American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2009-2010

quiz # 2 - solution

1. Let $X \rightsquigarrow b(2, p)$ and $Y \rightsquigarrow b(4, p)$. If $P(X \geq 1) = \frac{5}{9}$, find $P(Y \geq 1)$.

$P(X = 0) = (1-p)^2 = 1 - P(X \geq 1) = 4/9$, then $1 - p = 2/3$, and $p = 1/3$; hence

$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1-p)^4 = 1 - (\frac{2}{3})^4 = \frac{65}{81} = 0.802$

2. Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$. Find the probability that 3 or more accidents occur today if at least one accident occurs today.

$$P(X \geq 3 | X \geq 1) = \frac{P(X \geq 3)}{P(X \geq 1)} = \frac{1 - 0.423}{1 - 0.05} = 0.60618 \quad \text{(from Poisson table)}$$

3. A type of missile has failure probability 0.02. Find the probability that there will be at least 4 failures in the first 200 launches.

Let X be the number of failures in 200 launches; $X \rightsquigarrow b(200, 0.02)$, and $P(X \geq 4) = 0.568$ (by using the Binomial formula)

X can also be approximated by a Poisson distribution $\mathcal{P}(4)$, and $P(X \geq 4) \simeq 0.566$ (by using the Poisson formula)

4. The pdf of a random variable X is $f(x) = 2x$, $0 < x < 1$. Find $Var(X)$.

$E(X) = \displaystyle\int_0^1 2x^2\,dx = \frac{2}{3}$ and $E(X^2) = \displaystyle\int_0^1 2x^3\,dx = \frac{1}{2}$, then $Var(X) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

5. A student takes a multiple choice test with 10 questions, each question has five possible answers from which exactly one is correct. The student blindly guesses and gets one question correct. Find the probability that the correct question was one of the first 4.

$C_4^1 \times (\frac{1}{5}) \times (\frac{4}{5})^9 = (\frac{4}{5})^{10} = 0.107$

6. Let X be a random variable with pdf

$$f(x) = \begin{cases} cx & 0 < x < 1 \\ 1/2 & 1 < x < 2 \end{cases}$$

- find the value the constant c

$\displaystyle\int_0^1 cx\,dx + \int_1^2 \frac{1}{2}\,dx = \frac{c}{2} + \frac{1}{2} = 1$ cause f is a pdf; then $c = 1$

- find $E(X)$

$$E(X) = \int_0^1 x^2\,dx + \int_1^2 \frac{1}{2}\,x\,dx = \frac{13}{12}$$

7. If independent trials, each resulting in a success with probability 2/3, are performed, find the probability of 5 successes occurring before 2 failures.

Let A={event the first 5 trials are all successes}, and B={event one failure in the first 5 trials and the 6th trial is a success}

$p = P(A) + P(B) = (\frac{2}{3})^5 + C_5^1 \times (\frac{1}{3}) \times (\frac{2}{3})^5 = (1 + \frac{5}{3})(\frac{2}{3})^5 = \frac{8}{3} .(\frac{2}{3})^5 = \frac{2^8}{3^6} = \frac{256}{729} \simeq 0.35$

8. Each game you play is a win with probability 2/3. You plan to play 5 games, but if you win the fifth game, then you will keep on playing until you lose. Find the expected number of games that you lose.

Let X be the number of games that you lose in the first 4 games; $X \rightsquigarrow b(4, 1/3)$, and $E(X) = 4/3$

you expect to lose one more game, then the expected number of games that you lose is $4/3 + 1 = 7/3$.

9. Let $X \rightsquigarrow \mathcal{G}(\frac{1}{3})$. Find $P(X\ is\ even)$.

$$P(X\,is\,even) = \sum_{k=1}^{\infty} P(X = 2k) = \sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{2k-1} \times \frac{1}{3} = \frac{1}{2} \sum_{k=1}^{\infty} \left(\frac{4}{9}\right)^{k} = \frac{1}{2} \times \frac{4}{9} \times \frac{1}{1 - \frac{4}{9}} = \frac{2}{5}$$

10. The mgf of a random variable X is $M(t) = e^{\frac{t^2}{8}}$. Find $E(X^n)$, the moment of order n of X.

(hint: use the Maclaurin series of e^x)

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, and

$M(t) = e^{\frac{t^2}{8}} = \sum_{n=0}^{\infty} \frac{t^{2n}}{8^n . n!} = \sum_{n=0}^{\infty} \frac{E(X^n)}{n!} t^n$, then by comparison, we have

$E(X^{2n+1}) = 0$, and $E(X^{2n}) = \frac{(2n)!}{8^n . n!}$