8}\left(\frac{s}{10^4}\right) + 1}$$
$$= \frac{s^2 + 10^8}{s^2 + 1250s + 10^8}$$

P 15.15 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s + \frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2+1)}{Q(s^2+1) + s} = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For prototype circuit (b):

$$H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}}$$
$$= \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

P 15.16 From the solution to Problem 14.15, $\omega_o = 100$ krad/s and $\beta = 12.5$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(200 \times 10^3)}{100 \times 10^3} = 4\pi$$

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{1}{4\pi}\frac{10 \times 10^{-9}}{2.5 \times 10^{-9}} = \frac{1}{\pi}$$

Thus,

$$R' = k_m R = \frac{8000}{\pi} = 2546.48\,\Omega \qquad L' = \frac{k_m}{k_f}L = \frac{1/\pi}{4\pi}(10 \times 10^{-3}) = 253.303\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega_{c1}' = k_f\omega_{c1} = 4\pi(93.95 \times 10^3) = 1180.6 \text{ krad/s}$$

$$\omega_{c2}' = k_f\omega_{c2} = 4\pi(106.45 \times 10^3) = 1337.7 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega_{c2}' - \omega_{c1}' = 157.1 \text{ krad/s} = 4\pi\beta \text{ (Checks!)}$$

P 15.17 From the solution to Problem 14.24, $\omega_o = 10^6$ rad/s and $\beta = 2\pi(10.61)$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{50 \times 10^3}{10^6} = 0.05$$

$$k_m = \frac{k_f L'}{L} = \frac{0.05(200 \times 10^{-6})}{50 \times 10^{-6}} = 0.2$$

Thus,

$$R' = k_m R = (0.2)(750) = 150\,\Omega \qquad C' = \frac{C}{k_m k_f} = \frac{20 \times 10^{-9}}{(0.2)(0.05)} = 2\,\mu\text{F}$$

Calculate the bandwidth:

$$\beta' = k_f \beta = (0.05)[2\pi(10.61 \times 10^3)] = 3333 \text{ rad/s}$$

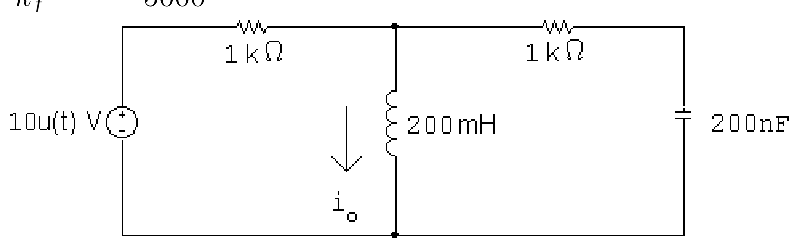
To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{10^6}{2\pi(10.61 \times 10^3)} = 15$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{50 \times 10^3}{3333} = 15 \text{ (Checks)}$$

P 15.18 **[a]** $k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{C}{k_m C'} = \frac{1}{(1000)(200 \times 10^{-9})} = 5000$

$$L' = \frac{k_m}{k_f}(L) = \frac{1000}{5000}(1) = 200\,\text{mH}$$



[b] $\frac{V - 10/s}{1000} + \frac{V}{0.2s} + \frac{V}{1000 + (5 \times 10^6/s)} = 0$

$$V\left(\frac{1}{1000} + \frac{5}{s} + \frac{s}{1000s + 5 \times 10^6}\right) = \frac{1}{100s}$$

$$V = \frac{10(s + 5000)}{2s^2 + 10{,}000s + 25 \times 10^6} = \frac{5(s + 5000)}{s^2 + 5000s + 12.5 \times 10^6}$$

$$I_o = \frac{V}{0.2s} = \frac{25(s+5000)}{s(s^2+5000s+12.5\times10^6)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2500-j2500} + \frac{K_2^*}{s+2500+j2500}$$

$K_1 = 0.01; \qquad K_2 = -0.005$

$i_o(t) = 10 - 10e^{-2500t}\cos 2500t\,\text{mA}$

Since $k_m = 1000$ and the source voltage didn't change, the amplitude of the current is reduced by a factor of 1000. Since $k_f = 5000$ the coefficients of t are multiplied by 5000.

P 15.19 $k_m = \dfrac{R'}{R} = \dfrac{5000}{50} = 100; \qquad k_f = \dfrac{\omega_o'}{\omega_o} = 5000$

$$C' = \frac{C}{k_m k_f} = \frac{4\times10^{-3}}{(100)(5000)} = 8\,\text{nF}$$

$50\,\Omega \rightarrow 5\,\text{k}\Omega; \qquad 700\,\Omega \rightarrow 70\,\text{k}\Omega$

$$L' = \frac{k_m}{k_f}L = \frac{100}{5000}(20) = 0.4\,\text{H}$$

$$0.05v_\phi \rightarrow \frac{0.05}{100}v_\phi = 5\times10^{-4}v_\phi$$

The original expression for the current:

$i_o(t) = 1728 + 2880e^{-20t}\cos(15t + 126.87^\circ)\,\text{mA}$

The frequency components will be multiplied by $k_f = 5000$:

$20 \rightarrow 20(5000) = 10^5; \qquad 15 \rightarrow 15(5000) = 75{,}000$

The magnitudes will be reduced by $k_m = 100$:

$1728 \rightarrow 1728/100 = 17.28; \qquad 2880 \rightarrow 2880/100 = 28.80$

The expression for the current in the scaled circuit is thus,

$i_o(t) = 17.28 + 28.80e^{-10^5 t}\cos(75{,}000t + 126.87^\circ)\,\text{mA}$

P 15.20 **[a]** From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

where $K = \frac{R_2}{R_1}, \quad \omega_c = \frac{1}{R_2C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s + \omega_c'}$$

where $K' = \frac{R_2'}{R_1'} \qquad \omega_c' = \frac{1}{R_2'C'}$

By hypothesis $R_1' = k_mR_1; \qquad R_2' = k_mR_2,$

and $C' = \frac{C}{k_fk_m}$. It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$, therefore

$$H'(s) = \frac{-Kk_f\omega_c}{s + k_f\omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \frac{-K}{(s+1)}$

[c] $H'(s) = \frac{-K}{\left(\frac{s}{k_f}\right) + 1} = \frac{-Kk_f}{s + k_f}$

P 15.21 **[a]** From Eq. 15.4

$H(s) = \frac{-Ks}{s + \omega_c}$ where $K = \frac{R_2}{R_1}$ and

$$\omega_c = \frac{1}{R_1C}$$

$\therefore \quad H'(s) = \frac{-K's}{s + \omega_c'}$ where $K' = \frac{R_2'}{R_1'}$

and $\omega_c' = \frac{1}{R_1'C'}$

By hypothesis

$$R_1' = k_mR_1; \qquad R_2' = k_mR_2; \qquad C' = \frac{C}{k_mk_f}$$

It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$

$$\therefore \quad H'(s) = \frac{-Ks}{s + k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-Ks}{(s+1)}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{(s+k_f)}$

P 15.22 **[a]** $H_{\rm hp} = \dfrac{-s}{s+1}; \qquad k_f = \dfrac{\omega_o'}{\omega} = \dfrac{1000(2\pi)}{1} = 2000\pi$

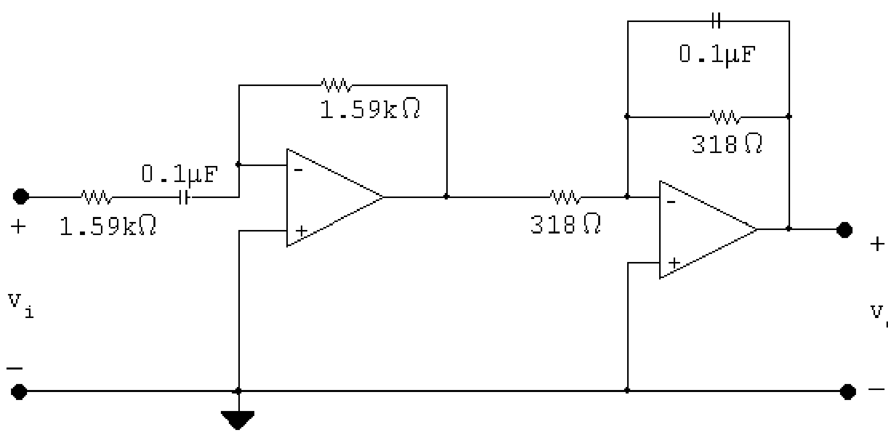
$$\therefore \quad H'_{\rm hp} = \frac{-s}{s+2000\pi}$$

$$\frac{1}{R_HC_H} = 2000\pi; \qquad \therefore \quad R_H = \frac{1}{(2000\pi)(0.1\times10^{-6})} = 1.59\,\text{k}\Omega$$

$$H_{\rm lp} = \frac{-1}{s+1}; \qquad k_f = \frac{\omega_o'}{\omega} = \frac{5000(2\pi)}{1} = 10{,}000\pi$$

$$\therefore \quad H'_{\rm lp} = \frac{-10{,}000\pi}{s+10{,}000\pi}$$

$$\frac{1}{R_LC_L} = 10{,}000\pi; \qquad \therefore \quad R_L = \frac{1}{(10{,}000\pi)(0.1\times10^{-6})} = 318.3\,\Omega$$



[b] $H'(s) = \dfrac{-s}{s+2000\pi}\cdot\dfrac{-10{,}000\pi}{s+10{,}000\pi}$

$$= \frac{10{,}000\pi s}{(s+2000\pi)(s+10{,}000\pi)}$$

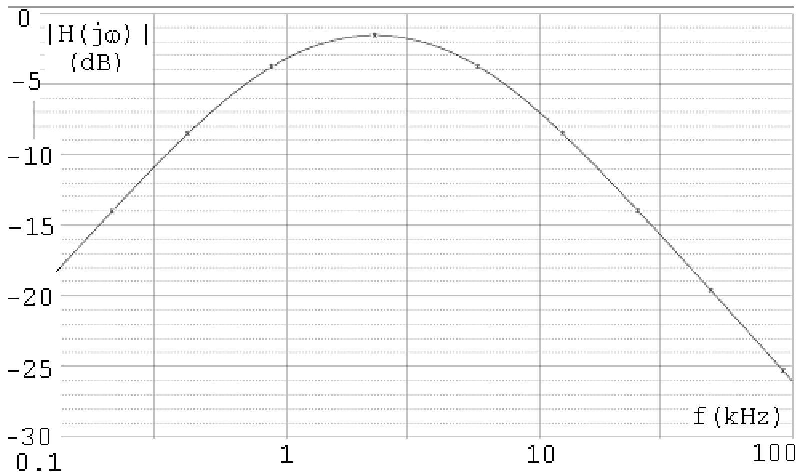
[c] $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(2000\pi)(10{,}000\pi)} = 1000\pi\sqrt{20}$ rad/s

$$H'(j\omega_o) = \frac{(10{,}000\pi)(j1000\pi\sqrt{20})}{(2000\pi+j1000\pi\sqrt{20})(10{,}000\pi+j1000\pi\sqrt{20})}$$

$$= \frac{j10\sqrt{20}}{(2+j\sqrt{20})(10+j\sqrt{20})} = 0.8333\underline{/0^\circ}$$

[d] $G = 20\log_{10}(0.8333) = -1.58$ dB

[e]



P 15.23 **[a]** For the high-pass section:

$$k_f = \frac{\omega_o'}{\omega} = \frac{4000(2\pi)}{1} = 8000\pi$$

$$H'(s) = \frac{-s}{s + 8000\pi}$$

$$\therefore \qquad \frac{1}{R_1(10\times 10^{-9})} = 8000\pi; \qquad R_1 = 3.98\,\text{k}\Omega \quad \therefore \qquad R_2 = 3.98\,\text{k}\Omega$$

For the low-pass section:

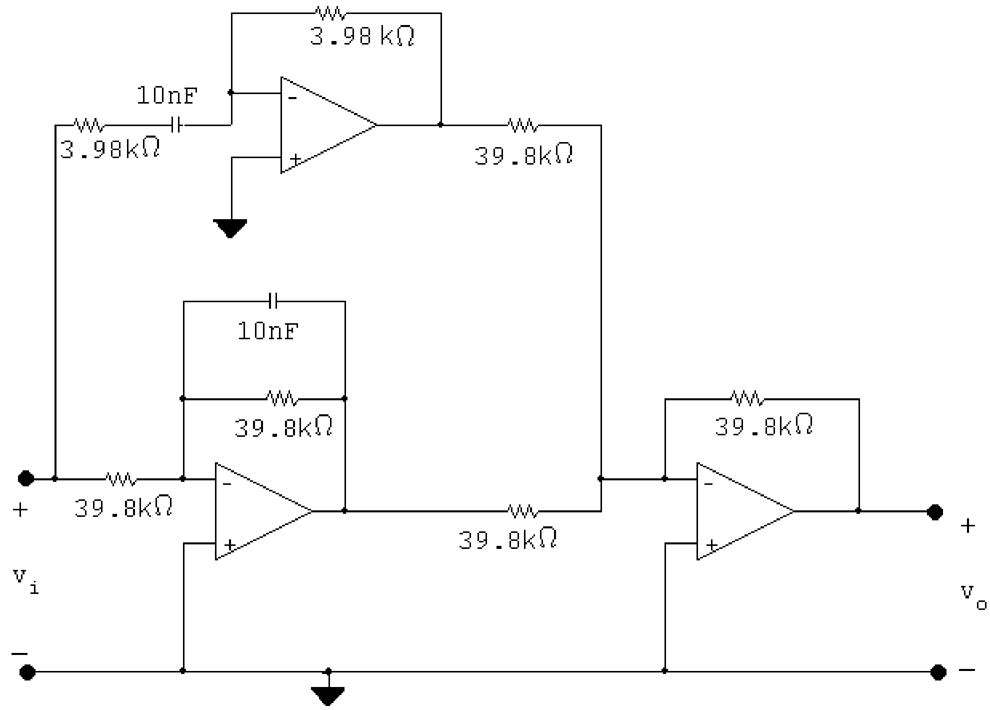
$$k_f = \frac{\omega_o'}{\omega} = \frac{400(2\pi)}{1} = 800\pi$$

$$H'(s) = \frac{-800\pi}{s + 800\pi}$$

$$\therefore \qquad \frac{1}{R_2(10\times 10^{-9})} = 800\pi; \qquad R_2 = 39.8\,\text{k}\Omega \quad \therefore \qquad R_1 = 39.8\,\text{k}\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose R_f and R_i so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 39.8\,\text{k}\Omega$.

[b]



[c] $H'(s) = \dfrac{s}{s + 8000\pi} + \dfrac{800\pi}{s + 800\pi}$

$$= \frac{s^2 + 1600\pi s + 64 \times 10^5\pi^2}{(s + 800\pi)(s + 8000\pi)}$$

[d] $\omega_o = \sqrt{(8000\pi)(800\pi)} = 800\pi\sqrt{10}$

$$H'(j800\pi\sqrt{10}) = \frac{-(800\pi\sqrt{10})^2 + 1600\pi(j800\pi\sqrt{10}) + 64 \times 10^5\pi^2}{(800\pi + j800\pi\sqrt{10})(8000\pi + j800\pi\sqrt{10})}$$

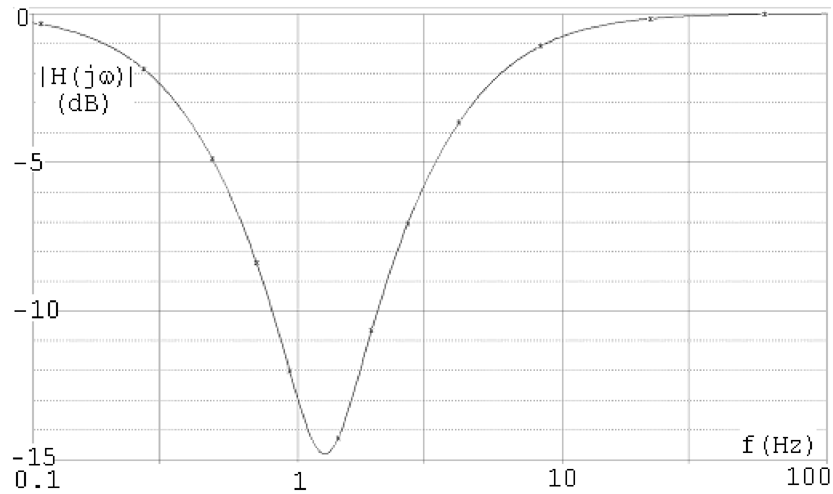
$$= \frac{j128 \times 10^4\sqrt{10}\pi^2}{(800\pi)^2(1 + j\sqrt{10})(10 + j\sqrt{10})}$$

$$= \frac{j2\sqrt{10}}{(1 + j\sqrt{10})(10 + j\sqrt{10})}$$

$$= 0.1818\underline{/0^\circ}$$

[e] $G = 20\log_{10} 0.1818 = -14.81$ dB

[f]



P 15.24 **[a]** $H(s) = \dfrac{(1/sC)}{R + (1/sC)} = \dfrac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)^2}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_i}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_i}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$V_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_i}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC + 2R_1R_2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_i}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2V_i}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_i} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

$$\therefore \; H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1 + R_2) + j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1 + R_2)^2 + \omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1 + R_2)^2 + 4R_1^2R_2^2C^2\omega^2}$$

P 15.25 $\omega_o = 2\pi f_o = 400\pi$ rad/s

$\beta = 2\pi(1000) = 2000\pi$ rad/s

$\therefore \quad \omega_{c_2} - \omega_{c_1} = 2000\pi$

$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 400\pi$

Solve for the cutoff frequencies:

$\omega_{c_1}\omega_{c_2} = 16 \times 10^4\pi^2$

$$\omega_{c_2} = \frac{16 \times 10^4\pi^2}{\omega_{c_1}}$$

$$\therefore \quad \frac{16 \times 10^4\pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

or $\omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16 \times 10^4\pi^2 = 0$

$$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16 \times 10^6\pi^2}$$

$$\omega_{c_1} = 1000\pi(-1 \pm \sqrt{1.16}) = 242.01\,\text{rad/s}$$

$$\therefore \quad \omega_{c_2} = 2000\pi + 242.01 = 6525.19\,\text{rad/s}$$

Thus, $f_{c1} = 38.52$ Hz and $f_{c2} = 1038.52$ Hz

Check: $\beta = f_{c2} - f_{c1} = 1000\text{Hz}$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65\,\Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43\,\Omega$$

P 15.26 $\omega_o = 1000\,\text{rad/s}$; GAIN $= 6$

$\beta = 4000$ rad/s; $C = 0.2\,\mu$F

$$\beta = \omega_{c_2} - \omega_{c_1} = 4000$$

$$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 1000$$

Solve for the cutoff frequencies:

$$\therefore \quad \omega_{c_1}^2 + 4000\omega_{c_1} - 10^6 = 0$$

$$\omega_{c_1} = -2000 \pm 1000\sqrt{5} = 236.07\,\text{rad/s}$$

$\omega_{c_2} = 4000 + \omega_{c_1} = 4236.07$ rad/s

Check: $\beta = \omega_{c2} - \omega_{c1} = 4000$ rad/s

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore \quad R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.81\,\text{k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18\,\text{k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 6R_i = 6\,\text{k}\Omega$

P 15.27 **[a]** $y = 20\log_{10}\dfrac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1+\omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1+\omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1+\omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore \quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \qquad \frac{d\omega}{dx} = \omega\ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\ \text{dB/decade}$$

at $\omega = \omega_c = 1$ rad/s

$$\frac{dy}{dx} = -10n\ \text{dB/decade.}$$

[b] $y = 20\log_{10}\dfrac{1}{[\sqrt{1+\omega^2}]^n} = -10n\log_{10}(1+\omega^2)$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10n}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20n\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \frac{dy}{dx} = \frac{-20n\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1} \qquad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}} \text{ dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	-10
2	-20
3	-30
4	-40
∞	$-\infty$

For the cascade of identical sections

n	dy/dx (dB/decade)
1	-10
2	-11.72
3	-12.38
4	-12.73
∞	-13.86

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.28 **[a]** $n \cong \dfrac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$$\therefore \quad n = 3$$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(7000/2000)^6}} = -32.65$ dB

P 15.29 **[a]** For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2 + \frac{2}{R'C_1'}s + \frac{1}{(R')^2C_1'C_2'}}$$

where

$$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/R^2C_1C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2C_1C_2}}$$

$$= \frac{1/R^2C_1C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2C_1C_2}}$$

P 15.30 **[a]** $H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

[b] $f_c = 2000\,\text{Hz}; \qquad \omega_c = 4000\pi$ rad/s; $\qquad k_f = 4000\pi$

$$H'(s) = \frac{1}{(\frac{s}{k_f}+1)[(\frac{s}{k_f})^2 + \frac{s}{k_f} + 1]}$$

$$= \frac{k_f^3}{(s+k_f)(s^2+k_fs+k_f^2)}$$

$$= \frac{(4000\pi)^3}{(s+4000\pi)[s^2+4000\pi s+(4000\pi)^2]}$$

[c] $H'(j14{,}000\pi) = \dfrac{64}{(4+j14)(-180+j56)}$

$$= 0.02332\underline{/-236.77^\circ}$$

Gain $= 20\log_{10}(0.02332) = -32.65$ dB

P 15.31 **[a]** In the first-order circuit $R = 1\,\Omega$ and $C = 1$ F.

$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

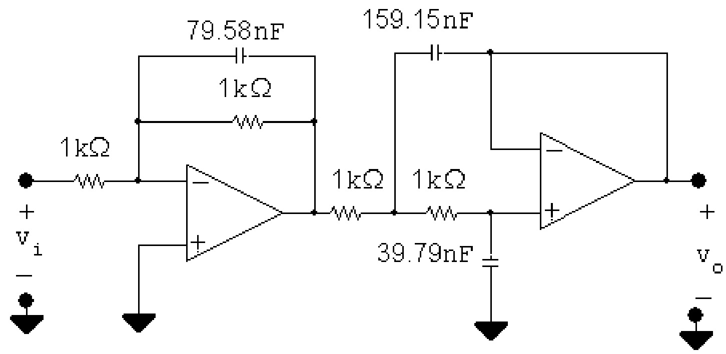
$$R' = k_mR = 1000\,\Omega; \qquad C' = \frac{C}{k_mk_f} = \frac{1}{(1000)(4000\pi)} = 79.58\,\text{nF}$$

In the second-order circuit $R = 1\,\Omega$, $2/C_1 = 1$ so $C_1 = 2$ F, and $C_2 = 1/C_1 = 0.5$ F. Therefore in the scaled second-order circuit

$$R' = k_mR = 1000\,\Omega; \qquad C_1' = \frac{C_1}{k_mk_f} = \frac{2}{(1000)(4000\pi)} = 159.15\,\text{nF}$$

$$C_2' = \frac{C_2}{k_mk_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79\,\text{nF}$$

[b]



P 15.32 **[a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(2000/500)} = 3.99 \qquad \therefore \quad n = 4$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{s^2}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \qquad R_2 = 2.61\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.383\,\Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{s^2}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \qquad R_2 = 1.082\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.9240\,\Omega$$

The scaling factors are:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

$$C' = \frac{C}{k_m k_f} \qquad \therefore \qquad 10 \times 10^{-9} = \frac{1}{4000\pi k_m}$$

$$\therefore \qquad k_m = \frac{1}{4000\pi(10 \times 10^{-9})} = 7957.75$$

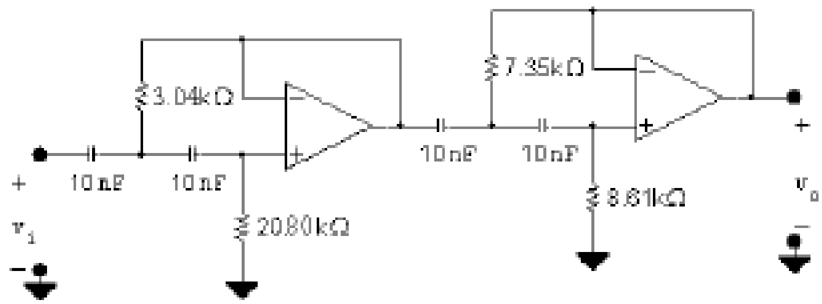
Therefore in the first section

$$R_1' = k_m R_1 = 3.04\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 20.80\,\text{k}\Omega$$

In the second section

$$R_1' = k_m R_1 = 7.35\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 8.61\,\text{k}\Omega$$

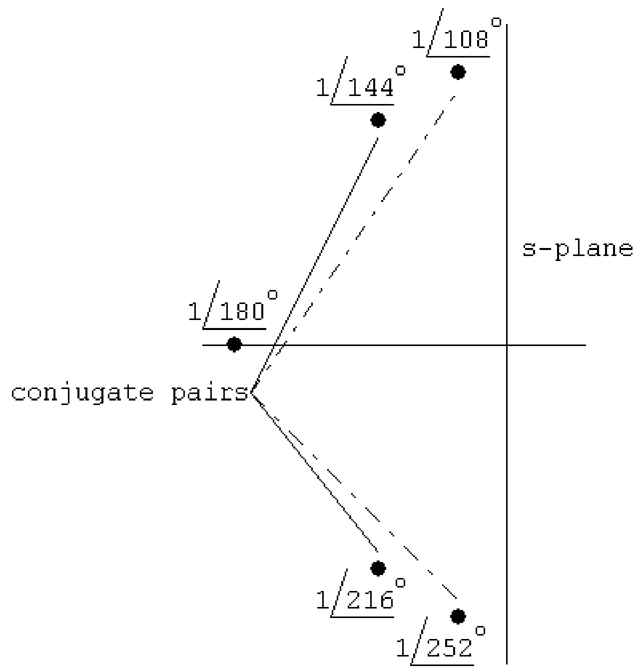
[b]



P 15.33 $n = 5$: $1 + (-1)^5 s^{10} = 0$; $\qquad s^{10} = 1$

$s^{10} = 1\underline{/0 + 36^\circ k}$

k	s_{k+1}
0	$1\underline{/0^\circ}$
1	$1\underline{/36^\circ}$
2	$1\underline{/72^\circ}$
3	$1\underline{/108^\circ}$
4	$1\underline{/144^\circ}$
5	$1\underline{/180^\circ}$
6	$1\underline{/216^\circ}$
7	$1\underline{/252^\circ}$
8	$1\underline{/288^\circ}$
9	$1\underline{/324^\circ}$



Group by conjugate pairs to form denominator polynomial.

$$(s+1)[s - (\cos 108^\circ + j\sin 108^\circ)][s - (\cos 252^\circ + j\sin 252^\circ)]$$

$$\cdot [s - (\cos 144^\circ + j\sin 144^\circ)][s - (\cos 216^\circ + j\sin 216^\circ)]$$

$$= (s+1)(s + 0.309 - j0.951)(s + 0.309 + j0.951)\cdot$$

$$(s + 0.809 - j0.588)(s + 0.809 + j0.588)$$

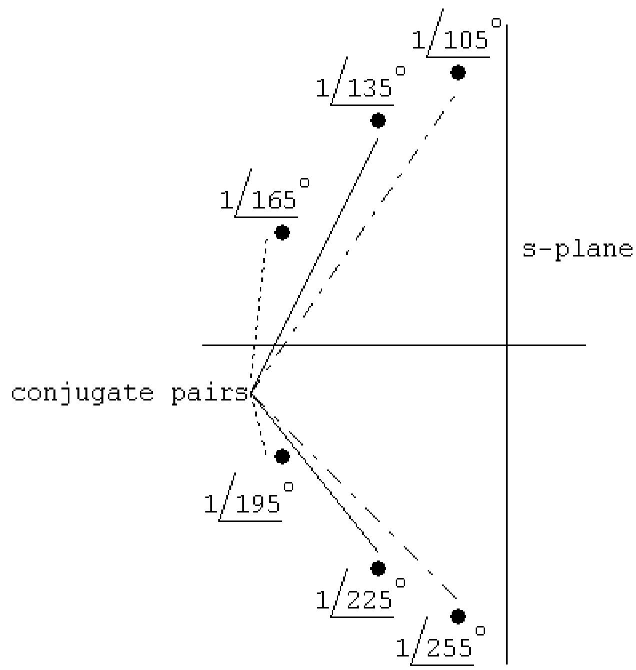
which reduces to

$$(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$s^{12} = 1\underline{/15^\circ + 36^\circ k}$

k	s_{k+1}
0	$1\underline{/15^\circ}$
1	$1\underline{/45^\circ}$
2	$1\underline{/75^\circ}$
3	$1\underline{/105^\circ}$
4	$1\underline{/135^\circ}$
5	$1\underline{/165^\circ}$
6	$1\underline{/195^\circ}$
7	$1\underline{/225^\circ}$
8	$1\underline{/255^\circ}$
9	$1\underline{/285^\circ}$
10	$1\underline{/315^\circ}$
11	$1\underline{/345^\circ}$



Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$

P 15.34 $H'(s) = \dfrac{s^2}{s^2 + \frac{2}{k_m R_2 (C/k_m k_f)} s + \frac{1}{k_m R_1 k_m R_2 (C^2/k_m^2 k_f^2)}}$

$$H'(s) = \frac{s^2}{s^2 + \frac{2k_f}{R_2 C} s + \frac{k_f^2}{R_1 R_2 C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \frac{2}{R_2 C}\left(\frac{s}{k_f}\right) + \frac{1}{R_1 R_2 C^2}}$$

P 15.35 **[a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(32/8)} = 3.99 \qquad \therefore \qquad n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \qquad \therefore \qquad C_1 = 2.61\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\text{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \qquad \therefore \qquad C_1 = 1.08\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\text{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 16{,}000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{2.61}{(16{,}000\pi)(1000)} = 52.01\,\text{nF}$$

$$C_2 = \frac{0.38}{(16{,}000\pi)(1000)} = 7.61\,\text{nF}$$

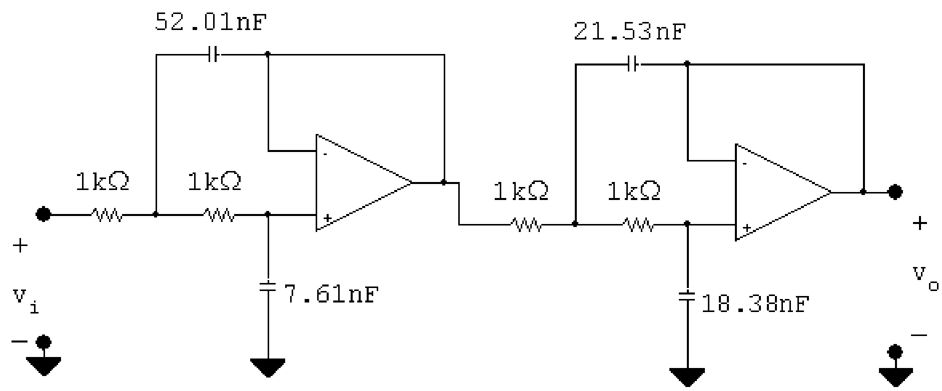
The scaled values for the second stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{1.08}{(16{,}000\pi)(1000)} = 21.53\,\text{nF}$$

$$C_2 = \frac{0.92}{(16{,}000\pi)(1000)} = 18.38\,\text{nF}$$

[b]



P 15.36 **[a]** The cascade connection is a bandpass filter.

[b] The cutoff frequencies are 2 kHz and 8 kHz.
The center frequency is $\sqrt{(2)(8)} = 4$ kHz.
The Q is $4/(8-2) = 2/3 = 0.67$

[c] For the high pass section $k_f = 4000\pi$. The prototype transfer function is

$$H_{\rm hp}(s) = \frac{s^4}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

$$\therefore \quad H'_{\rm hp}(s) = \frac{(s/4000\pi)^4}{[(s/4000\pi)^2+0.765(s/4000\pi)+1]} \cdot \frac{1}{[(s/4000\pi)^2+1.848(s/4000\pi)+1]}$$

$$= \frac{s^4}{(s^2+3060\pi s+16\times10^6\pi^2)(s^2+7392\pi s+16\times10^6\pi^2)}$$

For the low pass section $k_f = 16{,}000\pi$

$$H_{\rm lp}(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

$$\therefore \quad H'_{\rm lp}(s) = \frac{1}{[(s/16{,}000\pi)^2+0.765(s/16{,}000\pi)+1]} \cdot \frac{1}{[(s/16{,}000\pi)^2+1.848(s/16{,}000\pi)+1]}$$

$$= \frac{(16{,}000\pi)^4}{([s^2+12{,}240\pi s+(16{,}000\pi)^2)][s^2+29{,}568\pi s+(16{,}000\pi)^2]}$$

The cascaded transfer function is

$$H'(s) = H'_{\rm hp}(s)H'_{\rm lp}(s)$$

For convenience let

$$D_1 = s^2 + 3060\pi s + 16 \times 10^6\pi^2$$

$$D_2 = s^2 + 7392\pi s + 16 \times 10^6\pi^2$$

$$D_3 = s^2 + 12{,}240\pi s + 256 \times 10^6\pi^2$$

$$D_4 = s^2 + 29{,}568\pi s + 256 \times 10^6\pi^2$$

Then

$$H'(s) = \frac{65{,}536 \times 10^{12}\pi^4 s^4}{D_1D_2D_3D_4}$$

[d] $\omega_o = 2\pi(4000) = 8000\pi$ rad/s

$$s = j8000\pi$$

$$s^4 = 4096 \times 10^{12}\pi^4$$

$$D_1 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(3060\pi)$$

$$= 10^6\pi^2(-48 + j24.48) = 10^6\pi^2(53.88\underline{/152.98^\circ})$$

$$D_2 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(7392\pi)$$

$$= 10^6\pi^2(-48 + j59.136) = 10^6\pi^2(76.16\underline{/129.07^\circ})$$

$$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(12{,}240\pi)$$

$$= 10^6\pi^2(192 + j97.92) = 10^6\pi^2(215.53\underline{/27.02^\circ})$$

$$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(29{,}568\pi)$$

$$= 10^6\pi^2(192 + j236.544) = 10^6\pi^2(304.66\underline{/50.93^\circ})$$

$$H'(j\omega_o) = \frac{(65{,}536)(4096)\pi^8 \times 10^{24}}{(\pi^8 \times 10^{24})[(53.88)(76.16)(215.53)(304.66)\underline{/360^\circ}]}$$

$$= 0.996\underline{/-360^\circ} = 0.996\underline{/0^\circ}$$

P 15.37 **[a]** From the statement of the problem, $K = 10$ ($= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6\,\Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502}\,\Omega$$

$$R_3 = 2Q = 32\,\Omega$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(20 \times 10^{-9})(12{,}800\pi)} = 1243.40$$

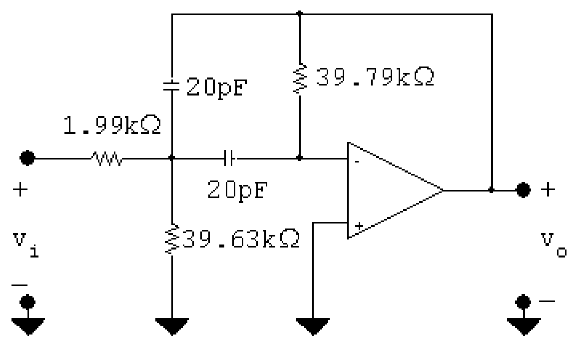
Therefore,

$$R_1' = k_m R_1 = (1.6)(1243.40) = 1.99\,\text{k}\Omega$$

$$R_2' = k_m R_2 = (16/502)(1243.40) = 39.63\,\Omega$$

$$R_3' = k_m R_3 = 32(1243.40) = 39.79\,\text{k}\Omega$$

[b]



P 15.38 From Eq 15.58 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1+R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1 R_2 R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

P 15.39 **[a]** First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 500 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-20)}{\log_{10}(500/200)} = 2.51$$

$$\therefore \quad n = 3$$

$$H_{\rm hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\text{F}$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(500) = 1000\pi$$

$$k_m = \frac{C}{C' k_f} = \frac{1}{(15 \times 10^{-9})(1000\pi)} = \frac{10^6}{15\pi}$$

In the scaled first-order section

$$R_1' = R_2' = k_m R_1 = \frac{10^6}{15\pi}(1) = 21.22\,\text{k}\Omega$$

$$C' = 15 \text{ nF}$$

In the scaled second-order section

$$R_1' = 0.5k_m = 10.61\,\text{k}\Omega$$

$$R_2' = 2k_m = 42.44\,\text{k}\Omega$$

$C' = 15\,\text{nF}$

For the low-pass section the cut-off frequency is 4500 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-20)}{\log_{10}(11{,}250/4500)} = 2.51; \qquad \therefore \quad n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 10^4; \qquad k_f = \frac{\omega_o'}{\omega_o} = (4500)(2\pi) = 9000\pi$$

For the scaled first-order section

$$R_1' = R_2' = 10\,\text{k}\Omega; \qquad C' = \frac{C}{k_f k_m} = \frac{1}{(9000\pi)(10^4)} = 3.54\,\text{nF}$$

For the scaled second-order section

$$R_1' = R_2' = 10\,\text{k}\Omega$$

$$C_1' = \frac{C_1}{k_f k_m} = \frac{2}{(9000\pi)(10^4)} = 7.07\,\text{nF}$$

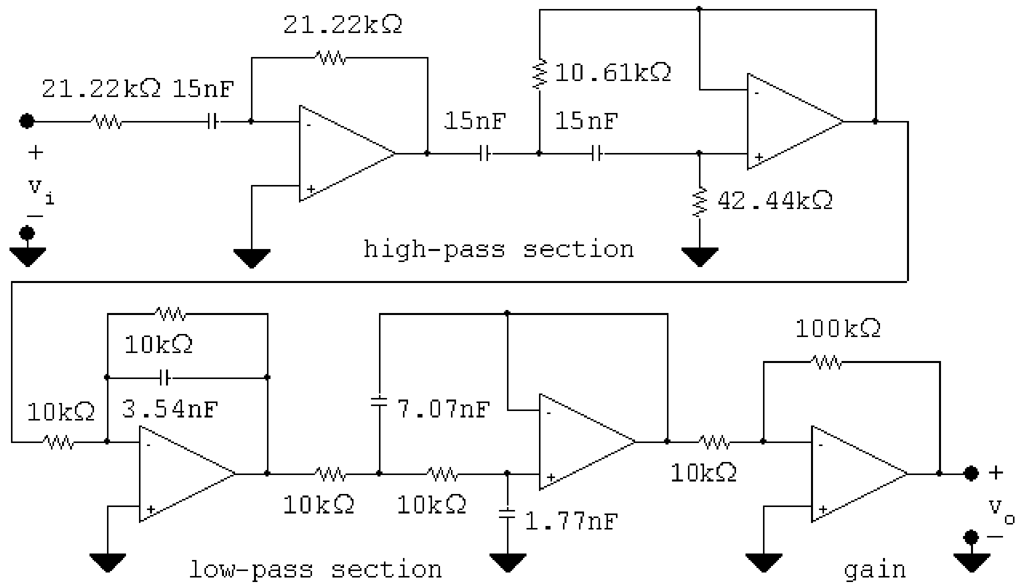
$$C_2' = \frac{C_2}{k_f k_m} = \frac{0.5}{(9000\pi)(10^4)} = 1.77\,\text{nF}$$

GAIN AMPLIFIER

$$20\log_{10} K = 20 \text{ dB}, \qquad \therefore \quad K = 10$$

Since we are using $10\,\text{k}\Omega$ resistors in the low-pass stage, we will use $R_f = 100\,\text{k}\Omega$ and $R_i = 10\,\text{k}\Omega$ in the inverting amplifier stage.

[b]



P 15.40 **[a]** Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 1000\pi$. Therefore the scaled transfer function is

$$H'_{hp}(s) = \frac{(s/1000\pi)^3}{\left(\frac{s}{1000\pi}+1\right)\left[\left(\frac{s}{1000\pi}\right)^2 + \frac{s}{1000\pi}+1\right]} = \frac{s^3}{(s+1000\pi)[s^2+1000\pi s+10^6\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 9000\pi$. Therefore the scaled transfer function is

$$H'_{lp}(s) = \frac{1}{\left(\frac{s}{9000\pi}+1\right)\left[\left(\frac{s}{9000\pi}\right)^2 + \left(\frac{s}{9000\pi}\right)+1\right]} = \frac{(9000\pi)^3}{(s+9000\pi)(s^2+9000\pi s+81\times 10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 10H'_{hp}(s)H'_{lp}(s) = \frac{729\times 10^{10}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$D_1 = s + 1000\pi$

$D_2 = s + 9000\pi$

$D_3 = s^2 + 1000\pi s + 10^6\pi^2$

$D_4 = s^2 + 9000\pi s + 81 \times 10^6\pi^2$

[b] At $f = 200\,\text{Hz}$ $\qquad \omega = 400\pi$ rad/s

$D_1(j400\pi) = 400\pi(2.5 + j1)$

$D_2(j400\pi) = 400\pi(22.5 + j1)$

$D_3(j400\pi) = 4 \times 10^5\pi^2(2.1 + j1.0)$

$D_4(j400\pi) = 4 \times 10^5\pi^2(202.1 + j9)$

Therefore

$D_1D_2D_3D_4(j400\pi) = 256\pi^6 10^{14}(28{,}534.82\underline{/52.36^\circ})$

$$H'(j400\pi) = \frac{(729\pi^3 \times 10^{10})(64 \times 10^6\pi^3)}{256\pi^6 \times 10^{14}(28{,}534.82\underline{/52.36^\circ})}$$

$$= 0.639\underline{/-52.36^\circ}$$

$\therefore \quad 20\log_{10}|H'(j400\pi)| = 20\log_{10}(0.639) = -3.89$ dB

At $f = 1500\,\text{Hz}$, $\qquad \omega = 3000\pi$ rad/s

Then

$D_1(j3000\pi) = 1000\pi(1 + j3)$

$D_2(j3000\pi) = 3000\pi(3 + j1)$

$D_3(j3000\pi) = 10^6\pi^2(-8 + j3)$

$D_4(j3000\pi) = 9 \times 10^6\pi^2(8 + j3)$

$$H'(j3000\pi) = \frac{(729 \times \pi^3 \times 10^{10})(27 \times 10^9\pi^3)}{27 \times 10^{18}\pi^6(730\underline{/270^\circ})}$$

$$= 9.99\underline{/90^\circ}$$

$\therefore \quad 20\log_{10}|H'(j3000\pi)| = 19.99$ dB

[c] From the transfer function the gain is down $19.99 + 3.89$ or 23.88 dB at 200 Hz. Because the upper cut-off frequency is nine times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 200 Hz. For a 3nd order Butterworth

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1+(500/200)^6}} = -23.89 \text{ dB}.$$

1500 Hz is in the passband for this bandpass filter, and is in fact the center frequency. Hence we expect the gain at 1500 Hz to equal, or nearly equal, 20 dB as specified in Problem 15.39. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.41 **[a]** From Table 15.1

$$H_{lp}(s) = \frac{1}{(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)}$$

$$H_{hp}(s) = \frac{1}{\left(\frac{1}{s^2}+0.518\left(\frac{1}{s}\right)+1\right)\left(\frac{1}{s^2}+\sqrt{2}\left(\frac{1}{s}\right)+1\right)\left(\frac{1}{s^2}+1.932\left(\frac{1}{s}\right)+1\right)}$$

$$H_{hp}(s) = \frac{s^6}{(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)}$$

P 15.42 **[a]** $k_f = 25{,}000$

$$H'_{\text{hp}}(s) = \frac{(s/25{,}000)^6}{[(s/25{,}000)^2+0.518(s/25{,}000)+1]}$$

$$\cdot\frac{1}{[(s/25{,}000)^2+1.414s/25{,}000+1][(s/25{,}000)^2+1.932s/25{,}000+1]}$$

$$= \frac{s^6}{(s^2+12{,}950s+625\times10^6)(s^2+35{,}350s+625\times10^6)}$$

$$\cdot\frac{1}{(s^2+48{,}300s+625\times10^6)}$$

[b] $$H'(j25{,}000) = \frac{-(25{,}000)^6}{[12{,}950(j25{,}000)][35{,}350(j25{,}000)][48{,}300(j25{,}000)]}$$

$$= \frac{-(25{,}000)^3}{(12{,}950)(25{,}350)(48{,}300)j^3}$$

$$= 0.7067\underline{/-90^\circ}$$

$$20\log_{10}|H'(j25{,}000)| = -3.02 \text{ dB}$$

P 15.43 **[a]** At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3} V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1 G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2 G_3}$$

$$= \frac{-G_1 G_3}{s^2 C_1 C_2 + (G_1 + G_2 + G_3)C_1 s + G_2 G_3}$$

$$= \frac{-G_1 G_3 / C_1 C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-\frac{G_1 G_2 G_3}{G_2 C_1 C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-K b_o}{s^2 + b_1 s + b_o}$$

where $K = \frac{G_1}{G_2}$; $\qquad b_o = \frac{G_2 G_3}{C_1 C_2}$

and $b_1 = \frac{G_1 + G_2 + G_3}{C_2}$

[c] Equating coefficients we see that

$$G_1 = K G_2$$

$$G_3 = \frac{b_o C_1 C_2}{G_2} = \frac{b_o C_1}{G_2}$$

since by hypothesis $C_2 = 1\,\text{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1+K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate G_2 (R_2)

4. Calculate G_1 (R_1); $G_1 = KG_2$

5. Calculate G_3 (R_3); $G_3 = b_oC_1/G_2$

P 15.44 **[a]** In the second order section of a third order Butterworth filter $b_o = b_1 = 1$
Therefore,

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05\,\text{F}$$

$$\therefore \qquad C_1 = 0.05\,\text{F} \quad \text{(limiting value)}$$

[b] $G_2 = \dfrac{1}{2(1+4)} = 0.1\,\text{S}$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5\,\text{S}$$

$$G_1 = 4(0.1) = 0.4\,\text{S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5\,\Omega; \qquad R_2 = \frac{1}{G_2} = 10\,\Omega; \qquad R_3 = \frac{1}{G_3} = 2\,\Omega$$

[c] $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(2500) = 5000\pi$

$$k_m = \frac{C_2}{C_2'k_f} = \frac{1}{(10 \times 10^{-9})k_f} = 6366.2$$

$$C_1' = \frac{0.05}{k_f k_m} = 0.5 \times 10^{-9} = 500\,\text{pF}$$

$$R_1' = (2.5)(6366.2) = 15.92\,\text{k}\Omega$$

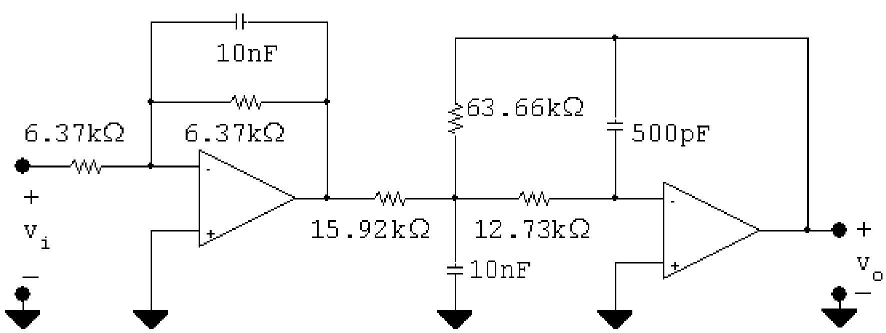
$$R_2' = (10)(6366.2) = 63.66\,\text{k}\Omega$$

$$R_3' = (2)(6366.2) = 12.73\,\text{k}\Omega$$

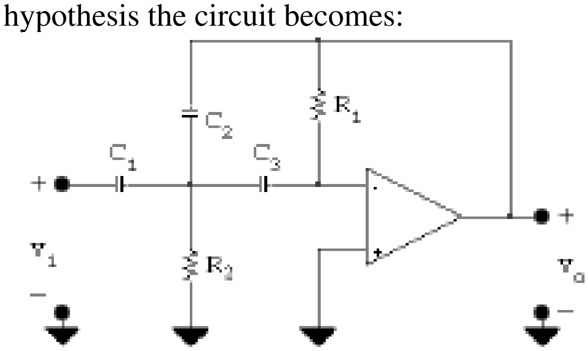
[d] $R_1' = R_2' = (6366.2)(1) = 6.37\,\text{k}\Omega$

$$C' = \frac{C}{k_f k_m} = \frac{1}{10^8} = 10\,\text{nF}$$

[e]



P 15.45 **[a]** By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{[C_2C_3s^2 + G_1(C_1 + C_2 + C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore \quad G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K + 2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.10\,\Omega$$

P 15.46 **[a]** Low-pass filter:

$$n = \frac{(-0.05)(-30)}{\log_{10}(1000/400)} = 3.77; \qquad \therefore \quad n = 4$$

In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1\,\text{F}$

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} \leq \frac{(0.765)^2}{(4)(2)} \leq 0.0732$$

choose $C_1 = 0.03\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.338\ \text{S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 2.96\,\Omega$$

$$G_1 = KG_2 = 0.338\ \text{S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 2.96\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \qquad R_3 = 1/G_3 = 11.3\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96\,\Omega; \qquad R_3 = 11.3\,\Omega$$

$$C_1 = 0.03\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit: $b_1 = 1.848,\ b_0 = 1,\ C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \leq \frac{(1.848)^2}{8} \leq 0.427$$

choose $C_1 = 0.30\,\text{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139 \text{ S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 1.4008\,\Omega$$

$$G_1 = KG_2 = 0.7139 \text{ S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 1.4008\,\Omega$$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202 \text{ S} \quad \therefore \qquad R_3 = 1/G_3 = 2.3796\,\Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(400) = 800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10 \times 10^{-9})k_f} = \frac{125{,}000}{\pi}$$

Therefore in the first scaled second-order section

$$R_1' = R_2' = 2.96k_m = 118\,\text{k}\Omega$$

$$R_3' = 11.3k_m = 450\,\text{k}\Omega$$

$$C_1' = \frac{0.03}{k_f k_m} = 300\,\text{pF}$$

$$C_2' = 10\,\text{nF}$$

In the second scaled second-order section

$$R_1' = R_2' = 1.4008k_m = 55.74\,\text{k}\Omega$$

$$R_3' = 2.3796k_m = 94.68\,\text{k}\Omega$$

$$C_1' = \frac{0.3}{k_f k_m} = 3\,\text{nF}$$

$$C_2' = 10\,\text{nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-30)}{\log_{10}(6400/2560)} = 3.77; \qquad n = 4.$$

In the first prototype second-order section: $b_1 = 0.765;\ b_o = 1;\ C_2 = C_3 = 1\,\text{F}$

$C_1 = K = 1\,\text{F}$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10\times 10^{-9})(12{,}800\pi)} = \frac{7812.5}{\pi}$$

In the first scaled second-order section

$$R_1' = 3.92k_m = 9.75\,\text{k}\Omega$$

$$R_2' = 0.255k_m = 634\,\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the second scaled second-order section

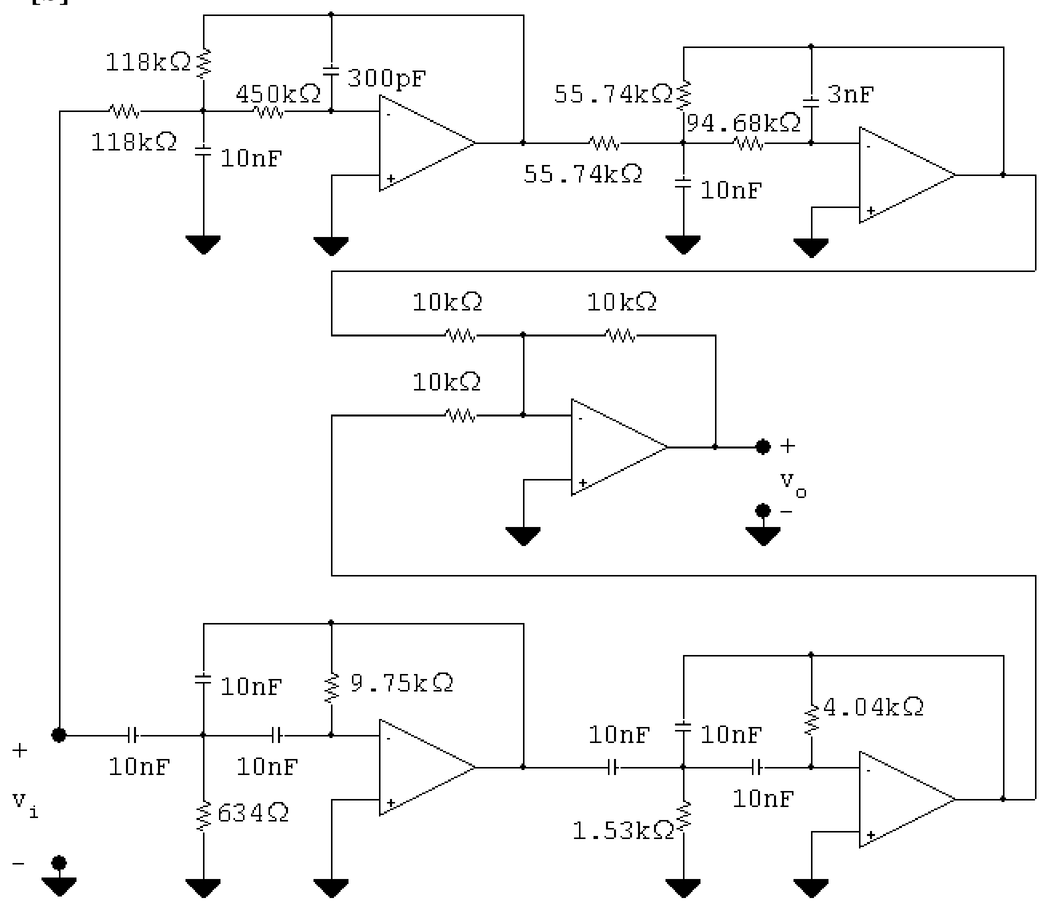
$$R_1' = 1.623k_m = 4.04\,\text{k}\Omega$$

$$R_2' = 0.616k_m = 1.53\,\text{k}\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 10\,\text{k}\Omega$.

[b]



P 15.47 **[a]** The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(400) = 800\pi$$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{800\pi}\right)^2 + \frac{0.765s}{800\pi} + 1\right]\left[\left(\frac{s}{800\pi}\right)^2 + \frac{1.848s}{800\pi} + 1\right]}$$
$$= \frac{4096 \times 10^8\pi^4}{[s^2 + 612\pi s + (800\pi)^2]\,[s^2 + 1478.4\pi s + (800\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(6400) = 12{,}800\pi$$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/12{,}800\pi)^4}{\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{0.765s}{12{,}800\pi} + 1\right]\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{1.848s}{12{,}800\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 9792\pi s + (12{,}800\pi)^2][s^2 + 23{,}654.4\pi s + (12{,}800\pi)^2]}$$

The transfer function for the filter is

$$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right]$$

[b] $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{(400)(6400)} = 1600\,\text{Hz}$

$\omega_o = 2\pi f_o = 3200\pi$ rad/s

$(j\omega_o)^2 = -1024 \times 10^4\pi^2$

$(j\omega_o)^4 = 1{,}048{,}576 \times 10^8\pi^4$

$$H'_{lp}(j\omega_o) = \frac{4096 \times 10^8\pi^4}{[-960 \times 10^4\pi^2 + j612(3200\pi^2)]} \times$$

$$\frac{1}{[-960 \times 10^4\pi^2 + j1478.4(3200\pi^2)]}$$

$$= \frac{40{,}000}{(-3000 + j612)(-3000 + j1478.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/37.76^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{1{,}048{,}576 \times 10^8\pi^4}{[15{,}360 \times 10^4\pi^2 + j9792(3200\pi^2)]}$$

$$\frac{1}{[15{,}360 \times 10^4\pi^2 + j23{,}654.4(3200\pi^2)]}$$

$$= \frac{10.24 \times 10^6}{(48{,}000 + j9792)(48{,}000 + j23{,}654.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/-37.76^\circ}$$

$$\therefore \quad H'(j\omega_o) = -3906.2 \times 10^{-6}(1\underline{/37.76^\circ} + 1\underline{/-37.76^\circ})$$

$$= -3906.2 \times 10^{-6}(1.58\underline{/0^\circ}) = -6176.35 \times 10^{-6}\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(6176.35 \times 10^{-6}) = -44.19\text{ dB}$$

P 15.48 **[a]** At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307\,\Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$R_2 = 1/G_2 = 0.765\,\Omega$

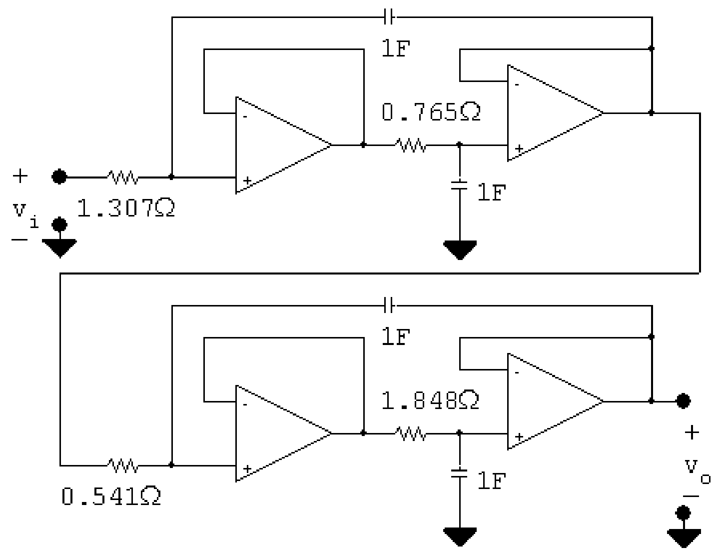
In the second section $b_o = 1, \quad b_1 = 1.848$

$\therefore \quad G_1 = 1.848\,\text{S}$

$R_1 = 1/G_1 = 0.541\,\Omega$

$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\text{S}$

$R_2 = 1/G_2 = 1.848\,\Omega$



P 15.49 **[a]** $k_f = \frac{\omega_o'}{\omega_o} = 2\pi(3000) = 6000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$

In the first section

$R_1' = 1.307k_m = 14.75\,\text{k}\Omega$

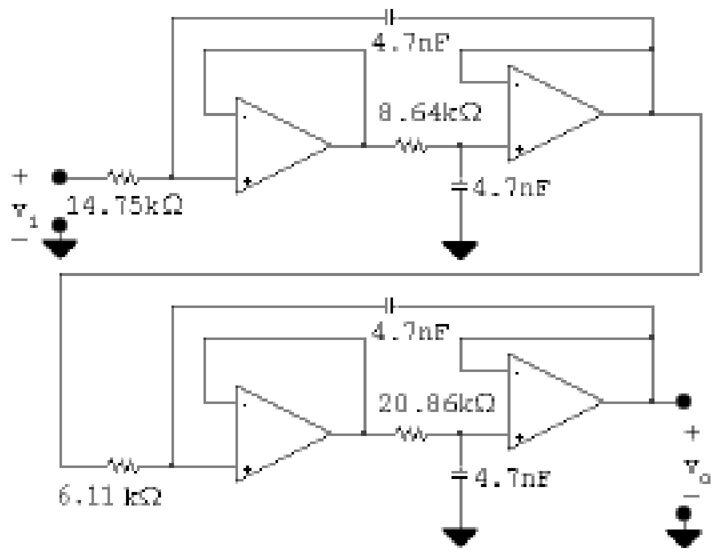
$R_2' = 0.765k_m = 8.64\,\text{k}\Omega$

In the second section

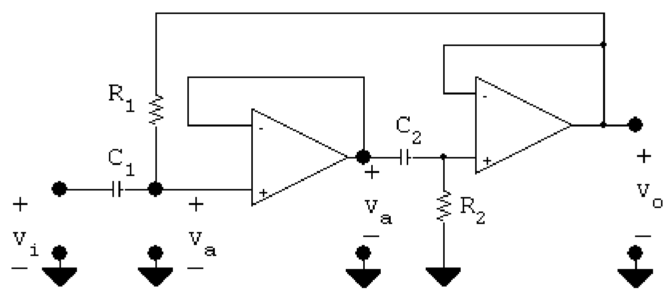
$R_1' = 0.541k_m = 6.11\,\text{k}\Omega$

$R_2' = 1.848k_m = 20.86\,\text{k}\Omega$

[b]



P 15.50 **[a]** Interchanging the Rs and Cs yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2}s + \frac{G_1G_2}{C_1C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o = \frac{G_1G_2}{C_1C_2}; \qquad b_1 = \frac{G_2}{C_2}$

$$\therefore \quad G_2 = b_1C_2; \qquad R_2 = \frac{1}{b_1C_2}$$

$$G_1 = \frac{b_o}{b_1}C_1 \therefore \quad R_1 = \frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

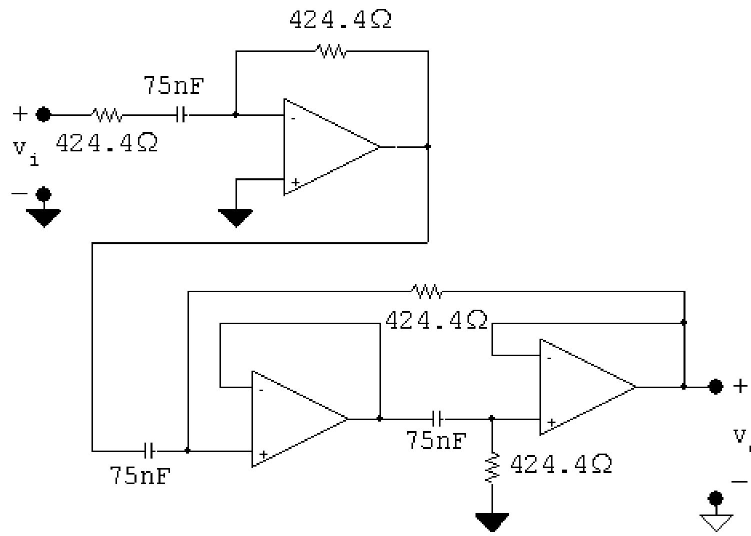
P 15.51 **[a]** $k_f = \frac{\omega_o'}{\omega_o} = 10^4\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(75 \times 10^{-9})(10^4\pi)} = \frac{10^5}{75\pi}$$

$$C_1 = C_2 = 75\,\text{nF}; \qquad R_1' = R_2' = k_mR = 424.4\,\Omega$$

[b] $R = 424.4\,\Omega; \qquad C = 75\,\text{nF}$

[c]



[d] $H_{\rm hp}(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$

$$H'_{\rm hp}(s) = \frac{(s/10^4\pi)^3}{[(s/10^4\pi)+1][(s/10^4\pi)^2+(s/10^4\pi)+1]}$$

$$= \frac{s^3}{(s+10^4\pi)(s^2+10^4\pi s+10^8\pi^2)}$$

[e] $H'_{\rm hp}(j10^4\pi) = \dfrac{(j10^4\pi)^3}{(j10^4\pi+10^4\pi)[(j10^4\pi)^2+10^4\pi(j10^4\pi)+10^8\pi^2]} = 0.7071\underline{/135^\circ}$

$$\therefore \qquad |H'_{\rm hp}| = 0.7071 = -3 \text{ dB}$$

P 15.52 **[a]** It follows directly from Eq 15.64 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[b] For Example 15.13, $\omega_o = 5000$ rad/s and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\dfrac{1}{5}\left(\dfrac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times10^6}{s^2+1000s+25\times10^6}$$

P 15.53 **[a]** $\omega_o = 2000\pi$ rad/s

$$\therefore \qquad k_f = \frac{\omega'_o}{\omega_o} = 2000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(15\times10^{-9})(2000\pi)} = \frac{10^5}{3\pi}$$

$$R' = k_m R = \frac{10^5}{3\pi}(1) = 10{,}610\,\Omega$$

$$\frac{R'}{2} = 5{,}305\,\Omega$$

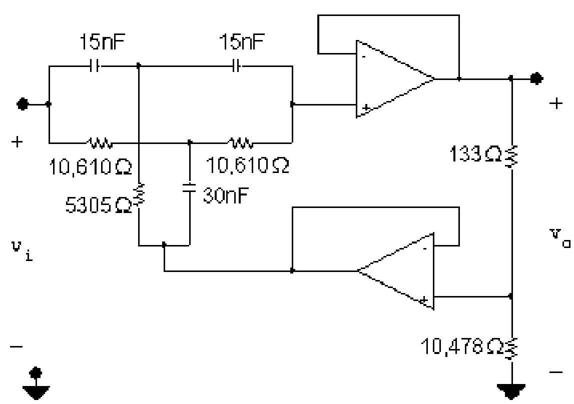
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(20)} = 0.9875$$

$$\sigma R' = 10{,}478\,\Omega; \qquad (1-\sigma)R' = 133\,\Omega$$

$$C' = 15\,\text{nF}$$

$$2C' = 30\,\text{nF}$$

[b]



[c] $k_f = 2000\pi$

$$H(s) = \frac{(s/2000\pi)^2 + 1}{(s/2000\pi)^2 + \frac{1}{20}(s/2000\pi) + 1}$$

$$= \frac{s^2 + 4\times10^6\pi^2}{s^2 + 100\pi s + 4\times10^6\pi^2}$$

P 15.54 To satisfy the gain specification of 20 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 10 \qquad \text{or} \qquad R_2 = 9R_1$$

Choose a standard resistor of $11.1\,\text{k}\Omega$ for R_1 and a $100\,\text{k}\Omega$ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(40)(10^5)} = 39.79\text{ nF}$$

Choose a standard capacitor value of 39 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{111.1}{11.1}\right)_{\alpha=1} = 20.01 \text{ dB}$$

When $C_1 = 39$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{10^5(39)} = 256.41 \text{ rad/s} = 40.81 \text{ Hz}$$

The magnitude of the transfer function at 256.41 rad/s is

$$|H(j256.41)|_{\alpha=1} = \frac{|111.1\times 10^3 + j256.41(11.1)(100)(39)10^{-3}|}{|11.1\times 10^3 + j256.41(11.1)(100)(39)10^{-3}|} = 7.11$$

Therefore the gain at 40.81 Hz is

$$20\log_{10}(7.11)_{\alpha=1} = 17.04 \text{ dB}$$

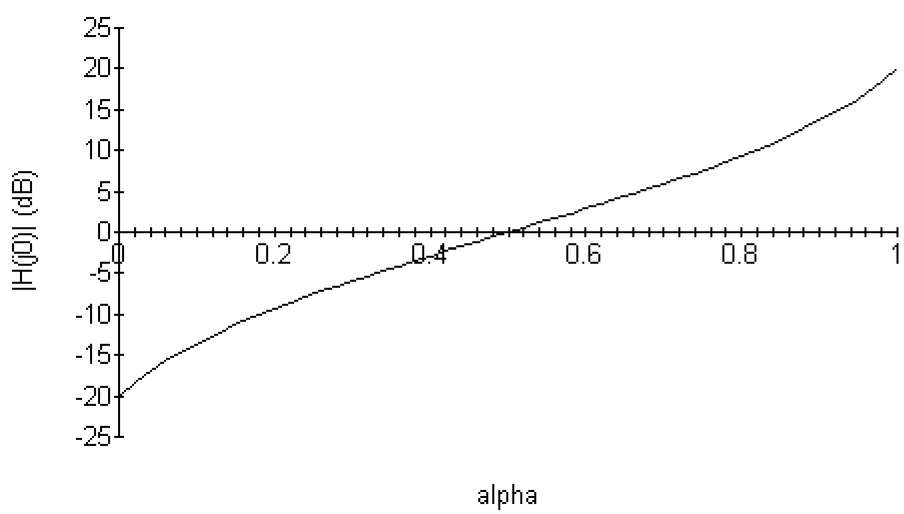
P 15.55 $$20\log_{10}\left(\frac{R_1+R_2}{R_1}\right) = 13.98$$

$$\therefore\quad \frac{R_1+R_2}{R_1} = 5; \qquad \therefore\quad R_2 = 4R_1$$

Choose $R_1 = 100\,\text{k}\Omega$. Then $R_2 = 400\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 100\pi \text{ rad/s}; \qquad \therefore\quad C_1 = \frac{1}{(100\pi)(400\times 10^3)} = 7.96\,\text{nF}$$

P 15.56 **[a]** |

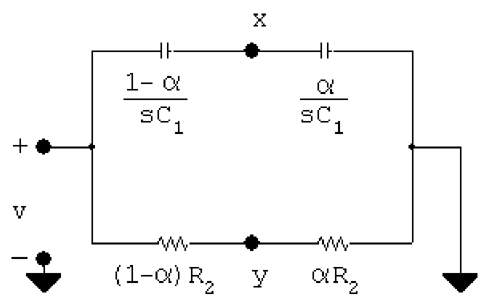


P 15.57 **[a]** Combine the impedances of the capacitors in series in Fig. P15.53(b) to get

$$C_{\text{eq}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.53(a).

[b]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha = V_x$$

[c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. 15.53(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot\alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.58 As $\omega \to 0$

$$|H(i\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_o} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.59 **[a]** $|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)} = \frac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$\therefore$ maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

[b] $|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$

$\therefore$ maximum cut $= -21.93$ dB

[c] $R_4 = 500\,\text{k}\Omega; \qquad R_o = R_1 + R_3 + 2R_5 = 65.9\,\text{k}\Omega$

$\therefore \; R_4 = 7.59R_o$

Yes, R_4 is significantly greater than R_o.

[d] $|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}{(2R_3 + R_4) + j(R_4 + R_o)}\right|$

$$= \left|\frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9}\right|$$

$$= 7.44$$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

[e] When $\beta = 0$

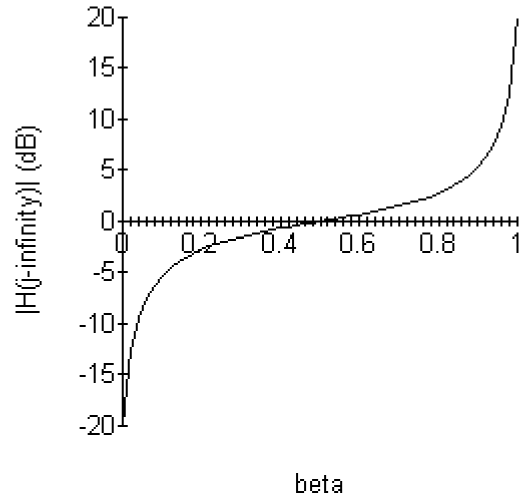
$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3 + R_4) + j(R_4 + R_o)}{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}$$

Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore \; 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.60 $$|H(j\infty)| = \frac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta R_4 + R_3][\beta R_4 + R_o]}$$



16

Fourier Series

Assessment Problems

AP 16.1 $a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\,dt = \frac{7}{9}V_m = 7\pi\,\text{V}$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1 - \cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1 - \cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 **[a]** $a_v = 7\pi = 21.99\,\text{V}$

[b] $a_1 = -5.196 \quad a_2 = 2.598 \quad a_3 = 0 \quad a_4 = -1.299 \quad a_5 = 1.039$

$b_1 = 9 \quad b_2 = 4.5 \quad b_3 = 0 \quad b_4 = 2.25 \quad b_5 = 1.8$

[c] $\omega_0 = \left(\frac{2\pi}{T}\right) = 50\,\text{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\text{Hz}$

[e] $v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\cos 100t + 4.5\sin 100t$
$-1.3\cos 200t + 2.25\sin 200t + 1.04\cos 250t + 1.8\sin 250t + \cdots\,\text{V}$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

$b_k = 0 \quad \text{for } k \text{ even}$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right)t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt$$

$$= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right)$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t\ \text{V}$$

AP 16.4 **[a]** Using the results from AP 16.2, and Equation (16.39),

$$A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}; \qquad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$$

$$A_3 = 0; \qquad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$$

$$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$$

$\theta_1 = -120^\circ; \qquad \theta_2 = -60^\circ; \qquad \theta_3$ not defined;

$$\theta_4 = -120^\circ; \qquad \theta_5 = -60^\circ$$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$

$$+\, 2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots\ \text{V}$$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

$$\therefore \qquad v_i = 2250 \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \cos 10nt \text{ mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ} \text{ mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ} \text{ mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ} \text{ mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ} \text{ mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ} \text{ mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ} \text{ mV}$$

$$\therefore \qquad v_o = 2238.33\cos(10t - 5.71^\circ) + 239.46\cos(30t - 16.70^\circ)$$

$$+ 80.50\cos(50t - 26.57^\circ) + \ldots \text{ mV}$$

AP 16.6 **[a]** The Fourier series of the input voltage is

$$v_g = \frac{4A}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin n\omega_0(t + T/4)$$

$$= 42 \sum_{n=1,3,5}^{\infty} \left[\frac{1}{n} \sin\left(\frac{n\pi}{2}\right)\right] \cos 2000nt \text{ V}$$

From the circuit we have

$$V_o sC + \frac{V_o}{sL} + \frac{V_o - V_g}{R} = 0$$

$$\therefore \qquad \frac{V_o}{V_g} = H(s) = \frac{s/RC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values yields

$$H(s) = \frac{500s}{s^2 + 500s + 10^8}$$

$\mathbf{V}_{g1} = 42\underline{/0^\circ} \qquad \omega_0 = 2000$ rad/s

$\mathbf{V}_{g3} = 14\underline{/180^\circ} \qquad 3\omega_0 = 6000$ rad/s

$\mathbf{V}_{g5} = 8.4\underline{/0^\circ} \qquad 5\omega_0 = 10{,}000$ rad/s

$\mathbf{V}_{g7} = 6\underline{/180^\circ} \qquad 7\omega_0 = 14{,}000$ rad/s

$$H(j2000) = \frac{500(j2000)}{10^8 - 4\times 10^6 + 500(j2000)} = \frac{j1}{96 + j1} = 0.01042\underline{/89.40^\circ}$$

$H(j6000) = 0.04682\underline{/87.32^\circ}$

$H(j10{,}000) = 1\underline{/0^\circ}$

$H(j14{,}000) = 0.07272\underline{/-85.83^\circ}$

Thus,

$\mathbf{V}_{o1} = (42\underline{/0^\circ})(0.01042\underline{/89.40^\circ}) = 0.4375\underline{/89.40^\circ}$ V

$\mathbf{V}_{o3} = 0.6555\underline{/-92.68^\circ}$ V

$\mathbf{V}_{o5} = 8.4\underline{/0^\circ}$ V

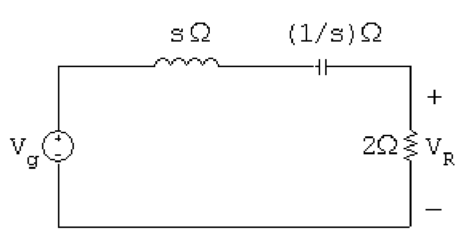
$\mathbf{V}_{o7} = 0.4363\underline{/94.17^\circ}$ V

Therefore,

$$v_o = 0.4375\cos(2000t + 89.40^\circ) + 0.6555\cos(6000t - 92.68^\circ)$$
$$+ 8.4\cos(10{,}000t) + 0.4363\cos(14{,}000t + 94.17^\circ) + \ldots \text{ V}$$

[b] The 5th harmonic, that is, the term at $10{,}000$ rad/s, dominates the output voltage. The circuit is a bandpass filter with a center frequency of $10{,}000$ rad/s and a bandwidth of 500 rad/s. Thus, Q is 20 and the filter is quite selective. This causes the attenuation of the fundamental, third, and seventh harmonic terms in the output signal.

AP 16.7 $\omega_0 = \dfrac{2\pi\times 10^3}{2094.4} = 3$ rad/s



$j\omega_0 k = j3k$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g_1} = 25.98\sin\omega_0 t\,\text{V}; \qquad V_{g_1} = 25.98\underline{/0^\circ}\,\text{V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R_1} = 15.588\underline{/-53.13^\circ}\,\text{V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75\,\text{W}$$

$$v_{g_3} = 0, \quad \text{therefore} \quad P_3 = 0\,\text{W}$$

$$v_{g_5} = -1.04\sin 5\omega_0 t\,\text{V}; \qquad V_{g_5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R_5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ}\,\text{mV}$$

$$P_5 = \frac{(0.138/\sqrt{2})^2}{2} = 4.76\,\text{mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75\,\text{W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2\sin\,k\omega_0 t\,dt + \frac{8}{T}\int_{T/8}^{T/4} 8\sin k\omega_0 t\,dt$$

$$= \left(\frac{8}{\pi k}\right)\left[1+3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

Therefore $\quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1+3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$

AP 16.9 **[a]** $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8} - \frac{T}{8}\right)\right]} = \sqrt{34} = 5.831\,\text{A}$

[b] $C_1 = \frac{-j12.5}{\pi}; \quad C_3 = \frac{j1.5}{\pi}; \quad C_5 = \frac{j0.9}{\pi};$

$$C_7 = \frac{-j1.8}{\pi}; \quad C_9 = \frac{-j1.4}{\pi}; \quad C_{11} = \frac{j0.4}{\pi}$$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2 + 0.4^2\right)}$$

$$\cong 5.777\,\text{A}$$

[c] $\%\text{ Error} = \frac{5.777 - 5.831}{5.831} \times 100 = -0.93\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2\right)}$$

$$\cong 5.774\,\text{A}$$

$$\%\text{ Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the $\%$ error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi}\sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{-j(n+1)(\pi/2)}e^{jn\omega_0 t}$$

Problems

P 16.1 **[a]** $\omega_{\text{oa}} = \dfrac{2\pi}{200 \times 10^{-6}} = 31,415.93$ rad/s

$$\omega_{\text{ob}} = \frac{2\pi}{40 \times 10^{-6}} = 157.080 \text{ krad/s}$$

[b] $f_{\text{oa}} = \dfrac{1}{T} = \dfrac{1}{200 \times 10^{-6}} = 5000\,\text{Hz}$; $\qquad f_{\text{ob}} = \dfrac{1}{40 \times 10^{-6}} = 25{,}000\,\text{Hz}$

[c] $a_{\text{va}} = 0$; $\qquad a_{\text{vb}} = \dfrac{100(10 \times 10^{-6})}{40 \times 10^{-6}} = 25\,\text{V}$

[d] The periodic function in Fig. P16.1(a) has half-wave symmetry. Therefore,

$a_{\text{va}} = 0$; $\quad a_{\text{ka}} = 0$ for k even; $\quad b_{\text{ka}} = 0$ for k even

For k odd,

$$\begin{aligned}
a_{\text{ka}} &= \frac{4}{T}\int_0^{T/4} 40\cos\frac{2\pi kt}{T}\,dt + \frac{4}{T}\int_{T/4}^{T/2} 80\cos\frac{2\pi kt}{T}\,dt \\
&= \frac{160}{T}\frac{T}{2\pi k}\sin\frac{2\pi kt}{T}\Big|_0^{T/4} + \frac{320}{T}\frac{T}{2\pi k}\sin\frac{2\pi kt}{T}\Big|_{T/4}^{T/2} \\
&= \frac{80}{\pi k}\sin\frac{\pi k}{2} + \frac{160}{\pi k}\left(\sin\pi k - \sin\frac{\pi k}{2}\right) \\
&= -\frac{80}{\pi k}\sin\frac{\pi k}{2}, \quad k \text{ odd}
\end{aligned}$$

$$\begin{aligned}
b_{\text{ka}} &= \frac{4}{T}\int_0^{T/4} 40\sin\frac{2\pi kt}{T}\,dt + \frac{4}{T}\int_{T/4}^{T/2} 80\sin\frac{2\pi kt}{T}\,dt \\
&= \frac{-160}{T}\frac{T}{2\pi k}\cos\frac{2\pi kt}{T}\Big|_0^{T/4} - \frac{320}{T}\frac{T}{2\pi k}\cos\frac{2\pi kt}{T}\Big|_{T/4}^{T/2} \\
&= \frac{-80}{\pi k}(0-1) - \frac{160}{\pi k}(-1-0) \\
&= \frac{240}{\pi k}
\end{aligned}$$

The periodic function in Fig. P16.1(b) is even; therefore, $b_k = 0$ for all k. Also,

$a_{\text{vb}} = 25\,\text{V}$

$$\begin{aligned}
a_{\text{kb}} &= \frac{4}{T}\int_0^{T/8} 100\cos\frac{2\pi kt}{T}\,dt \\
&= \frac{400}{T}\frac{T}{2\pi k}\sin\frac{2\pi k}{T}t\Big|_0^{T/8} \\
&= \frac{200}{\pi k}\sin\frac{\pi k}{4}
\end{aligned}$$

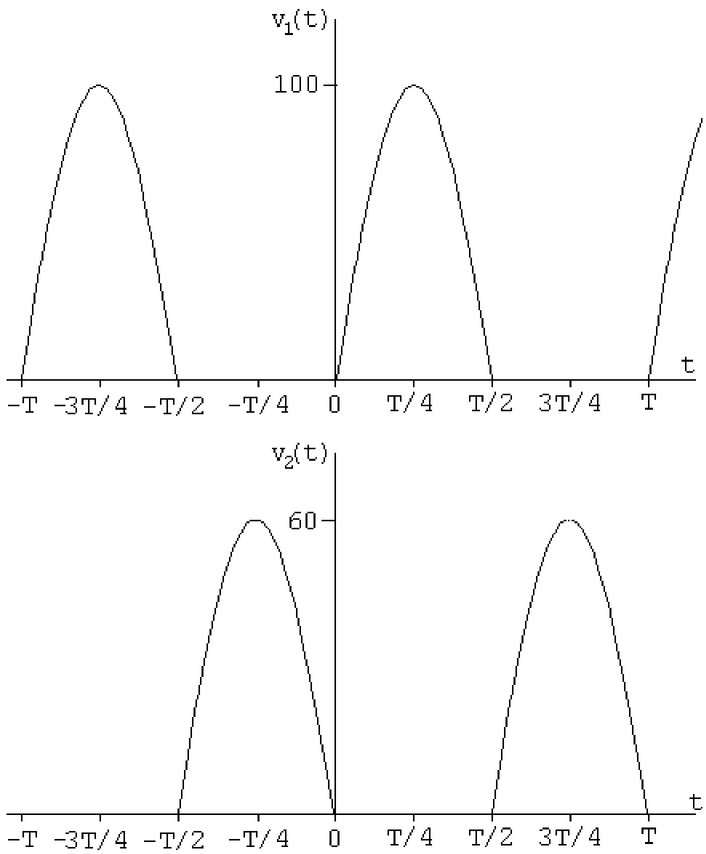
[e] For the periodic function in Fig. P16.1(a),

$$v(t) = \frac{80}{\pi} \sum_{n=1,3,5}^{\infty} \left(-\frac{1}{n} \sin\frac{n\pi}{2} \cos n\omega_o t + \frac{3}{n} \sin n\omega_o t \right) \text{ V}$$

For the periodic function in Fig. P16.1(b),

$$v(t) = 25 + \frac{200}{\pi} \sum_{n=1}^{\infty} \left(\frac{1}{n} \sin\frac{n\pi}{4} \cos n\omega_o t \right) \text{ V}$$

P 16.2 In studying the periodic function in Fig. P16.2 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.3(c).



$$v_1(t) = \frac{100}{\pi} + 50 \sin\omega_o t - \frac{200}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o t}{(n^2 - 1)} \text{ V}$$

$$v_2(t) = \frac{60}{\pi} + 30 \sin\omega_o (t - T/2) - \frac{120}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o (t - T/2)}{(n^2 - 1)} \text{ V}$$

Observe the following, noting that n is even:

$$\sin\omega_o(t - T/2) = \sin\left(\omega_o t - \frac{2\pi}{T}\frac{T}{2}\right) = \sin(\omega_o t - \pi) = -\sin\omega_o t$$

$$\cos n\omega_o(t - T/2) = \cos\left(n\omega_o t - \frac{2\pi n}{T}\frac{T}{2}\right) = \cos(n\omega_o t - n\pi) = \cos n\omega_o t$$

Using the observations above,

$$v_2(t) = \frac{60}{\pi} - 30\sin\omega_o t - \frac{120}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\omega_o t)}{(n^2-1)}\ \text{V}$$

Thus,

$$v(t) = v_1(t) + v_2(t) = \frac{160}{\pi} + 20\sin\omega_o t - \frac{320}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\omega_o t)}{(n^2-1)}\ \text{V}$$

P 16.3 **[a]** Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m \sin k\omega_0 t\, dt = \frac{4V_m}{k\pi}, \qquad k \text{ odd}$$

and $\quad v(t) = \dfrac{4V_m}{\pi}\displaystyle\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_0 t\ \text{V}$

[b] Even function: $\quad b_k = 0 \quad$ for $\quad k$

$$a_v = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\, dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\cos k\omega_0 t\, dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

and $\quad v(t) = \dfrac{2V_m}{\pi}\left[1 + 2\displaystyle\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\ \text{V}$

[c] $a_v = \dfrac{1}{T}\displaystyle\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\, dt = \frac{V_m}{\pi}$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\, dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

Note: $\quad a_k = 0 \quad$ for k-odd, $\quad a_k = \dfrac{2V_m}{\pi(1-k^2)} \quad$ for k even,

$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\sin k\omega_0 t\, dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $\quad b_1 = \dfrac{V_m}{2}; \qquad$ therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t\ \text{V}$$

P 16.4 Starting with Eq. (16.2),

$$f(t)\sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\,dt = 0 + 0 + b_k\left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\,dt$$

P 16.5 **[a]** $I_6 = \int_{t_o}^{t_o+T} \sin m\omega_0 t\,dt = -\frac{1}{m\omega_0}\cos m\omega_0 t\,\Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o + T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T} \cos m\omega_0 t_o\,dt = \frac{1}{m\omega_0}[\sin m\omega_0 t]\,\Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o + T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T} [\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\,dt$

But $(m + n)$ and $(m - n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\,dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\,dt = \frac{T}{2} - 0 = \frac{T}{2}$$

[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t\,dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t]\,dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\,dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\,dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\,dt + \int_{0}^{T/2} f(t)\,dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2}$ when $t = \frac{-T}{2}$

and $x = 0$ when $t = 0$

Therefore $\frac{1}{T}\int_{-T/2}^{0} f(t)\,dt = \frac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\frac{1}{T}\int_{0}^{T/2} f(x)\,dx$

Therefore $a_v = -\frac{1}{T}\int_{0}^{T/2} f(t)\,dt + \frac{1}{T}\int_{0}^{T/2} f(t)\,dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x\,dx$$

Therefore $a_k = 0$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\,dx$$

Therefore we have $b_k = \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

P 16.7 $b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also $\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x \cos k\pi$. Therefore

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x \cos k\pi\,dx \quad \text{and}$$

$$b_k = \frac{2}{T}(1 - \cos k\pi)\int_{0}^{T/2} f(x)\sin k\omega_0 t\,dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt \quad \text{when } k \text{ is odd}$$

P 16.8 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \le t \le T/2$;
thus we write

$$a_k = \frac{4}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt = -\frac{4}{T}\int_0^{T/4} f(x)\cos k\pi\cos k\omega_0 x\,dx$$

Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi)\int_0^{T/4} f(t)\cos k\omega_0 t\,dt$$

But k is odd, hence

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

P 16.9 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \le t \le T/2$. Thus we have

$$b_k = \frac{4}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt = -\frac{4}{T}\cos k\pi\int_0^{T/4} f(x)(\sin k\omega_0 x)\,dx$$

But k is odd, therefore the expression becomes

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt$$

P 16.10 **[a]** $f = \dfrac{1}{T} = \dfrac{1}{16 \times 10^{-3}} = 62.5\,\text{Hz}$

[b] no, because $f(3\text{ ms}) = 10\text{ mA}$ but $f(-3\text{ ms}) = -10\text{ mA}$.

[c] yes, because $f(-t) = -f(t)$ for all t.

[d] yes

[e] yes

[f] $a_v = 0,$ function is odd

$a_k = 0,$ for all k; the function is odd

$b_k = 0,$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} 5t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} 0.01\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 5\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= 5\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4} - \frac{0.625T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = 0.01\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{-0.01}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{0.01}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{0.01}{k\omega_o} - \frac{0.625T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$$0.625T = 0.625(16 \times 10^{-3}) = 0.01$$

$$\therefore \quad \text{Int1} + \text{Int2} = \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{5}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{0.16}{\pi^2k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

$$i(t) = \frac{160}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\,\text{mA}$$

P 16.11 **[a]** $T = 1; \quad \omega_o = \frac{2\pi}{T} = 2\pi$ rad/s

[b] yes

[c] no

[d] no

P 16.12 **[a]** $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m \cos k\omega_0 t\,dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for k-even, $b_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\,dt = -\frac{8V_p}{\pi^2 k^2}$$

Therefore $\quad v(t) = -\frac{8V_p}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$ V

P 16.13 **[a]** $i(t)$ is even, therefore $b_k = 0$ for all k.

$$a_v = \frac{1}{2}\cdot\frac{T}{4}\cdot I_m\cdot 2\cdot\frac{1}{T} = \frac{I_m}{4}\text{ A}$$

$$a_k = \frac{4}{T}\int_0^{T/4}\left(I_m - \frac{4I_m}{T}t\right)\cos k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\,dt - \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\,dt$$

$$= \text{Int}_1 - \text{Int}_2$$

$$\text{Int}_1 = \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\,dt = \frac{2I_m}{\pi k}\sin\frac{k\pi}{2}$$

$$\text{Int}_2 = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{1}{k^2\omega_o^2}\cos k\omega_o t + \frac{t}{k\omega_o}\sin k\omega_o t\right\}\Bigg|_0^{T/4}$$

$$= \frac{4I_m}{\pi^2 k^2}\left(\cos\frac{k\pi}{2} - 1\right) + \frac{2I_m}{k\pi}\sin\frac{k\pi}{2}$$

$$\therefore\quad a_k = \frac{4I_m}{\pi^2 k^2}\left(1 - \cos\frac{k\pi}{2}\right)\text{ A}$$

$$\therefore\quad i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{1-\cos(n\pi/2)}{n^2}\cos n\omega_o t\text{ A}$$

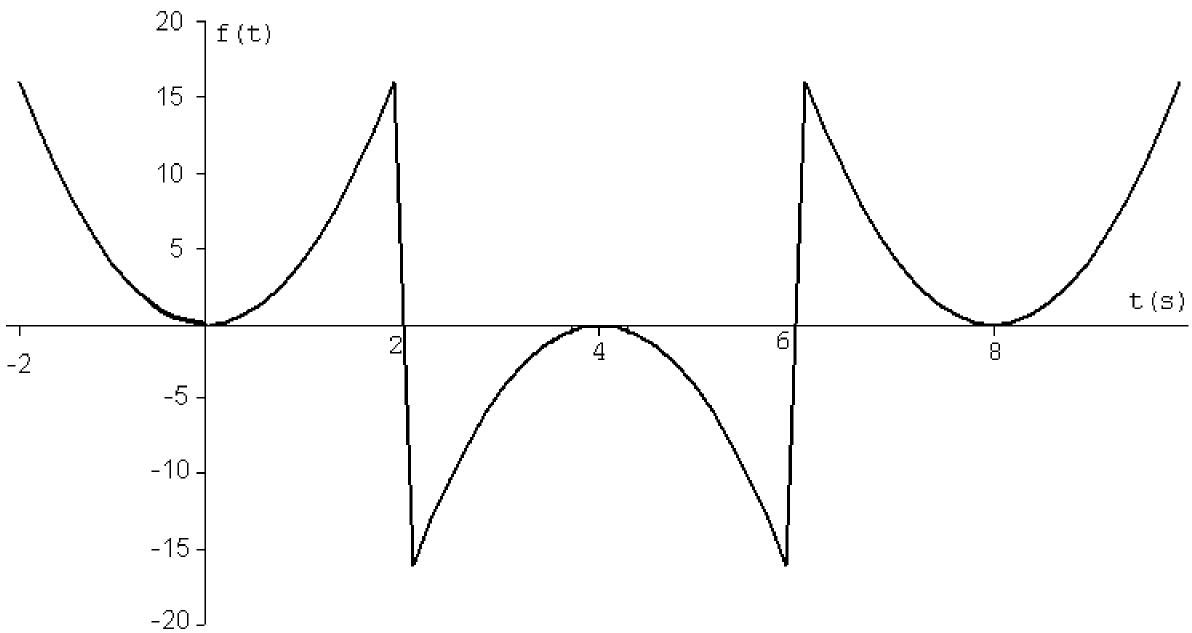
[b] Shifting the reference axis to the left is equivalent to shifting the periodic function to the right:

$$\cos n\omega_o(t - T/2) = \cos n\pi\cos n\omega_o t$$

Thus

$$i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{(1-\cos(n\pi/2))\cos n\pi}{n^2}\cos n\omega_o t\text{ A}$$

P 16.14 **[a]**



[b] Even, since $f(t) = f(-t)$

[c] Yes, since $f(t) = -f(T/2 - t)$ in the interval $0 < t < 4$.

[d] $a_v = 0,\quad a_k = 0,\quad$ for k even (half-wave symmetry)

$b_k = 0,\quad$ for all k (function is even)

Because of the quarter-wave symmetry, the expression for a_k is

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt,\qquad k\text{ odd}$$

$$= \frac{8}{8}\int_0^2 4t^2\cos k\omega_0 t\,dt = 4\left[\frac{2t}{k^2\omega_0^2}\cos k\omega_0 t + \frac{k^2\omega_0^2 t^2 - 2}{k^3\omega_0^3}\sin k\omega_0 t\right]_0^2$$

$$k\omega_0(2) = k\left(\frac{2\pi}{8}\right)(2) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0,$ since k is odd

$$\therefore \quad a_k = 4\left[0 + \frac{4k^2\omega_0^2 - 2}{k^3\omega_0^3}\sin(k\pi/2)\right] = \frac{16k^2\omega_0^2 - 8}{k^3\omega_0^3}\sin(k\pi/2)$$

$$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}; \qquad \omega_0^2 = \frac{\pi^2}{16}; \qquad \omega_0^3 = \frac{\pi^3}{64}$$

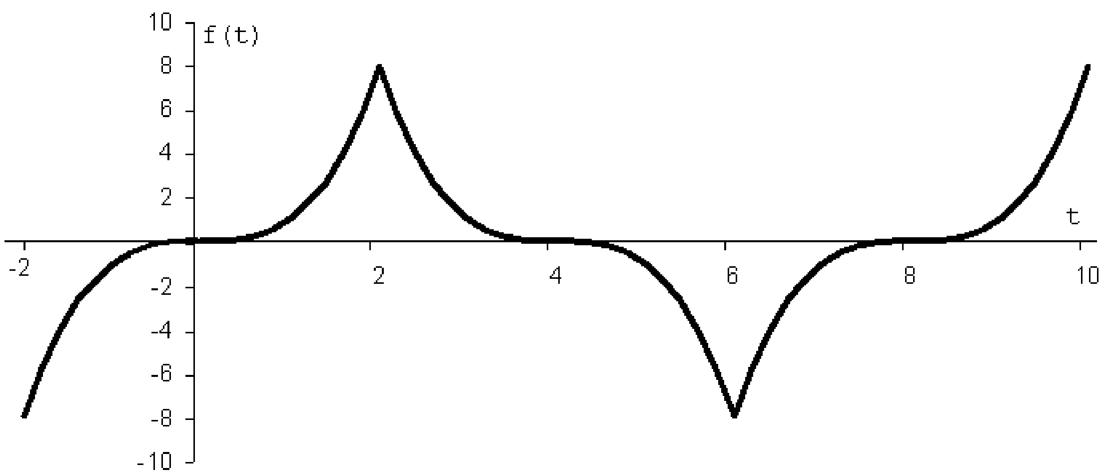
$$a_k = \left(\frac{k^2\pi^2 - 8}{k^3\pi^3}\right)(64)\sin(k\pi/2)$$

$$f(t) = 64\sum_{n=1,3,5}^{\infty}\left[\frac{n^2\pi^2 - 8}{\pi^3 n^3}\right]\sin(n\pi/2)\cos(n\omega_0 t)$$

[e] $\cos n\omega_0(t-2) = \cos(n\omega_0 t - \pi/2) = \sin n\omega_0 t \sin(n\pi/2)$

$$f(t) = 64\sum_{n=1,3,5}^{\infty}\left[\frac{n^2\pi^2 - 8}{\pi^3 n^3}\right]\sin^2(n\pi/2)\sin(n\omega_0 t)$$

P 16.15 **[a]**



[b] Odd, since $f(-t) = -f(t)$

[c] $f(t)$ has quarter-wave symmetry, since $f(T/2 - t) = f(t)$ in the interval $0 < t < 4$.

[d] $a_v = 0,$ (half-wave symmetry); $a_k = 0,$ for all k (function is odd)

$b_k = 0,$ for k even (half-wave symmetry)

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\, dt, \quad k \text{ odd}$$

$$= \frac{8}{8}\int_0^2 t^3 \sin k\omega_0 t\, dt$$

$$= \left[\frac{3t^2}{k^2\omega_0^2}\sin k\omega_0 t - \frac{6}{k^4\omega_0^4}\sin k\omega_0 t - \frac{t^3}{k\omega_0}\cos k\omega_0 t + \frac{6t}{k^3\omega_0^3}\cos k\omega_0 t\right]_0^2$$

$$k\omega_0(2) = k\left(\frac{2\pi}{8}\right)(2) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0,\quad$ since k is odd

$$\therefore \qquad b_k = \left[\frac{12}{k^2\omega_0^2}\sin(k\pi/2) - \frac{6}{k^4\omega_0^4}\sin(k\pi/2)\right]$$

$$k\omega_0 = k\left(\frac{2\pi}{8}\right) = \frac{k\pi}{4}; \qquad k^2\omega_0^2 = \frac{k^2\pi^2}{16}; \qquad k^4\omega_0^4 = \frac{k^4\pi^4}{256}$$

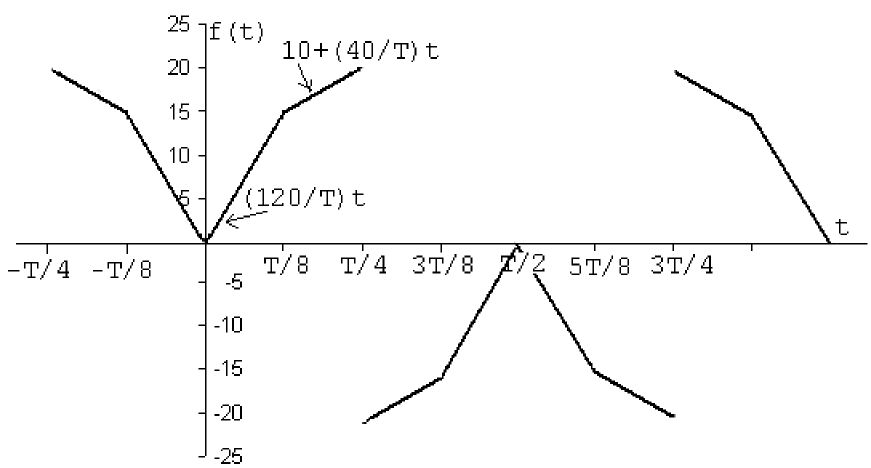
$$\therefore \qquad b_k = \frac{192}{\pi^2 k^2}\left[1 - \frac{8}{\pi^2 k^2}\right]\sin(k\pi/2), \quad k \text{ odd}$$

$$f(t) = \frac{192}{\pi^2}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\sin(n\pi/2)\right]\sin n\omega_0 t$$

[e] $\sin n\omega_0(t-2) = \sin(n\omega_0 t - \pi/2) = -\cos n\omega_0 t \sin(n\pi/2)$

$$f(t) = \frac{-192}{\pi^2}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\sin^2(n\pi/2)\right]\cos n\omega_0 t$$

P 16.16 **[a]**



[b] $a_v = 0; \qquad a_k = 0, \quad$ for k even; $\qquad b_k = 0, \quad$ for all k

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\, dt, \quad \text{for } k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/8}\frac{120t}{T}\cos k\omega_0 t\, dt + \frac{8}{T}\int_{T/8}^{T/4}\left(10 + \frac{40}{T}t\right)\cos k\omega_0 t\, dt$$

$$= \frac{960}{T^2}\int_0^{T/8} t\cos k\omega_0 t\, dt + \frac{80}{T}\int_{T/8}^{T/4}\cos k\omega_0 t\, dt + \frac{320}{T^2}\int_{T/8}^{T/4} t\cos k\omega_0 t\, dt$$

$$= \frac{960}{T^2}\left[\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t\sin k\omega_0 t}{k\omega_0}\right]_0^{T/8} + \frac{80}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_{T/8}^{T/4}$$

$$+ \frac{320}{T^2}\left[\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t\sin k\omega_0 t}{k\omega_0}\right]_{T/8}^{T/4}$$

$$k\omega_0\frac{T}{4} = \frac{k\pi}{2}; \qquad k\omega_0\frac{T}{8} = \frac{k\pi}{4}$$

$$b_k = \frac{960}{T^2}\left[\frac{\cos(k\pi/4)}{k^2\omega_0^2} + \frac{T}{8k\omega_0}\sin(k\pi/4) - \frac{1}{k^2\omega_0^2}\right] + \frac{80}{k\omega_0 T}\left[\sin(k\pi/2) - \sin(k\pi/4)\right]$$

$$+ \frac{320}{T^2}\left[\frac{\cos(k\pi/2)}{k^2\omega_0^2} + \frac{T}{4}\frac{\sin(k\pi/2)}{k\omega_0} - \frac{\cos(k\pi/4)}{k^2\omega_0^2} - \frac{T\sin(k\pi/4)}{8k\omega_0}\right]$$

$$= \frac{640}{(k\omega_0 T)^2}\cos(k\pi/4) + \frac{160}{k\omega_0 T^2}\sin(k\pi/2) - \frac{960}{(k\omega_0 T)^2}$$

$$k\omega_0 T = 2k\pi; \qquad (k\omega_0 T)^2 = 4k^2\pi^2$$

$$a_k = \frac{160}{\pi^2 k^2}\cos(k\pi/4) + \frac{80}{\pi k}\sin(k\pi/2) - \frac{240}{\pi^2 k^2}$$

[c] $a_k = \frac{80}{\pi^2 k^2}[2\cos(k\pi/4) + \pi k\sin(k\pi/2) - 3]$

$$a_1 = \frac{80}{\pi^2}[2\cos(\pi/4) + \pi k\sin(\pi/2) - 3] \cong 12.61$$

$$a_3 = \frac{80}{9\pi^2}[2\cos(3\pi/4) + \pi k\sin(3\pi/2) - 3] \cong -12.46$$

$$a_5 = \frac{80}{25\pi^2}[2\cos(5\pi/4) + \pi k\sin(5\pi/2) - 3] \cong 3.66$$

$$f(t) = 12.61\cos(\omega_0 t) - 12.46\cos(3\omega_0 t) + 3.66\cos(5\omega_0 t) + \dots$$

[d] $t = \frac{T}{4}; \qquad \omega_0 t = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$

$$f(T/4) \cong 12.61\cos(\pi/2) - 12.46\cos(3\pi/2) + 3.66\cos(5\pi/2) = 0$$

The result would have been non-trivial for $t = T/8$ or if the function had been specified as odd.

P 16.17 Let $f(t) = v_2(t - T/6)$.

$a_v = -(2V_m/3)(T/3)(1/T) = -(2V_m/9)$ and $b_k = 0$ since $f(t)$ is even

$$a_k = \frac{4}{T}\int_0^{T/6}\left(-\frac{2V_m}{3}\right)\cos k\omega_o t dt = -\frac{4}{T}\frac{2V_m}{3}\frac{1}{k\omega_o}\sin k\omega_o t\Big|_0^{T/6}$$
$$= -\frac{8V_m}{3k2\pi}\sin\left(k\frac{\pi}{3}\right) = -\frac{4V_m}{3k\pi}\sin\left(k\frac{\pi}{3}\right)$$

Therefore, $\quad v_2(t - T/6) = -\dfrac{2V_m}{9} - \dfrac{4V_m}{3\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o t$

and $\quad v_2(t) = -\dfrac{2V_m}{9} - \dfrac{4V_m}{3\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o(t + T/6)$

Then, $v(t) = v_1(t) + v_2(t)$. Simplifying,

$$v(t) = \frac{7V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right)\right]\cos n\omega_o t$$
$$+ \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin^2\left(\frac{n\pi}{3}\right)\right]\sin n\omega_o t \text{ V}$$

If $\quad V_m = 9\pi \qquad$ then $\quad a_v = 7\pi = 21.99 \quad$ (Checks)

$$a_k = -\left(\frac{12}{n}\right)\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right) = -\left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\sin\left(\frac{2n\pi}{3}\right) = \left(\frac{6}{n}\right)\sin\left(\frac{4n\pi}{3}\right)$$

$$b_k = \left(\frac{12}{n}\right)\sin^2\left(\frac{n\pi}{3}\right) = \left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\left[1 - \cos\left(\frac{2n\pi}{3}\right)\right] = \left(\frac{6}{n}\right)\left[1 - \cos\left(\frac{4n\pi}{3}\right)\right]$$

$a_1 = 6\sin(4\pi/3) = -5.2; \qquad b_1 = 6[1 - \cos(4\pi/3)] = 9$

$a_2 = 3\sin(8\pi/3) = 2.6; \qquad b_2 = 3[1 - \cos(8\pi/3)] = 4.5$

$a_3 = 2\sin(12\pi/3) = 0; \qquad b_3 = 2[1 - \cos(12\pi/3)] = 0$

$a_4 = 1.5\sin(16\pi/3) = -1.3; \qquad b_4 = 1.5[1 - \cos(16\pi/3)] = 2.25$

$a_5 = 1.2\sin(20\pi/3) = 1.04; \qquad b_5 = 1.2[1 - \cos(20\pi/3)] = 1.8$

All coefficients check!

P 16.18 **[a]** The voltage has half-wave symmetry. Therefore,

$$a_v = 0; \qquad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$a_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\cos k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} I_m \cos k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\cos k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 T\right]_0^{T/2}$$

$$= 0 - \frac{8I_m}{T^2}\left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2}\right]$$

$$= \left(\frac{8I_m}{T^2}\right)\left(\frac{1}{k^2\omega_0^2}\right)(1 - \cos k\pi)$$

$$= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\sin k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\right]_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\right]_0^{T/2}$$

$$= \frac{4I_m}{T}\left[\frac{1 - \cos k\pi}{k\omega_0}\right] - \frac{8I_m}{T^2}\left[\frac{-T\cos k\pi}{2k\omega_0}\right]$$

$$= \frac{8I_m}{k\omega_0 T}\left[1 + \frac{1}{2}\cos k\pi\right]$$

$$= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd}$$

$$a_k - jb_k = \frac{20}{k^2} - j\frac{10\pi}{k} = \frac{10}{k}\left(\frac{2}{k} - j\pi\right) = \frac{10}{k^2}\sqrt{\pi^2 k^2 + 4}\underline{/-\theta_k}$$

$$\text{where} \qquad \tan\theta_k = \frac{\pi k}{2}$$

$$i(t) = 10\sum_{n=1,3,5}^{\infty}\frac{\sqrt{(n\pi)^2 + 4}}{n^2}\cos(n\omega_0 t - \theta_n)$$

[b] $A_1 = 10\sqrt{4+\pi^2} \cong 37.24\,\text{A} \qquad \tan\theta_1 = \frac{\pi}{2} \qquad \theta_1 \cong 57.52^\circ$

$$A_3 = \frac{10}{9}\sqrt{4+9\pi^2} \cong 10.71\,\text{A} \qquad \tan\theta_3 = \frac{3\pi}{2} \qquad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25}\sqrt{4+25\pi^2} \cong 6.33\,\text{A} \qquad \tan\theta_5 = \frac{5\pi}{2} \qquad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49}\sqrt{4+49\pi^2} \cong 4.51\,\text{A} \qquad \tan\theta_7 = \frac{7\pi}{2} \qquad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81}\sqrt{4+81\pi^2} \cong 3.50\,\text{A} \qquad \tan\theta_9 = \frac{9\pi}{2} \qquad \theta_9 \cong 85.95^\circ$$

$$i(t) \cong 37.24\cos(\omega_o t - 57.52^\circ) + 10.71\cos(3\omega_o t - 78.02^\circ)$$
$$+ 6.33\cos(5\omega_o t - 82.74^\circ) + 4.51\cos(7\omega_o t - 84.80^\circ)$$
$$+ 3.50\cos(9\omega_o t - 85.95^\circ) + \ldots$$

$$i(T/4) \cong 37.24\cos(90 - 57.52^\circ) + 10.71\cos(270 - 78.02^\circ)$$
$$+ 6.33\cos(450 - 82.74^\circ) + 4.51\cos(630 - 84.80^\circ)$$
$$+ 3.50\cos(810 - 85.95^\circ) \cong 26.22\,\text{A}$$

Actual value:

$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67\,\text{A}$$

P 16.19 The function has half-wave symmetry, thus $a_k = b_k = 0$ for *k*-even, $a_v = 0$; for *k*-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\rho = \left[1 + e^{-T/2RC}\right]$.

Upon integrating we get

$$a_k = \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$
$$-\frac{8V_m}{\rho T}\cdot\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0\sin k\omega_0 t\right]\Big|_0^{T/2}$$
$$= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]}$$

$$b_k = \frac{4}{T}\int_0^{T/2} V_m \sin k\omega_0 t\, dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC} \sin k\omega_0 t\, dt$$

$$= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\frac{-e^{-t/RC}}{(1/RC)^2+(k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC} + k\omega_0\cos k\omega_0 t\right]\Big|_0^{T/2}$$

$$= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2C^2}{T[1+(k\omega_0 RC)^2]}$$

P 16.20 **[a]** $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 RC a_k\right)^2$

$$= a_k^2\left[1+(k\omega_0 RC)^2\right] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RC a_k\right]$$

But $\quad a_k = \dfrac{-8V_m RC}{T\left[1+(k\omega_0 RC)^2\right]}$

Therefore $\quad a_k^2 = \dfrac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]^2}, \quad$ thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]} + \frac{16V_m^2}{\pi^2k^2} - \frac{64V_m^2 k\omega_0 R^2C^2}{\pi k T[1+(k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_m^2/\pi^2k^2(1+\alpha^2)$. It follows that

$$\sqrt{a_k^2+b_k^2} = \frac{4V_m}{\pi k\sqrt{1+(k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 RC a_k + \dfrac{4V_m}{\pi k}$

Thus $\quad \dfrac{b_k}{a_k} = k\omega_0 RC + \dfrac{4V_m}{\pi k a_k} = \alpha - \dfrac{1+\alpha^2}{\alpha} = -\dfrac{1}{\alpha}$

Therefore $\quad \dfrac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.21 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5}^{\infty} \frac{4V_m}{n\pi} \frac{1}{\sqrt{1 + (n\omega_0 RC)^2}} \cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.57, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90^\circ + \beta_n$ and $\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90^\circ) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.22 **[a]** $e^{-x} \cong 1 - x$ for small x; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o \cong V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$\cong \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_m T}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right) V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.23 **[a]** Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin n\omega_0 t$$

The dc component of the current is $V_m/2R$, and with $\sin n\omega_0 t = \cos(n\omega_0 t - 90^\circ)$ the kth harmonic phase current is

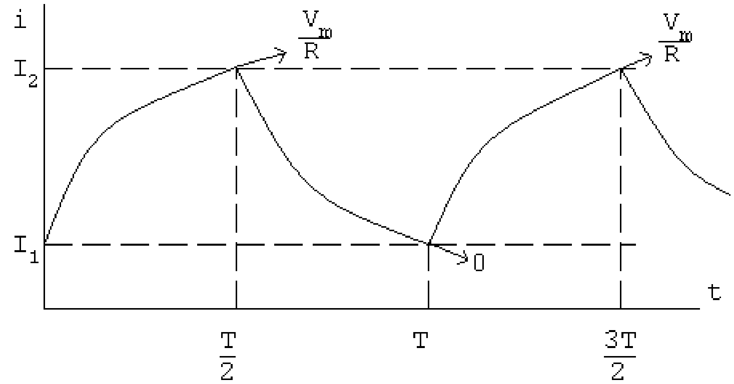
$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L\underline{/-90^\circ}} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}}\underline{/-90^\circ - \theta_k}$$

$$\text{where} \quad \theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi} \sum_{n=1,3,5}^{\infty} \frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}} \text{ A}$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1 + e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1 + e^{-x})}\right]e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1 + e^{-x})}\right]e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$\begin{aligned}
I_{\text{avg}} &= \frac{1}{T}\left\{\int_0^{T/2}\left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau}\right]dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\,dt\right\} \\
&= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1 - e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\} \\
&= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R}
\end{aligned}$$

P 16.24 $v_i = \frac{4A}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_0(t+T/4)$

$$= \frac{4A}{\pi}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$\omega_0 = \frac{2\pi}{4\pi}\times 10^3 = 500 \text{ rad/s}; \qquad \frac{4A}{\pi} = 60$$

$$v_i = 60\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos 500nt \text{ V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R+j\omega L}\cdot j\omega L = \frac{j\omega}{R/L+j\omega}\mathbf{V}_i = \frac{j\omega}{1000+j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 60\underline{/0^\circ} \text{ V}; \qquad \omega = 500 \text{ rad/s}$$

$$\mathbf{V}_{i3} = -20\underline{/0^\circ} = 20\underline{/180^\circ} \text{ V}; \qquad 3\omega = 1500 \text{ rad/s}$$

$$\mathbf{V}_{i5} = 12\underline{/0^\circ} \text{ V}; \qquad 5\omega = 2500 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{j500}{1000+j500}(60\underline{/0^\circ}) = 26.83\underline{/63.43^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \frac{j1500}{1000+j1500}(20\underline{/180^\circ}) = 16.64\underline{/-146.31^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \frac{j2500}{1000+j2500}(12\underline{/0^\circ}) = 11.14\underline{/21.80^\circ} \text{ V}$$

$$\therefore \qquad v_o = 26.83\cos(500t+63.43^\circ) + 16.64\cos(1500t-146.31^\circ)$$

$$+\,11.14\cos(2500t+21.80^\circ) + \ldots \text{ V}$$

P 16.25 **[a]** From the solution to Assessment Problem 16.6 the Fourier series for the input voltage is

$$v_g = 42\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\right]\cos 2000nt \text{ V}$$

Also from the solution to Assessment Problem 16.6 we have

$$\mathbf{V}_{g1} = 42\underline{/0^\circ} \qquad \omega_0 = 2000 \text{ rad/s}$$

$\mathbf{V}_{g3} = 14\underline{/180^\circ} \qquad 3\omega_0 = 6000$ rad/s

$\mathbf{V}_{g5} = 8.4\underline{/0^\circ} \qquad 5\omega_0 = 10{,}000$ rad/s

$\mathbf{V}_{g7} = 6\underline{/180^\circ} \qquad 7\omega_0 = 14{,}000$ rad/s

From the circuit in Fig. P16.26 we have

$$\frac{V_o}{R} + \frac{V_o - V_g}{sL} + (V_o - V_g)sC = 0$$

$$\therefore \quad \frac{V_o}{V_g} = H(s) = \frac{s^2 + 1/LC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values gives

$$H(s) = \frac{s^2 + 10^8}{s^2 + 500s + 10^8}$$

$$H(j2000) = \frac{96}{96 + j1} = 0.9999\underline{/-0.60^\circ}$$

$$H(j6000) = \frac{64}{64 + j3} = 0.9989\underline{/-2.68^\circ}$$

$$H(j10{,}000) = 0$$

$$H(j14{,}000) = \frac{96}{96 - j7} = 0.9974\underline{/4.17^\circ}$$

$\mathbf{V}_{o1} = (42\underline{/0^\circ})(0.9999\underline{/-0.60^\circ}) = 41.998\underline{/-0.60^\circ}$ V

$\mathbf{V}_{o3} = (14\underline{/180^\circ})(0.9989\underline{/-2.68^\circ}) = 13.985\underline{/177.32^\circ}$ V

$\mathbf{V}_{o5} = 0$ V

$\mathbf{V}_{o7} = (6\underline{/180^\circ})(0.9974\underline{/4.17^\circ}) = 5.984\underline{/184.17^\circ}$ V

$$v_o = 41.998\cos(2000t - 0.60^\circ) + 13.985\cos(6000t + 177.32^\circ)$$
$$+ 5.984\cos(14{,}000t + 184.17^\circ) + \ldots \text{ V}$$

[b] The 5th harmonic at the frequency $\sqrt{1/LC} = 10{,}000$ rad/s has been eliminated from the output voltage by the circuit, which is a bandreject filter with a center frequency of 10,000 rad/s.

P 16.26 **[a]** Note – find $i_o(t)$

$$\frac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \frac{V_0}{1000} = 0$$

$$V_0\left[\frac{1}{16s} + 12.5 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2 n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6}\underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7}\underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7}\underline{/-178.48^\circ}$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \ldots\right]$$

$$i_0 = \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi}(8.82 \times 10^{-6})\cos(\omega_0 t - 173.89^\circ)$$

$$- \frac{1360}{15\pi}(2.20 \times 10^{-6})\cos(2\omega_0 t - 176.96^\circ)$$

$$- \frac{1360}{35\pi}(9.78 \times 10^{-7})\cos(3\omega_0 t - 177.97^\circ)$$

$$- \frac{1360}{63\pi}(5.5 \times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \ldots$$

$$= 216.45 \times 10^{-3} - 1.27 \times 10^{-3}\cos(\omega_0 t - 173.89^\circ)$$

$$- 6.35 \times 10^{-5}\cos(2\omega_0 t - 176.96^\circ)$$

$$- 1.21 \times 10^{-5}\cos(3\omega_0 t - 177.97^\circ)$$

$$- 3.8 \times 10^{-6}\cos(4\omega_0 t - 178.48^\circ) - \ldots$$

$i_0 \cong 216.45 - 1.27\cos(\omega_0 t - 173.89°)$ mA

Note that the sinusoidal component is very small compared to the dc component, so

$i_0 \cong 216.45\,\text{mA}$ (a dc current)

[b] Yes, the solution makes sense. The circuit is a low-pass filter which nearly eliminates all but the dc component.

P 16.27 The function is odd with half-wave and quarter-wave symmetry. Therefore,

$a_k = 0,\qquad$ for all k; the function is odd

$b_k = 0,\qquad$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/10} 500t\sin k\omega_o t\,dt + \int_{T/10}^{T/4} \sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 500\int_0^{T/10} t\sin k\omega_o t\,dt$$

$$= 500\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/10}\right]$$

$$= \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5} - \frac{50T}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int2} = \int_{T/10}^{T/4}\sin k\omega_o t\,dt = \frac{-1}{k\omega_o}\cos k\omega_o t\,\Big|_{T/10}^{T/4} = \frac{1}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int1} + \text{Int2} = \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5} + \left(\frac{1}{k\omega_o} - \frac{50T}{k\omega_o}\right)\cos\frac{k\pi}{5}$$

$$50T = 50(20\times 10^{-3}) = 1$$

$$\therefore\quad \text{Int1} + \text{Int2} = \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{500}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{5} = \frac{20}{\pi^2k^2}\sin\frac{k\pi}{5}, \qquad k \text{ odd}$$

$$i(t) = \frac{20}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\pi/5)}{n^2}\sin n\omega_o t \text{ A}$$

From the circuit,

$$H(s) = \frac{V_o}{I_g} = Z_{\text{eq}}$$

$$Y_{\text{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + sL} + sC$$

$$Z_{\text{eq}} = \frac{1/C(s + R_2/L)}{s^2 + s(R_1R_2C + L)/R_1LC + (R_1 + R_2)/R_1LC}$$

Therefore,

$$H(s) = \frac{320 \times 10^4(s + 32 \times 10^4)}{s^2 + 32.8 \times 10^4 s + 28.8 \times 10^8}$$

We want the output for the third harmonic:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{20 \times 10^{-3}} = 100\pi; \qquad 3\omega_0 = 300\pi$$

$$I_{g3} = \frac{20}{9\pi^2}\sin\frac{3\pi}{5\sin 3\omega_0 t} = 0.214\underline{/-90^\circ}$$

$$H(j300\pi) = \frac{320 \times 10^4(j300\pi + 32 \times 10^4)}{(j300\pi)^2 + 32.8 \times 10^4(j300\pi) + 28.8 \times 10^8} = 353.6\underline{/-5.96^\circ}$$

Therefore,

$$V_{o3} = H(j300\pi)I_{g3} = (353.6\underline{/-5.96^\circ})(0.214\underline{/-90^\circ}) = 75.7\underline{/-90^\circ - 5.96^\circ}\text{ V}$$

$$v_{o3} = 75.7\sin(300\pi t - 5.96^\circ)\text{ V}$$

P 16.28 $$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 200 \text{ krad/s}$$

$$\therefore \quad n = \frac{3 \times 10^6}{0.2 \times 10^6} = 15; \qquad n = \frac{5 \times 10^6}{0.2 \times 10^6} = 25$$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{RC} = \frac{10^{12}}{(250 \times 10^3)(4)} = 10^6; \qquad \frac{1}{LC} = \frac{(10^3)(10^{12})}{(10)(4)} = 25 \times 10^{12}$$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 25 \times 10^{12}}$$

$$H(j\omega) = \frac{j\omega \times 10^6}{(25 \times 10^{12} - \omega^2) + j10^6\omega}$$

15th harmonic input:

$$v_{g15} = (150)(1/15)\sin(15\pi/2)\cos 15\omega_o t = -10\cos 3 \times 10^6 t \text{ V}$$

$$\therefore \quad \mathbf{V}_{g15} = 10\underline{/-180^\circ} \text{ V}$$

$$H(j3 \times 10^6) = \frac{j3}{16 + j3} = 0.1843\underline{/79.38^\circ}$$

$$\mathbf{V}_{o15} = (10)(0.1843)\underline{/-100.62^\circ} \text{ V}$$

$$v_{o15} = 1.84\cos(3 \times 10^6 t - 100.62^\circ) \text{ V}$$

25th harmonic input:

$$v_{g25} = (150)(1/25)\sin(25\pi/2)\cos 5 \times 10^6 t = 6\cos 5 \times 10^6 t \text{ V}$$

$$\therefore \quad \mathbf{V}_{g25} = 6\underline{/0^\circ} \text{ V}$$

$$H(j5 \times 10^6) = \frac{j5}{0 + j5} = 1\underline{/0^\circ}$$

$$\mathbf{V}_{o25} = 6\underline{/0^\circ} \text{ V}$$

$$v_{o25} = 6\cos 5 \times 10^6 t \text{ V}$$

P 16.29 **[a]** $a_v = \frac{T}{2}\left[\frac{1}{2}\left(\frac{T}{2}\right)I_m + \frac{T}{2}I_m\right] = \frac{3I_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t\cos k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m \cos k\omega_o t\,dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t \sin k\omega_o t\, dt + \frac{2}{T}\int_{T/2}^{T} I_m \sin k\omega_o t\, dt$$

$$= \frac{-I_m}{\pi k}$$

$$a_v = \frac{3I_m}{4}, \quad a_1 = \frac{-2I_m}{\pi^2}, \quad a_2 = 0$$

$$a_3 = \frac{-2I_m}{9\pi^2}$$

$$b_1 = \frac{-I_m}{\pi}, \qquad b_2 = \frac{-I_m}{2\pi}$$

$$\therefore \qquad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040 I_m \qquad \text{(Eq. 16.81)}$$

$$I_{\text{rms}} = 192.95\,\text{mA}$$

$$P = (0.19295)^2(1000) = 37.23\,\text{W}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2} t\, dt + I_m^2 \frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\Big|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2 T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3} T I_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}T I_m^2} = \sqrt{\frac{2}{3}} I_m = 195.96\,\text{mA}$$

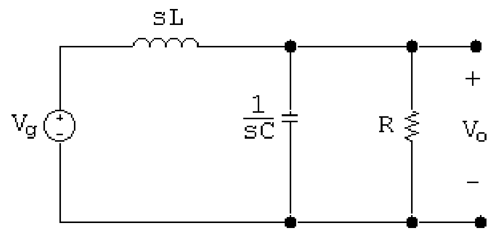
$$P = (0.19596)^2 1000 = 38.4\,\text{W}$$

[c] Error $= \left(\frac{37.23}{38.40} - 1\right)(100) = -3.05\%$

P 16.30 $v_g = 10 + \frac{80}{\pi^2}\sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\cos n\omega_o t$ V

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} \times 10^3 = 500 \text{ rad/s}$$

$$v_g = 10 + \frac{80}{\pi^2}\cos 500t + \frac{80}{9\pi^2}\cos 1500t + \ldots$$



$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701\underline{/-43.31^\circ}$$

$$H(j1500) = 0.4061\underline{/-120.51^\circ}$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701)\cos(500t - 43.31^\circ)$$

$$+ \frac{80}{9\pi^2}(0.4061)\cos(1500t - 120.51^\circ) + \ldots$$

$$v_o = 10 + 7.86\cos(500t - 43.31^\circ) + 0.3658\cos(1500t - 120.51^\circ) + \ldots$$

$$V_{\rm rms} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44\,\text{V}$$

$$P \cong \frac{V_{\rm rms}^2}{50\sqrt{2}} = 1.85\,\text{W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580)\left(\frac{80}{25\pi^2}\right)\cos(2500t - 146.04^\circ) = 0.0512\cos(2500t - 146.04^\circ)\,\text{V}$$

P 16.31 **[a]** $a_v = \dfrac{2\left(\frac{1}{2}\,\frac{T}{4}V_m\right)}{T} = \dfrac{V_m}{4}$

$$a_k = \frac{4}{T}\int_0^{T/4}\left[V_m - \frac{4V_m}{T}t\right]\cos k\omega_o t\,dt$$

$$= \frac{4V_m}{\pi^2 k^2}\left[1 - \cos\frac{k\pi}{2}\right]$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{60}{4} = 15\,\text{V}$$

$$a_1 = \frac{240}{\pi^2}$$

$$a_2 = \frac{240}{4\pi^2}(1 - \cos\pi) = \frac{120}{\pi^2}$$

$$V_{\rm rms} = \sqrt{(15)^2 + \frac{1}{2}\left[\left(\frac{240}{\pi^2}\right)^2 + \left(\frac{120}{\pi^2}\right)^2\right]} = 24.38\,\text{V}$$

$$P = \frac{(24.38)^2}{10} = 59.46\,\text{W}$$

[b] Area under v^2; $\quad 0 \le t \le T/4$

$$v^2 = 3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2$$

$$A = 2\int_0^{T/4}\left[3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2\right]\,dt = 600T$$

$$V_{\rm rms} = \sqrt{\frac{1}{T}600T} = \sqrt{600} = 24.49\,\text{V}$$

$$P = \sqrt{600}^2/10 = 60\,\text{W}$$

[c] $\text{Error} = \left(\frac{59.46}{60.00} - 1\right) 100 = -0.9041\%$

P 16.32 **[a]** $v = 15 + 400\cos 500t + 100\cos(1500t - 90°)$ V

$$i = 2 + 5\cos(500t - 30°) + 3\cos(1500t - 15°)\ \text{A}$$

$$P = (15)(2) + \frac{1}{2}(400)(5)\cos(30°) + \frac{1}{2}(100)(3)\cos(-75°) = 934.85\ \text{W}$$

[b] $V_{\text{rms}} = \sqrt{(15)^2 + \left(\frac{400}{\sqrt{2}}\right)^2 + \left(\frac{100}{\sqrt{2}}\right)^2} = 291.93\ \text{V}$

[c] $I_{\text{rms}} = \sqrt{(2)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 4.58\ \text{A}$

P 16.33 **[a]** Area under $v^2 = A = 4\int_0^{T/6} \frac{36V_m^2}{T^2} t^2\, dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{2V_m^2 T}{9} + \frac{V_m^2 T}{3}\right)} = V_m\sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356\ \text{V}$

[b] $v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots$ V

Therefore $V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\ \text{V}$

P 16.34 **[a]** $v(t) = \frac{480}{\pi}\{\sin\omega_o t + \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t + \frac{1}{7}\sin 7\omega_o t + \frac{1}{9}\sin 9\omega_o t + \cdots\}$

$$V_{\text{rms}} \cong \frac{480}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2}$$

$$= \frac{480}{\pi\sqrt{2}}\sqrt{1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}}$$

$$\cong 117.55\ \text{V}$$

[b] $\%\ \text{error} = \left(\frac{117.55}{120} - 1\right)(100) = -2.04\%$

[c] $v(t) = \frac{960}{\pi^2}\left\{\sin\omega_o t + \frac{1}{9}\sin 3\omega_o t + \frac{1}{25}\sin 5\omega_o t\right.$

$$\left. + \frac{1}{49}\sin 7\omega_o t + \frac{1}{81}\sin 9\omega_o t - \cdots\right\}$$

$$V_{\text{rms}} \cong \frac{960}{\pi^2\sqrt{2}}\sqrt{1 + \frac{1}{81} + \frac{1}{625} + \frac{1}{2401} + \frac{1}{6561}}$$

$$\cong 69.2765\ \text{V}$$

$$V_{\rm rms} = \frac{120}{\sqrt{3}} = 69.2820\,\text{V}$$

$$\%\ \text{error} = \left(\frac{69.2765}{69.2820} - 1\right)(100) = -0.0081\%$$

P 16.35 **[a]** $v(t) \approx \frac{340}{\pi} - \frac{680}{\pi}\left\{\frac{1}{3}\cos\omega_o t + \frac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\rm rms} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\,\text{V}$$

[b] $V_{\rm rms} = \frac{170}{\sqrt{2}} = 120.2082$

$$\%\ \text{error} = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

[c] $v(t) \approx \frac{170}{\pi} + 85\sin\omega_o t - \frac{340}{3\pi}\cos 2\omega_o t$

$$V_{\rm rms} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\rm rms} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error} = -0.23\%$$

P 16.36 **[a]** Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k. For k odd,

$$a_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\right\}\Bigg|_0^{T/4}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right],$$

$$b_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\right\}\Bigg|_0^{T/4} = \frac{4I_m}{\pi^2 k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $a_k - jb_k = \dfrac{2I_m}{\pi k}\left\{\left[\sin\left(\dfrac{k\pi}{2}\right) - \dfrac{2}{\pi k}\right] - \left[j\dfrac{2}{\pi k}\sin\left(\dfrac{k\pi}{2}\right)\right]\right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+ 0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $I_g = \sqrt{\displaystyle\sum_{n=1,3,5}^{\infty}\left(\frac{A_n^2}{2}\right)}$

$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] Area under $i_g^2 = 2\displaystyle\int_0^{T/4}\left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] % error $= \left(\dfrac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\dfrac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$

P 16.37 **[a]** v has half-wave symmetry, quarter-wave symmetry, and is odd

$$\therefore\ \ a_v = 0,\ a_k = 0 \text{ all } k,\ b_k = 0\ k\text{-even}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \quad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8}\frac{V_m}{4}\sin k\omega_o t\,dt + \int_{T/8}^{T/4} V_m\sin k\omega_o t\,dt\right\}$$

$$= \frac{8V_m}{4T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_0^{T/8}\right] + \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4}\right]$$

$$= \frac{8V_m}{4k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{\frac{1}{4} - \frac{1}{4}\cos\frac{k\pi}{4} + \cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{\frac{1}{4} + 0.75\cos\frac{k\pi}{4}\right\} = \frac{1}{k}[10 + 30\cos(k\pi/4)]$$

$$b_1 = 10 + 30\cos(\pi/4) = 31.21$$

$$b_3 = \frac{1}{3}[10 + 30\cos(3\pi/4)] = -3.74$$

$$b_5 = \frac{1}{5}[10 + 30\cos(5\pi/4)] = -2.24$$

$$b_7 = \frac{1}{7}[10 + 30\cos(7\pi/4)] = 4.46$$

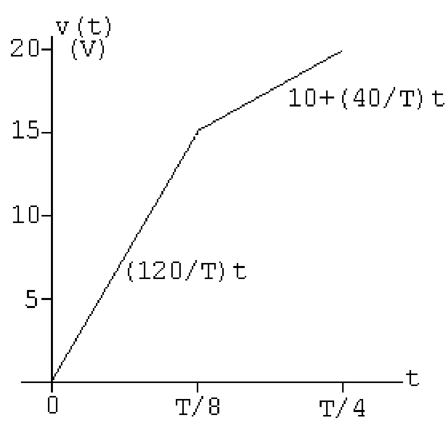
$$V(\text{rms}) \approx V_m\sqrt{\frac{31.21^2 + 3.74^2 + 2.24^2 + 4.46^2}{2}} = 22.51$$

[b] Area under $v^2 = 2\left[2(2.5\pi)^2\left(\frac{T}{8}\right) + 100\pi^2\left(\frac{T}{4}\right)\right] = 53.125\pi^2 T$

$$V(\text{rms}) = \sqrt{\frac{1}{T}(53.125\pi^2)T} = \sqrt{53.125}\pi = 22.90$$

[c] % Error $= \left(\frac{22.51}{22.90} - 1\right)(100) = -1.7\%$

P 16.38 **[a]** From Problem 16.16,



The area under v^2:

$$A = 4\left[\int_0^{T/8}\frac{14{,}400}{T^2}t^2\,dt + \int_{T/8}^{T/4}\left(10 + \frac{40t}{T}\right)^2\,dt\right]$$

$$= \frac{57{,}600}{T^2}\frac{t^3}{3}\Big|_0^{T/8} + 400t\Big|_{T/8}^{T/4} + \frac{3200}{T}\frac{t^2}{2}\Big|_{T/8}^{T/4} + \frac{6400}{T^2}\frac{t^3}{3}\Big|_{T/8}^{T/4}$$

$$= \frac{57{,}600}{1536}T + 400\frac{T}{8} + 1600\frac{3T}{64} + 6400\frac{7T}{1536} = \frac{575}{3}T$$

$$V_{\rm rms} = \sqrt{\frac{1}{T}\left(\frac{575}{3}T\right)} = \sqrt{\frac{575}{3}} = 13.84\,\text{V}$$

[b] $P = \dfrac{V_{\rm rms}^2}{15} = 12.78\,\text{W}$

[c] From Problem 16.16,

$$b_1 = \frac{80}{\pi^2}(2\cos 45^\circ + \pi\sin 90^\circ - 3) = 12.61\,\text{V}$$

$$v_g \cong 12.61\sin\omega_0 t\,\text{V}$$

$$P = \frac{(19.57/\sqrt{2})^2}{15} = 5.30\,\text{W}$$

[d] % error $= \left(\dfrac{5.30}{13.84} - 1\right)(100) = -61.71\%$

P 16.39 Figure P16.39(b): $t_a = 0.2$ s; $t_b = 0.6$ s

$$v = 50t \quad 0 \le t \le 0.2$$

$$v = -50t + 20 \quad 0.2 \le t \le 0.6$$

$$v = 25t - 25 \quad 0.6 \le t \le 1.0$$

Area 1 under $v^2 = A_1 = \displaystyle\int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$

Area 2 $= A_2 = \displaystyle\int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$

Area 3 $= A_3 = \displaystyle\int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\rm rms} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.39(c): $t_a = t_b = 0.4$ s

$$v(t) = 25t \quad 0 \leq t \leq 0.4$$

$$v(t) = \frac{50}{3}(t-1) \quad 0.4 \leq t \leq 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\textbf{V}.$$

Figure P16.39 (d): $t_a = t_b = 1$

$$v = 10t \quad 0 \leq t \leq 1$$

$$A_1 = \int_0^{1} 100t^2\,dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\textbf{V}.$$

P 16.40
$$c_n = \frac{1}{T}\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt = \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_0^{T/4}\right]$$
$$= \frac{V_m}{Tn\omega_o}[j(e^{-jn\pi/2} - 1)] = \frac{V_m}{2\pi n}\sin\frac{n\pi}{2} + j\frac{V_m}{2\pi n}\left(\cos\frac{n\pi}{2} - 1\right)$$
$$= \frac{V_m}{2\pi n}\left[\sin\frac{n\pi}{2} - j\left(1 - \cos\frac{n\pi}{2}\right)\right]$$

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

$$c_o = a_v = \frac{1}{T}\int_0^{T/4} V_m\,dt = \frac{V_m}{4}$$

or

$$c_o = \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{\sin(n\pi/2)}{n} - j\frac{1-\cos(n\pi/2)}{n}\right]$$

$$= \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{(\pi/2)\cos(n\pi/2)}{1} - j\frac{(\pi/2)\sin(n\pi/2)}{1}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\pi}{2} - j0\right] = \frac{V_m}{4}$$

Note it is much easier to use $c_o = a_v$ than to use L'Hopital's rule to find the limit of $0/0$.

P 16.41 $c_o = a_v = \frac{V_m T}{2}\cdot\frac{1}{T} = \frac{V_m}{2}$

$$c_n = \frac{1}{T}\int_0^T \frac{V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn\omega_0 t}}{-n^2\omega_0^2}(-jn\omega_0 t - 1)\right]_0^T$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn2\pi T/T}}{-n^2\omega_0^2}\left(-jn\frac{2\pi}{T}T - 1\right) - \frac{1}{-n^2\omega_0^2}(-1)\right]$$

$$= \frac{V_m}{T^2}\left[\frac{1}{n^2\omega_0^2}(1 + jn2\pi) - \frac{1}{n^2\omega_0^2}\right]$$

$$= j\frac{V_m}{2n\pi}, \quad n = \pm 1, \pm 2, \pm 3, \ldots$$

P 16.42 **[a]** $V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}\right)^2 t^2\,dt}$

$$= \sqrt{\frac{V_m^2}{T^3}\frac{t^3}{3}\Big|_0^T}$$

$$= \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}}$$

$$P = \frac{(120/\sqrt{3})^2}{10} = 480\,\text{W}$$

[b] From the solution to Problem 16.41

$$c_0 = \frac{120}{2} = 60\,\text{V}; \qquad c_4 = j\frac{120}{8\pi} = j\frac{15}{\pi}$$

$$c_1 = j\frac{120}{2\pi} = j\frac{60}{\pi}; \qquad c_5 = j\frac{120}{10\pi} = j\frac{12}{\pi}$$

$$c_2 = j\frac{120}{4\pi} = j\frac{30}{\pi}; \qquad c_6 = j\frac{120}{12\pi} = j\frac{10}{\pi}$$

$$c_3 = j\frac{120}{6\pi} = j\frac{20}{\pi}; \qquad c_7 = j\frac{120}{14\pi} = j\frac{8.57}{\pi}$$

$$V_{\rm rms} = \sqrt{c_o^2 + 2\sum_{n=1}^{\infty} |c_n|^2}$$

$$= \sqrt{60^2 + \tfrac{2}{\pi^2}(60^2 + 30^2 + 20^2 + 15^2 + 12^2 + 10^2 + 8.57^2)}$$

$$= 68.58\,\text{V}$$

[c] $P = \dfrac{(68.58)^2}{10} = 470.29\,\text{W}$

$$\%\text{ error } = \left(\frac{470.29}{480} - 1\right)(100) = -2.02\%$$

P 16.43 **[a]** $C_o = a_v = \dfrac{(1/2)(T/2)V_m}{T} = \dfrac{V_m}{4}$

$$C_n = \frac{1}{T}\int_0^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\right]_0^{T/2}$$

$$= \frac{V_m}{2n^2\pi^2}[e^{-jn\pi}(jn\pi + 1) - 1]$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2}(\cos n\pi - 1) + j\frac{V_m}{2n\pi}\cos n\pi$$

[b] $C_o = \dfrac{54}{4} = 13.5\,\text{V}$

$$C_{-1} = \frac{-54}{\pi^2} + j\frac{27}{\pi} = 10.19\underline{/122.48^\circ}\,\text{V}$$

$$C_1 = 10.19\underline{/-122.48^\circ}\,\text{V}$$

$$C_{-2} = -j\frac{13.5}{\pi} = 4.30\underline{/-90^\circ}\,\text{V}$$

$$C_2 = 4.30\underline{/90^\circ}\,\text{V}$$

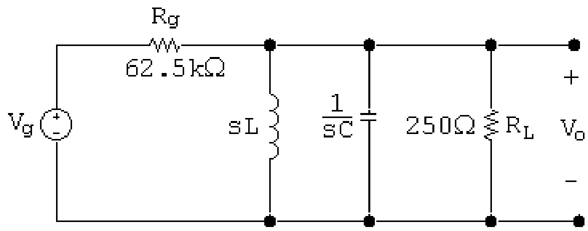
$$C_{-3} = \frac{-6}{\pi^2} + j\frac{9}{\pi} = 2.93\underline{/101.98^\circ}\,\text{V}$$

$$C_3 = 2.93\underline{/-101.98^\circ}\,\text{V}$$

$$C_{-4} = -j\frac{6.75}{\pi} = 2.15\underline{/-90^\circ}\text{ V}$$

$$C_4 = 2.15\underline{/90^\circ}\text{ V}$$

[c]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5 \times 10^3} = 0$$

$$\therefore \qquad (250LCs^2 + 1.004sL + 250)V_o = 0.004sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(1/62,500C)s}{s^2 + 1/249C + 1/LC}$$

$$H(s) = \frac{16s}{s^2 + 1/249Cs + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{jk}{12,500(1-k^2) + j251k}$$

Therefore,

$$H_{-1} = 0.0398\underline{/0^\circ}; \qquad H_1 = 0.0398\underline{/0^\circ}$$

$$H_{-2} = \frac{-j2}{-37,500 - j20} = 5.33 \times 10^{-5}\underline{/86.23^\circ}; \qquad H_2 = 5.33 \times 10^{-5}\underline{/-89.23^\circ}$$

$$H_{-3} = \frac{-3j}{-10^{-5} - j753} = 3.00 \times 10^{-5}\underline{/89.57^\circ}; \qquad H_2 = 3.00 \times 10^{-5}\underline{/-89.57^\circ}$$

$$H_{-4} = \frac{-4j}{-187,500 - j1004} = 2.13 \times 10^{-5}\underline{/89.69^\circ}; \qquad H_2 = 2.13 \times 10^{-5}\underline{/-89.69^\circ}$$

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19\underline{/122.48^\circ})(0.00398\underline{/0^\circ}) = 0.0406\underline{/122.48^\circ}\text{ V}$$

$$C_1 = 0.0406\underline{/-122.48^\circ}\ \text{V}$$

$$C_{-2} = (4.30\underline{/-90^\circ})(5.33\times10^{-5}\underline{/86.23^\circ}) = 2.29\times10^{-4}\underline{/-3.77^\circ}\ \text{V}$$

$$C_2 = 2.29\times10^{-4}\underline{/3.77^\circ}\ \text{V}$$

$$C_{-3} = (2.93\underline{/101.98^\circ})(3.00\times10^{-5}\underline{/89.57^\circ}) = 8.79\times10^{-5}\underline{/191.55^\circ}\ \text{V}$$

$$C_3 = 8.79\times10^{-5}\underline{/-191.55^\circ}\ \text{V}$$

$$C_{-4} = (2.15\underline{/-90^\circ})(2.13\times10^{-5}\underline{/89.69^\circ}) = 4.58\times10^{-5}\underline{/-0.31^\circ}\ \text{V}$$

$$C_4 = 4.58\times10^{-5}\underline{/0.31^\circ}\ \text{V}$$

[d] $V_{\text{rms}} \cong \sqrt{C_o^2 + 2\sum_{n=1}^{4}|C_n|^2} \cong \sqrt{2\sum_{n=1}^{4}|C_n|^2}$

$$\cong \sqrt{2(0.0406^2 + (2.29\times10^{-4})^2 + (8.79\times10^{-5})^2 + (4.58\times10^{-5})^2} \cong 0.0574\ \text{V}$$

$$P = \frac{(0.0574)^2}{250} = 13.2\,\mu\text{W}$$

P 16.44 **[a]** $V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2V_m}{T}t\right)^2 dt}$

$$= \sqrt{\frac{1}{T}\left[\frac{4V_m^2}{T^2}\frac{t^3}{3}\right]_0^{T/2}}$$

$$= \sqrt{\frac{4V_m^2}{(3)(8)}} = \frac{V_m}{\sqrt{6}}$$

$$V_{\text{rms}} = \frac{54}{\sqrt{6}} = 22.05\ \text{V}$$

[b] From the solution to Problem 16.43

$C_0 = 13.5$; $|C_3| = 2.93$

$|C_1| = 10.19$; $|C_4| = 2.15$

$|C_2| = 4.30$

$$V_g(\text{rms}) \cong \sqrt{13.5^2 + 2(10.19^2 + 4.30^2 + 2.93^2 + 2.15^2)} \cong 21.29\ \text{V}$$

[c] % Error $= \left(\frac{21.29}{22.05} - 1\right)(100) = -3.44\%$

P 16.45 **[a]** From Example 16.3 we have:

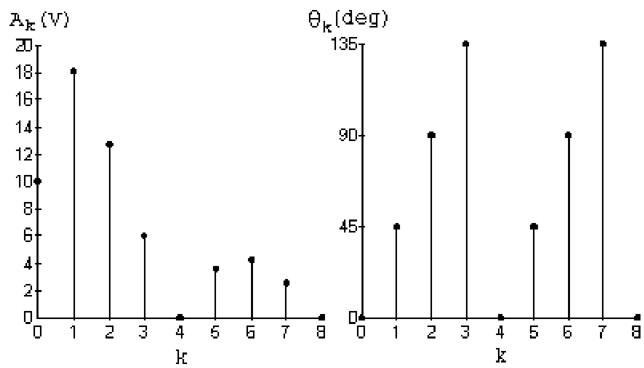
$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

$$A_1 = 18.01\,\text{V} \qquad \theta_1 = -45^\circ, \qquad A_2 = 12.73\,\text{V}, \qquad \theta_2 = -90^\circ$$

$$A_3 = 6\,\text{V}, \qquad \theta_3 = -135^\circ, \qquad A_4 = 0, \qquad A_5 = 3.6\,\text{V}, \qquad \theta_5 = -45^\circ$$

$$A_6 = 4.24\,\text{V}, \qquad \theta_6 = -90^\circ, \qquad A_7 = 2.57\,\text{V}, \qquad \theta_7 = -135^\circ$$



[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

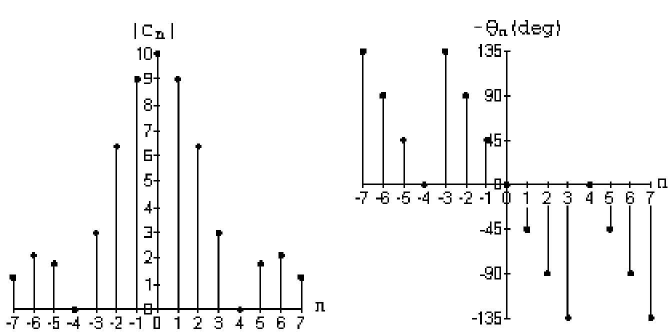
$$C_0 = a_v = 10\,\text{V} \qquad C_3 = 3\underline{/135^\circ}\,\text{V} \qquad C_6 = 2.12\underline{/90^\circ}\,\text{V}$$

$$C_1 = 9\underline{/45^\circ}\,\text{V} \qquad C_{-3} = 3\underline{/-135^\circ}\,\text{V} \qquad C_{-6} = 2.12\underline{/-90^\circ}\,\text{V}$$

$$C_{-1} = 9\underline{/-45^\circ}\,\text{V} \qquad C_4 = C_{-4} = 0 \qquad C_7 = 1.29\underline{/135^\circ}\,\text{V}$$

$$C_2 = 6.37\underline{/90^\circ}\,\text{V} \qquad C_5 = 1.8\underline{/45^\circ}\,\text{V} \qquad C_{-7} = 1.29\underline{/-135^\circ}\,\text{V}$$

$$C_{-2} = 6.37\underline{/-90^\circ}\,\text{V} \qquad C_{-5} = 1.8\underline{/-45^\circ}\,\text{V}$$



P 16.46 **[a]** From the solution to Problem 16.29 we have

$$A_k = a_k - jb_k = \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1) + j\frac{I_m}{\pi k}$$

$$A_0 = 0.75 I_m = 180\,\text{mA}$$

$$A_1 = \frac{240}{\pi^2}(-2) + j\frac{240}{\pi} = 90.56\underline{/122.48^\circ}\,\text{mA}$$

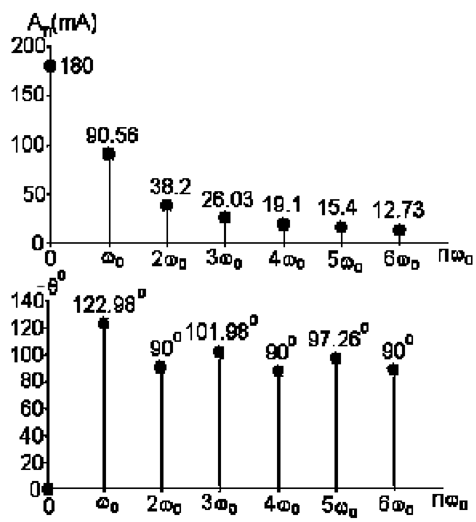
$$A_2 = j\frac{240}{2\pi} = 38.20\underline{/90^\circ}\,\text{mA}$$

$$A_3 = \frac{240}{9\pi^2}(-2) + j\frac{240}{3\pi} = 26.03\underline{/101.98^\circ}\,\text{mA}$$

$$A_4 = j\frac{240}{4\pi} = 19.10\underline{/90^\circ}\,\text{mA}$$

$$A_5 = \frac{240}{25\pi^2}(-2) + j\frac{240}{5\pi} = 15.40\underline{/97.26^\circ}\,\text{mA}$$

$$A_6 = j\frac{240}{6\pi} = 12.73\underline{/90^\circ}\,\text{mA}$$



[b] $C_0 = A_0 = 180\,\text{mA}$

$$C_1 = \frac{1}{2}A_1\underline{/-\theta_1} = 45.28\underline{/122.48^\circ}\,\text{mA}$$

$$C_{-1} = 45.28\underline{/-122.48^\circ}\,\text{mA}$$

$$C_2 = \frac{1}{2}A_2\underline{/-\theta_2} = 19.1\underline{/90^\circ}\,\text{mA}$$

$$C_{-2} = 19.1\underline{/-90^\circ}\,\text{mA}$$

$$C_3 = \frac{1}{2}A_3\underline{/-\theta_3} = 13.02\underline{/101.98^\circ}\text{ mA}$$

$$C_{-3} = 13.02\underline{/-101.98^\circ}\text{ mA}$$

$$C_4 = \frac{1}{2}A_4\underline{/-\theta_4} = 9.55\underline{/90^\circ}\text{ mA}$$

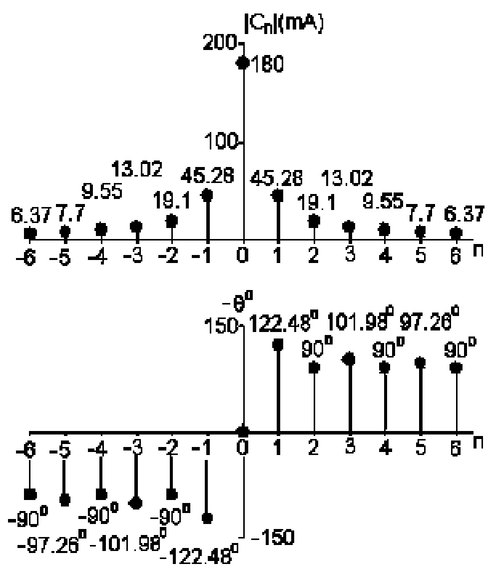
$$C_{-4} = 9.55\underline{/-90^\circ}\text{ mA}$$

$$C_5 = \frac{1}{2}A_5\underline{/-\theta_5} = 7.70\underline{/97.26^\circ}\text{ mA}$$

$$C_{-5} = 7.70\underline{/-97.26^\circ}\text{ mA}$$

$$C_6 = \frac{1}{2}A_6\underline{/-\theta_6} = 6.37\underline{/90^\circ}\text{ mA}$$

$$C_{-6} = 6.37\underline{/-90^\circ}\text{ mA}$$



P 16.47 **[a]** $v = A_1\cos(\omega_o t + 90^\circ) + A_3\cos(3\omega_o t - 90^\circ)$

$$+A_5\cos(5\omega_o t + 90^\circ) + A_7\cos(7\omega_o t - 90^\circ)$$

$$v = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

[b] $v(-t) = A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$

$\therefore\quad v(-t) = -v(t);$ odd function

[c] $v(t - T/2) = -A_1\sin(\omega_o t - \pi) + A_3\sin(3\omega_o t - 3\pi)$

$$-A_5\sin(5\omega_o t - 5\pi) + A_7\sin(7\omega_o t - 7\pi)$$

$$= A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$$

$\therefore\quad v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$$f(T/2 - t) = f(t)$$

$$\begin{aligned} f(T/2 - t) &= -A_1 \sin(\pi - \omega_o t) + A_3 \sin(3\pi - 3\omega_o t) \\ &\qquad A_5 \sin(5\pi - 5\omega_o t) + A_7 \sin(7\pi - 7\omega_o t) \\ &= -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t \end{aligned}$$

$\therefore \quad f(T/2 - t) = f(t)$ and the voltage has quarter-wave symmetry

P 16.48 **[a]** $i = 11{,}025 \cos 10{,}000t + 1225 \cos(30{,}000t - 180°) + 441 \cos(50{,}000t - 180°)$

$$+ 225 \cos 70{,}000t \; \mu\text{A}$$

$$= 11{,}025 \cos 10{,}000t - 1225 \cos 30{,}000t - 441 \cos 50{,}000t$$

$$+ 225 \cos 70{,}000t \; \mu\text{A}$$

[b] $i(t) = i(-t)$, Function is even

[c] Yes, $A_0 = 0$, $A_n = 0$ for n even

[d] $I_{\text{rms}} = \sqrt{\dfrac{11{,}025^2 + 1225^2 + 441^2 + 225^2}{2}} = 7.85\,\text{mA}$

[e] $A_1 = 11{,}025\underline{/0°}\;\mu\text{A}$; $C_1 = 5512.50\underline{/0°}\;\mu\text{A}$

$A_3 = 1225\underline{/180°}\;\mu\text{A}$; $C_3 = 612.5\underline{/180°}\;\mu\text{A}$

$A_5 = 441\underline{/180°}\;\mu\text{A}$; $C_5 = 220.5\underline{/180°}\;\mu\text{A}$

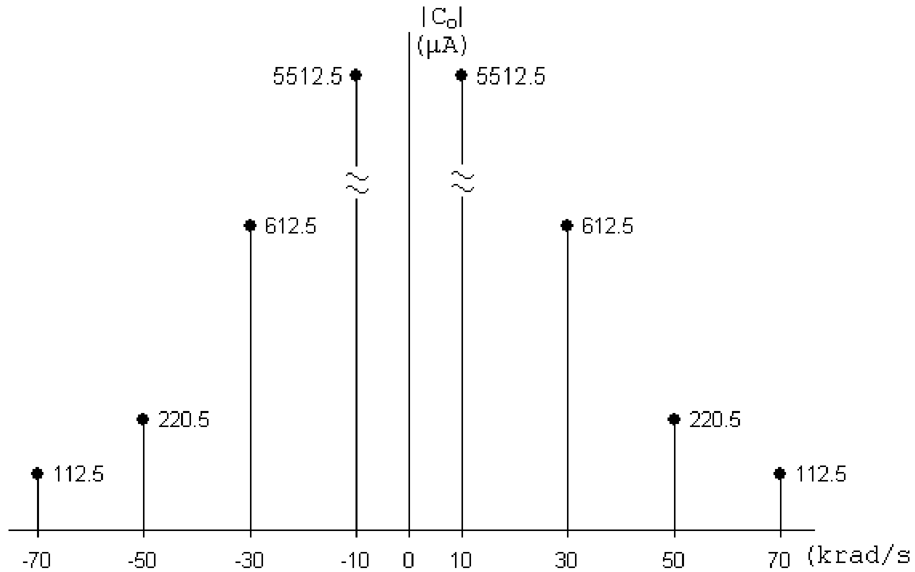
$A_7 = 225\underline{/0°}\;\mu\text{A}$; $C_7 = 112.50\underline{/0°}\;\mu\text{A}$

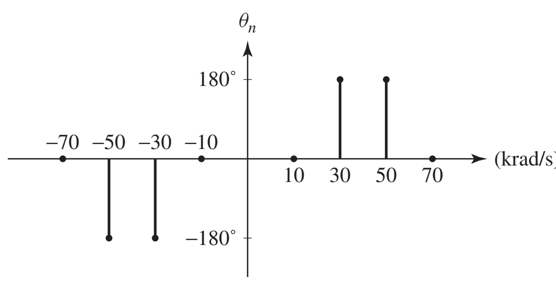
$C_{-1} = 5512.50\underline{/0°}\;\mu\text{A}$; $C_{-3} = 612.5\underline{/-180°}\;\mu\text{A}$

$C_{-5} = 220.5\underline{/-180°}\;\mu\text{A}$; $C_{-7} = 112.50\underline{/0°}\;\mu\text{A}$

$$\begin{aligned} i &= 112.5e^{-j70{,}000t} + 220.5e^{-j180°}e^{-j50{,}000t} + 612.5e^{-j180°}e^{-j30{,}000t} \\ &\quad + 5512.5e^{-j10{,}000t} + 5512.5e^{j10{,}000t} + 612.5e^{j180°}e^{j30{,}000t} \\ &\quad + 220.5e^{j180°}e^{j50{,}000t} + 112.5e^{j70{,}000t}\;\mu\text{A} \end{aligned}$$

[f]





P 16.49 From Table 15.1 we have

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{10^6}{(s+100)(s^2+100s+10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{5\pi} \times 10^3 = 400 \text{ rad/s}$$

$$\therefore \quad H'(jn\omega_o) = \frac{1}{(1+j4n)[(1-16n^2)+j4n]}$$

It follows that

$$H(j0) = 1\underline{/0^\circ}$$

$$H(j\omega_o) = \frac{1}{(1+j4)(-15+j4)} = 0.0156\underline{/-241.03^\circ}$$

$$H(j2\omega_o) = \frac{1}{(1+j8)(-63+j8)} = 0.00195\underline{/-255.64^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_o t - \frac{2A}{\pi}\sum_{n=2,4,6,}^{\infty}\frac{\cos n\omega_o t}{n^2-1}$$

$$= 54 + 27\pi\sin\omega_o t - 36\cos 2\omega_o t - \cdots \text{ V}$$

$$\therefore \quad v_o = 54 + 1.33\sin(400t - 241.03^\circ) - 0.07\cos(800t - 255.64^\circ) - \cdots \text{ V}$$

P 16.50 Using the technique outlined in Problem 16.17 we can derive the Fourier series for $v_g(t)$. We get

$$v_g(t) = 100 + \frac{800}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

The transfer function of the prototype second-order low pass Butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}, \qquad \text{where } \omega_c = 1 \text{ rad/s}$$

Now frequency scale using $k_f = 2000$ to get $\omega_c = 2$ krad/s:

$$H(s) = \frac{4\times 10^6}{s^2 + 2000\sqrt{2}s + 4\times 10^6}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{4\times 10^6}{(j5000)^2 + 2000\sqrt{2}(j5000)^2 + 4\times 10^6} = 0.1580\underline{/-146.04^\circ}$$

$$H(j15{,}000) = \frac{4\times 10^6}{(j15{,}000)^2 + 2000\sqrt{2}(j15{,}000)^2 + 4\times 10^6} = 0.0178\underline{/-169.13^\circ}$$

$$\mathbf{V}_{\text{dc}} = 100\,\text{V}$$

$$\mathbf{V}_{g1} = \frac{800}{\pi^2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_{g3} = \frac{800}{9\pi^2}\underline{/0^\circ}\,\text{V}$$

$$V_{odc} = 100(1) = 100\,\text{V}$$

$$\mathbf{V}_{o1} = \frac{800}{\pi^2}(0.1580\underline{/-146.04^\circ}) = 12.81\underline{/-146.04^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = \frac{800}{9\pi^2}(0.0178\underline{/-169.13^\circ}) = 0.16\underline{/-169.13^\circ}\,\text{V}$$

$$v_o(t) = 100 + 12.81\cos(5000t - 146.04^\circ) + 0.16\cos(15{,}000t - 169.13^\circ) + \cdots\,\text{V}$$

P 16.51 **[a]** Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of $R_1, R_2, R_3, C_1,$ and C_2 we have

$$-\frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right) = -\frac{R_3}{2R_1} = -\frac{400}{313}$$

$$\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 2000$$

$$\frac{R_1 + R_2}{R_1R_2R_3C_1C_2} = 0.16 \times 10^{10} = 16 \times 10^8$$

$$H(s) = \frac{-(400/313)(2000)s}{s^2 + 2000s + 16 \times 10^8}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{50\pi} \times 10^6 = 4 \times 10^4 \text{ rad/s}$$

$$H(jn\omega_o) = \frac{-(400/313)(2000)jn\omega_o}{16 \times 10^8 - n^2\omega_o^2 + j2000n\omega_o}$$

$$= \frac{-j(20/313)n}{(1 - n^2) + j0.05n}$$

$$H(j\omega_o) = \frac{-j(20/313)}{j(0.050)} = -\frac{400}{313} = -1.28$$

$$H(j3\omega_o) = \frac{-j(20/313)(3)}{-8 + j0.15} = 0.0240\underline{/91.07^\circ}$$

$$H(j5\omega_o) = \frac{-j(100/313)}{-24 + j0.25} = 0.0133\underline{/90.60^\circ}$$

$$v_g(t) = \frac{4A}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin(n\pi/2)\cos n\omega_o t$$

$$A = 15.65\pi \text{ V}$$

$$v_g(t) = 62.60\cos\omega_o t - 20.87\cos 3\omega_o t + 12.52\cos 5\omega_o t - \cdots$$

$$v_o(t) = -80\cos\omega_o t - 0.50\cos(3\omega_o t + 91.07^\circ)$$
$$+ 0.17\cos(5\omega_o t + 90.60^\circ) - \cdots \text{ V}$$

17

The Fourier Transform

Assessment Problems

AP 17.1 **[a]** $F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\,dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\,dt$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$
$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2}\right]$$
$$= \frac{-j2A}{\omega}[1 - \cos\frac{\omega\tau}{2}]$$

[b] $F(\omega) = \int_{0}^{\infty} te^{-at}e^{-j\omega t}\,dt = \int_{0}^{\infty} te^{-(a+j\omega)t}\,dt = \frac{1}{(a+j\omega)^2}$

AP 17.2 $f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\,d\omega + \int_{-2}^{2} e^{jt\omega}\,d\omega + \int_{2}^{3} 4e^{jt\omega}\,d\omega\right\}$

$$= \frac{1}{j2\pi t}\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 **[a]** $F(\omega) = F(s)\mid_{s=j\omega} = \mathcal{L}\{e^{-at}\sin\omega_0 t\}_{s=j\omega}$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2}\Big|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $F(\omega) = \mathcal{L}\{f(-t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$

[c] $f^+(t) = te^{-at}, \qquad f^-(t) = te^{at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(-t)\} = \frac{-1}{(s+a)^2}$$

Therefore $F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 **[a]** $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \quad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \quad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau}\delta(t) + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega) = -\omega^2F(\omega); \qquad$ therefore $F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5 $v(t) = V_m\left[u\left(t+\dfrac{\tau}{2}\right) - u\left(t-\dfrac{\tau}{2}\right)\right]$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

AP 17.6 **[a]** $I_g(\omega) = \mathcal{F}\{10\text{sgn}\,t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)\,\text{V}$

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\text{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18\,\text{A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\text{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\text{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\text{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80\,\text{V}$

AP 17.7 **[a]** $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned}
V_a(\omega) &= H(\omega)V_g(\omega) \\
&= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\
&= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \pi\delta(\omega)
\end{aligned}$$

Therefore $\quad v_a(t) = \left[\frac{1}{4}e^t u(-t) + \frac{1}{6}\text{sgn}\, t - \frac{1}{12}e^{-3t}u(t) + \frac{1}{6}\right]$ V

[b] $v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}$ V

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\text{ V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\text{ V}$$

AP 17.8 $v(t) = 4te^{-t}u(t); \qquad V(\omega) = \frac{4}{(1+j\omega)^2}$

Therefore $\quad |V(\omega)| = \frac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}}\left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\text{ J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\text{ J}$$

Therefore $\quad \% = \frac{3.769}{4}(100) = 94.23\%$

AP 17.9 $|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega, \qquad 0 \le \omega \le 2000\pi$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2\times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36\times 10^{-6}}{4\pi^2}\omega^2\right] d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36\times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36\times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\mathbf{kJ}$$

$$W_{6\text{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\mathbf{J}$$

Problems

P 17.1 **[a]** $F(\omega) = \int_{-2}^{2}\left[A\sin\left(\frac{\pi}{2}\right)t\right]e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $F(\omega) = \int_{-\tau/2}^{0}\left(\frac{2A}{\tau}t + A\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(\frac{-2A}{\tau}t + A\right)e^{-j\omega t}\,dt$

$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.2 **[a]** $F(\omega) = \int_{-\tau/2}^{\tau/2}\frac{2A}{\tau}te^{-j\omega t}\,dt$

$$= \frac{2A}{\tau}\left[\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2}\left(\frac{j\omega\tau}{2} + 1\right) - e^{j\omega\tau/2}\left(\frac{-j\omega\tau}{2} + 1\right)\right]$$

$$F(\omega) = \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2} + j\frac{\omega\tau}{2}\left(e^{-j\omega\tau/2} + e^{j\omega\tau/2}\right)\right]$$

$$F(\omega) = j\frac{2A}{\tau}\left[\frac{\omega\tau\cos(\omega\tau/2) - 2\sin(\omega\tau/2)}{\omega^2}\right]$$

[b] Using L'Hopital's rule,

$$F(0) = \lim_{\omega\to 0} j2A\left[\frac{\omega\tau(\tau/2)(-\sin\omega\tau/2) + \tau\cos\omega(\tau/2) - 2(\tau/2)\cos(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} j2A\left[\frac{-\omega\tau(\tau/2)\sin(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} j2A\left[\frac{-\tau\sin(\omega\tau/2)}{4}\right] = 0$$

$$\therefore \qquad F(0) = 0$$

[c] When $A = 1$ and $\tau = 1$

$$F(\omega) = j2\left[\frac{\omega\cos(\omega/2) - 2\sin(\omega/2)}{\omega^2}\right]$$

$$|F(\omega)| = \left|\frac{2\omega\cos(\omega/2) - 4\sin(\omega/2)}{\omega^2}\right|$$

$$F(0) = 0$$

$$|F(2)| = \left|\frac{4\cos 1 - 4\sin 1}{4}\right| = 0.30$$

$$|F(4)| = \left|\frac{8\cos 2 - 4\sin 2}{16}\right| = 0.44$$

$$|F(6)| = \left|\frac{12\cos 3 - 4\sin 3}{36}\right| = 0.35$$

$$|F(8)| = \left|\frac{16\cos 4 - 4\sin 4}{64}\right| = 0.12$$

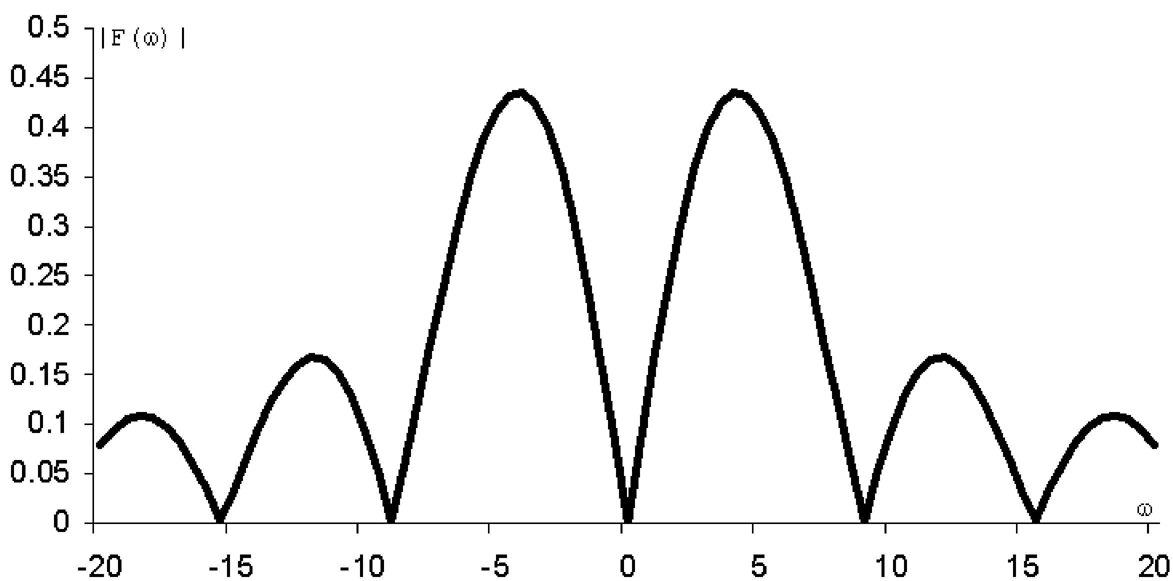
$$|F(9)| = \left|\frac{18\cos 4.5 - 4\sin 4.5}{81}\right| \cong 0$$

$$|F(10)| = \left|\frac{20\cos 5 - 4\sin 5}{100}\right| = 0.10$$

$$|F(12)| = \left|\frac{24\cos 6 - 4\sin 6}{144}\right| = 0.17$$

$$|F(14)| = \left|\frac{28\cos 7 - 4\sin 7}{196}\right| = 0.09$$

$$|F(15.5)| = \left|\frac{31\cos 7.75 - 4\sin 7.75}{240.25}\right| \cong 0$$



P 17.3 **[a]** $F(\omega) = A + \frac{2A}{\omega_o}\omega, \quad -\omega_o/2 \le \omega \le 0$

$$F(\omega) = A - \frac{2A}{\omega_o}\omega, \quad 0 \le \omega \le \omega_o/2$$

$F(\omega) = 0$ elsewhere

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{0}\left(A + \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$+\frac{1}{2\pi}\int_{0}^{\omega_o/2}\left(A - \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega + \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right.$$

$$\left.+\int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega - \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right]$$

$$= \frac{1}{2\pi}[\,\text{Int1} \;+\; \text{Int2} \;+\; \text{Int3} \;-\; \text{Int4}\,]$$

$$\text{Int1} \;= \int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(1 - e^{-jt\omega_o/2})$$

$$\text{Int2} \;= \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(1 - j\frac{t\omega_o}{2}e^{-jt\omega_o/2} - e^{-jt\omega_o/2})$$

$$\text{Int3} \;= \int_{0}^{\omega_o/2} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(e^{jt\omega_o/2} - 1)$$

$$\text{Int4} \;= \int_{0}^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(-j\frac{t\omega_o}{2}e^{jt\omega_o/2} + e^{jt\omega_o/2} - 1)$$

$$\text{Int1} \;+\; \text{Int3} \;= \frac{2A}{t}\sin(\omega_o t/2)$$

$$\text{Int2} \;-\; \text{Int4} \;= \frac{4A}{\omega_o t^2}[1 - \cos(\omega_o t/2)] - \frac{2A}{t}\sin(\omega_o t/2)$$

$$\therefore \qquad f(t) = \frac{1}{2\pi}\left[\frac{4A}{\omega_o t^2}\left(1 - \cos(\omega_o t/2)\right)\right]$$

$$= \frac{2A}{\pi\omega_o t^2}\left[2\sin^2(\omega_o t/4)\right]$$

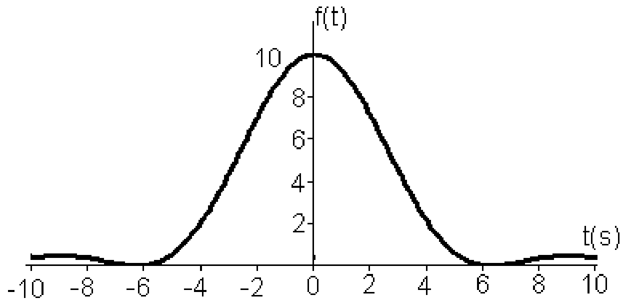
$$= \frac{4\omega_o A}{\pi\omega_o^2 t^2}\sin^2(\omega_o t/4)$$

$$= \frac{\omega_o A}{4\pi}\left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)}\right]^2$$

[b] $f(0) = \frac{\omega_o A}{4\pi}(1)^2 = 79.58 \times 10^{-3}\omega_o A$

[c] $A = 20\pi; \qquad \omega_o = 2$ rad/s

$$f(t) = 10\left[\frac{\sin(t/2)}{(t/2)}\right]^2$$



P 17.4 **[a]** $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2 + 4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3e^{-at}\} = \dfrac{6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{6}{(a+j\omega)^4} - \frac{6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0}$$

$$+ \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0}$$

$$= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-ja}{a^2+(\omega-\omega_0)^2} + \frac{ja}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.5

$$\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\frac{e^{-j\omega_0 t}}{2j}\right\}$$

$$= \frac{1}{2j}[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)]$$

$$= j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

P 17.6

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega$$

$$+ \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\,d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\,d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega};$ therefore $B(\omega) = \dfrac{-2}{\omega};$ thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\,d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\sin t\omega}{\omega}\,d\omega$

Therefore,

$$\left.\begin{aligned} f(t) &= \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \qquad\quad t > 0 \\ f(t) &= \frac{2}{\pi}\cdot\left(\frac{-\pi}{2}\right) = -1 \ t < 0 \end{aligned}\right\} \text{from a table of definite integrals}$$

Therefore $\quad f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.4[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $\quad F(\omega) \to \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt$

$$= \int_{-\infty}^{0} f(t)\cos\omega t\,dt + \int_{0}^{\infty} f(t)\cos\omega t\,dt$$

$$= 2\int_{0}^{\infty} f(t)\cos\omega t\,dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\,dt = -\int_{0}^{\infty} f(t)\sin\omega t\,dt$$

P 17.11 $A(\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\,dt + \int_{0}^{\infty} f(t)\cos\omega t\,dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$$B(\omega) = -2\int_{0}^{\infty} f(t)\sin\omega t\,dt, \quad \text{since } f(t)\sin\omega t \text{ is an even function.}$$

P 17.12 **[a]** $\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \frac{df(t)}{dt} e^{-j\omega t}\,dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}$; let $dv = [df(t)/dt]\,dt$, then $v = f(t)$.

Therefore $\quad \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\,dt]$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\}$, let $g(t) = \dfrac{df(t)}{dt}$

Then $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\} = \mathcal{F}\left\{\dfrac{dg(t)}{dt}\right\} = j\omega G(\omega)$

But $G(\omega) = \mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = j\omega F(\omega)$

Therefore we have $\mathcal{F}\left\{\dfrac{d^2 f(t)}{dt^2}\right\} = (j\omega)^2 F(\omega)$

Repeated application of this thought process gives

$$\mathcal{F}\left\{\frac{d^n f(t)}{dt^n}\right\} = (j\omega)^n F(\omega).$$

P 17.13 **[a]** $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right] e^{-j\omega t}\,dt$

Now let $u = \int_{-\infty}^{t} f(x)\,dx$, then $du = f(t)dt$

Let $dv = e^{-j\omega t}\,dt$, then $v = \dfrac{e^{-j\omega t}}{-j\omega}$

Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right] f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \dfrac{1}{a} \neq 0$

P 17.14 **[a]** $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at$, $du = a\,dt$, $u = \pm\infty$ when $t = \pm\infty$

Therefore,

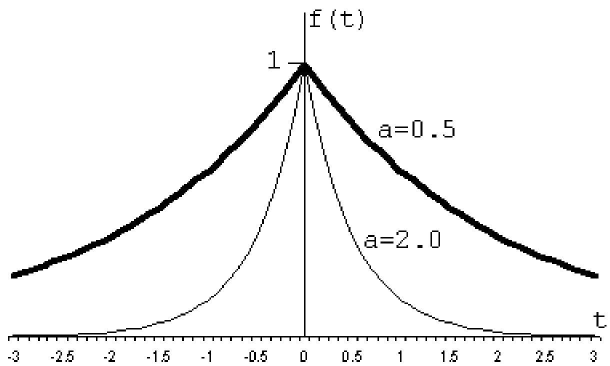
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{\omega}{a}\right), \qquad a > 0$$

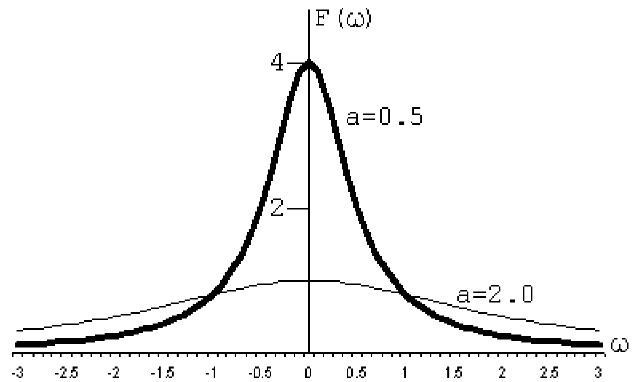
[b] $\mathcal{F}\{e^{-|t|}\} = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$

Therefore $\quad \mathcal{F}\{e^{-a|t|}\} = \frac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\quad \mathcal{F}\{e^{-0.5|t|}\} = \frac{4}{4\omega^2+1}, \qquad \mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "*a*" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.





P 17.15 **[a]** $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$.
Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda\right]e^{-j\omega t}\,dt$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(t-\lambda)e^{-j\omega t}\,dt\right]d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

$$\text{Therefore}\quad Y(\omega) = \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(u)e^{-j\omega(u+\lambda)}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[e^{-j\omega\lambda}\int_{-\infty}^{\infty} h(u)e^{-j\omega u}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)e^{-j\omega\lambda}H(\omega)\,d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)e^{jtu}du\right] f_2(t)e^{-j\omega t}\,dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} F_1(u)f_2(t)e^{-j\omega t}e^{jtu}\,du\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F_1(u)\int_{-\infty}^{\infty} f_2(t)e^{-j(\omega-u)t}\,dt\right]du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

P 17.18 **[a]** $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty}\frac{d}{d\omega}\left[f(t)e^{-j\omega t}\right]dt = -j\int_{-\infty}^{\infty} tf(t)e^{-j\omega t}\,dt = -j\mathcal{F}\{tf(t)\}$$

$$\text{Therefore}\quad j\frac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty}(-jt)(-jt)f(t)e^{-j\omega t}\,dt = (-j)^2\mathcal{F}\{t^2f(t)\}$$

$$\text{Note that}\quad (-j)^n = \frac{1}{j^n}$$

$$\text{Thus we have}\quad j^n\left[\frac{d^nF(\omega)}{d\omega^n}\right] = \mathcal{F}\{t^nf(t)\}$$

[b] (i) $\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega} = F(\omega);\qquad \frac{dF(\omega)}{d\omega} = \frac{-j}{(a+j\omega)^2}$

$$\text{Therefore}\quad j\left[\frac{dF(\omega)}{d\omega}\right] = \frac{1}{(a+j\omega)^2}$$

$$\text{Therefore}\quad \mathcal{F}\{te^{-at}u(t)\} = \frac{1}{(a+j\omega)^2}$$

(ii) $\mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 **[a]** $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

Thus $\quad F_2(u) = \dfrac{\tau\sin(u\tau/2)}{u\tau/2}$

Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the duration of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$.
The area under the $[\sin x]/x$ function is independent of τ, that is

$$\frac{\tau}{2}\int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$f_1(t)f_2(t) \to \cos\omega_0 t \quad$ and $\quad F(\omega) \to \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

P 17.20 **[a]** $v_g = 100u(t)$

$$V_g(\omega) = 100\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$H(s) = \frac{10}{5s+10} = \frac{2}{s+2}$$

$$H(\omega) = \frac{2}{j\omega+2}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{200\pi\delta(\omega)}{j\omega+2} + \frac{200}{j\omega(j\omega+2)}$$

$$= V_1(\omega) + V_2(\omega)$$

$$v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{200\pi e^{jt\omega}}{j\omega+2}\delta(\omega)\,d\omega = \frac{1}{2\pi}\left(\frac{200\pi}{2}\right) = 50 \text{ (sifting property)}$$

$$V_2(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega+2} = \frac{100}{j\omega} - \frac{100}{j\omega+2}$$

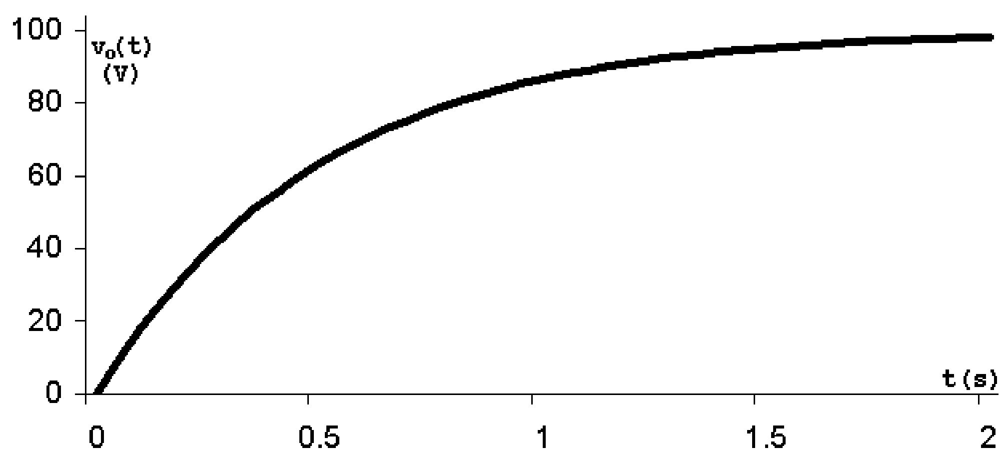
$$v_2(t) = 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$v_o(t) = v_1(t) + v_2(t) = 50 + 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$= 100u(t) - 100e^{-2t}u(t)$$

$$v_o(t) = 100(1 - e^{-2t})u(t)\text{ V}$$

[b]



P 17.21 **[a]** From the solution to Problem 17.20

$$H(\omega) = \frac{2}{j\omega + 2}$$

Now,

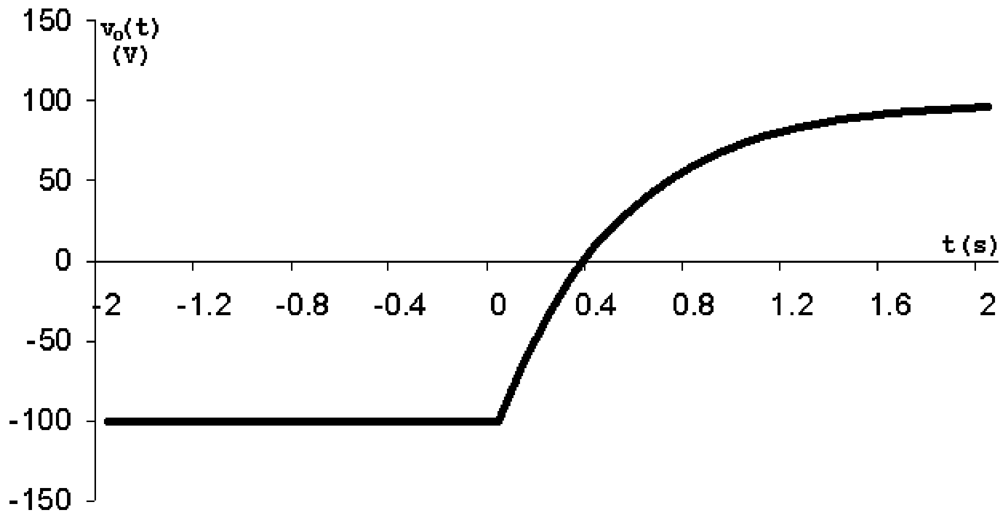
$$V_g(\omega) = \frac{200}{j\omega}$$

Then,

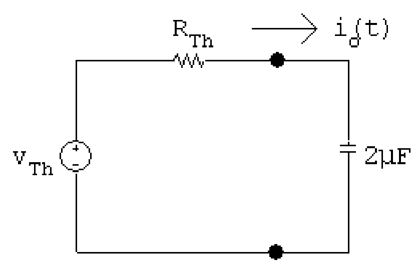
$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{400}{j\omega(j\omega + 2)} = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 2} = \frac{200}{j\omega} - \frac{200}{j\omega + 2}$$

$$\therefore \qquad v_o(t) = 100\text{sgn}(t) - 200e^{-2t}u(t)\,\text{V}$$

[b]



P 17.22 **[a]** Find the Thévenin equivalent with respect to the terminals of the capacitor:



$$v_{\text{Th}} = \frac{5}{6}v_g; \qquad R_{\text{Th}} = 60\|12 = 10\,\text{k}\Omega$$

$$I_o = \frac{V_{\text{Th}}}{10{,}000 + 10^6/2s} = \frac{2sV_{\text{Th}}}{20{,}000s + 10^6}$$

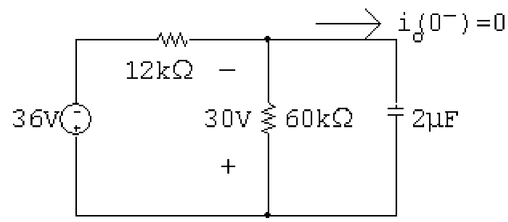
$$H(s) = \frac{I_o}{V_{\text{Th}}} = \frac{10^{-4}s}{s + 50}; \qquad H(\omega) = \frac{j\omega \times 10^{-4}}{j\omega + 50}$$

$$v_{\text{Th}} = \frac{5}{6}v_g = 30\,\text{sgn}(t); \qquad V_{\text{Th}} = \frac{60}{j\omega}$$

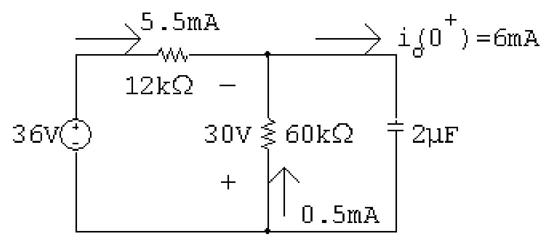
$$I_o = H(\omega)V_{\text{Th}}(\omega) = \left(\frac{60}{j\omega}\right)\left(\frac{j\omega \times 10^{-4}}{j\omega + 50}\right) = \frac{6 \times 10^{-3}}{j\omega + 50}$$

$$i_o(t) = 6e^{-50t}u(t)\,\text{mA}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



$$i_g(0^+) = \frac{30 + 36}{12} = 5.5\,\text{mA}$$

$$i_{60\text{k}}(0^+) = \frac{30}{60} = 0.5\,\text{mA}$$

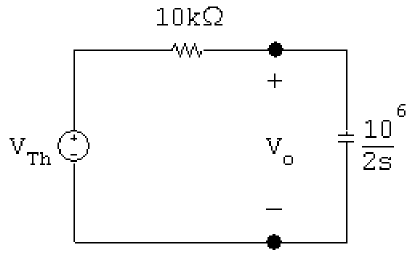
$$i_o(0^+) = 5.5 + 0.5 = 6\,\text{mA}$$

which agrees with our solution.
We also know $i_o(\infty) = 0$, which agrees with our solution.
The time constant with respect to the terminals of the capacitor is $R_{\text{Th}}C$ Thus,

$$\tau = (10{,}000)(2 \times 10^{-6}) = 20\,\text{ms}; \qquad \therefore \quad \frac{1}{\tau} = 50,$$

which also agrees with our solution.
Thus our solution makes sense in terms of known circuit behavior.

P 17.23 **[a]** From the solution of Problem 17.22 we have



$$V_o = \frac{V_{\rm Th}}{10^4 + (10^6/2s)} \cdot \frac{10^6}{2s}$$

$$H(s) = \frac{V_o}{V_{\rm Th}} = \frac{50}{s+50}$$

$$H(j\omega) = \frac{50}{j\omega + 50}$$

$$V_{\rm Th}(\omega) = \frac{60}{j\omega}$$

$$V_o(\omega) = H(j\omega)V_{\rm Th}(\omega) = \left(\frac{60}{j\omega}\right)\frac{50}{j\omega + 50}$$

$$= \frac{3000}{(j\omega)(j\omega+50)} = \frac{60}{j\omega} - \frac{60}{j\omega + 50}$$

$$v_o(t) = 30\text{sgn}(t) - 60e^{-50t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -30\,\text{V}$

$$v_o(0^+) = 30 - 60 = -30\,\text{V}$$

This makes sense because there cannot be an instantaneous change in the voltage across a capacitor.

$$v_o(\infty) = 30\,\text{V}$$

This agrees with $v_{\rm Th}(\infty) = 30$ V.
As in Problem 17.22 we know the time constant is 20 ms.

P 17.24 **[a]** $\dfrac{V_o}{V_g} = H(s) = \dfrac{4/s}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{400}{s^2 + 50s + 400} = \frac{400}{(s+10)(s+40)}$$

$$H(j\omega) = \frac{400}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{2400}{j\omega(j\omega+10)(j\omega+40)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega+10} + \frac{K_3}{j\omega+40}$$

$$K_1 = \frac{2400}{400} = 6; \qquad K_2 = \frac{2400}{(-10)(30)} = -8$$

$$K_3 = \frac{2400}{(-40)(-30)} = 2$$

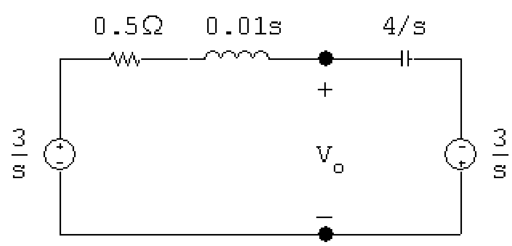
$$V_o(\omega) = \frac{6}{j\omega} - \frac{8}{j\omega+10} + \frac{2}{j\omega+40}$$

$$v_o(t) = 3\text{sgn}(t) - 8e^{-10t}u(t) + 2e^{-40t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -3\,\text{V}$

[c] $v_o(0^+) = 3 - 8 + 2 = -3\,\text{V}$

[d] For $t \geq 0^+$:



$$\frac{V_o - 3/s}{0.5 + 0.01s} + \frac{(V_o + 3/s)s}{4} = 0$$

$$V_o\left[\frac{100}{s+50} + \frac{s}{4}\right] = \frac{300}{s(s+50)} - 0.75$$

$$V_o = \frac{1200 - 3s^2 - 150s}{s(s+10)(s+40)} = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+40}$$

$$K_1 = \frac{1200}{400} = 3; \qquad K_2 = \frac{1200 - 300 + 1500}{(-10)(30)} = -8$$

$$K_3 = \frac{1200 - 4800 + 6000}{(-40)(-30)} = 2$$

$$v_o(t) = (3 - 8e^{-10t} + 2e^{-40t})u(t)\,\text{V}$$

[e] Yes.

P 17.25 **[a]** $I_o = \dfrac{V_g}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{I_o}{V_g} = \frac{100s}{s^2 + 50s + 400} = \frac{100s}{(s+10)(s+40)}$$

$$H(\omega) = \frac{100(j\omega)}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{600}{(j\omega + 10)(j\omega + 40)}$$

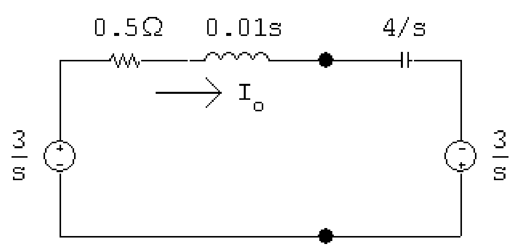
$$= \frac{20}{j\omega + 10} - \frac{20}{j\omega + 40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 0$

[d]



$$I_o = \frac{6/s}{0.5 + 0.01s + 4/s} = \frac{600}{s^2 + 50s + 400}$$

$$= \frac{600}{(s+10)(s+40)} = \frac{20}{s+10} - \frac{20}{s+40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[e] Yes.

P 17.26 **[a]** $I_o = \dfrac{I_g R}{R + 1/sC} = \dfrac{RCsI_g}{RCs + 1}$; $\qquad H(s) = \dfrac{I_o}{I_g} = \dfrac{s}{s + 1/RC}$

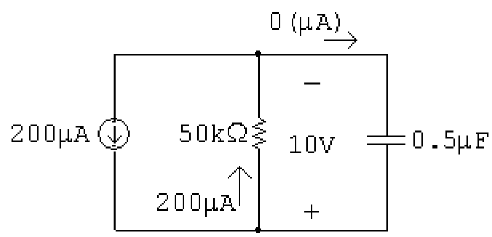
$$\frac{1}{RC} = \frac{10^6}{25 \times 10^3} = 40; \qquad H(\omega) = \frac{j\omega}{j\omega + 40}$$

$$i_g = 200\text{sgn}(t)\,\mu\text{A}; \qquad I_g = (200 \times 10^{-6})\left(\frac{2}{j\omega}\right) = \frac{400 \times 10^{-6}}{j\omega}$$

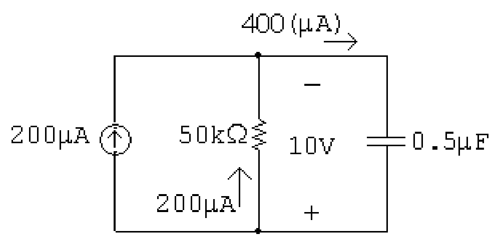
$$I_o = I_g[H(\omega)] = \frac{400 \times 10^{-6}}{j\omega} \cdot \frac{j\omega}{j\omega + 40} = \frac{400 \times 10^{-6}}{j\omega + 40}$$

$$i_o(t) = 400e^{-40t}u(t)\,\mu\text{A}$$

[b] Yes, at the time the source current jumps from $-200\,\mu$A to $+200\,\mu$A the capacitor is charged to $(200)(50) \times 10^{-3} = 10$ V, positive at the lower terminal. The circuit at $t = 0^-$ is



At $t = 0^+$ the circuit is



The time constant is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms.

$$\therefore \quad \frac{1}{\tau} = 40 \quad \therefore \quad \text{for } t > 0, \quad i_o = 400e^{-40t}\,\mu\text{A}$$

P 17.27 **[a]** $V_o = \dfrac{I_gR(1/sC)}{R + (1/sC)} = \dfrac{I_gR}{RCs + 1}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + (1/RC)} = \frac{2 \times 10^6}{s + 40}$$

$$H(\omega) = \frac{2 \times 10^6}{40 + j\omega}; \qquad I_g(\omega) = \frac{400 \times 10^{-6}}{j\omega}$$

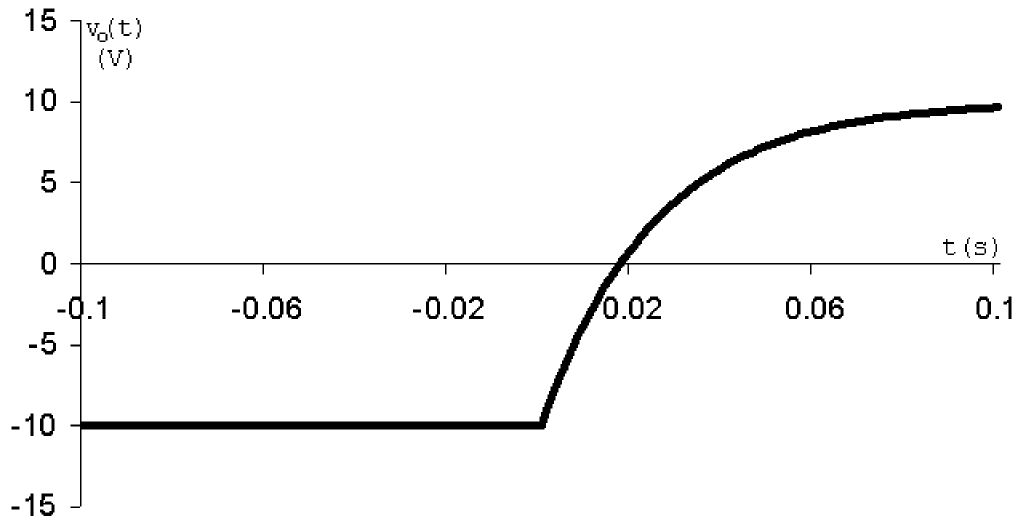
$$V_o(\omega) = H(\omega)I_g(\omega) = \left(\frac{400 \times 10^{-6}}{j\omega}\right)\left(\frac{2 \times 10^6}{40 + j\omega}\right)$$

$$= \frac{800}{j\omega(40 + j\omega)} = \frac{20}{j\omega} - \frac{20}{40 + j\omega}$$

$$v_o(t) = 10\text{sgn}(t) - 20e^{-40t}u(t)\,\text{V}$$

[b] Yes, at the time the current source jumps from -200 to $+200\,\mu$A the capacitor is charged to -10 V. That is, at $t = 0^-$,
$v_o(0^-) = (50 \times 10^3)(-200 \times 10^{-6}) = -10$ V.

At $t = \infty$ the capacitor will be charged to $+10$ V. That is,
$v_o(\infty) = (50 \times 10^3)(200 \times 10^{-6}) = 10$ V
The time constant of the circuit is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms, so $1/\tau = 40$. The function $v_o(t)$ is plotted below:



P 17.28 **[a]** $i_g = 3e^{-5|t|}$

$$\therefore \qquad I_g(\omega) = \frac{3}{j\omega + 5} + \frac{3}{-j\omega + 5} = \frac{30}{(j\omega + 5)(-j\omega + 5)}$$

$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g$$

$$\therefore \qquad \frac{V_o}{I_g} = H(s) = \frac{10}{s + 1}; \qquad H(\omega) = \frac{10}{j\omega + 1}$$

$$V_o(\omega) = I_g(\omega)H(\omega) = \frac{300}{(j\omega + 1)(j\omega + 5)(-j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{j\omega + 5} + \frac{K_3}{-j\omega + 5}$$

$$K_1 = \frac{300}{(4)(6)} = 12.5$$

$$K_2 = \frac{300}{(-4)(10)} = -7.5$$

$$K_3 = \frac{300}{(6)(10)} = 5$$

$$V_o(\omega) = \frac{12.5}{j\omega + 1} - \frac{7.5}{j\omega + 5} + \frac{5}{-j\omega + 5}$$

$$v_o(t) = [12.5e^{-t} - 7.5e^{-5t}]u(t) + 5e^{5t}u(-t) \text{ V}$$

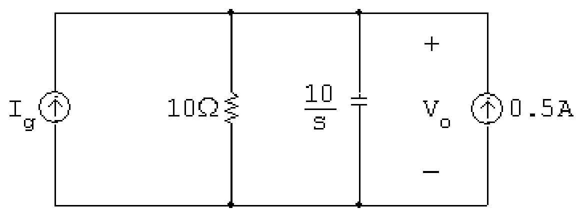
[b] $v_o(0^-) = 5\,\text{V}$

[c] $v_o(0^+) = 12.5 - 7.5 = 5\,\text{V}$

[d] $i_g = 3e^{-5t}u(t), \quad t \geq 0^+$

$$I_g = \frac{3}{s+5}; \qquad H(s) = \frac{10}{s+1}$$

$$v_o(0^+) = 5\,\text{V}; \qquad \gamma C = 0.5$$



$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g + 0.5$$

$$V_o(s+1) = \frac{30}{s+5} + 5$$

$$V_o = \frac{30}{(s+5)(s+1)} + \frac{5}{s+1}$$

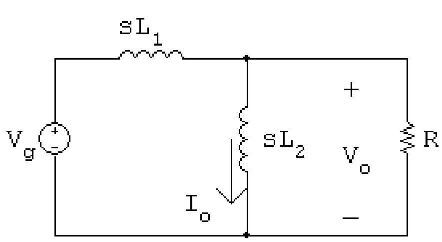
$$= \frac{-7.5}{s+5} + \frac{7.5}{s+1} + \frac{5}{s+1} = \frac{12.5}{s+1} - \frac{7.5}{s+5}$$

$$\therefore \quad v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

P 17.29 **[a]**



$$\frac{V_o - V_g}{sL_1} + \frac{V_o}{sL_2} + \frac{V_o}{R} = 0$$

$$\therefore \quad V_o = \frac{RV_g}{L_1\left[s + R\left(\frac{1}{L_1} + \frac{1}{L_2}\right)\right]}$$

$$I_o = \frac{V_o}{sL_2}$$

$$\therefore \quad \frac{I_o}{V_g} = H(s) = \frac{R/L_1L_2}{s(s + R[(1/L_1) + (1/L_2)])}$$

$$\frac{R}{L_1L_2} = 12 \times 10^5$$

$$R\left(\frac{1}{L_1} + \frac{1}{L_2}\right) = 3 \times 10^4$$

$$\therefore \quad H(s) = \frac{12 \times 10^5}{s(s + 3 \times 10^4)}$$

$$H(\omega) = \frac{12 \times 10^5}{j\omega(j\omega + 3 \times 10^4)}$$

$$V_g(\omega) = 125\pi[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{1500\pi \times 10^5[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]}{j\omega(j\omega + 3 \times 10^4)}$$

$$i_o(t) = \frac{1500\pi \times 10^5}{2\pi} \int_{-\infty}^{\infty} \frac{[\delta(\omega + 4 \times 10^4) + \delta(\omega - 4 \times 10^4)]e^{jt\omega}}{j\omega(j\omega + 3 \times 10^4)}\, d\omega$$

$$i_o(t) = 750 \times 10^5 \left\{ \frac{e^{-j40{,}000t}}{-j40{,}000(30{,}000 - j40{,}000)} + \frac{e^{j40{,}000t}}{j40{,}000(30{,}000 + j40{,}000)} \right\}$$

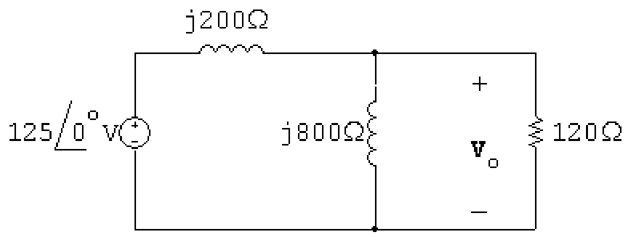
$$= \frac{75 \times 10^6}{4 \times 10^8} \left\{ \frac{e^{-j40{,}000t}}{-j(3 - j4)} + \frac{e^{j40{,}000t}}{j(3 + j4)} \right\}$$

$$= \frac{75}{400} \left\{ \frac{e^{-j40{,}000t}}{5\underline{/-143.13^\circ}} + \frac{e^{j40{,}000t}}{5\underline{/143.13^\circ}} \right\}$$

$$= 0.075\cos(40{,}000t - 143.13^\circ)\,\text{A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\,\text{mA}$$

[b] In the phasor domain:



$$\frac{\mathbf{V}_o - 125}{j200} + \frac{\mathbf{V}_o}{j800} + \frac{\mathbf{V}_o}{120} = 0$$

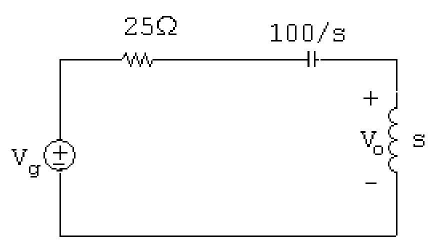
$$12\mathbf{V}_o - 1500 + 3\mathbf{V}_o + j20\mathbf{V}_o = 0$$

$$\mathbf{V}_o = \frac{1500}{15 + j20} = 60\underline{/-53.13^\circ}\,\text{V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{j800} = 75 \times 10^{-3}\underline{/-143.13^\circ}\,\text{A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\,\text{mA}$$

P 17.30 **[a]**



$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \qquad H(\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = -450e^{10t}u(-t) - 450e^{-10t}u(t)\,\text{V}$$

$$V_g = -\frac{450}{-j\omega + 10} - \frac{450}{j\omega + 10}$$

$$V_o(\omega) = H(\omega)V_g = \frac{-450(j\omega)^2}{(-j\omega+10)(j\omega+5)(j\omega+20)}$$

$$+\frac{-450(j\omega)^2}{(j\omega+10)(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{-j\omega+10} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20} + \frac{K_4}{j\omega+5} + \frac{K_5}{j\omega+10} + \frac{K_6}{j\omega+20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = -100 \qquad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = -50 \qquad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = 400 \qquad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

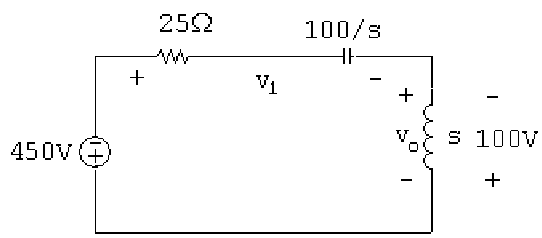
$$V_o(\omega) = \frac{-100}{-j\omega + 10} + \frac{-200}{j\omega + 5} + \frac{-800}{j\omega + 20} + \frac{900}{j\omega + 10}$$

$$v_o = -100e^{10t}u(-t) + [900e^{-10t} - 200e^{-5t} - 800e^{-20t}]u(t)\ \text{V}$$

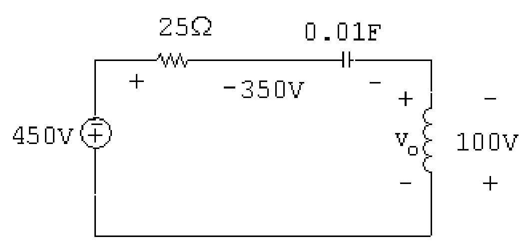
[b] $v_o(0^-) = -100\,\text{V}$

[c] $v_o(0^+) = 900 - 200 - 800 = -100\,\text{V}$

[d] At $t = 0^-$ the circuit is

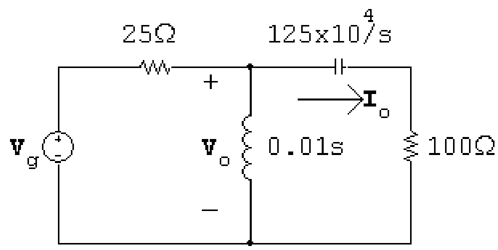


Therefore, the solution predicts $v_1(0^-)$ will be -350 V.
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the $25\,\Omega$ resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.
At $t = 0^+$ the circuit is



From the circuit at $t = 0^+$ we see that v_o must be -100 V, which is consistent with the solution for v_o obtained in part (c).

P 17.31



$$\frac{V_o - V_g}{25} + \frac{100V_o}{s} + \frac{V_o s}{100s + 125 \times 10^4} = 0$$

$$\therefore \quad V_o = \frac{s(100s + 125 \times 10^4)V_g}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$I_o = \frac{sV_o}{100s + 125 \times 10^4}$$

$$H(s) = \frac{I_o}{V_g} = \frac{s^2}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$H(\omega) = \frac{-8 \times 10^{-3}\omega^2}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$V_g(\omega) = 300\pi[\delta(\omega + 5000) + \delta(\omega - 5000)]$$

$$I_o(\omega) = H(\omega)V_g(\omega) = \frac{-2.4\pi\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$i_o(t) = \frac{-2.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega} e^{jt\omega}\, d\omega$$

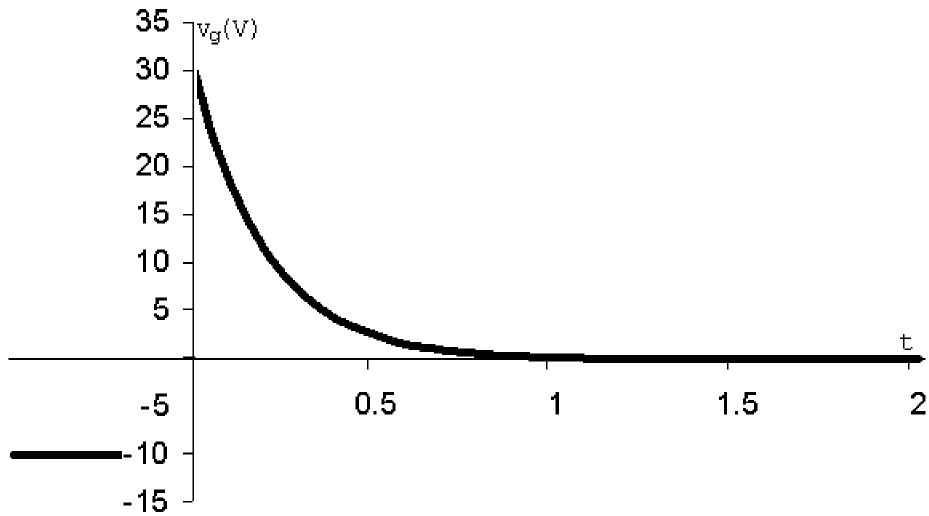
$$= -1.2\left\{\frac{25 \times 10^6 e^{-j5000t}}{-j(12{,}000)(5000)} + \frac{25 \times 10^6 e^{j5000t}}{j(12{,}000)(5000)}\right\}$$

$$= \frac{6}{12}\left\{\frac{e^{-j5000t}}{-j} + \frac{e^{j5000t}}{j}\right\}$$

$$= 0.5[e^{-j(5000t+90^\circ)} + e^{j(5000t+90^\circ)}]$$

$$i_o(t) = 1\cos(5000t + 90^\circ)\,\text{A}$$

P 17.32 **[a]**



From the plot of v_g note that v_g is -10 V for an infinitely long time before $t = 0$. Therefore

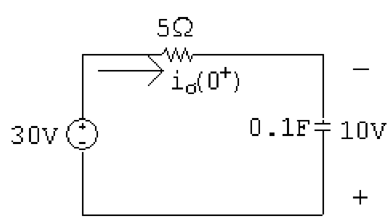
$$\therefore \qquad v_o(0^-) = -10\,\text{V}$$

There cannot be an instantaneous change in the voltage across a capacitor, so

$$v_o(0^+) = -10\,\text{V}$$

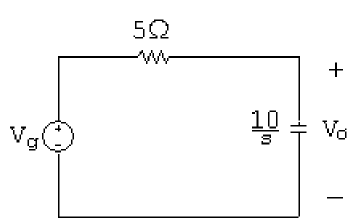
[b] $i_o(0^-) = 0\,\text{A}$

At $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{30 - (-10)}{5} = \frac{40}{5} = 8\,\text{A}$$

[c] The s-domain circuit is



$$V_o = \left[\frac{V_g}{5 + (10/s)}\right]\left(\frac{10}{s}\right) = \frac{2V_g}{s+2}$$

$$\frac{V_o}{V_g} = H(s) = \frac{2}{s+2}$$

$$H(\omega) = \frac{2}{j\omega + 2}$$

$$V_g(\omega) = 5\left(\frac{2}{j\omega}\right) - 5[2\pi\delta(\omega)] + \frac{30}{j\omega + 5} = \frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega + 5}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{2}{j\omega + 2}\left[\frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega + 5}\right]$$

$$= \frac{20}{j\omega(j\omega + 2)} - \frac{20\pi\delta(\omega)}{j\omega + 2} + \frac{60}{(j\omega + 2)(j\omega + 5)}$$

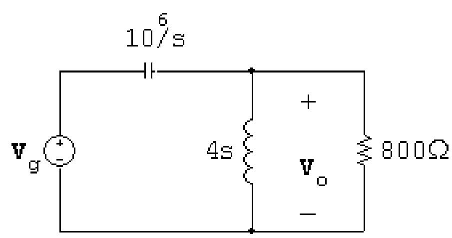
$$= \frac{K_0}{j\omega} + \frac{K_1}{j\omega + 2} + \frac{K_2}{j\omega + 2} + \frac{K_3}{j\omega + 5} - \frac{20\pi\delta(\omega)}{j\omega + 2}$$

$$K_0 = \frac{20}{2} = 10; \quad K_1 = \frac{20}{-2} = -10; \quad K_2 = \frac{60}{3} = 20; \quad K_3 = \frac{60}{-3} = -20$$

$$V_o(\omega) = \frac{10}{j\omega} + \frac{10}{j\omega + 2} - \frac{20}{j\omega + 5} - \frac{20\pi\delta(\omega)}{j\omega + 2} = \frac{10}{j\omega} + \frac{10}{j\omega + 2} + \frac{20}{j\omega + 5} - 10\pi\delta(\omega)$$

$$v_o(t) = 5\text{sgn}(t) + [10e^{-2t} - 20e^{-5t}]u(t) - 5\text{ V}$$

P 17.33 **[a]**



$$\frac{(V_o - V_g)s}{10^6} + \frac{V_o}{4s} + \frac{V_o}{800} = 0$$

$$\therefore \quad V_o = \frac{s^2 V_g}{s^2 + 1250s + 25 \times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s + 250)(s + 1000)}$$

$$H(\omega) = \frac{(j\omega)^2}{(j\omega + 250)(j\omega + 1000)}$$

$$v_g = 45e^{-500|t|}; \qquad V_g(\omega) = \frac{45{,}000}{(j\omega + 500)(-j\omega + 500)}$$

$$\therefore \quad V_o(\omega) = H(\omega)V_g(\omega) = \frac{45{,}000(j\omega)^2}{(j\omega + 250)(j\omega + 500)(j\omega + 1000)(-j\omega + 500)}$$

$$= \frac{K_1}{j\omega + 250} + \frac{K_2}{j\omega + 500} + \frac{K_3}{j\omega + 1000} + \frac{K_4}{-j\omega + 500}$$

$$K_1 = \frac{45{,}000(-250)^2}{(250)(750)(750)} = 20$$

$$K_2 = \frac{45{,}000(-500)^2}{(-250)(500)(1000)} = -90$$

$$K_3 = \frac{45{,}000(-1000)^2}{(-750)(-500)(1500)} = 80$$

$$K_4 = \frac{45{,}000(500)^2}{(750)(1000)(1500)} = 10$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) + 10e^{500t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 10\,\text{V}; \qquad V_o(0^+) = 20 - 90 + 80 = 10\,\text{V}$

$v_o(\infty) = 0\,\text{V}$

[c] $I_L = \dfrac{V_o}{4s} = \dfrac{0.25sV_g}{(s+250)(s+1000)}$

$$H(s) = \frac{I_L}{V_g} = \frac{0.25s}{(s+250)(s+1000)}$$

$$H(\omega) = \frac{0.25(j\omega)}{(j\omega+250)(j\omega+1000)}$$

$$I_L(\omega) = \frac{0.25(j\omega)(45{,}000)}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_4 = \frac{(0.25)(500)(45{,}000)}{(750)(1000)(1500)} = 5\,\text{mA}$$

$$i_L(t) = 5e^{500t}u(-t); \qquad \therefore \quad i_L(0^-) = 5\,\text{mA}$$

$$K_1 = \frac{(0.25)(-250)(45{,}000)}{(250)(750)(750)} = -20\,\text{mA}$$

$$K_2 = \frac{(0.25)(-500)(45{,}000)}{(-250)(500)(1000)} = 45\,\text{mA}$$

$$K_3 = \frac{(0.25)(-1000)(45{,}000)}{(-750)(-500)(1500)} = -20\,\text{mA}$$

$$\therefore \quad i_L(0^+) = K_1 + K_2 + K_3 = -20 + 45 - 20 = 5\,\text{mA}$$

Checks, i.e., $\quad i_L(0^+) = i_L(0^-) = 5\,\text{mA}$

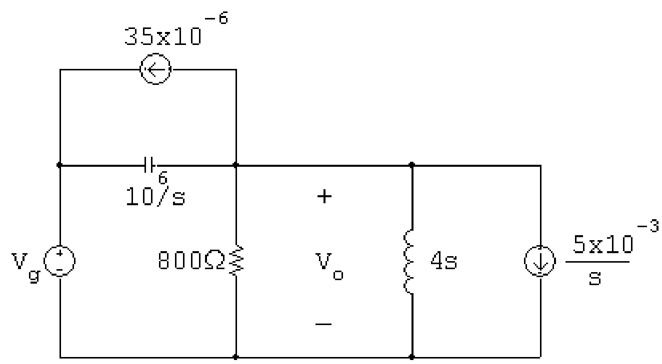
At $t = 0^-$:

$v_C(0^-) = 45 - 10 = 35\,\text{V}$

At $t = 0^+$:

$v_C(0^+) = 45 - 10 = 35\,\text{V}$

[d] We can check the correctness of our solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes



$$\frac{V_o}{800} + \frac{V_o}{4s} + \frac{(V_o - V_g)s}{10^6} + 35 \times 10^{-6} + \frac{5 \times 10^{-3}}{s} = 0$$

$$\therefore \quad (s^2 + 1250s + 25 \times 10^4)V_o = s^2 V_g - (35s + 5000)$$

$$v_g(t) = 45e^{-500t}u(t)\,\text{V}; \qquad V_g = \frac{45}{s + 500}$$

$$\therefore \quad (s + 250)(s + 1000)V_o = \frac{45s^2 - (35s + 5000)(s + 500)}{(s + 500)}$$

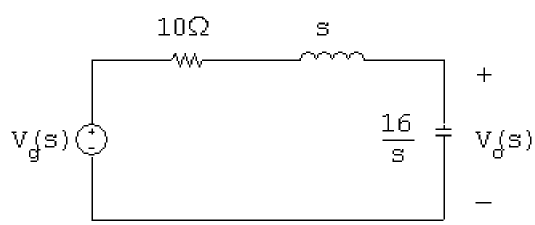
$$\therefore \quad V_o = \frac{10s^2 - 22{,}500s - 250 \times 10^4}{(s + 250)(s + 500)(s + 1000)}$$

$$= \frac{20}{s + 250} - \frac{90}{s + 500} + \frac{80}{s + 1000}$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.34 **[a]**



$$V_g(\omega) = \frac{36}{4 - j\omega} - \frac{36}{4 + j\omega} = \frac{72j\omega}{(4 - j\omega)(4 + j\omega)}$$

$$V_o(s) = \frac{(16/s)}{10 + s + (16/s)} V_g(s)$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{16}{s^2 + 10s + 16} = \frac{16}{(s+2)(s+8)}$$

$$H(\omega) = \frac{16}{(j\omega + 2)(j\omega + 8)}$$

$$V_o(\omega) = H(\omega) \cdot V_g(\omega) = \frac{1152 j\omega}{(4 - j\omega)(4 + j\omega)(2 + j\omega)(8 + j\omega)}$$

$$= \frac{K_1}{4 - j\omega} + \frac{K_2}{4 + j\omega} + \frac{K_3}{2 + j\omega} + \frac{K_4}{8 + j\omega}$$

$$K_1 = \frac{1152(4)}{(8)(6)(12)} = 8$$

$$K_2 = \frac{1152(-4)}{(8)(-2)(4)} = 72$$

$$K_3 = \frac{1152(-2)}{(6)(2)(6)} = -32$$

$$K_4 = \frac{1152(-8)}{(12)(-4)(-6)} = -32$$

$$\therefore \quad V_o(j\omega) = \frac{8}{4 - j\omega} + \frac{72}{4 + j\omega} - \frac{32}{2 + j\omega} - \frac{32}{8 + j\omega}$$

$$\therefore \quad v_o(t) = 8e^{4t}u(-t) + [72e^{-4t} - 32e^{-2t} - 32e^{-8t}]u(t)\text{V}$$

[b] $v_o(0^-) = 8$ V

[c] $v_o(0^+) = 72 - 32 - 32 = 8$ V

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

P 17.35 $$V_o(s) = \frac{10}{s} + \frac{30}{s+20} - \frac{40}{s+30} = \frac{600(s+10)}{s(s+20)(s+30)}$$

$$V_o(s) = H(s) \cdot \frac{15}{s}$$

$$\therefore \qquad H(s) = \frac{40(s+10)}{(s+20)(s+30)}$$

$$\therefore \qquad H(\omega) = \frac{40(j\omega + 10)}{(j\omega + 20)(j\omega + 30)}$$

$$\therefore \qquad V_o(\omega) = \frac{30}{j\omega} \cdot \frac{40(j\omega + 10)}{(j\omega + 20)(j\omega + 30)} = \frac{1200(j\omega + 10)}{j\omega(j\omega + 20)(j\omega + 30)}$$

$$v_o(\omega) = \frac{20}{j\omega} + \frac{60}{j\omega + 20} - \frac{80}{j\omega + 30}$$

$$v_o(t) = 10\text{sgn}(t) + [60e^{-20t} - 80e^{-30t}]u(t)\text{ V}$$

P 17.36 **[a]** $f(t) = \frac{1}{2\pi}\left\{\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_0^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \frac{1/\pi}{1+t^2}$

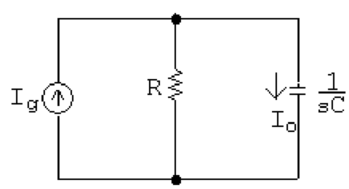
[b] $W = 2\int_0^{\infty} \frac{(1/\pi)^2}{(1+t^2)^2}\,dt = \frac{2}{\pi^2}\int_0^{\infty}\frac{dt}{(1+t^2)^2} = \frac{1}{2\pi}$ **J**

[c] $W = \frac{1}{\pi}\int_0^{\infty} e^{-2\omega}\,d\omega = \frac{1}{\pi}\frac{e^{-2\omega}}{-2}\Big|_0^{\infty} = \frac{1}{2\pi}$ **J**

[d] $\frac{1}{\pi}\int_0^{\omega_1} e^{-2\omega}\,d\omega = \frac{0.9}{2\pi}, \qquad 1 - e^{-2\omega_1} = 0.9, \qquad e^{2\omega_1} = 10$

$\omega_1 = (1/2)\ln 10 \cong 1.15$ rad/s

P 17.37



$$I_o = \frac{I_g R}{R + (1/sC)} = \frac{RCsI_g}{RCs + 1}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + (1/RC)}$$

$$RC = (100 \times 10^3)(1.25 \times 10^{-6}) = 125 \times 10^{-3}; \qquad \frac{1}{RC} = \frac{1}{0.125} = 8$$

$$H(s) = \frac{s}{s+8}; \qquad H(\omega) = \frac{j\omega}{j\omega + 8}$$

$$I_g(\omega) = \frac{30 \times 10^{-6}}{j\omega + 2}$$

$$I_o(\omega) = H(\omega)I_g(\omega) = \frac{30 \times 10^{-6} j\omega}{(j\omega + 2)(j\omega + 8)}$$

$$|I_o(\omega)| = \frac{\omega(30 \times 10^{-6})}{(\sqrt{\omega^2+4})(\sqrt{\omega^2+64})}$$

$$|I_o(\omega)|^2 = \frac{900 \times 10^{-12}\omega^2}{(\omega^2+4)(\omega^2+64)} = \frac{K_1}{\omega^2+4} + \frac{K_2}{\omega^2+64}$$

$$K_1 = \frac{(900 \times 10^{-12})(-4)}{(60)} = -60 \times 10^{-12}$$

$$K_2 = \frac{(900 \times 10^{-12})(-64)}{(-60)} = 960 \times 10^{-12}$$

$$|I_o(\omega)|^2 = \frac{960 \times 10^{-12}}{\omega^2+64} - \frac{60 \times 10^{-12}}{\omega^2+4}$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |I_o(\omega)|^2\,d\omega = \frac{960 \times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+64} - \frac{60 \times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2+4}$$

$$= \frac{120 \times 10^{-12}}{\pi}\tan^{-1}\frac{\omega}{8}\Big|_0^\infty - \frac{30 \times 10^{-12}}{\pi}\tan^{-1}\frac{\omega}{2}\Big|_0^\infty$$

$$= \left(\frac{120}{\pi}\cdot\frac{\pi}{2} - \frac{30}{\pi}\cdot\frac{\pi}{2}\right) \times 10^{-12} = (60-15) \times 10^{-12} = 45\,\text{pJ}$$

Between 0 and 4 rad/s

$$W_{1\Omega} = \left[\frac{120}{\pi}\tan^{-1}\frac{1}{2} - \frac{30}{\pi}\tan^{-1}2\right] \times 10^{-12} = 7.14\,\text{pJ}$$

$$\% = \frac{7.14}{45}(100) = 15.86\%$$

P 17.38 **[a]** $V_g(\omega) = \dfrac{60}{(j\omega+1)(-j\omega+1)}$

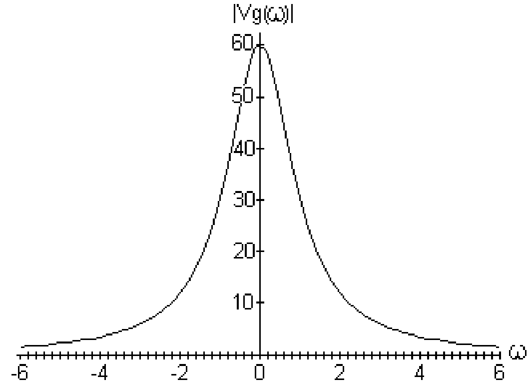
$$H(s) = \frac{V_o}{V_g} = \frac{0.4}{s+0.5}; \qquad H(\omega) = \frac{0.4}{(j\omega+0.5)}$$

$$V_o(\omega) = \frac{24}{(j\omega+1)(j\omega+0.5)(-j\omega+1)}$$

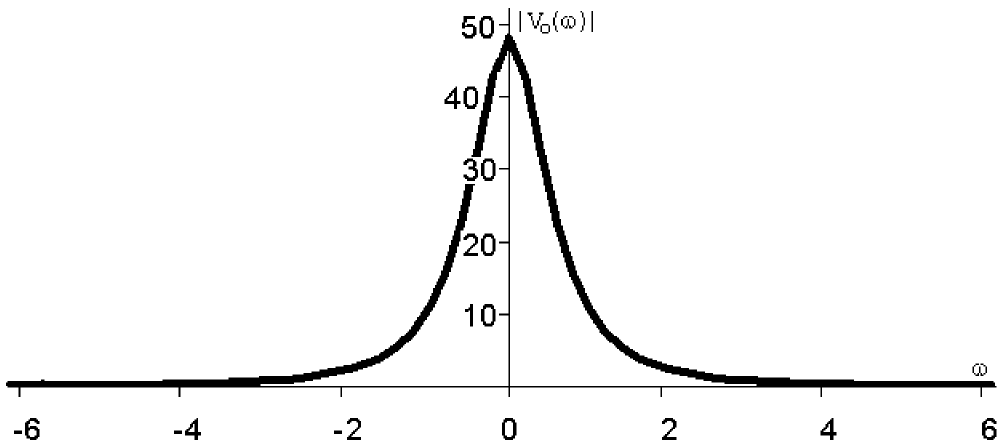
$$V_o(\omega) = \frac{-24}{j\omega+1} + \frac{32}{j\omega+0.5} + \frac{8}{-j\omega+1}$$

$$v_o(t) = [-24e^{-t} + 32e^{-t/2}]u(t) + 8e^{t}u(-t)\,\text{V}$$

[b] $|V_g(\omega)| = \dfrac{60}{(\omega^2 + 1)}$



[c] $|V_o(\omega)| = \dfrac{24}{(\omega^2 + 1)\sqrt{\omega^2 + 0.25}}$



[d] $W_i = 2\int_0^\infty 900e^{-2t}\,dt = 1800\,\dfrac{e^{-2t}}{-2}\Big|_0^\infty = 900\,\text{J}$

[e] $W_o = \int_{-\infty}^{0} 64e^{2t}\,dt + \int_0^\infty (-24e^{-t} + 32e^{-t/2})^2\,dt$

$= 32 + \int_0^\infty [576e^{-2t} - 1536e^{-3t/2} + 1024e^{-t}]\,dt$

$= 32 + 288 - 1024 + 1024 = 320\,\text{J}$

[f] $|V_g(\omega)| = \dfrac{60}{\omega^2+1}, \quad |V_g^2(\omega)| = \dfrac{3600}{(\omega^2+1)^2}$

$$W_g = \frac{3600}{\pi}\int_0^2 \frac{d\omega}{(\omega^2+1)^2}$$

$$= \frac{3600}{\pi}\left\{\frac{1}{2}\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)\Big|_0^2\right\}$$

$$= \frac{1800}{\pi}\left(\frac{2}{5} + \tan^{-1}2\right) = 863.53\,\text{J}$$

$$\therefore \quad \% = \left(\frac{863.53}{900}\right)\times 100 = 95.95\%$$

[g] $|V_o(\omega)|^2 = \dfrac{576}{(\omega^2+1)^2(\omega^2+0.25)}$

$$= \frac{1024}{\omega^2+0.25} - \frac{768}{(\omega^2+1)^2} - \frac{1024}{(\omega^2+1)}$$

$$W_o = \frac{1}{\pi}\left\{1024\cdot 2\cdot \tan^{-1}2\omega\Big|_0^2 - 768\left(\frac{1}{2}\right)\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)_0^2\right.$$

$$\left. -1024\tan^{-1}\omega\Big|_0^2\right\}$$

$$= \frac{2048}{\pi}\tan^{-1}4 - \frac{384}{\pi}\left(\frac{2}{5} + \tan^{-1}2\right) - \frac{1024}{\pi}\tan^{-1}2$$

$$= 319.2\,\text{J}$$

$$\% = \frac{319.2}{320}\times 100 = 99.75\%$$

P 17.39 $I_o = \dfrac{0.5sI_g}{0.5s+25} = \dfrac{sI_g}{s+50}$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s+50}$$

$$H(\omega) = \frac{j\omega}{j\omega+50}$$

$$I(\omega) = \frac{12}{j\omega+10}$$

$$I_o(\omega) = H(\omega)I(\omega) = \frac{12(j\omega)}{(j\omega+10)(j\omega+50)}$$

$$|I_o(\omega)| = \frac{12\omega}{\sqrt{(\omega^2+100)(\omega^2+2500)}}$$

$$|I_o(\omega)|^2 = \frac{144\omega^2}{(\omega^2+100)(\omega^2+2500)}$$

$$= \frac{-6}{\omega^2+100} + \frac{150}{\omega^2+2500}$$

$$W_o(\text{total}) = \frac{1}{\pi}\int_0^\infty \frac{150d\omega}{\omega^2+2500} - \frac{1}{\pi}\int_0^\infty \frac{6d\omega}{\omega^2+100}$$

$$= \frac{3}{\pi}\tan^{-1}\left(\frac{\omega}{50}\right)\Big|_0^\infty - \frac{0.6}{\pi}\tan^{-1}\left(\frac{\omega}{10}\right)\Big|_0^\infty$$

$$= 1.5 - 0.3 = 1.2\,\text{J}$$

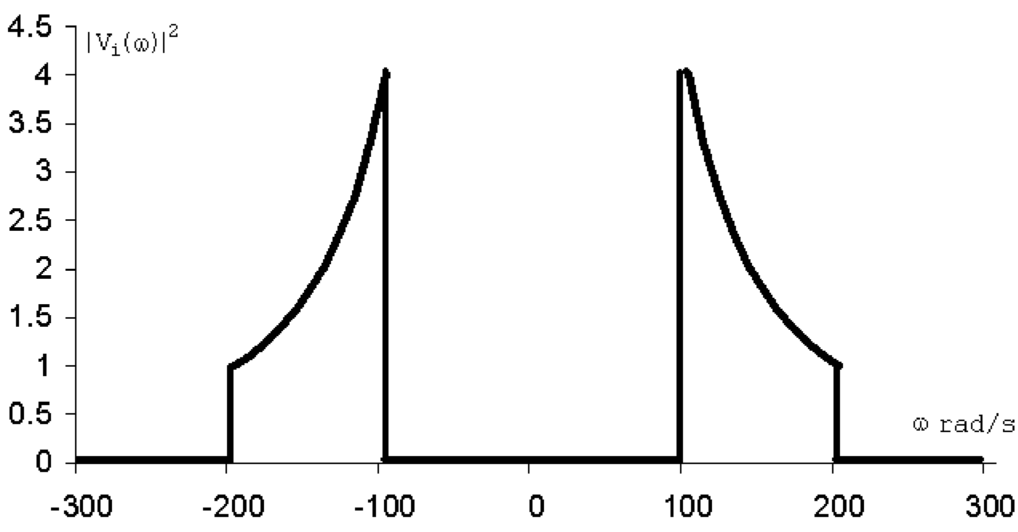
$$W_o(\text{0–100 rad/s}) = \frac{3}{\pi}\tan^{-1}(2) - \frac{0.6}{\pi}\tan^{-1}(10)$$

$$= 1.06 - 0.28 = 0.78\,\text{J}$$

Therefore, the percent between 0 and 100 rad/s is

$$\frac{0.78}{1.2}(100) = 64.69\%$$

P 17.40 **[a]** $|V_i(\omega)|^2 = \dfrac{4\times10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \dfrac{4\times10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \dfrac{4\times10^4}{200^2} = 1$



[b] $V_o = \frac{V_i R}{R + (1/sC)} = \frac{RCV_i}{RCs + 1}$

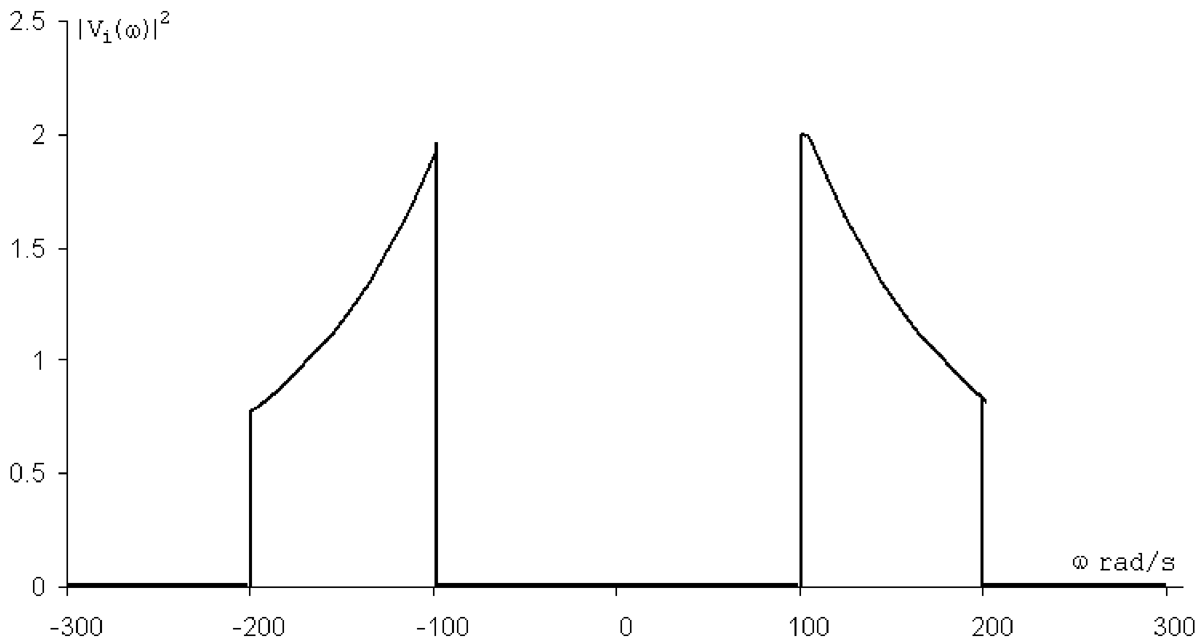
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(\omega) = \frac{j\omega}{j\omega + 100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|} \cdot \frac{|\omega|}{\sqrt{\omega^2 + 10^4}} = \frac{200}{\sqrt{\omega^2 + 10^4}}$$

$$|V_o(\omega)|^2 = \frac{4 \times 10^4}{\omega^2 + 10^4}, \quad 100 \leq \omega \leq 200 \text{ rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4 \times 10^4}{10^4 + 10^4} = 2; \qquad |V_o(200)|^2 = \frac{4 \times 10^4}{5 \times 10^4} = 0.8$$



[c] $W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2} \, d\omega = \frac{4 \times 10^4}{\pi} \left[-\frac{1}{\omega} \right]_{100}^{200}$

$$= \frac{4 \times 10^4}{\pi} \left[\frac{1}{100} - \frac{1}{200} \right] = \frac{200}{\pi} \cong 63.66 \text{ J}$$

[d] $W_{1\Omega} = \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2 + 10^4} \, d\omega = \frac{4 \times 10^4}{\pi} \cdot \tan^{-1} \frac{\omega}{100} \Big|_{100}^{200}$

$$= \frac{400}{\pi} [\tan^{-1} 2 - \tan^{-1} 1] \cong 40.97 \text{ J}$$

P 17.41 **[a]** $V_i(\omega) = \dfrac{A}{a + j\omega}; \qquad |V_i(\omega)| = \dfrac{A}{\sqrt{a^2 + \omega^2}}$

$$H(s) = \frac{s}{s+\alpha}; \qquad H(\omega) = \frac{j\omega}{\alpha + j\omega}; \qquad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2 + \omega^2}}$$

Therefore $\quad |V_o(\omega)| = \dfrac{\omega A}{\sqrt{(a^2+\omega^2)(\alpha^2+\omega^2)}}$

Therefore $\quad |V_o(\omega)|^2 = \dfrac{\omega^2 A^2}{(a^2+\omega^2)(\alpha^2+\omega^2)}$

$W_{\text{IN}} = \displaystyle\int_0^\infty A^2 e^{-2at}\,dt = \frac{A^2}{2a}; \qquad$ when $\alpha = a$ we have

$$W_{\text{OUT}}(a) = \frac{A^2}{\pi}\int_0^a \frac{\omega^2\,d\omega}{(\omega^2+a^2)^2} = \frac{A^2}{\pi}\left\{\int_0^a \frac{d\omega}{a^2+\omega^2} - \int_0^a \frac{a^2\,d\omega}{(a^2+\omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2} - 1\right)$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2+\omega^2)^2}\right] d\omega = \frac{A^2}{4a}$$

Therefore $\quad \dfrac{W_{\text{OUT}}(a)}{W_{\text{OUT}}(\text{total})} = 0.5 - \dfrac{1}{\pi} = 0.1817 \quad$ or $\quad 18.17\%$

[b] When $\alpha \neq a$ we have

$$W_{\text{OUT}}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2+\omega^2} + \frac{K_2}{\alpha^2+\omega^2}\right] d\omega\right\}$$

where $\quad K_1 = \dfrac{a^2}{a^2-\alpha^2} \quad$ and $\quad K_2 = \dfrac{-\alpha^2}{a^2-\alpha^2}$

Therefore

$$W_{\text{OUT}}(\alpha) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a+\alpha)}$$

Therefore $\quad \dfrac{W_{\text{OUT}}(\alpha)}{W_{\text{OUT}}(\text{total})} = \dfrac{2}{\pi(a-\alpha)} \cdot \left[a\tan^{-1}\left(\dfrac{\alpha}{a}\right) - \dfrac{\alpha\pi}{4}\right]$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.