American University of Beirut
STAT 230
Introduction to Probability and Random Variables
Spring 2010

quiz # 2 solution

1. It is known that 30% of all calls coming into a telephone exchange are long-distance calls. Find the probability that the first long-distance call is among the first 50 coming calls.

Let A ={event the first long-distance call in among the first 50}; then $\overline{A}$={no long-distance call is among the first 50}, and

$P(A) = 1 - P(\overline{A}) = 1 - (0.7)^{50} = 0.99$

2. A certain typing agency employs two typists. The average number of errors per article is 3 when typed by the first typist and 4.5 when typed by the second typist. If your article is equally likely to be typed by either typist, find the probability that it will have exactly one error.

The average number of error by typist 1 is $X \rightsquigarrow \mathcal{P}(3)$, and the average number of error by typist 2 is $Y \rightsquigarrow \mathcal{P}(4.5)$.

Let A ={event article have exactly one error}

$P(A) = \frac{1}{2}P(X = 1) + \frac{1}{2}P(Y = 1) = \frac{1}{2}(0.149 + 0.0499) = 0.099$ (from the Poisson table)

3. The probability that a machine produces a defective item is 0.01. Each item is checked as it is produced. Approximate the probability that at least 3 items are defective among the next 120 items checked.

Let X be the number of defective items among the 120; $X \rightsquigarrow b(120, 0.01)$.

X can be approximated by a Poisson distribution Y with parameter $\lambda = 1.2$

$P(X \geq 3) \simeq P(Y \geq 3) = 0.12$ (from the Poisson table)

4. The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Sam buys a used radio, find the probability that it will be working after an additional 8 years.

Let X be the lifetime (in years) of a radio; $X \rightsquigarrow \mathcal{E}(1/8)$, and

$$f(x) = \frac{1}{8}\,e^{-x/8} \quad 0 < x < \infty$$

Let y be the lifetime of the radio when Sam bought it,

$$P(X > 8 + y|X > y) = \frac{P(X > y + 8)}{P(X > y)} = \frac{\int_{y+8}^{\infty} \frac{1}{8}\,e^{-x/8}\,dx}{\int_{y}^{\infty} \frac{1}{8}\,e^{-x/8}\,dx} = \frac{e^{-(y+8)/8}}{e^{-y/8}} = e^{-1}$$

(you may also say it directly $P(X > 8)$ after mentioning that the exponential distribution is memory less)

5. A town has 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96. Find the probability that at least one fire engine is available when needed.

Let A ={event at least one fire engine is available}

$P(A) = 1 - P(\overline{A}) = 1 - (0.04)^2 = 0.9984$

6. You arrive at a bus stop at 10 o'clock, knowing that bus will arrive at some time uniformly distributed between 10 and 10:30. If at 10:15 the bus has not arrived yet, what is the probability that you will have to wait at least an additional 10 minutes ?

The arriving time X of the bus can be modeled by a uniform distribution on the interval $(0, 1)$ (here we consider that 10 o'clock is 0 and 10:30 is 1)

$P(X > 5/6|X > 1/2) = \frac{P(X>5/6)}{P(X>1/2)} = \frac{1/6}{1/2} = 1/3$

7. A roulette wheel has 18 red cases and 20 black cases. Suppose that you continue to make \$5 bet on red until you win 4 of these bets.

i Find the probability that you place a total of 9 bets.

Let X be the number of bets until you win 4 bets; X has a negative binomial distribution with parameters $r = 4$ and $p = 18/38$; then

$P(X = 9) = C_8^3(\frac{20}{38})^5(\frac{18}{38})^4$

ii Find your expected winning when you stop.

The expected number of bets until you win 4 is $r/p = 76/9$, hence you are expected to play $76/9$ games; out of theses games 4 are wining games and the rest is losing.

Your expected wining is then given by

$4 \times 5 - (\frac{76}{9} - 4) \times 5 = -\frac{20}{9}\$$

8. Let X be a random variable with pdf

$$f(x) = 3x^2 \quad , \quad 0 < x < 1$$

i Find $F(x)$, the cdf of X.

$$F(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

ii Find $E(X^n)$.

$$E(X^n) = \int_0^1 3x^{n+2}\,dx = \frac{3}{n+3}$$

9. Sam and Jad play a game in which Sam's chance of winning is $2/3$. Suppose that if Jad win a game then Sam must pay him 1 \$, and that if Sam win a game then Jad must pay him 0.75 \$. Find the expected gain of Jad in a series of 50 such games, supposedly independent.

Let X be the number of games won by Jad among the 50; $X \rightsquigarrow b(50, \frac{1}{3})$.

The expected number of games that Sam will win is $E(X) = 50/3$, hence the expected wining of Sam is

$1 \times \frac{50}{3} - 0.75 \times \frac{100}{3} = -\frac{25}{3}$