American University of Beirut

STAT 230

Introduction to Prob0ability and Random Variables

Summer 2008

Final Exam

Name: .... ID #: ....

Section: **4 (7:30)** **6 (12:30)**

Exercise 1 *(10 points)* The number of accidents in a period of one week follows a Poisson distribution with mean two. The number of accidents from week to week are independent. Find the probability of exactly seven accidents in a given three weeks.

Exercise 2 Let X be a continuous random variable with pdf $f(x) = kx^2 e^{-x^2/2}$, $0 < x < +\infty$

a. *(10 points)* find the value of the constant k

b. *(10 points)* find $E(X)$ and $Var(X)$

Exercise 3 *(20 points)* Let X_1 and X_2 be independent $\chi^2(2)$ distributions, i.e.

$$f(x) = \frac{1}{2}\, e^{-x/2} \qquad 0 < x < +\infty$$

find the joint pdf of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$, then find the marginal pdf of Y_2

Exercise 4 *(20 points)* Flip $n = 8$ fair coins and remove all that come up heads. Flip the other (tails) coins and remove the heads. Continue flipping the remaining coins until each has come up heads. We shall find the pdf of Y, the number of trials needed. Let X_i, equal the number of flips required to observe heads on coin $i, i = 1, 2, \ldots, 8$. Then $Y = \max(X_1, X_2, \ldots, X_8)$.

a. Show that $P(Y \le y) = [1 - (1/2)^y]^8$

b. Show that $P(Y = y) = [1 - (1/2)^y]^8 - [1 - (1/2)^{y-1}]^8$, $y = 1, 2, ..$

c. Find $E(Y)$

Exercise 5 *(15 points)* Suppose that the length of life in hours of a light bulb manufactured by company A is $\mathcal{N}(800, 14400)$ and the length of life in hours of a light bulb manufactured by company B is $\mathcal{N}(850, 2500)$. One bulb is selected from each company and is burned until death.

a. find the probability that length life of the bulb from company A exceeds the length of life of the bulb from company B by at least 15 hours.

b. find the probability that at least one of the bulbs lives for at least 920 hours.

Exercise 6 *(15 points)* On each bet, a gambler loses 1 with probability 0.7, loses 2 with probability 0.2, or wins 10 with probability 0.1. Approximate the probability that the gambler will be losing after the first 100 bets.