Stat 230 Problem Set 1

1. We roll a pair of dice; one is red and one is green. Consider the following events: $A =$ the sum of the numbers shown is 7, $B =$ at least one of the dice shows a 6, $C =$ both dice show odd numbers, $D =$ the dice show different numbers, $E =$ the green die shows a 4.

   (a) What is the probability of B?

   (b) How are A and D related? What consequences follow for their probabilities? And for their independence?

   (c) Give an example of two events which are independent but not mutually exclusive.

   (d) Give an example of two events which are mutually exclusive but not independent.

2. If A and B are independent events, show that A and B' are independent events.

3. Show that conditional probability satisfies the axioms of probability (hard question).

4. An elementary school is offering 3 language classes: one in Spanish, one in French and one in German. These classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both French and German and 6 that are in both Spanish and German. In addition, there are 2 students taking all 3 classes.

   (a) If a student is chosen randomly, what is the probability that he or she is not in any of these classes?

   (b) If a student is is chosen randomly, what is the probability that he or she is taking exactly one language class?

   (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

5. Find $P(A \cup (B' \cup C')')$ in each of the following four cases:

   (a) A, B and C are mutually exclusive events and $P(A) = 1/3$

   (b) $P(A) = 2P(B \cap C) = 4P(A \cap B \cap C) = 1/2$

   (c) $P(A) = 1/2$, $P(B \cap C) = 1/3$ and $P(A \cap C) = 0$.

   (d) $P(A' \cap (B' \cup C')) = 0.6$

6. Some years ago, the government of a small island nation in the North Atlantic Ocean decided to change from a constitutional hereditary monarchy to a constitutional appointed monarchy. Six applicants: Ahmed, Basmah, Carl, Dima, Eli and Fida presented themselves before the House of Commons Search Committee for interviews for the job. Subsequently, the Committee forwarded a short list of three names to the whole House of Commons for its decision.

   (a) How many different short lists could the Committee have selected?

    (b) Assume that all possible short lists are equally likely: What is the probability that both Basmah and Carl are on the short list?

    (c) What is the probability Ahmed and two women are on the short list?

7. Urn 1 has five white and seven black balls. Urn 2 has three white and twelve black balls. We flip a fair coin. If the outcome is heads then a ball from urn 1 is selected, while if the outcome is tails, a ball from urn 2 is selected. Suppose a white ball is selected. What is the probability that the coin landed on tails?

8. Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade, given that the second and third cards are spades.

9. Suppose 5% of men and 0.25% of women are color blind. A color blind person is chosen at random. What is the probability of this person being male? Assume there are an equal number of males and females. What if the population consisted of twice as many males as females?