American University of Beirut

Quiz I- Stat. 230 (Sec.1.1-2.3Hogg and Tanis(8th)) March 19, 2011

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NAME:..... I.D:..... Section Number: 1 (5)

WRITTEN	MULTIPLE CHOICE	
1 15 / 20 2. 13 / 30 GRADE OF WRITTEN: 28	Correct: 5 x 5 = 25	**QUIZ I GRADE** 53

PART I: ***Circle your choice of the correct answer. You get:***

i) A score of 4 points for each correct answer.

ii) Zero points for each wrong answer no answer, or multiple answer .

iv) If you strongly feel that the answer is missing, write your finding in the space next to option E.

If your answer is correct, you secure a full mark of 4.

REMARK: Some fractional answers are rounded to the recorded number of decimal places.

- Determine the coefficient K in the trinomial expansion:
  $(2\sqrt{x}+\sqrt[3]{y}-z^2)^{10}=.....+Kx^2yz^6+.....$
  A -42700 **B 25** **(C) -67200** **D 110** **E.....**

- One four sided die is cast three times. Find the probability that the sum of the three outcomes is 5, given that at least two of the dice turned up 1 each?
  (A) 2/11 **B 3/11** **C 3/10** **D 1/4** **E.....**

- The p.m.f. of a random variable, X, is given as: $f(x)=\begin{cases}0.13 & if \quad x=1\\ cx^2 & if \quad x=2,3,4\end{cases}$. Find E(X).
  A 3.1 **(B) 2.5** **C 4.2** **D 1.18** **E.....**

- A deck of cards is randomly arranged in a left to right row. Find the probability that as you go from left to right, you will meet an ace before any red face card.
  A 3/4 **B 2/3** **(C) 0.3** **D 0.4** **E.....**

- Each of four Boxes (I, II, III, IV) contains five balls respectively bearing the numbers 1, 2, 3,4, and5. One ball is randomly selected from each box. Find the probability that there will be at least one ball bearing an even digit.

A 0.3250 **B 0.5780** **(C) 0.8704** **D 0.4125** **E.....**

- A number, x, is randomly selected from the real number set [1, 5].Let A be the event that x belongs to the interval [1.8, 3.5] , B be the event that x>3, and C the event that X is an even digit. Find $P(A \cup B \cup C)$.

(A) 0.425 **B 0.8** **C 0.4** **D 0.325** **E.....**

- In a state lottery, five digits are drawn at random, one at a time and with replacement from the ten digits: 0, 1, 2, 3, …,9. If the drawn digits match your five selected digits in any order, then you are a winner. Find the probability that you win if you have selected 5, 5, 2, 2, and 0.

A 0.0005 **B 0.0012** **C 0.0004** **(D) 0.0003** **E.....**

- A code of six characters has three starting letters that are distinct letters of the alphabet. Its three finishing letters should be different from the starting letters, but may contain repetitions. Then the number of different codes are:....

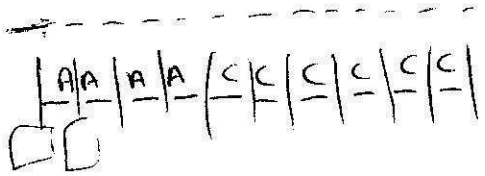
(A) 189805200 **B 832500** **C 4578000** **D 41225 000** **E.....**

- How many passwords can you form using an arrangement of four A's, five B's, and six C's , so that all the A's are before all the C's?

A 1225 **B 2850** **C 3003** **(D) 5005** **E.....**

- Given; $\mu = -2$, $\sigma^2 = 4$ and $E(X^3) = 6$. . Find $E[(X-\mu)^3]$.

A 110 **(B) 38** **C 42** **D 25** **E.....**



PART II: ***Write down the complete solution for the following questions***:

1.(4x5=20 Points)The p.m.f. of a discrete random variable, X, is given as:

$f(x)=\frac{c}{3^x}$; $x=2,3,4,....$

a) Show that $c=6$.

$f(x)=\frac{c}{3^x}$; $\sum_{x=2}^{\infty} f(x)=1 \Rightarrow \sum_{n=2}^{\infty}\frac{c}{3^n}=1 \Rightarrow c\sum_{n=2}^{\infty}\frac{1}{3^n}=1$

$\sum_{n=2}^{\infty}\frac{1}{3^n}=\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots\right)$ (sum of GP) $=\frac{1/3^2}{1-1/3}=\frac{1/9}{2/3}=\frac{3}{18}=\frac{1}{6}$

$c\times\frac{1}{6}=1$

$\boxed{c=6}$

5

b) Find P(X >5).

$P(X>5)=1-[P(2)+P(3)+P(4)+P(5)]$

$P(2)=\frac{6}{3^2}=\frac{6}{9}$ $P(3)=\frac{6}{3^3}=\frac{6}{27}$ $P(4)=\frac{6}{3^4}=\frac{6}{81}$

$=1-\left[\frac{54}{81}+\frac{18}{81}+\frac{6}{81}\right]$

$=1-\frac{78}{81}=\frac{3}{81}$ ✗

3

c) Calculate E(X).

$\mu=E(X)=\sum_{2}^{\infty} x f(x)=\sum_{2}^{\infty}\frac{x\times 6}{3^x}=6\sum_{2}^{\infty}\frac{x}{3^x}$

$\mu-\frac{1}{3}\mu=6\sum_{2}^{\infty}\frac{x}{3^x}-\frac{1}{3}\times 6\sum\frac{x}{3^x}$

$=6\left[\left(\frac{2}{3^2}+\frac{3}{3^3}+\frac{4}{3^4}+\cdots\right)-\left[\frac{2}{3^3}+\frac{3}{3^4}+\frac{4}{3^5}+\cdots\right]\right)$ ✓2

$=6\left[\frac{2}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\cdots\right]=\left(6\times\frac{2}{3^2}\times\left[\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}+\cdots\right]\right.$

$=\frac{12}{9}\left(\times\frac{1/3^3}{1-1/3}\right)=\frac{12}{9}\times\frac{1/27}{2/3}=\frac{12\times 3}{9\times 27\times 2}=\frac{2}{27}$

d) Find P(A), where A is the event that X is a multiple of 3.

$P(A)=\left(\frac{6}{3^3}+\frac{6}{3^6}+\frac{6}{3^9}+\cdots\right)$

$=6\left(\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}+\cdots\right)=6\times\frac{1/3^3}{1-1/3^3}=6\times\frac{1/27}{26/27}=\frac{6}{26}$

5

6- 6x (2+3) 3

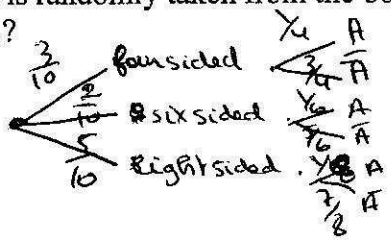
6x5=30

6x2 (3)

12x3

2.(5x6=30 Points)A box contains 3 four-sided , 2 six-sided and 5 eight sided dice.

a) One die is randomly taken from the box and is cast. What is the probability that the cast die shows 1?



A: shows 1.

$$P(A) = \frac{3}{10} \times \frac{1}{4} + \frac{2}{10} \times \frac{1}{6} + \frac{5}{10} \times \frac{1}{8}$$

$$= \frac{3}{40} + \frac{2}{60} + \frac{5}{80} =$$

b) One die is randomly taken from the box and is cast. If the die shows 1, what is the probability that it was six-sided?

B: six sided.

$$P(\text{6 six sided} / A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{2}{10} \times \frac{1}{6}}{\frac{3}{40} + \frac{2}{60} + \frac{5}{80}} = \frac{\frac{2}{60}}{\frac{41}{240}} = \frac{8}{41}$$

c) The five eight-sided dice are cast. What is the probability that there will be a full house?

full house: 3 same 2 same.

$$P(\text{full house}) = \frac{\binom{5}{2} \times \binom{2}{1} \times \binom{8}{3} \times \binom{5}{2}}{(5)8}$$

d) If all the ten dice are cast, what is the probability that the sum is 11?

sum 11: 1 2 8 2 1 8.
sum 11: 1 3 7.
1 4 6.
1 5 5
1 6 4.

e) One die is randomly selected from the box. You are offered as many dollars as the number of sides on the die. The price per trial of this game is set at \$6.25.BrieflyComment on the fairness of this price? X : number of dollars + price of trial (on the number of die).

3 four
2 six
5 eight ✓

support of X = {

X	1	2	3	4	5	6	7	8
P(X = x)	30/192	30/192	10/192	10/192	4/192	7/192	5/192	5/192

if 1: −5.25\$
if 2: −4.25\$
if 3: −3.25\$
4: −2.25\$
5: −1.25\$
6: −0.25\$
7: 0.75\$
8: 1.75\$

It's not a fair price because only in two cases you can win: if you get a 7 or a 8 which consist of a very little probability and cost.

0

f) (BONUUS OF 5 OR NONE)One die is randomly selected from the box. You are offered as many dollars as the number on the die. What is the expected value of this game?

X : number of dollars.

S = {1, 2, 3, 4, 5, 6, 7, 8}.

X	1	2	3	4	5	6	7	8
P(x) = P(X = x)	41/240	41/240	41/240	41/240	2/15 ~~32/240~~	~~32/240~~	1/10 ~~24/240~~	~~24/240~~

C: get 2

3/10 4 sided: 1/4 C, 3/4 C̄
2/10 6 sided: 1/6 C, 5/6 C̄
5/10 8 sided: 1/8 C, 7/8 C̄

$$P(1) = \frac{3}{40} + \frac{2}{60} + \frac{5}{80} = \frac{41}{240}$$

$$E(X) = 1 \times \frac{41}{240} + 2 \times \frac{41}{240} + 3 \times \frac{41}{240} + 4 \times \frac{41}{240} + 5 \times \frac{32}{240} + 6 \times \frac{32}{240} + 7 \times \frac{24}{240} + 8 \times \frac{24}{240} \ldots$$

D: get 5
3/10 4 sided: 0
2/10 6 sided: 1/6, 5/6
8/10 8 sided: 1/8, 7/8

$$P(\text{get }5) = \frac{2}{10} \times \frac{1}{6} + \frac{8}{10} \times \frac{1}{8}$$

Six same as 5.

5

E: get 7:
3/10
2/10 6 sided
8/10 8 sided: 1/8, 7/8
get 7 same as get 8

3