Stat 230 Problem Set 2

1. If the temperature at which a certain compound melts is a random variable with mean 130°C and standard deviation 3°C, what are the mean temperature and standard deviation measured in °F? [Hint: °F=1.8°C+32.]

2. Let X be the total medical expenses (in 1000s of dollars) incurred by a particular individual during a given year. Although X is a discrete random variable, its distribution is quite well approximated by a continuous distribution with pdf $f(x) = k(1 + x/2.5)^{-7}$ for $x \geq 0$.

   (a) What is the value of k?

   (b) Graph the pdf of X.

   (c) What are the expected value and standard deviation of total medical expenses?

   (d) This individual is covered by an insurance plan that entails a $500 deductible provision (so the first $500 worth of expenses are paid by the individual). Then the plan will pay 80% of any additional expenses exceeding $500, and the maximum payment by the individual (including the deductible amount) is $2500. Let Y denote the amount of this individual's medical expenses paid by the insurance company. What is the expected value of Y? [Hint: First figure out what value of X corresponds to the maximum out-of-pocket expense of $2500. Then write an expression for Y as a function of X (which involves several different pieces) and calculate the expected value of this function.]

3. Use the fact that
$$\sum_x (x - \mu)^2 p(x) \geq \sum_{x:\ |x-\mu| \geq k\sigma} (x - \mu)^2 p(x)$$
   for any mass function $p(\cdot)$ to prove Chebyshev's inequality.

4. Show that $\mathrm{cov}(X + Y, Z) = \mathrm{cov}(X, Z) + \mathrm{cov}(Y, Z)$.

5. If the cumulative distribution of X is given by
$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{1}{2} & 0 \leq b < 1 \\ \frac{3}{5} & 1 \leq b < 2 \\ \frac{4}{5} & 2 \leq b < 3 \\ \frac{9}{10} & 3 \leq b < 3.5 \\ 1 & b \geq 3.5 \end{cases}$$
   calculate the probability mass function of X.

6. If $E(X) = 1$ and $\mathrm{var}(X) = 5$, find
   (a) $E\big[(2 + X)^2\big]$
   (b) $\mathrm{var}(4 + 3x)$