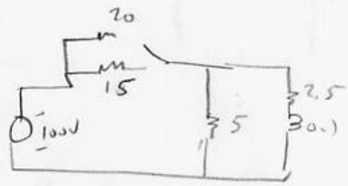
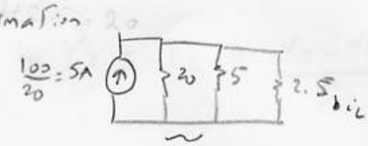
AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE210 – FALL 2004
QUIZ 2 SOLUTIONS

PROBLEM 1



1) for $t < 0$
L is a short

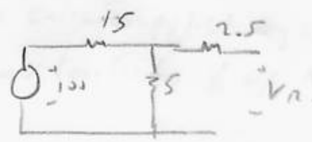
source transformation



$R_1 = 20 // 5 = \frac{20 \cdot 5}{25} = 4\Omega$

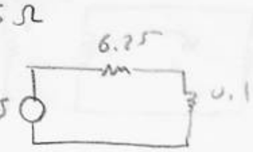
$\Rightarrow i_L = \frac{4 * 5}{4 + 2.5} = \frac{20}{6.5} = \frac{200}{65}$ A.

2) for $t \geq 0$, let's find Thevenin equivalent:



$V_{th} = \frac{5 \times 100}{20} = 25$ V;

$R_{th} = 5 // 15 + 2.5 = \frac{75}{20} + 2.5$
$= 3.75 + 2.5 = 6.25\,\Omega$



$i_L(t) = [i_L(0) - i_L(\infty)]e^{-\frac{t}{\tau}} + i_L(\infty)$

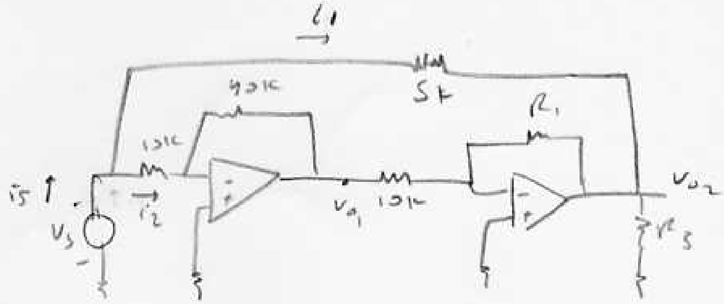
$i_L(0) = i_L(0^-) = \frac{200}{65}$; $i_L(\infty) = \frac{25}{6.25}$

$\tau = \frac{L}{R_{th}} = \frac{0.1}{6.25}$ sec

$\Rightarrow i_L(t) = \frac{200}{65} e^{-62.5t} + 4(1 - e^{-62.5t})$ Amps $t \geq 0$

W_{max} occurs for $i_{Lmax} = 0.4$ A $\Rightarrow W_{max} = \frac{1}{2} L I_{max}^2$
$= \frac{1}{2} * 0.1 * (4)^2$
$= 0.8$ J

PROBLEM 2



$V_{o1} = -\frac{40}{10} \cdot V_s = -4V_s$

$V_{o2} = -\frac{R_1}{10} \cdot V_{o1} = +0.4 R_1 V_s$ .. (1)

for $i_s = 0 \Rightarrow i_1 = -i_2 \Rightarrow \frac{V_S - V_{o2}}{5} = -\frac{V_S}{10} \Rightarrow V_{o2} = V_s + \frac{5}{10} V_s$

$\boxed{V_{o2} = 1.5 V_s}$ - (2)

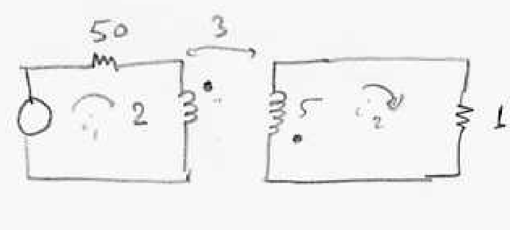
(1) + (2) $\Rightarrow$ $0.4 R_1 V_s = 1.5 V_s$

$\Rightarrow R_1 = \frac{1.5}{0.4} = \frac{15}{4} = 3.75 \, K\Omega$

R_3 does not affect The current supplied by source hence only constrained by power limitations of Op-Amp.

PROBLEM 3

(a)



$-V(t) + 50 i_1 + 2\frac{di_1}{dt} + 3\frac{di_2}{dt} = 0$

and

$i_2 + 5\frac{di_2}{dt} + 3\frac{di_1}{dt} = 0$

(b) substitute currents into equations:

$\boxed{i_1(t) = 0.24 - 4e^{-t} \; ; \; i_2(t) = 3e^{-t}}$ $V(t) = 12 - 201e^{-t}$

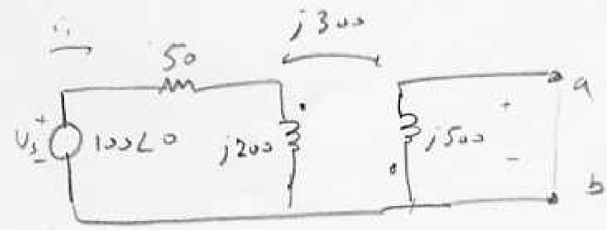
$\Rightarrow -12 + 201e^{-t} + 50(0.24 - 4e^{-t}) + 2(4)e^{-t} + 3(-3e^{-t})$

$= -12 + 12 + (201 - 200 + 8 - 9)e^{-t} = 0$

$i's$: $3e^{-t} + 5(-3e^{-t}) + 3(4e^{-t}) = (3 + 12 - 15)e^{-t} = 0$

PROBLEM 3 -(b)

phasor ckt shown:

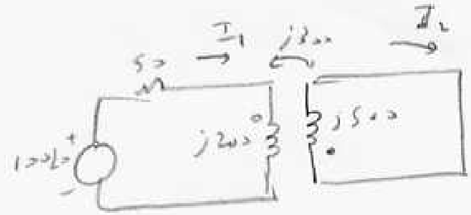


$V_{oc} = V_R = -j300 \cdot I_1$

$I_1 = \dfrac{V_s}{50+j200} = \dfrac{100\angle 0}{50+j200}$

$\Rightarrow V_{oc} = -j\dfrac{3\times 10^4}{50+j200} = \dfrac{3\times 10^4 \angle -\frac{\pi}{2}}{206.15\angle 1.32} = \underline{\underline{145.52\angle -165.96^\circ}}$ Volts

$Z_{th} = \dfrac{V_{oc}}{I_{sc}} \Rightarrow$ find I_{sc}



$I_{sc} = I_2$

$j500 I_2 + j300 I_1 = 0$

$\Rightarrow \left[I_1 = -\frac{5}{3} I_2 \right]$

$-100\angle 0 + 50 I_1 + j200 I_1 + j300 I_2 = 0$

$\Rightarrow \left[(50+j200)\left(-\frac{5}{3}\right) + j300 \right] I_2 = 100\angle 0$

$(-250 - j1000 + j900) I_2 = 300\angle 0$

$\Rightarrow I_2 = \dfrac{300\angle 0}{-250-j100} = I_{sc}$

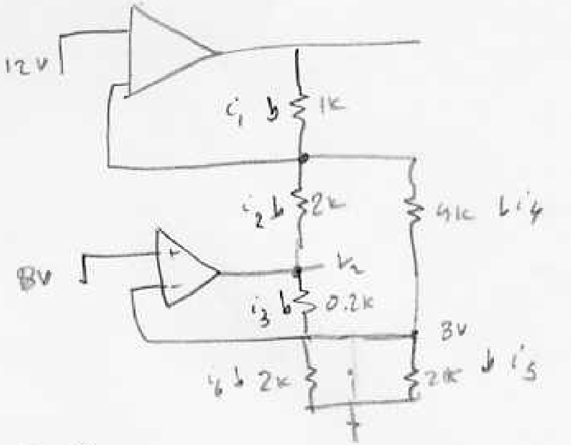
$Z_{th} = \dfrac{V_{oc}}{I_{sc}} = 145.52\angle -165.96 \times \left(\dfrac{-250-j100}{300}\right)$

$= 130.608 \angle -324.15$

$= 130.608 \angle +35.84\ \Omega$

$\boxed{Z_{th} = 105.9 + j76.5\ \Omega}$

PROBLEM 4



$i_4 = \frac{12-8}{4K} = 1\ mA$

$i_5 = \frac{8}{2} = 4\ mA$

$i_6 = \frac{8}{2} = 4\ mA$

$i_5 + i_6 = i_4 + i_3$

$\Rightarrow i_3 = 7\ mA \quad \Rightarrow \quad V_2 = 0.2 * 7 + 8$

$\boxed{V_2 = 9.4\ V}$

$i_2 = \frac{12-9.4}{2K} = \frac{2.6}{2} = 1.3\ mA$

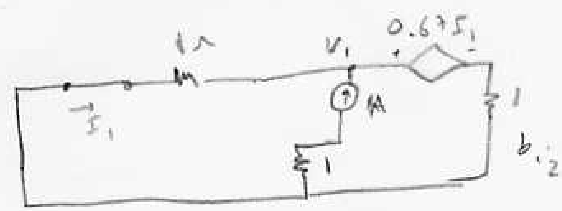
$i_1 = i_2 + i_4 = 1.3 + 1 = 2.3\ mA \Rightarrow V_1 = (2.3)*(1) + 12$

$\boxed{V_1 = 14.3\ V}$

PROBLEM 5

Solve by superposition:

for DC source: L short, C open $\} \Rightarrow$



Nodal

$\frac{V_1}{1} - 1 + \frac{V_1 - 0.67 I_1}{1} = 0$

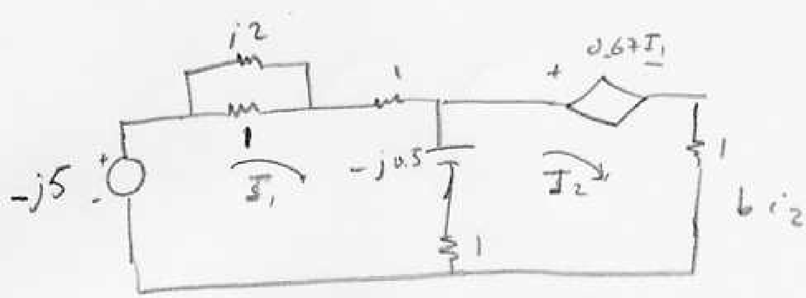
$I_1 = -\frac{V_1}{1} = -V_1 \quad \Rightarrow \quad V_1 - 1 + V_1 + 0.67 V_1 = 0$

$\Rightarrow V_1 = \frac{1}{2.67}$ Volts

$\Rightarrow i_{2(1A)} = \frac{V_1 - 0.67 I_1}{1} = 1.67 V_1 = \frac{1.67}{2.67}$ A ↓

FOR AC SOURCE:

phasor ckt:



mesh analysis

$$+j5 + \left(\frac{j2}{1+j2}\right) I_1 + I_1 + (1-0.5j)(I_1 - I_2) = 0 \quad \cdots (1)$$

$$(1-0.5j)(I_2 - I_1) + 0.67 I_1 + I_2 = 0 \quad \cdots (2)$$

$$(1) \Rightarrow \left(\frac{j2}{1+j2} + 2 - 0.5j\right) I_1 + (0.5j - 1) I_2 = -j5$$

$$(2) \Rightarrow (-0.33 + 0.5j) I_1 + (2 - 0.5j) I_2 = 0$$

$$(2) \Rightarrow I_1 = \left[\frac{0.5j - 2}{0.5j - 0.33}\right] I_2$$

substitute in (1) $\Rightarrow \left[\left(\frac{j2 + 2 + j4 - 0.5j + 1}{1+j2}\right)\left(\frac{0.5j - 2}{0.5j - 0.33}\right) + (0.5j - 1)\right] I_2 = j5$

$$\left[\frac{(3 + j5.5)(0.5j - 2)}{(1+j2)(0.5j - 0.33)} + (0.5j - 1)\right] I_2 = -j5$$

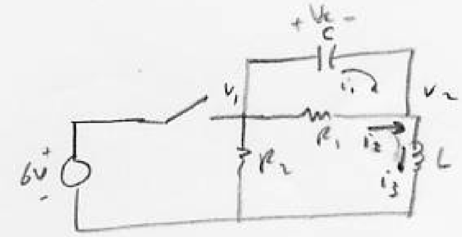
$$\Rightarrow I_2 = -0.394 - j0.369$$

$$= 0.5393 \angle -2.38 \quad A$$

$$I_2 = 0.539 \angle -136.87 \quad A$$

$$\therefore \left[\begin{array}{l} i_2(t) = 0.539 \cos(2t - 136.87) + \frac{1.67}{2.67} \quad A \\ i_2(t) = 0.6255 + 0.539 \cos(2t - 2.33) \quad A \end{array}\right]$$

PROBLEM 6



for $t < 0$ $\quad i_L(0) = V_c(0) = 0$

for $t \geq 0$

$$i_3 = i_2 + i_1 \qquad i_1 = C\frac{dV_c}{dt} = C\frac{d(V_1 - V_2)}{dt}$$

$$i_2 = \frac{V_1 - V_2}{R_1}$$

$$i_3 = \frac{1}{L}\int v_2(\lambda)\,d\lambda$$

$$\Rightarrow \frac{di_3}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\Rightarrow \frac{1}{L}v_2 = C\frac{d^2}{dt^2}(v_1 - v_2) + \frac{1}{R_1}\frac{d(v_1 - v_2)}{dt}$$

$V_1 = 6V = \text{constant}$

$$\Rightarrow \quad -C\frac{d^2V_2}{dt^2} - \frac{1}{R_1}\frac{dV_2}{dt} = \frac{1}{L}v_2$$

OR $\left[\dfrac{d^2V_2}{dt^2} + \dfrac{1}{R_1C}\dfrac{dV_2}{dt} + \dfrac{1}{LC}V_2 = 0 \right]$ O.D.E. (similar to what we know)

characteristic polyn.: $\quad s^2 + \frac{1}{R_1C}s + \frac{1}{LC} = 0 \quad \Rightarrow s = \frac{-1}{2R_1C} \pm \sqrt{\left(\frac{1}{2R_1C}\right)^2 - \frac{1}{LC}}$

a) for $R_1 = 3$, $L = 5$, $C = 4$, $R_2 = 2\Omega$

$$s = -\frac{1}{2\cdot3\cdot4} \pm \sqrt{\left(\frac{1}{24}\right)^2 - \frac{1}{20}} = \frac{-1}{24} \pm \sqrt{\left(\frac{1}{24}\right)^2 - \frac{1}{20}}$$

$$s_{1,2} = -0.0417 \pm j0.2197$$

$v_c(t) = v_{cN}(t) + v_{cF}(t)$

$V_{cF}(t) = K$

Note now that $V_C = V_1 - V_2 \Rightarrow V_2 = V_1 - V_C$

$$\Rightarrow \frac{d^2}{dt^2}(V_1 - V_C) + \frac{1}{R_1 C}\frac{d(V_1 - V_C)}{dt} + \frac{1}{LC}(V_1 - V_C) = 0$$

$V_1 = 6V$ $\Rightarrow \boxed{\frac{d^2 V_C}{dt^2} + \frac{1}{R_1 C}\frac{dV_C}{dt} + \frac{1}{LC}V_C = \frac{1}{LC}V_1}$

$V_{CF}(t) = K_0 \Rightarrow \frac{K_0}{LC} = \frac{1}{LC}V_1 \Rightarrow K_0 = V_1$

$V_{CN}(t)$ = natural response where $S_{1,2} = -0.0417 \pm j0.2197$

$$V_{CN}(t) = Ae^{-0.0417t}\cos(0.2197t) + Be^{-0.0417t}\sin(0.2197t)$$

so

$$V_C(t) = 6 + e^{-0.0417t}\left(A\cos(0.2197t) + B\sin(0.2197t)\right)$$

apply initial conditions:

$V_C(0) = 0 \Rightarrow 6 + A = 0 \Rightarrow \boxed{A = -6}$

$i_L(0) = 0$ but $i_L = i_C + i_{R_1} = C\frac{dV_C}{dt} + \frac{V_C}{R_1}$

$$i_L(0) = C\frac{dV_C}{dt}(0) + \frac{V_C(0)}{R_1}$$

$$\Rightarrow C\frac{dV_C}{dt}(0) = i_L(0) - \frac{V_C(0)}{R_1} = 0$$

$$\Rightarrow \frac{dV_C}{dt}(0) = 0$$

$$\left.\frac{dV_C}{dt}\right|_{t=0} = -0.0417A + 0.2197B = 0$$

$$\Rightarrow B = \frac{0.0417A}{0.2197} = \frac{0.0417 \cdot 6}{0.2197} = 1.14$$

$\therefore V_C(t) = 6 + e^{-0.0417t}\left(-6\cos 0.2197t + 1.14\sin 0.2197t\right)$ Volts

$t \geq 0$

(b) since $s_{1,2}$ are complex, response is oscillatory
$\Rightarrow$ it is underdamped.

(c) for $L = 5$; $C = 4$

need $\left(\frac{1}{2R_1C}\right)^2 = \frac{1}{LC} \Rightarrow \frac{1}{2R_1C} = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{R_1} = 2\sqrt{\frac{C}{L}}$

OR $\boxed{R_1 = \frac{1}{2}\sqrt{\frac{L}{C}}}$

* $R_1 = \frac{1}{2}\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} = 0.559\ \Omega$

* R_2 has no effect on the response.