

American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Summer 2010

Final Exam

1. Let X and Y be a couple of random variables with joint pdf

$$f(x, y) = c, \quad 0 < x < 1, \quad x^2 < y < 1$$

Find the value of the constant c .

2. Refer to question 1, find $E(X)$.
3. Lori just bought a new set of 4 tires for her car. The life of each tire is normally distributed with a mean of 45000 miles and a standard deviation of 3200 miles. Find the probability that all 4 tires will last at least 46000 miles. Assume the life of each of these tires is independent of the lives of other tires.
4. A television store owner figures that 45 percent of the customers entering his store will purchase an ordinary television set, 15 percent will purchase a plasma television set, and 40 percent will just be browsing. If 5 customers enter his store on a given day, what is the probability that he will sell exactly 2 ordinary sets and 1 plasma set on that day?
5. A study found that 80% of students are too young to vote. Find the probability that at least 175 students can vote in a random sample of 800 students.
6. Suppose that in a community the distributions of heights of men and women (in centimeters) are $\mathcal{N}(173, 40)$ and $\mathcal{N}(160, 20)$, respectively. Calculate the probability that the average height of 10 randomly selected men is at least 5 centimeters larger than the average height of six randomly selected women.

7. Let (X, Y) be a couple of random variables with joint pdf

$$f(x, y) = e^{-(x+y)}, \quad x > 0, \quad y > 0$$

Let $U = X + Y$ and $V = X/Y$.

- a. Find the joint pdf of the couple (U, V) .
- b. Find the marginal pdf of U and V .
8. Otto is trying out for the javelin throw to compete in the olympics. The lengths of his javelin throws is normally distributed with a mean of 290 feet and a standard deviation of 10 feet. Find the probability that the longest of three of his throws is 320 feet or more.
9. The time it takes for a student to finish an aptitude test (in hours) has the pdf

$$f(x) = 6(x - 1)(2 - x), \quad 1 < x < 2$$

Approximate the probability that the average length of time it takes for a random sample of 15 students to complete the test is less than 1 hour and 25 minutes.