Final solution, Summer 2010

Exercise

Lori just bought a new set of 4 tires for her car. The life of each tire is normally distributed with a mean of 45000 miles and a standard deviation of 3200 miles. Find the probability that all 4 tires will last at least 46000 miles. Assume the life of each of these tires is independent of the lives of other tires.

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Solution

The probability that one tire lives more than 46000 *miles is* 0.377*, and the probability that all* 4 *tires live more than* 46000 *each is* $(0.377)^4 = 0.02$.

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Suppose that in a community the distributions of heights of men and women (in centimeters) are $\mathcal{N}(173, 40)$ and $\mathcal{N}(160, 20)$, respectively. Calculate the probability that the average height of 10 randomly selected men is at least 5 centimeters larger than the average height of six randomly selected women.

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$\overline{X}$ *has a normal distribution with mean* 173 *and variance* 4 *and* $\overline{Y}$ *has a normal distribution with mean* 160 *and variance* $10/3$, *and then* $\overline{X} - \overline{Y}$ *has a normal distribution with mean* 13 *and variance* $22/3$

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$P(\overline{X} - \overline{Y} > 5) = 0.86$ *(from the standard normal table)*

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$$= 1 - (P(Z < 3))^3 = 1 - (0.998)^3 = 0.004\ .$$

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$P(\overline{X} < 1.42) \simeq P(Z < -1.38) = 0.083$