American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Spring 2011

quiz # 1

Name: solution

ID:

Section: **12:30 PM** **2 PM**

1. In how many ways can three different prizes be awarded among nine contestants if no person is to receive more than one prize ?

$$9 \times 8 \times 7 = {}_9P_3 = 504$$

2. One of the five elevators in a building starts with seven passengers and stops at nine floors. Assuming that it is equally likely that a passenger gets off at any of these nine floors, find the probability that at least two of these passengers will get off at the same floor.

$A = \{$at least two passengers get off at the same floor$\}$

$$P(A) = 1 - P(\bar{A})$$

$$= 1 - \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{9^7}$$

$$= 1 - \frac{{}_9P_7}{9^7} = 0.96$$

3. Chips are drawn at random, one at a time and without replacement from an urn that contains 5 pink chips and 7 white chips, until only those of the same color are left. Find the probability that exactly 7 draws are needed.

$\boxed{7W}$ or $\boxed{4P\ 2W}\ \boxed{P}$

$$\frac{C_7^7}{C_{12}^7} + \frac{C_5^4 \times C_7^2}{C_{12}^6} \times \frac{1}{6}$$

$$= \frac{2}{99}$$

4. Four couples are to be seated in a row. Find the probability that each husband sits next to his wife.

let C_1, C_2, C_3, C_4 be the 4 couples

C1	C2	C3	C4

$$\frac{4! \times 2^4}{8!} = \frac{1}{105}$$

$4!$: permutation of the 4 couples

2^4 : permutation of each couple ($\times 4$)

$8!$: total nb. of permutation

5. Let A, B and C three independent events with probabilities $1/2, 1/6, 1/4$. Find $P((\overline{A} \cap \overline{B}) \cup C)$.

$$P((\bar{A} \cap \bar{B}) \cup C) = P(\bar{A} \cap \bar{B}) + P(C) - P(\bar{A} \cap \bar{B} \cap C)$$

$$\overset{\text{ind.}}{=} P(\bar{A})P(\bar{B}) + P(C) - P(\bar{A})P(\bar{B})P(C)$$

$$= \frac{1}{2} \times \frac{5}{6} + \frac{1}{4} - \frac{1}{2} \times \frac{5}{6} \times \frac{1}{4}$$

$$= \frac{27}{48} = \frac{9}{16} = 0.5625$$

6. The pdf of a random variable X is given in the following table

x	-2	1	4
$P(X=x)$	1/8	2/8	5/8

Find $E(2X-1)$

$$E(2X-1) = -5 \times \frac{1}{8} + 1 \times \frac{2}{8} + 7 \times \frac{5}{8}$$

$$= 4$$

7. An absentminded professor wrote 15 letters and sealed them in envelopes before writing the addresses on the envelopes. Then he wrote the ~~~~ 15 addresses on the envelopes at random. What is the probability that no letter was addressed correctly ?

Matching formula without replacement

$A = \{$no letter was add. correctly$\}$

$$P(A) = 1 - P(\bar{A})$$

$$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \cdots - \frac{1}{15!}$$

8. A computer is instructed to generate a random sequence using the digits 0 through 9; repetitions are allowed. Find the shortest length the sequence can be and still have at least 70% probability of containing at least one 4.

Let n be the length of the sequence

$A = \{$event sequence containing at least one 4$\}$

$$P(A) = 1 - P(\bar{A}) = 1 - \left(\frac{9}{10}\right)^n$$

$$1 - \left(\frac{9}{10}\right)^n \geq 0.7 \underset{\text{solve it !!}}{\Longrightarrow} n = 12$$

9. In a squash tournament between three players A, B and C, each player plays the others once (ie. A plays B, A plays C and B plays C). Assume the following probabilities: $P(A\ \textit{beats}\ B) = 0.6$; $P(A\ \textit{beats}\ C) = 0.7$; $P(B\ \textit{beats}\ C) = 0.6$. Assuming independence of the match results, calculate the probability that A wins at least as many games as any other player.

$W = \{$ event A wins at least as many games as any other player $\}$

$\underline{b}$ means beats

$$P(W) = P((A\,\underline{b}\,B) \cap (A\,\underline{b}\,C)) + P((A\,\underline{b}\,B) \cap (B\,\underline{b}\,C) \cap (C\,\underline{b}\,A)) + P((A\,\underline{b}\,C) \cap (C\,\underline{b}\,B) \cap (B\,\underline{b}\,A))$$

$$\underset{\text{ind.}}{=} 0.7 \times 0.6 + 0.6 \times 0.6 \times 0.3 + 0.7 \times 0.4 \times 0.4$$

$$= 0.42 + 0.108 + 0.112$$

$$= 0.64$$

10. Let A and B be two events with $P(B) = 0.7$ and $P(\overline{A}|B) = 0.4$. Find $P(A \cup \overline{B})$.

$$P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B})$$

$$= P(A) + P(\overline{B}) - (P(A) - P(A \cap B))$$

$$= P(\overline{B}) + P(A \cap B)$$

$$= P(\overline{B}) + P(A|B) \times P(B)$$

$$= 0.3 + 0.6 \times 0.7$$

$$= 0.72$$

11. On a multiple-choice exam with four choices for each question, a student either knows the answer to a question or marks it at random. If the probability that he or she knows the answers is 2/3, what is the probability that an answer that was marked correctly was not marked randomly ?

Let $C = \{$event question answered correctly$\}$

$R = \{$question answered by random$\}$

$$P(\bar{R}|C) = \frac{P(C|\bar{R}) \times P(\bar{R})}{P(C|R) \times P(R) + P(C|\bar{R}) \times P(\bar{R})}$$

Bayes rule

$$= \frac{1 \times \frac{2}{3}}{\frac{1}{4} \times \frac{1}{3} + 1 \times \frac{2}{3}} = \frac{8}{9}$$

12. Marwan, Jad and Kate simultaneously toss coins. The coin tossed by each one turns up head with probability 1/2, 1/4 and 1/3 respectively. If one person gets an outcome different from those of the other two, then he is the odd man out. If there is no odd man out, the players flip again and continue to do so until they get an odd man out.

a) Find the probability that Marwan is out in the first round.

$$P(\text{Marwan out in first round}) = P(HTT) + P(THH)$$

$$\overset{\text{ind}}{=} \frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{4} + \frac{1}{24}$$

$$= \frac{7}{24}$$

b) What is the probability that Marwan will be the odd man out ?

$M_i = \{$event Marwan out at the i^{th} round$\}$

$$P(\text{Marwan out}) = P(M_1) + P(M_2) + P(M_3) + \cdots$$

$$= \frac{7}{24} + \underbrace{\left(\frac{7}{24}\right)}_{\text{no one out in first round}} \cdot \frac{7}{24} + \underbrace{\left(\frac{7}{24}\right)^2}_{\text{no one out in second round}} \cdot \frac{7}{24} + \cdots$$

$$= \frac{7}{24} \times \frac{1}{1 - \frac{7}{24}} = \frac{7}{17}.$$