

AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 210

Electric Circuits

Fall 2005-2006

Prof. Kabalan

QUIZ II

December 9, 2005

OPEN BOOK

DURATION: 90 minutes

NO PROGRAMMABLE CALCULATOR IS ALLOWED

NAME:

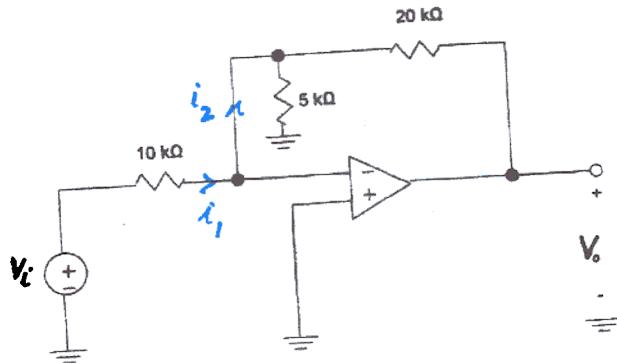
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INSTRUCTIONS:

1. 8 problems
 2. 14 pages
 3. Total Grade 50.
 4. Provide your answers on the question sheet only.
 5. The scratch sheet will not be considered in grading.
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Problem 1 (4 points)

Find the voltage gain V_o / V_i .



$$i_1 = \frac{V_i - V_n}{10^4}$$

$$V_n = V_P = 0 \Rightarrow i_1 = \frac{V_i}{10^4}$$

$$i_2 = -\frac{V_o}{20 \times 10^3}$$

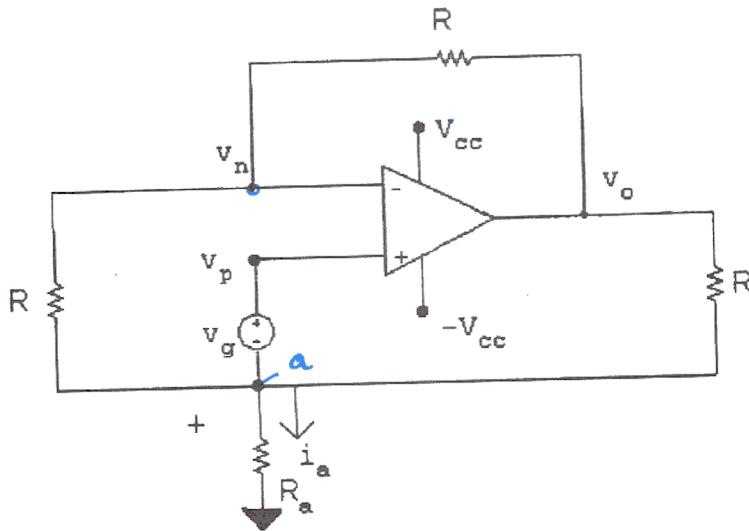
$$i_1 = i_2 \quad (i_n = 0) \Rightarrow \frac{V_o}{20 \times 10^3} = -\frac{V_i}{10^4}$$

$$\Rightarrow V_o = -2 V_i$$

$$\Rightarrow \frac{V_o}{V_i} = -2$$

Problem 2 (6 pts)

Determine a relation between i_a and v_g



$$\text{at the - terminal; } \frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0 \Rightarrow 2v_n - v_a = v_o$$

$$\begin{aligned} \text{at node } a; \quad & \frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0 \Rightarrow v_a \left(\frac{1}{R_a} + \frac{2}{R} \right) = \frac{v_n}{R} + \frac{v_o}{R} \\ & \Rightarrow v_a \left(2 + \frac{R}{R_a} \right) - v_n = v_o \end{aligned}$$

$$\text{also; } v_n = v_p = v_g + v_a$$

From the above 3 equations;

$$2v_n - v_a = 2(v_g + v_a) - v_a = v_a \left(2 + \frac{R}{R_a} \right) - (v_g + v_a)$$

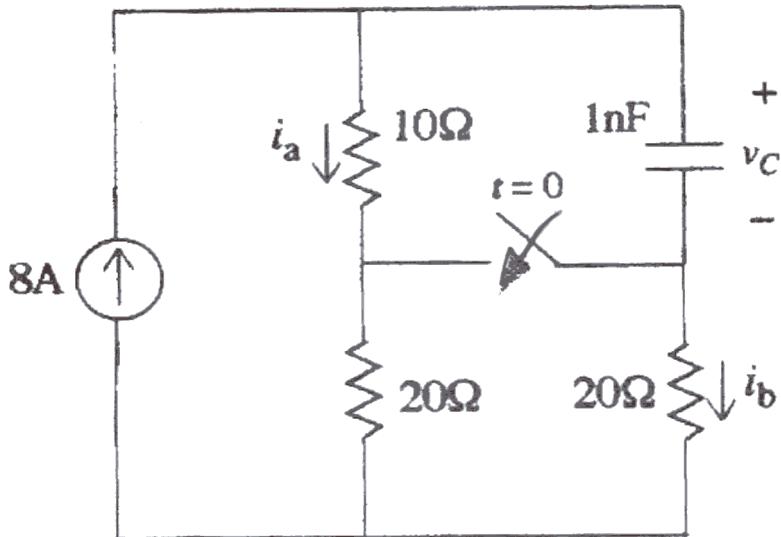
$$\Rightarrow 2v_g + v_a = v_a \left(1 + \frac{R}{R_a} \right) - v_g$$

$$3v_g = \frac{R}{R_a} v_a, \text{ but } \frac{v_a}{R_a} = i_a$$

$$\boxed{i_a = \frac{3}{R} v_g}$$

Problem 3 (9 pts)

The switch in the circuit below has been open for a long time before closing at $t=0$.



- a. Find $v_c(0^-)$, $i_a(0^-)$, and $i_b(0^-)$ (3 pts)

$$v_c(0^-) = 240V$$

$$i_a(0^-) = 8A$$

$$i_b(0^-) = 0A$$

- b. Find $v_c(\infty)$, $i_a(\infty)$, and $i_b(\infty)$ (3 pts)

$$v_c(\infty) = 80V$$

$$i_a(\infty) = 8A$$

$$i_b(\infty) = 4A$$

c. Find a differential equation for the voltage $v_c(t)$. (3 pts)

$$i_a(t) + i_c(t) = 8$$

$$\frac{V_c(t)}{R} + C \frac{dV_c(t)}{dt} = 8$$

$$\frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = \frac{8}{C}$$

$$C \frac{dV_c(t)}{dt} + 10^8 V_c(t) = 8 \times 10^9$$

d. Solve part (c) for $v_c(t)$. (2 pt)

$$V_c(t) = (A e^{-10^8 t} + B) u(t)$$

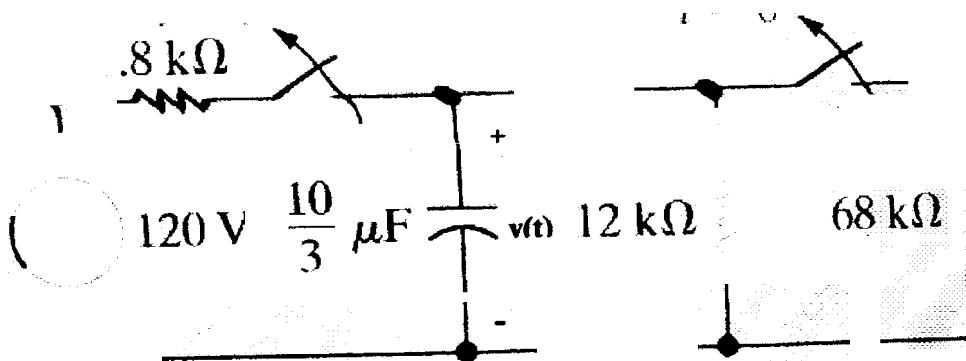
$$V_c(0) = 80 \Rightarrow B = 80$$

$$V_c(0) = 240 \Rightarrow A = 160$$

$$V_c(t) = (160 e^{-10^8 t} + 80) u(t) \text{ Volts}$$

Problem 4 (6 pts)

In the circuit shown below, both switches operate together; that is, they either open or close at the same time. The switches are closed for a long time before opening at $t=0$ s. Let $v(t)$ be the voltage across the capacitor.



- a. Determine the initial value of the voltage across the capacitor. (2 pts)

$$R_{eq} = \frac{12 \cdot 68}{12 + 68} = 10.2 \text{ k}\Omega \Rightarrow I_T = \frac{120}{(10.2 + 1.8) \times 10^3} = 10 \text{ mA}$$

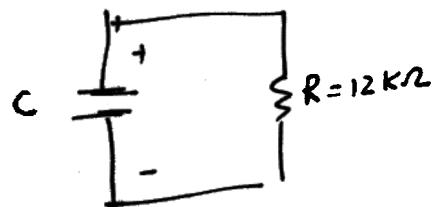
$$v(0) = -10 \times 10^{-3} \times 10.2 \times 10^3 = -102 \text{ Volts}$$

- b. Determine the final value of $v(t)$. (1 pt)

For very large value of t , $v(t) = 0$ Volts

$$v(\infty) = \frac{120}{(1.8 + 12) \times 10^{-3}} = 12 \times 10^3 = 12 \text{ kVolts}$$

c. Determine a differential equation for this voltage. (2 pts)



$$V(t) + R i(t) = 0 \quad \text{where } i(t) = C \frac{dV(t)}{dt}$$

$$\Rightarrow \frac{dV(t)}{dt} + \frac{1}{RC} V(t) = 0 .$$

d. Solve for v(t). (1 pt)

$$V(t) = A e^{-\frac{1}{RC}t} u(t)$$

$$\frac{1}{RC} = \frac{1}{12 \times 10^3 \times \frac{10}{3} \times 10^{-6}} = \frac{10^3}{40} = 25$$

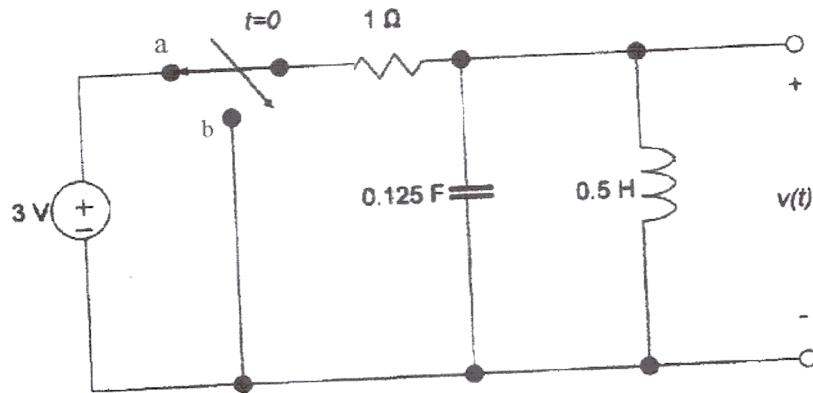
$$V(t) = A e^{-25t} \text{ volt}$$

$$V(0) = -102 = A$$

$$V(t) = -102 e^{-25t} \text{ volt}$$

Problem 5 (8 pts)

In the figure below, the switch has been at position (a) for a long time. At $t=0$ s, the switch moves to position b.



a. Determine $v(0)$ and $\left.\frac{dv(t)}{dt}\right|_{t=0}$ (2 pts)

$$v(0) = 0 \quad (\text{Inductor is a short-circuit})$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = -24 \text{ Volts}$$

b. Determine the differential equation for $v(t)$. (2 pts)

$$i(t) = 0 = i_R(t) + i_L(t) + i_C(t)$$

$$\Rightarrow \frac{v(t)}{R} + \frac{1}{L} \int v(s) ds + C \frac{dv(t)}{dt} = 0$$

Taking the derivative, we obtain

$$\frac{d^2 v(t)}{dt^2} + \frac{1}{RC} \frac{dv(t)}{dt} + \frac{1}{LC} v(t) = 0$$

$$\boxed{\frac{1}{RC} = \frac{1}{1 \times 0.125} = 8} \quad ; \quad \boxed{\frac{1}{LC} = 16}$$

$$\frac{d^2 v(t)}{dt^2} + 8 \frac{dv(t)}{dt} + 16 v(t) = 0$$

c. Solve for $v(t)$. (Be Careful) (3 pts)

$$s^2 + 8s + 16 = 0 \Rightarrow s_1 = s_2 = -4$$

$$v(t) = (A e^{-4t} + B t e^{-4t}) u(t)$$

$$\text{at } t=0 ; A=0 \Rightarrow v(t) = B t e^{-4t} u(t)$$

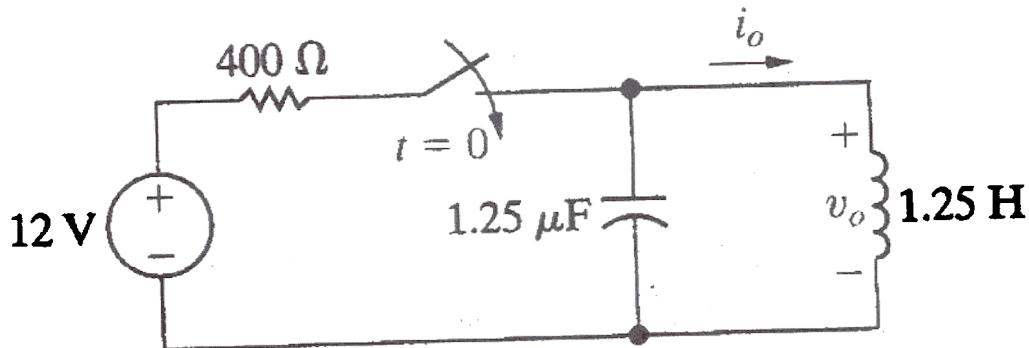
$$\frac{dv(t)}{dt} = B e^{-4t} - 4B t e^{-4t}$$

$$\left. \frac{dv(t)}{dt} \right|_{t=0} = B = -24$$

$$v(t) = -24 t e^{-4t} u(t)$$

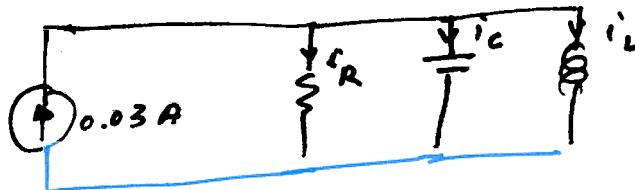
Problem 6 (4 pts)

There is no energy stored in the circuit shown below when the switch is closed at time $t=0$ s.



- a. Find a differential equation for the voltage $v_0(t)$ (2 pts)

Convert the voltage source into a current source



$$i_R + i_C + i_L = 0.03$$

$$\begin{aligned} i_L &= \frac{1}{L} \int_{-\infty}^t v_0(\tau) d\tau \\ i_C &= C \frac{d v_0(t)}{dt} \\ i_R &= \frac{v_0(t)}{R} \end{aligned}$$

$$\frac{v_0(t)}{R} + e \frac{d v_0(t)}{dt} + \frac{1}{L} \int_{-\infty}^t v_0(\tau) d\tau = 0.03$$

Taking the derivative,

$$\frac{d^2 v_0(t)}{dt^2} + \frac{1}{RC} \frac{d v_0(t)}{dt} + \frac{1}{LC} v_0(t) = 0$$

$$\frac{1}{RC} = 2000 ; \quad \frac{1}{LC} = 640,000$$

$$\frac{d^2 v_0(t)}{dt^2} + 2000 \frac{d v_0(t)}{dt} + 640,000 v_0(t) = 0$$

b. Solve $v_0(t)$ for $t \geq 0$ (2 pts)

$$s^2 + 2000s + 640,000 = 0$$

$$\Rightarrow s = -1000 \pm \frac{400}{2}$$

$$\Rightarrow v_0(t) = A e^{-1000t} + B t e^{-1000t} \quad (pt)$$

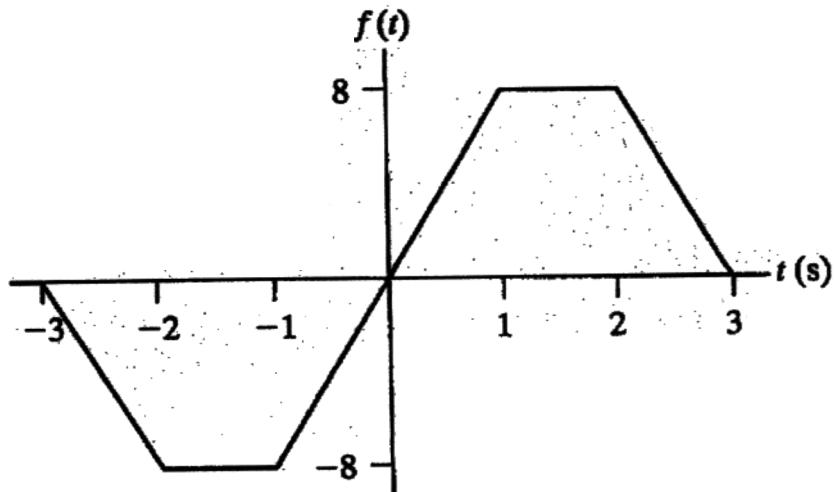
$$v_0(0) = 0 \Rightarrow A + B = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow B = -A$$

$$\left. \begin{array}{l} \frac{dv_0(t)}{dt} = 0 \\ t=0 \end{array} \right\} \Rightarrow A - 4B = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow A = 16$$

$$v_0(t) = (16e^{-1000t} - 4e^{-1000t}) u(t) \text{ Volts} \quad (pt)$$

Problem 7 (6 pts)

- a. Use step functions to write the derivative of the function $f(t)$ shown below (2 pts)



$$\frac{df(t)}{dt} = -8[u(t+3) - u(t+2)] + 8[u(t+1) - u(t-1)] \\ 8[u(t-2) - u(t-3)]$$

- b. Determine the Laplace transform of the first derivative of $f(t)$. (2 pt)

$$LT\left\{\frac{df(t)}{dt}\right\} = \frac{8}{s} \left[-e^{3s} + e^{2s} + 8e^s - 8e^{-s} - 8e^{-2s} + 8e^{-3s} \right]$$

c. Determine the second derivative of the function $f(t)$. (2 pts)

$$\frac{d^2 f(t)}{dt^2} = -8 [\delta(t+3) - 8\delta(t+2) - 8\delta(t+1) + \delta(t-1) \\ - 8\delta(t-2) + \delta(t-3)]$$

d. Determine the Laplace transform of the second derivative of $f(t)$. (1 pt)

$$L\{ \frac{d^2 f(t)}{dt^2} \} = -8 [e^{3s} - e^{2s}] + 8[e^s - e^{-1}] - 8[e^{2s} - e^{-3s}]$$

Problem 8 (7 pts)

Let $F(s)$ be the Laplace Transform of the function $f(t)$. Find, as a function of $F(s)$, the Laplace Transform of the following functions

a. $g(t) = \frac{d(tf(t))}{dt}$ (3 pts)

let $g(t) = f(t)$
 $G(s) = -\frac{dF(s)}{ds}$

$$\begin{aligned}\mathcal{L}\{ \frac{d g(t)}{dt} \} &= sG(s) - g(0) \\ &= -s \frac{dF(s)}{ds} - 0 = -s \frac{dF(s)}{ds}\end{aligned}$$

b. $h(t) = e^{-at} f(t-b)$ where a and b are positive constants. (3 pts)

let $g(t) = f(t-b)$
 $G(s) = e^{-bs} F(s)$

$$\mathcal{L}\{ e^{-at} g(t) \} = G(s+a) = e^{-b/(s+a)} F(s+a)$$