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FALL TERM 2007-08

QUIZ #2

Name:....................................
ID:....................................
Professor:..........................

Dec. 17, 2007

******TEST ID UNKNOWN *****

(EECE 210) ELECTRIC CIRCUITS

CLOSED BOOK (1 ½ HRS)

1. Programmable Calculators are not allowed
2. Provide your answers on the computer's card only
3. Return the computer's card attached to the question sheet
4. Mark with a pencil your last name.
5. Mark your AUB ID NO.
6. The test ID No. is your exam version. Mark it in the box titled ' Test ID".
7. Use pencil for marking your answers
8. When using eraser, be sure that you have erased well

Problems 1 and 2 below refer to the circuit shown in the figure below.

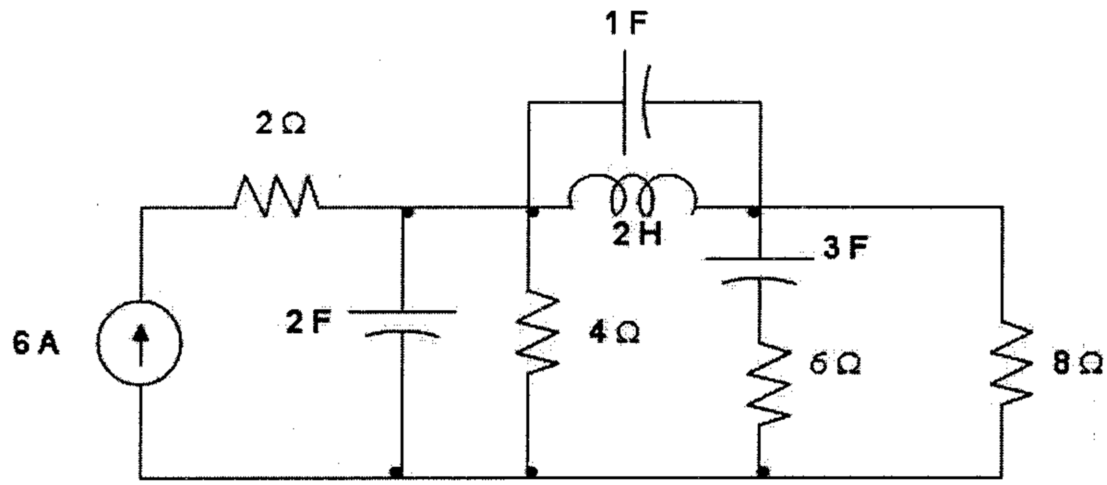


Figure Pb 1

Problem 1

The total energy stored in the inductor is

(a) 16 J
(b) 8 J
(c) 4 J
(d) 36 J
(e) None of the above

Problem 2

The total energy stored in **all** the capacitors (C= 1 Farad, 2 F and 3 F) is

(a) 680 J
(b) 384 J
(c) 642 J
(d) 640J
(e) None of the above

Problems 3, 4 and 5 below refer to the circuit shown in the figure below. The switch was at position (a) for a long time. At time t=0, the switch is moved to position (b).

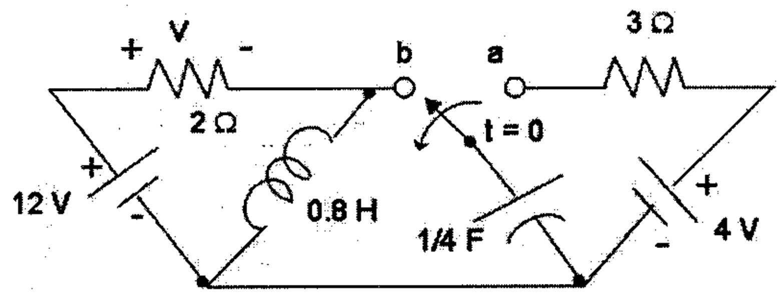


Figure Pb 2

<u>Problem 3</u>

The initial conditions, after the switch moves from ***a*** to ***b***, on the capacitor, inductor and the voltage across the 2 Ω resistor are:

(a) $i_L(0+) = 6$ A; $V_C(0+) = 4$ V; $V(0+) = 4$ V
(b) $i_L(0+) = 6$ A; $V_C(0+) = 4$ V; $V(0+) = 8$ V
(c) $i_L(0+) = 12/5$ A; $V_C(0+) = 4$ V; $V(0+) = 8$ V
(d) $i_L(0+) = 6$ A; $V_C(0+) = 4$ V; $V(0+) = 0$ V
(e) None of the above

<u>Problem 4</u>

The differential equation governing the voltage across the 2 Ω resistor is :

(a) $\frac{d^2V}{dt^2} + 8\frac{dV}{dt} + 5V = 60$

(b) $\frac{d^2V}{dt^2} + 2\frac{dV}{dt} + 5V = 60$

(c) $\frac{d^2V}{dt^2} + 2\frac{dV}{dt} + 5V = 12$

(d) $\frac{d^2V}{dt^2} + 3\frac{dV}{dt} + 5V = 60$

(e) None of the above

Problem 5

The voltage across the 2 Ω resistor after the switch moved from a to b is :

(a) $v(t) = 12 - 4e^{-t} \qquad t \geq 0$

(b) $v(t) = 12 - 4e^{-t}\cos(2t) + e^{-t}\sin(2t) \qquad t \geq 0$

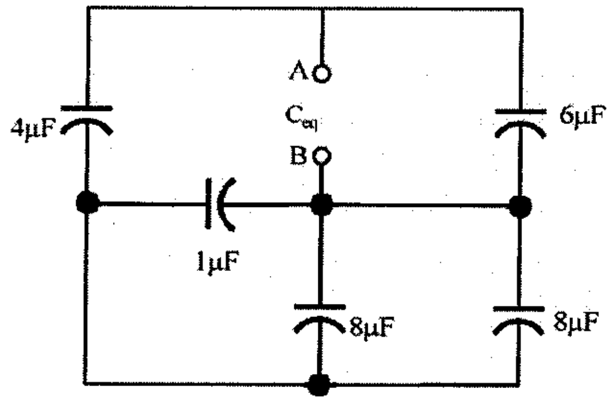
(c) $v(t) = 12 - 4e^{-t}\cos(2t) + 2\,e^{-t}\sin(2t) \qquad t \geq 0$

(d) $v(t) = 4e^{-t}\cos(2t) + 2e^{-t}\sin(2t) \qquad t \geq 0$

(e) none of the above.

Problem 6

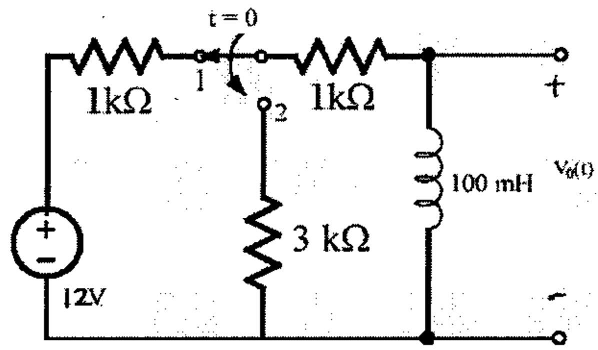
Find C_{eq} the equivalent capacitance at terminals A-B.



A) 10.941μF
B) 9.238 μF
C) 3.600 μF
D) 2.667 μF
E) none of the above.

Problem 7

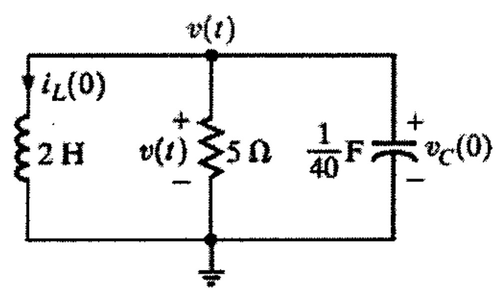
The switch has been in position 1 for a long time before it moves to position 2 at t=0. Find $V_0(t)$ for t >0



A) $-6\ e^{-10000t}$ V
B) $18\ e^{-40000t}$ V
C) $-24\ e^{-40000t}$ V
D) $-24\ e^{-10000t}$ V
E) none of the above.

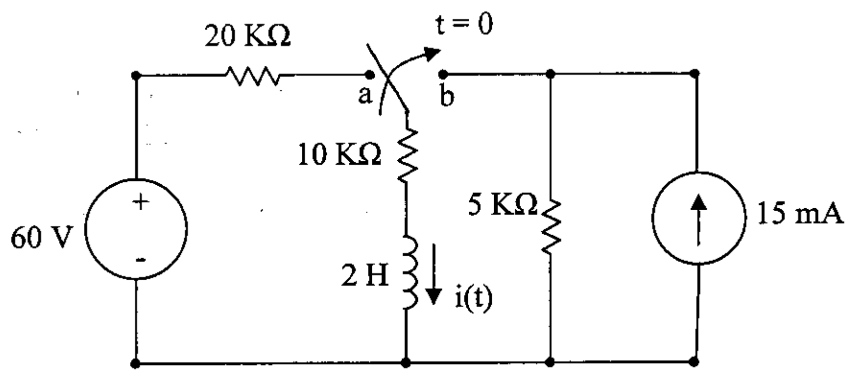
Problem 8

Determine the voltage v(t) if the initial conditions on the storage elements are $i_L(0)$=1A and $V_c(0)$=10V.



A) $e^{-4t}(10\cos 2t - 40\sin 2t)$ V
B) $e^{-4t}(10\cos 2t + 80\sin 2t)$ V
C) $e^{-4t}(-40\cos 2t + 10\sin 2t)$ V
D) $-80te^{-4t} + 10e^{-4t}$ V
E) none of the above.

Problems 9 to 12 depend on the circuit shown below



After being in the "a" position for a "long time", the switch in the circuit is moved to position b at t = 0.

Problem 9

Determine the initial condition for the current i(t); $i(0^+)$

a. 2 mA
b. 3 mA
c. 6 mA
d. 0 mA
e. None of the above

Problem 10

Determine the final value for the current i(t).

a. 15 mA
b. 10 mA
c. 5 mA
d. 1.5 mA
e. None of the above

Problem 11

Determine the Differential Equation for the current i(t) for t≥0.

a. $\frac{di(t)}{dt} + 15000i(t) = 75$

b. $\frac{di(t)}{dt} + 7500i(t) = 0$

c. $\frac{di(t)}{dt} + 15000i(t) = 0$

d. $\frac{di(t)}{dt} + 7500i(t) = 37.5$

e. None of the above

Problem 12

Solve for i(t) for t≥0.

a. $i(t) = -3e^{-7500t}mA$

b. $i(t) = 2e^{-7500t}mA$

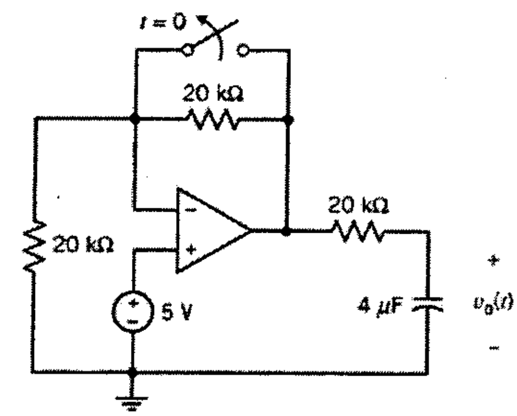
c. $i(t) = (-3e^{-7500t} + 5)mA$

d. $i(t) = (2e^{-7500t} + 3)mA$

e. None of the above

Problem 13

The switch in the circuit shown below opens at t=0. Determine $v_0(t)$ for t>0.



a. $v_0(t) = 5e^{-4000t}$ Volts

b. $v_0(t) = 5e^{-4000t} + 12$ Volts

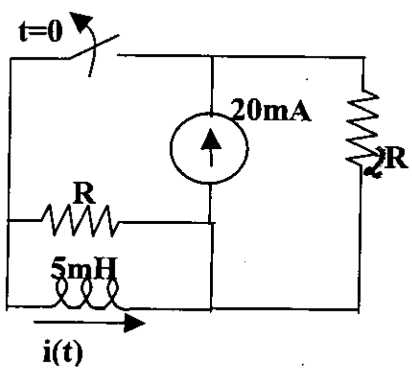
c. $v_0(t) = 5e^{-2000t} + 12$ Volts

d. $v_0(t) = 5e^{-1000t} + 12$ Volts

e. None of the above

Problem 14

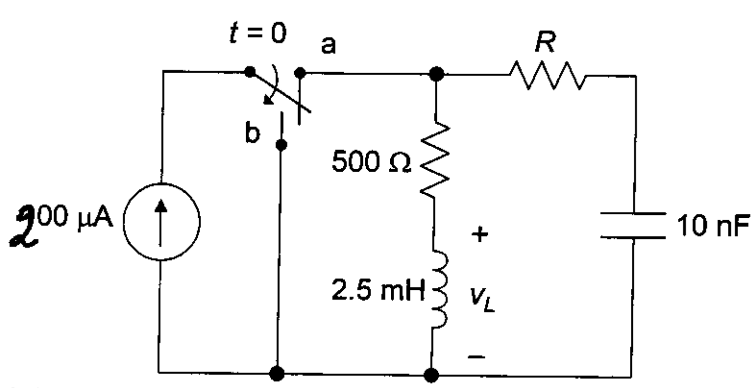
Considering the circuit below, the switch was closed for a long time before it opens at t=0, find the current i(t) flowing through the inductance for t>0. (R=100Ω).



a) 10 exp(– 40000t) mA, t > 0
b) 20 exp(– 40000t) mA, t > 0
c) 10 exp(– 20000t) mA, t > 0
d) 20 exp(– 20000t) mA, t > 0
e) None of the above

Problem 15.

The switch in the figure is moved to position 'b' at $t = 0$, after being in position 'a' for a long time. Choose R so that the response is critically damped and determine the initial value of v_L.



A. 50 mV
B. -50 mV
C. -100 mV
D. -150 mV
E. None on the above

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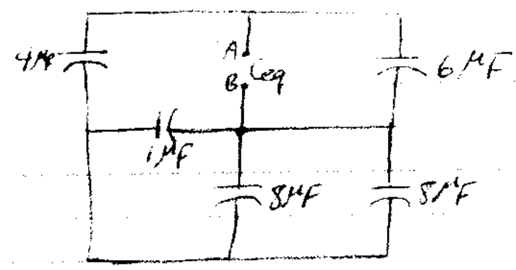
solution

#1. Ceq.



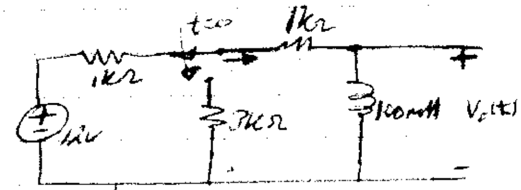
8μF // 8μF = 16μF
16μF // 1μF = 17μF

17μF series 4μF = $\frac{4\times17}{4+17}$ = 3.23809 μF // 6μF

⇒ Ceq = 9.23809 μF

A) 9.238 μF
B) 2.667 μF
C) 3.600 μF
D) 10.941 μF

#9 $V_o(t)$ for $t > 0$



RL circuit.

$V_o(t) = V_f + (V_{(0)} - V_f)e^{-t/\tau}$

$\tau = \frac{L}{R_{eq}} = \frac{100m}{4K} = 25\times10^{-6}$ sec

$i(0^-) = i(0^+) = $ 6mA ⇒ $V(0) = -6\times4$ (Voltage across 1kΩ & 3kΩ)
$= 24v$

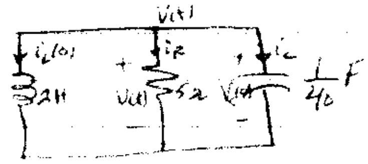
$V_f = 0$ ⇒ $V_o(t) = -24e^{-4\times10^4 t}$

B) $-24e^{-10^4 t}$
C) $18e^{-4\times10^4 t}$
D) $-6e^{-10^4 t}$

1

\# 15 $v(t)$ if $i_L(0) = 1A$, $V_C(0) = 10V$

RLC in parallel natural response



$\omega_0 = \frac{1}{\sqrt{LC}} = 4.4721$

$\alpha = \frac{1}{2RC} = 4$

$t=0^+$ $i_L + i_R + i_C = 0$

$1 + \frac{V_C(0)}{R} + i_C = 0$

$i_C = -3$

$\omega_0^2 > \alpha^2 \Rightarrow$ underdamped

$V(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2$

$V(0) = B_1 = 10$

$\frac{dV(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0^+)}{C} = \frac{-3}{\frac{1}{40}} = -120$

$-40 + 2B_2 = -120$

$B_2 = -40$

$V(t) = e^{-4t}(10 \cos 2t - 40 \sin 2t)$

$V(t) = e^{-4t}(10 \cos 2t + 80 \sin 2t)$

$V(t) = -80te^{-4t} + 10e^{-4t}$

$V(t) = e^{-4t}(-40 \cos 2t + 10 \sin 2t)$

Error: $V_0 = D_2 = 10$

$\frac{dV(0)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2$

$-120 = D_1 - 40$

$\Rightarrow D_1 = -80$

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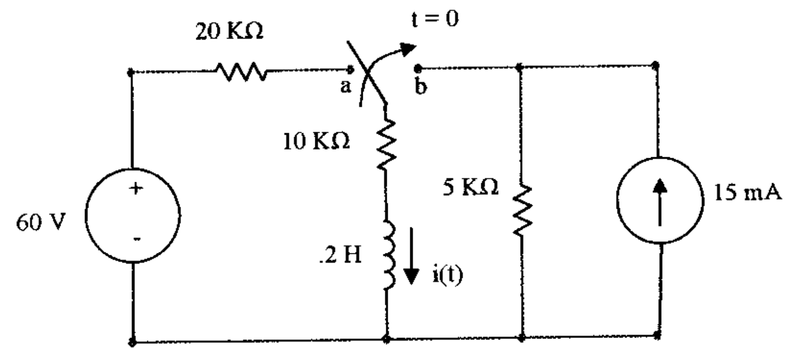
2 TO 5

Problems 2 to 5 depend on the circuit shown below



After being in the "a" position for a "long time", the switch in the circuit is moved to position b at t = 0.

Problem 2

Determine the initial condition for the current i(t); $i(0^+)$

a. 2 mA
b. 3 mA
c. 6 mA
d. 0 mA
e. None of the above

Solution: When the switch is at position a for a long period of time, the inductor is a short circuit. The current is 60/30000 = 2 mA. As the current is continuous at all time t, then i(0+)=i(0-)=2 mA

Problem 3

Determine the final value for the current i(t).

a. 5 mA
b. 10 mA
c. 15 mA
d. 1.5 mA
e. None of the above

Solution: When the switch is at position b for a long period of time, the inductor again acts as a short circuit. Using current divider rule, we obtain I(t)=15*5/15=5 mA

Problem #4

Determine the DE for the current i(t) for t≥0.

a. $\frac{di(t)}{dt} + 7500i(t) = 37.5$

b. $\frac{di(t)}{dt} + 15000i(t) = 75$

c. $\frac{di(t)}{dt} + 7500i(t) = 0$

d. $\frac{di(t)}{dt} + 15000i(t) = 0$

e. None of the above

Solution: Using source conversion, we obtain a voltage source of 75 Volts in series with a resistor of 15 KΩ and an inductor of 2H. Using KVL, we obtain

$$15 * 10^3 i(t) + 2\frac{di(t)}{dt} = 75 \quad \text{or} \quad \frac{di(t)}{dt} + 7500i(t) = 37.5$$

Problem #5

Solve for i(t) for t≥0.

a. $i(t) = (-3e^{-7500t} + 5)mA$

b. $i(t) = -3e^{-7500t}mA$

c. $i(t) = 2e^{-7500t}mA$

d. $i(t) = (2e^{-7500t} + 3)mA$

e. None of the above

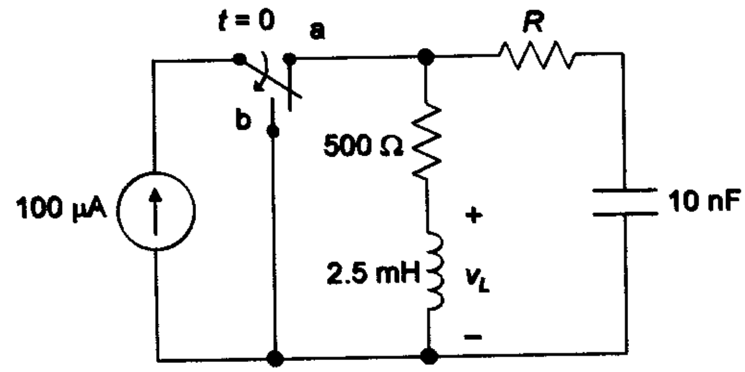
Solution: $i(t) = Ae^{-7500t} + B$

Using the initial and final conditions, we obtain: A+B=2 and B=5. Therefore, A=-3. The solution is:

$$i(t) = (-3e^{-7500t} + 5)mA$$

11.

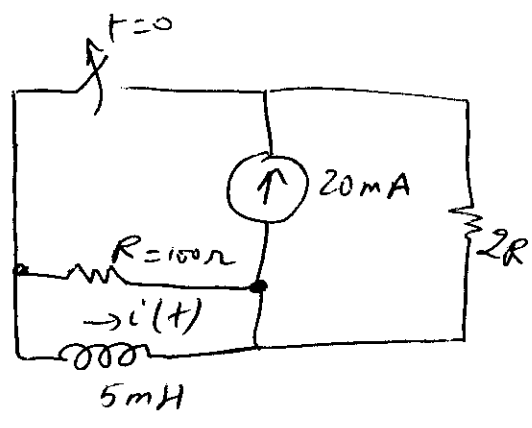
The switch in the figure is moved to position 'b' at $t = 0$, after being in position 'a' for a long time. Choose R so that the response is critically damped and determine the initial value of v_L.



A. 50 mV
B. -50 mV
C. -100 mV
D. -150 mV
E. None on the above

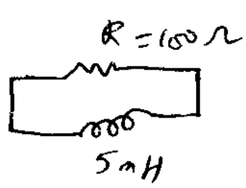
Solution: For critical damping, $\frac{R_s}{2L} = \omega_0 = \frac{1}{\sqrt{10^{-8} \times 2.5 \times 10^{-3}}} = 2 \times 10^5$ rad/s, where $R_s = R + 500\ \Omega$; $R_s = 2 \times 2.5 \times 10^{-3} \times 2 \times 10^5 = 1{,}000\ \Omega$. Hence, $R = 500\ \Omega$. When the switch is in position a, i_L is 100 μA and v_C is (100 μA)×(500 Ω) = 50 mV. When the switch is moved to position b, i_L and v_C at $t = 0^+$ remain the same. From KVL: $50 = 0.1 \times 1000 + v_L$, so that $v_L = -50$ mV.

6.



$t = 0^-$; $i_L(0^-) = i_L(0^+) = 20\text{ mA}$

$t = 0^+$

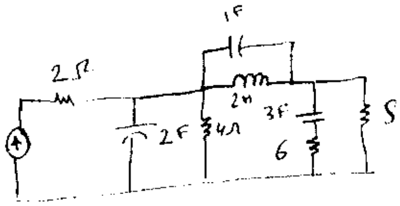


$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{100}$

$i(t) = I_0 e^{-\frac{t}{\tau}} = 20\, e^{-\frac{100 \times 10^{3}}{5} t}\text{ mA} = 20\, e^{-20000t}\text{ mA},\ t > 0.$

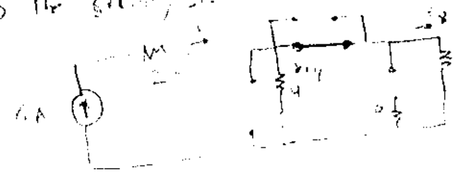
5)

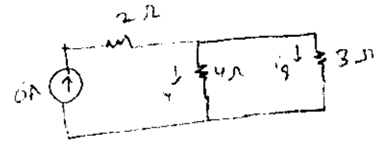
Problem 7



Under steady state conditions, capacitor will act as an open circuit and inductor will act [illegible] short circuit.

⇒ The steady state circuit will look like.





The current through [illegible] is i_g

$\Rightarrow i_g = \frac{4}{4+3} \cdot [illegible] = \frac{24}{12} = 2\,A$

⇒ Energy stored in inductor (2H) is $W_{2H} = \frac{1}{2} L i_g^2 = \frac{1}{2}(2)(2)^2$

$= 4\,J$

⇒ [redacted]

Problem 8

The 1F [illegible] by that inductor

[illegible] $i_{c,1F} = 0 \Rightarrow W_{c,1F} = 0\,J$

The [illegible] voltage as the 3Ω resistor

$V_{c,3F}$ [illegible] 16 Volts

$W_{c,3F} = \frac{1}{2} C V_{c,3F}^2 = \frac{1}{2}(3)(16)^2 = 384\,J$

The [illegible] has [illegible] voltage [illegible] 4Ω resistor

[illegible] V [illegible] 16 Volts

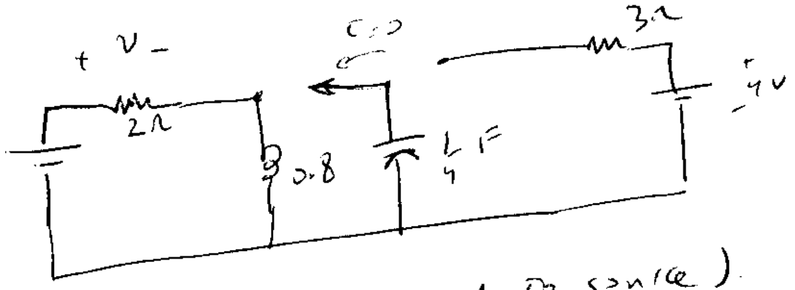
[illegible] W_c [illegible] 256 J

$\therefore W_c = W_{c,1F} + W_{c,2F} + W_{c,3F} = 256 + 384$

$= 640\,J$

answer is ([redacted])

problem 12



for t < 0, $V_c = 4V$ (fully charged to source)

$i_L = \frac{12}{2} = 6A$ (fully charged)

for t ≥ 0 $V_c(0^+) = V_c(0^-) = 4V$

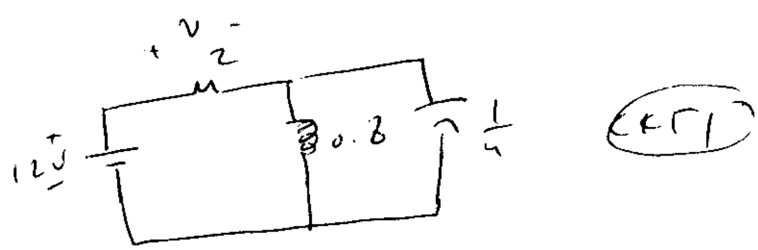
$i_L(0^+) = i_L(0^-) = 6A$

$V(0^+) = 12 - V_c(0^+) = 12 - 4 = 8V$

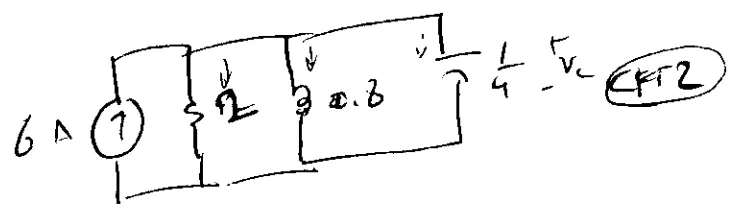
∴ ANSWER is [redacted]

problem 13

circuit looks like



redraw to be parallel RLC



find V_c first, then V is simply

$V = 12 - V_c$. t ≥ 0

in (CKT 2), V_c is governed by

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{1}{(2)(\frac{1}{4})}\frac{dV_c}{dt} + \frac{1}{(\frac{1}{4})(0.8)}V_c = 0$$

$$\Rightarrow \frac{d^2V_c}{dt^2} + 2\frac{dV_c}{dt} + 5V_c = 0$$

replace V_c by $12 - V$ $\Rightarrow \frac{d^2(12-V)}{dt^2} + 2\frac{d(12-V)}{dt} + 5(12-V) = 0$

7/

∴ $\frac{d^2v}{dt^2} + \frac{2dv}{dt} + 5v = 60$ ⇒ Answer is ([redacted])

problem 14

Two ways to find $v(t)$: find $v_c(t)$ then $v(t) = 12 - v_c(t)$
or find directly from O.D.E.

Using $v_c(t)$:

$v_c(0^-) = v_c(0^+) = 4V$

$i_L(0^-) = i_L(0^+) = 6A$

$\alpha = \frac{1}{2RC} = \frac{2}{2} = 1$ $\quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(\frac{1}{4})(0.8)}}$

$\alpha < \omega_0$ ⇒ underdamped.

$\omega_d = \sqrt{-\alpha^2 + \omega_0^2} = \sqrt{-1 + \frac{1}{(0.8)(\frac{1}{4})}} = \sqrt{4} = 2$

∴ $v_c(t) = Ae^{-t}\cos 2t + Be^{-t}\sin 2t \quad t \geq 0$

$v_c(0^+) = 4 = A$

$\frac{dv_c}{dt}\Big|_{t=0^+} = \frac{1}{C} i_c(0^+) = \frac{1}{C}\left[-i_L(0^+) - \frac{v_c(0^+)}{R} + 6\right]$
$= 4\left[-6 - \frac{4}{2} + 6\right] = -8$

$\frac{dv_c}{dt}\Big|_{t=0^+} = -A + 2B = -8$

$\Rightarrow B = \frac{-8+A}{2} = \frac{-8+4}{2} = -2$

∴ $v_c(t) = 4e^{-t}\cos 2t - 2e^{-t}\sin 2t$

$v(t) = 12 - 4e^{-t}\cos 2t + 2e^{-t}\sin 2t \quad t \geq 0$

answer = [redacted]